Fluctuations and the QCD critical point

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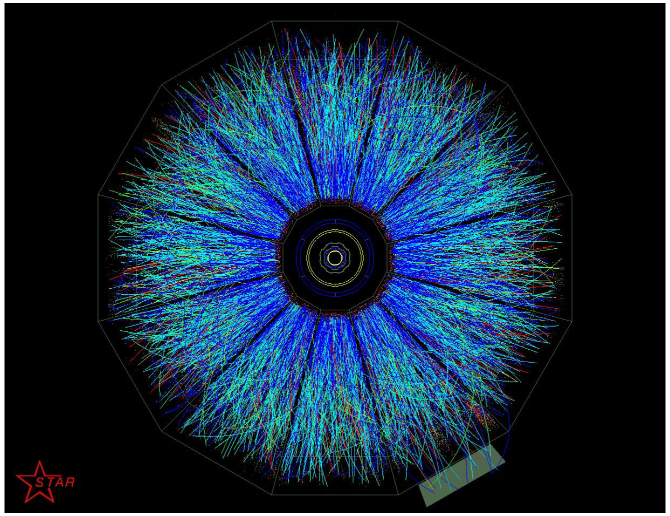
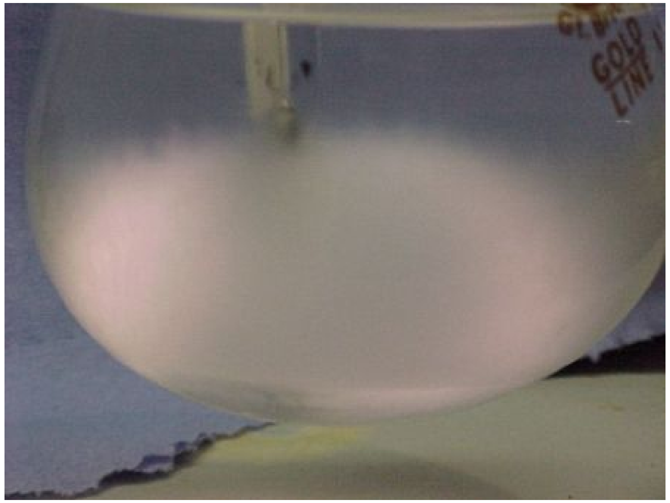
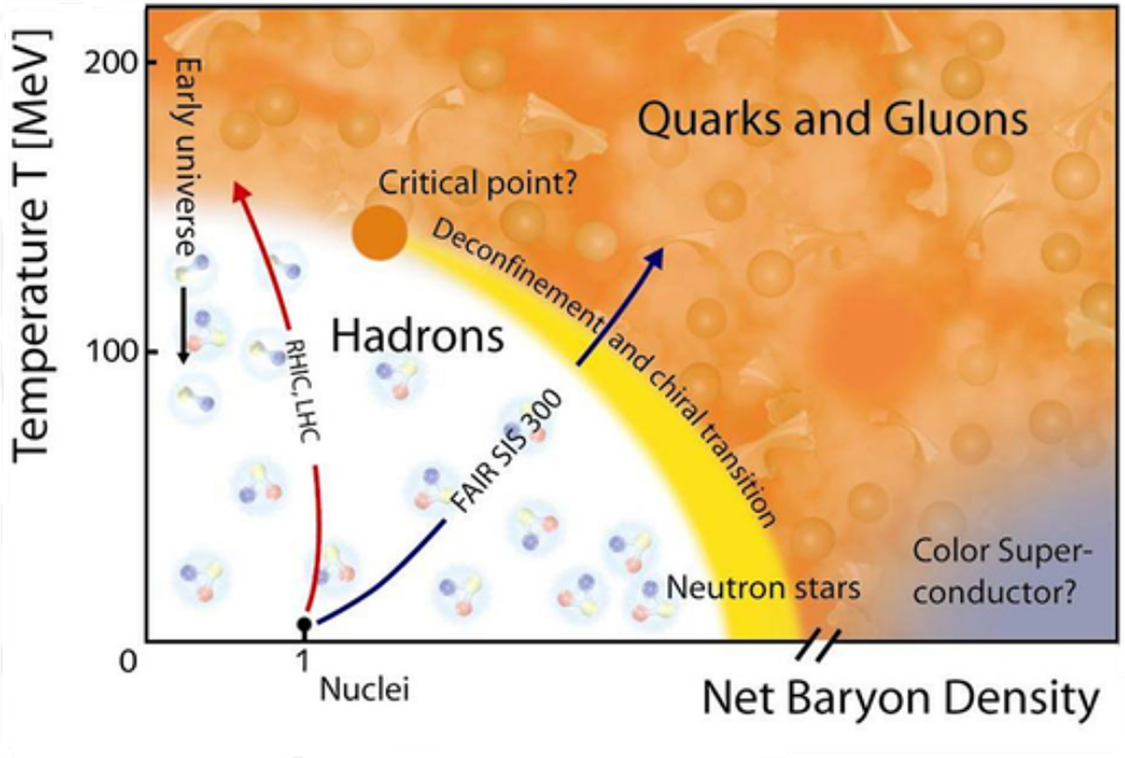
BEST COLLABORATION



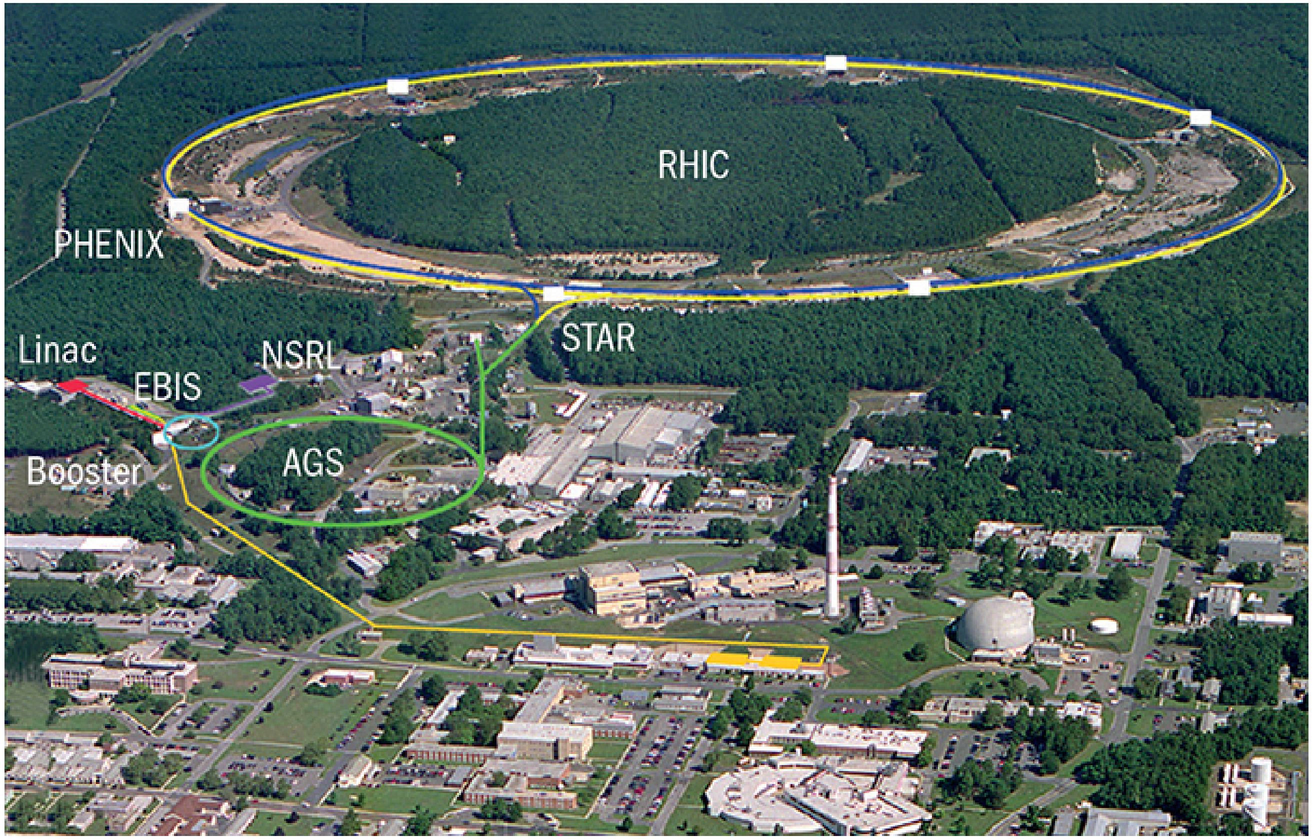


The phase diagram of QCD

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4g^2}G^a_{\mu\nu}G^a_{\mu\nu}$$



2000: Dawn of the collider era at RHIC

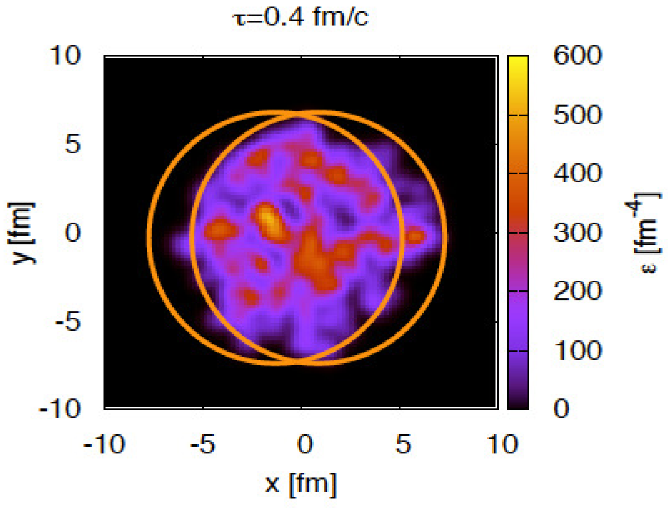


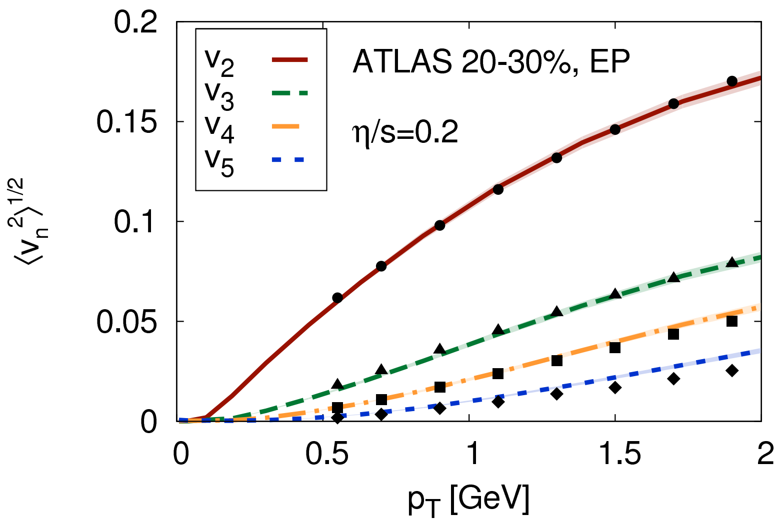
$Au + Au$ @200 $AGeV$

What did we find?

Heavy ion collisions at RHIC are described by a very simple theory:

$\pi\alpha\nu\tau\alpha\ \rho\varepsilon\iota$ (*everything flows*)

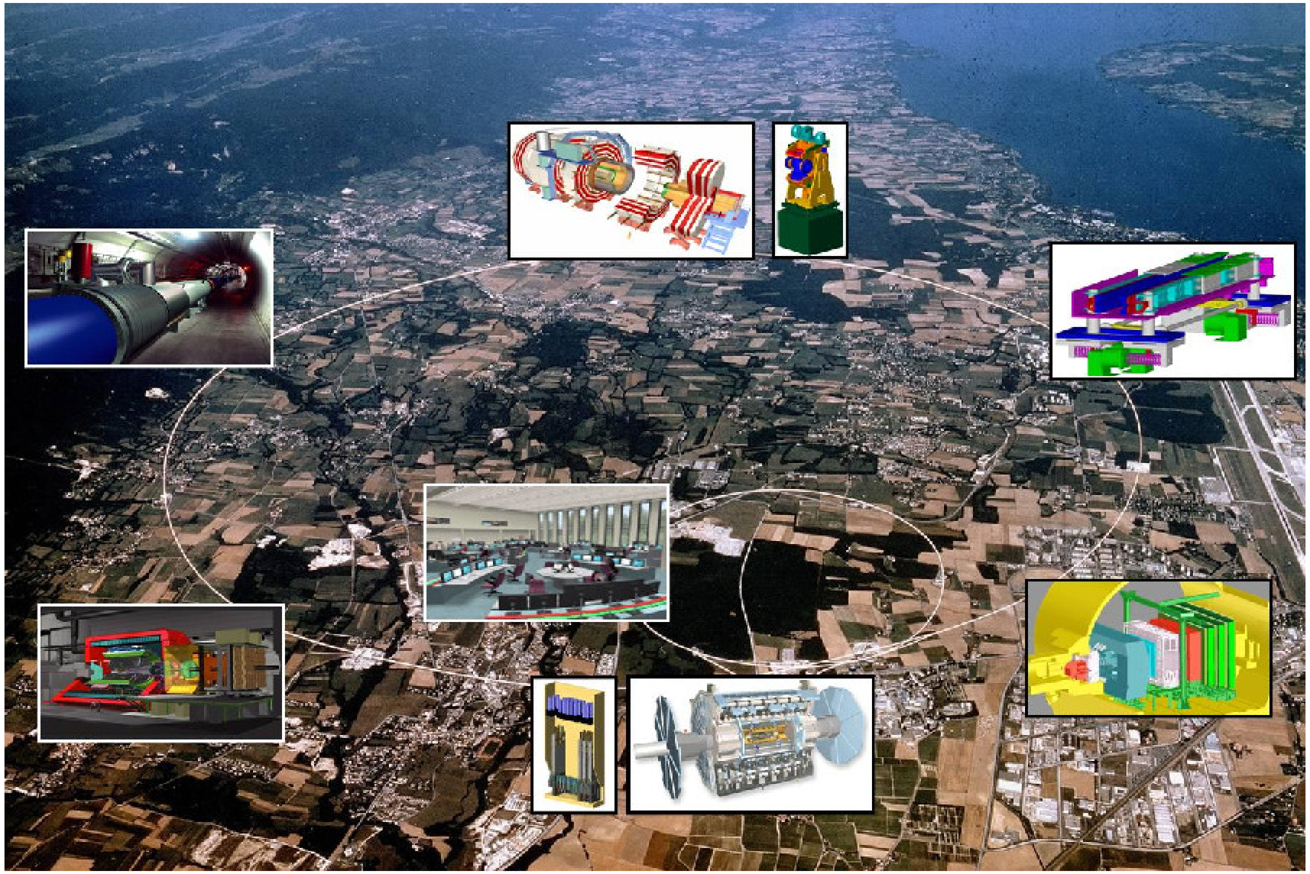




B. Schenke C. Gale et al.

Hydro converts initial state geometry, including fluctuations, to flow. Attenuation coefficient is small, $\eta/s \simeq 0.08\hbar/k_B$, indicating that the plasma is strongly coupled.

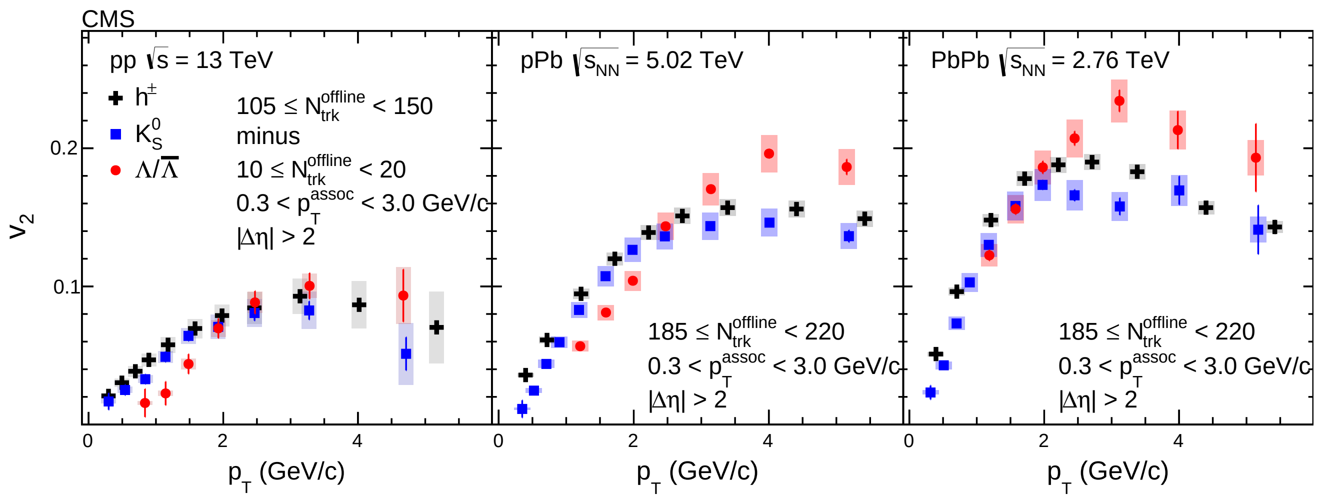
2010: The energy frontier at LHC



$Pb + Pb$ @$2.76\, ATeV$, now $5.5\, ATeV$

What did we find?

Even the smallest droplets of QGP fluid produced in (high multiplicity) pp and pA collisions exhibit collective flow.

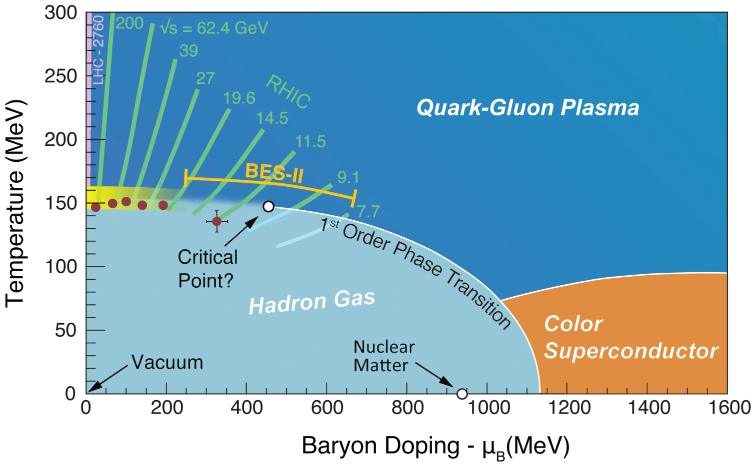


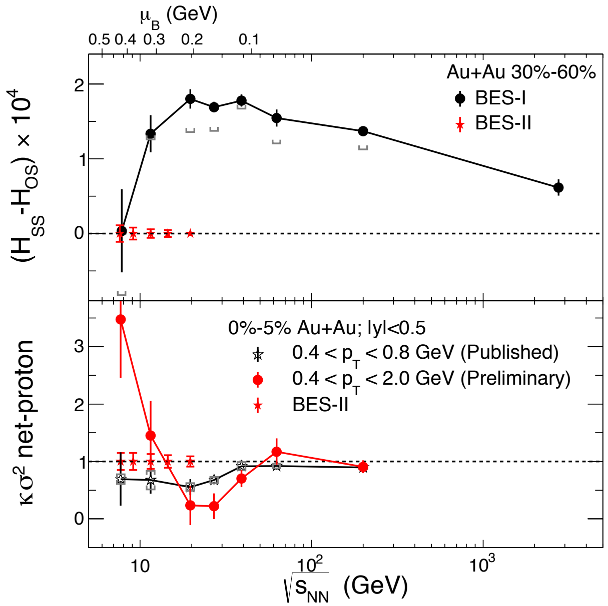
Small viscosity $\eta/s \simeq 0.08\hbar/k_B$ implies short mean free path and rapid hydrodynamization.

The next step (2010-21):

RHIC beam energy scan (BES I/II)

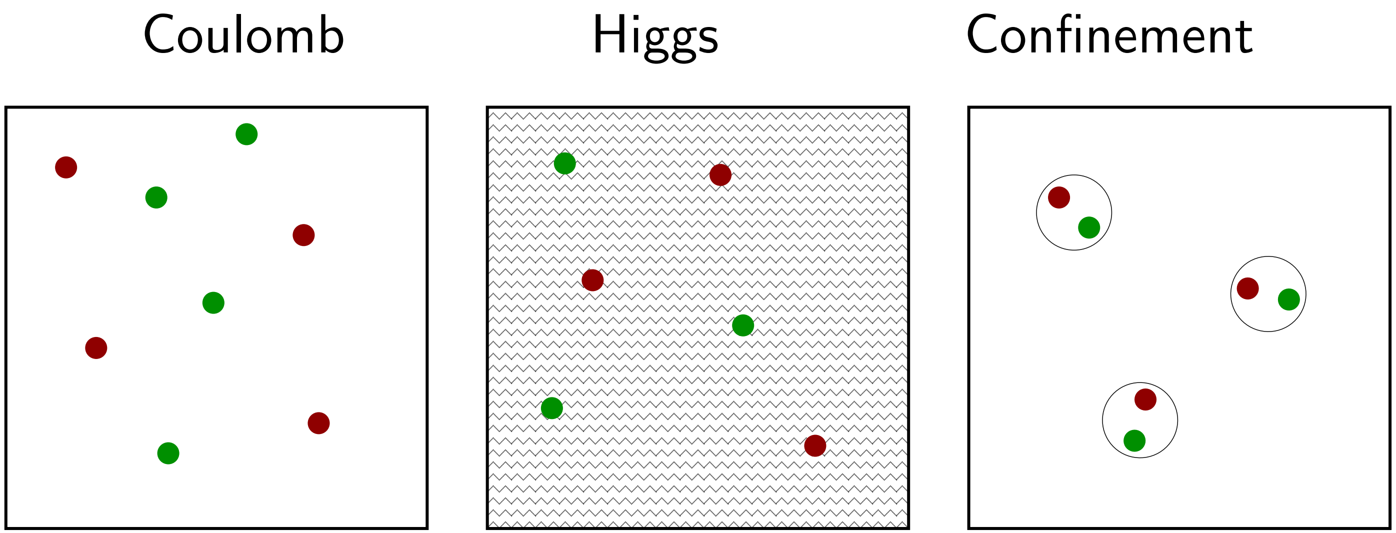
Can we locate the phase transition itself, either by locating a critical point, or identifying a first order transition?





BEST COLLABORATION

What is a Phase of QCD? Phases of Gauge Theories

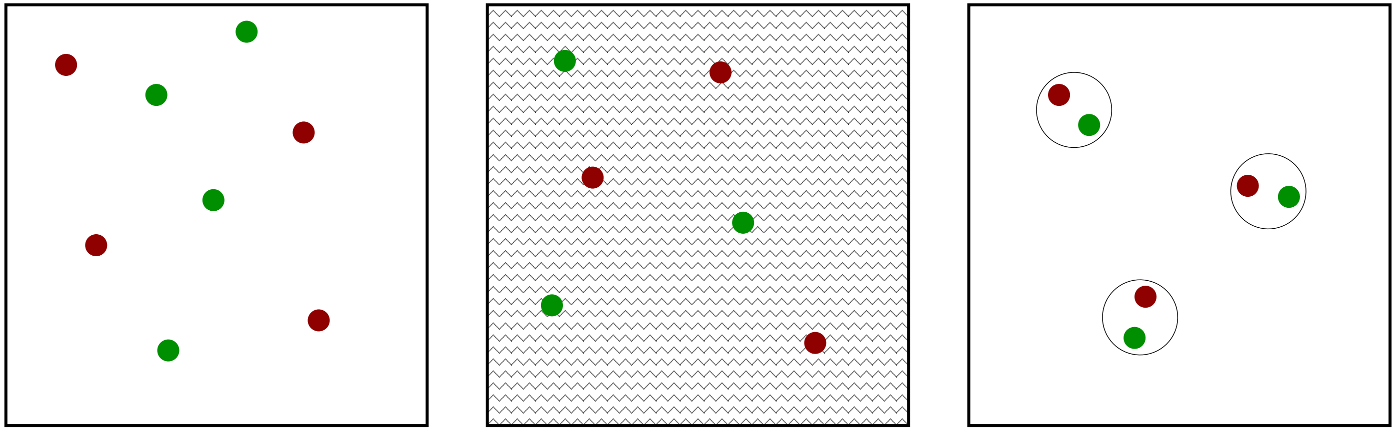


$$V(r) \sim -\frac{e^2}{r} \qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

Standard Model: $U(1) \times SU(2) \times SU(3)$

What is a Phase of QCD? Phases of Gauge Theories

Coulomb | Higgs | Confinement



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QCD: High T phase | High μ phase | Low T, μ phase

What is a Phase of QCD? Phases of Gauge Theories

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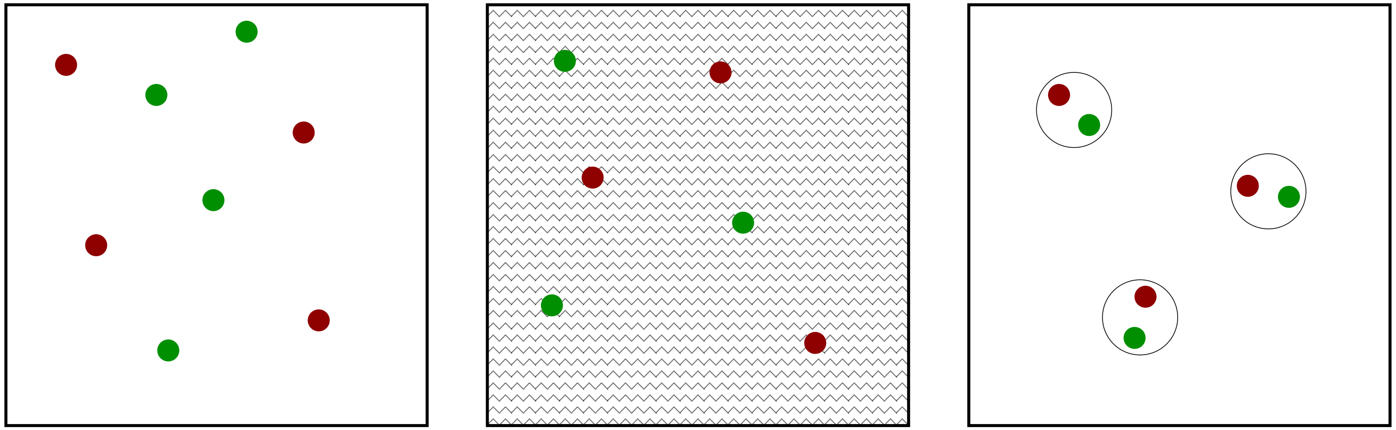


$$V(r) \sim -\frac{e^2}{r} \qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

No local order parameters: Phases can be continuously connected.

Phases of QCD: Global symmetries

Local order parameters and change of symmetry: Sharp phase transitions.

$$\vec{M} \to \hat{R}\vec{M} \qquad \langle \vec{M} \rangle \neq 0 \quad \Longrightarrow \quad \textit{Broken Symmetry}$$

QCD: Approximate chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

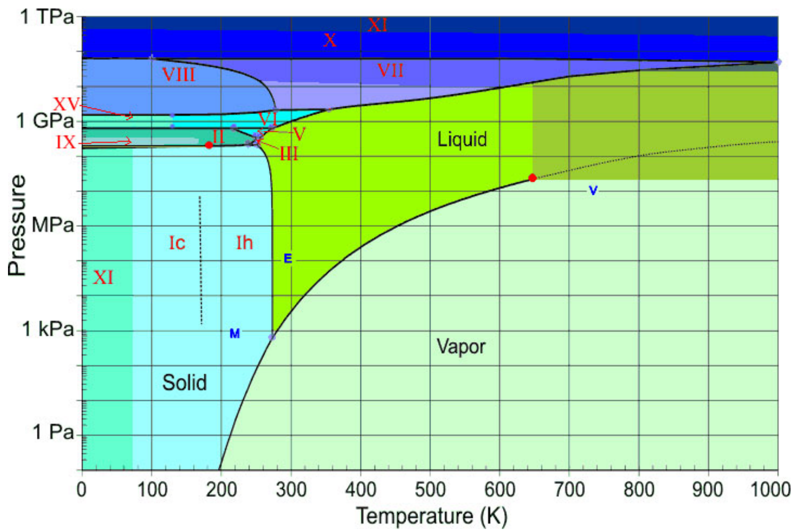
$$\psi^a_{L,f} \to L_{fg}\psi^a_{L,g}, \qquad \psi^a_{R,f} \to R_{fg}\psi^a_{R,g}$$

Broken explicitly by quark masses $m_f \ll \Lambda_{QCD}$, spontaneously by quark condensate

$$\langle \bar{\psi}_{f,L}\psi_{g,R} + \bar{\psi}_{f,R}\psi_{g,L} \rangle \simeq -\delta_{fg}\Sigma$$

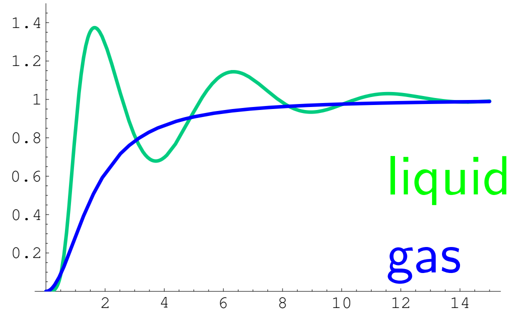
Transitions without change of symmetry: Liquid-Gas

Phase diagram of water

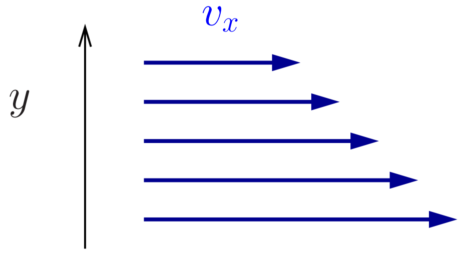


Characteristics of a liquid

Pair correlation function

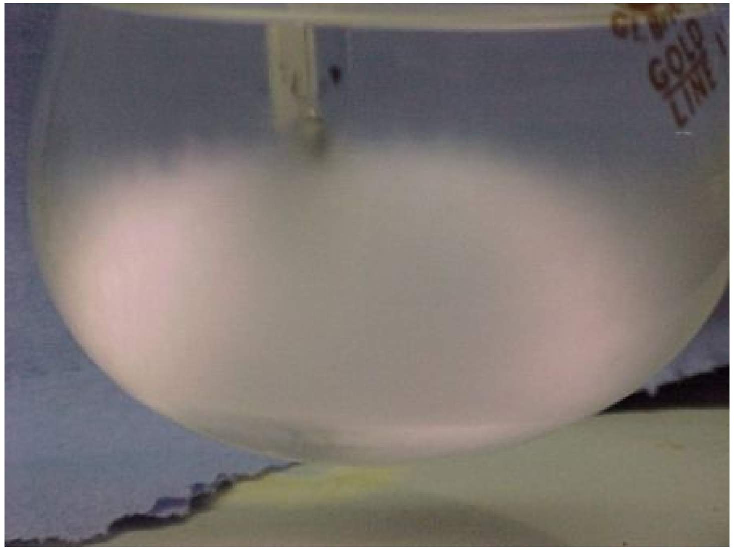


Good fluid: low viscosity



$$F_x = \eta A \frac{\partial v_x}{\partial y}$$

Signatures of the critical endpoint



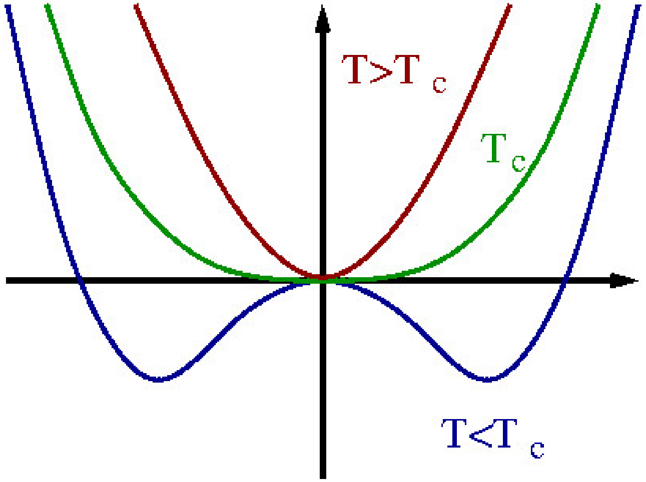
Correlation length diverges

Critical opalescence

Scalar order parameter $\phi = \rho - \rho_0$

$$F[\phi] = \int d^3x \, \{\kappa(\nabla\phi)^2 + r\phi^2 + \lambda\phi^4 + h\phi\}$$

Free energy functional:



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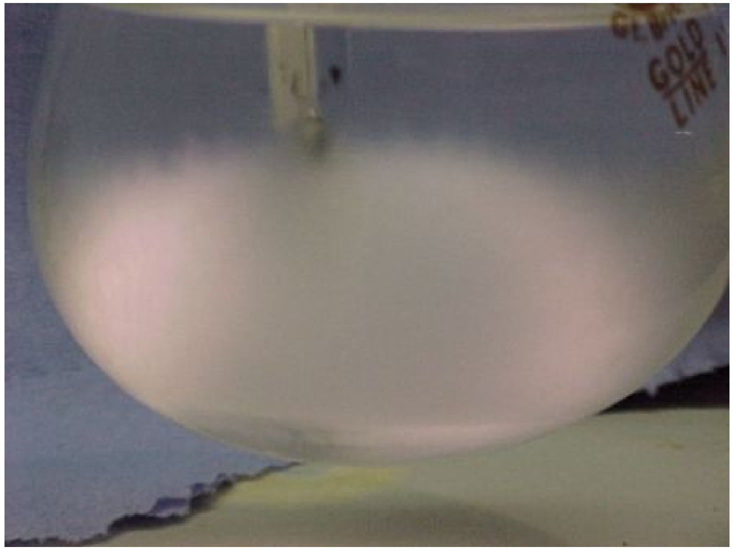

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Critical opalescence

Scalar order parameter $\phi = \rho - \rho_0$

$$F[\phi] = \int d^3x \left\{ \kappa (\nabla \phi)^2 + r\phi^2 + \lambda \phi^4 + h\phi \right\}$$

$F[\phi]$ universal, ϕ could be the magnetization of a spin system.

$$\xi \sim t^{-\nu} \qquad t = \frac{T - T_c}{T_c}$$

$$\nu = \frac{3}{4}\epsilon + O(\epsilon^2) \simeq 0.58$$

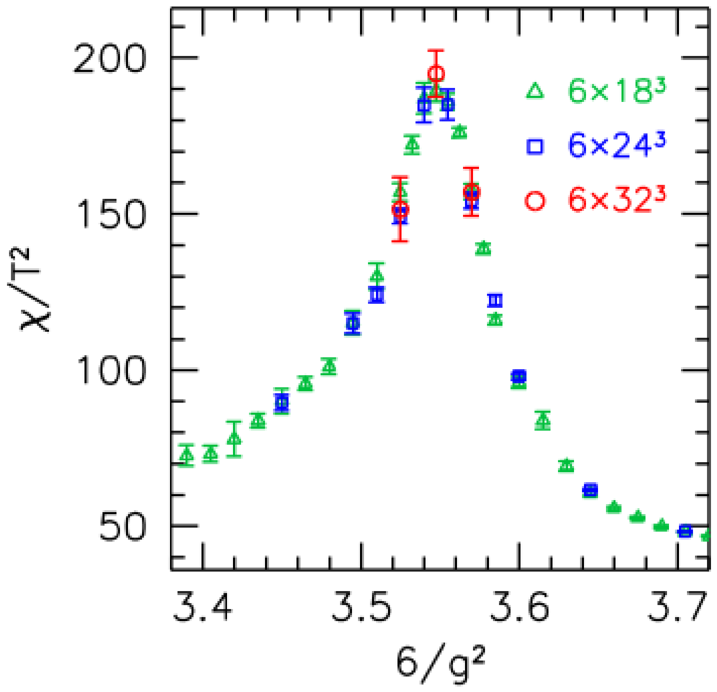
Classical fluids are in the universality class of the $3d$ Ising model.

Critical endpoint in QCD?

Quarks have finite masses. $\rightarrow$ No sharp phase transitions required, but first order transitions could be present.

Lattice QCD:

The $\mu = 0$ transition is a crossover.

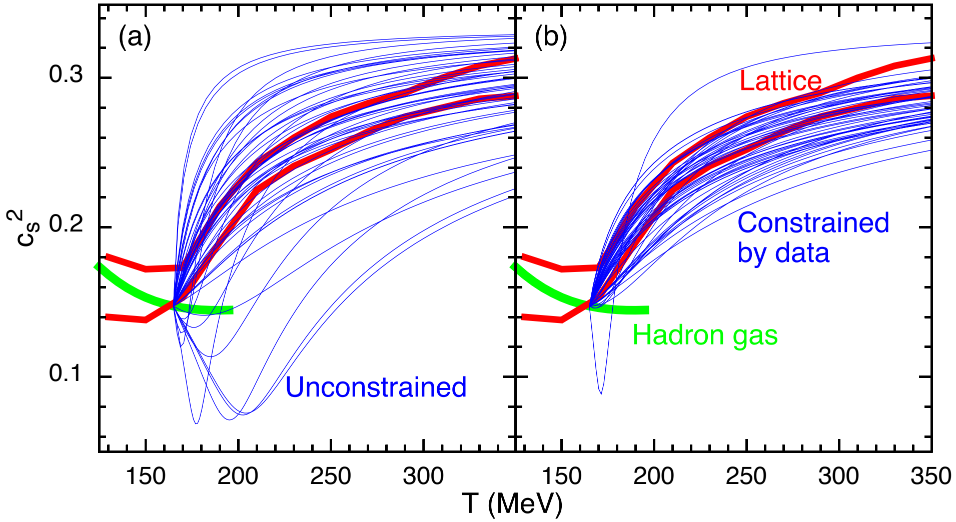


Temperature

Aoki et al., Nature (2006)

Crossover: Experimental indications

The speed of sound $c_s^2 = \frac{\partial P}{\partial \mathcal{E}}$ determines the acceleration history of the fireball. Sharp phase transition: $c_s^2 = 0$. Crossover: Soft point $c_s^2(min) > 0$



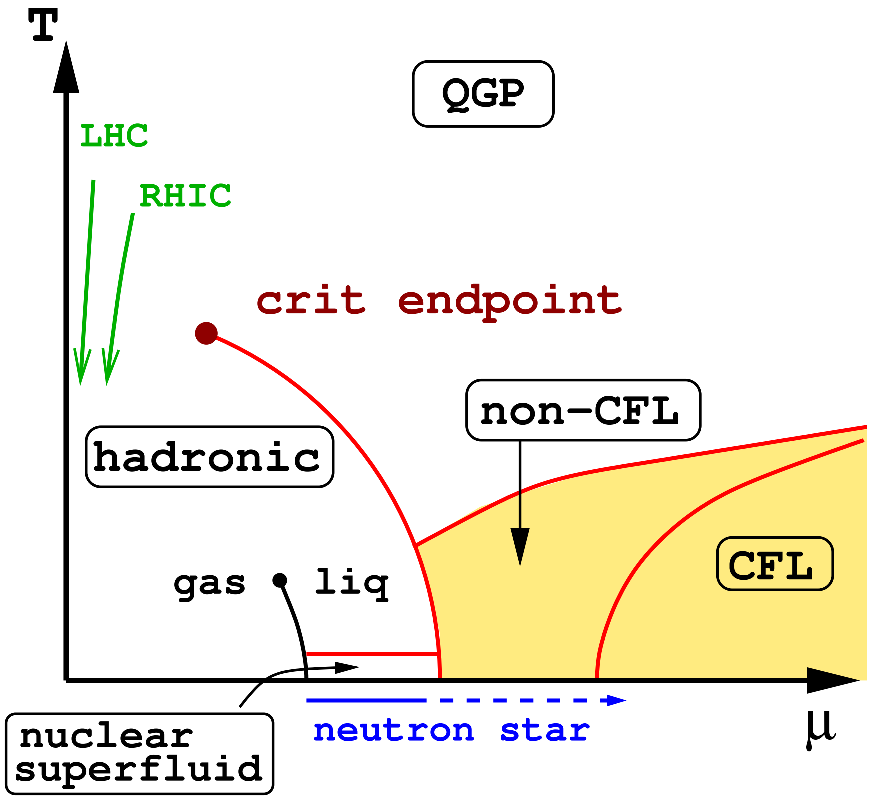
Pratt et al, PRL (2013)

Reconstruct sound speed from particle spectra, HBT source sizes and emission duration

Critical endpoint in QCD?

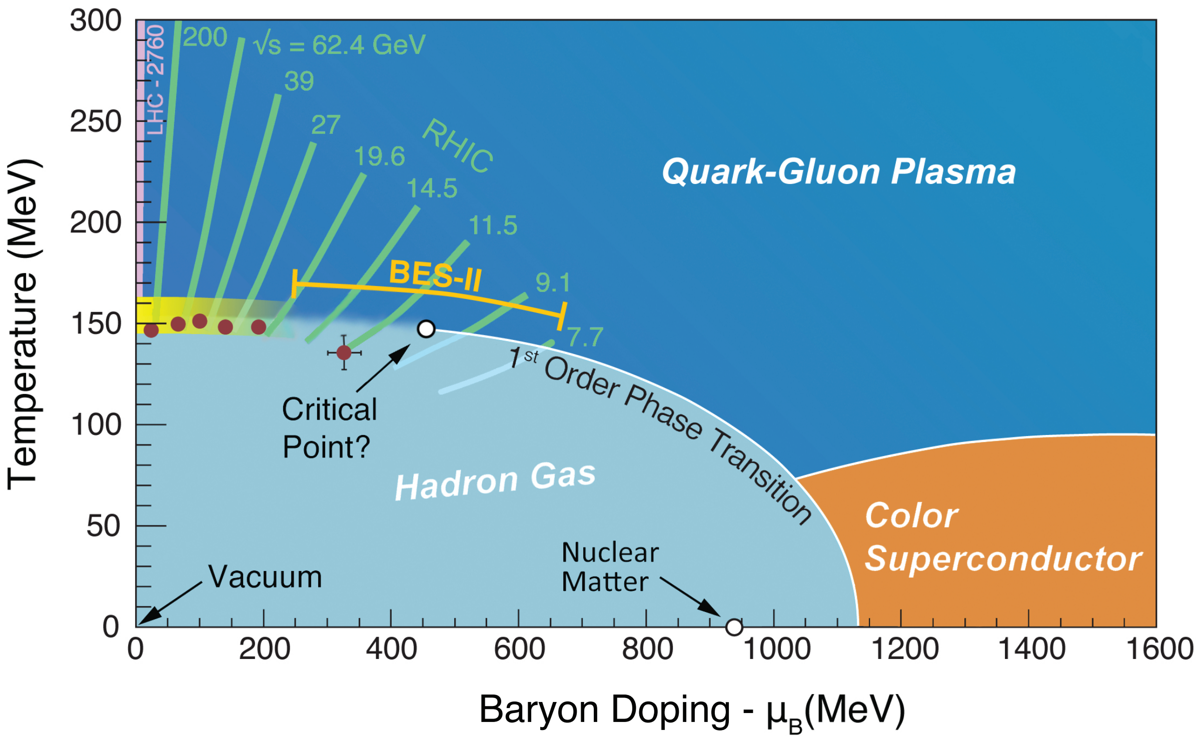
What happens for $\mu \neq 0$? Lattice calculations cannot tell (the QCD sign problem). Two options: The transition weakens, or it strengthens.

If the transition strengthens for $\mu > 0$ (as suggested by models) then there is a critical endpoint.



How would we know?

Basic Idea: Control μ via beam energy (change number of stopped nucleons)



Study fluctuation observables such as $\langle (\Delta N_p)^2 \rangle$

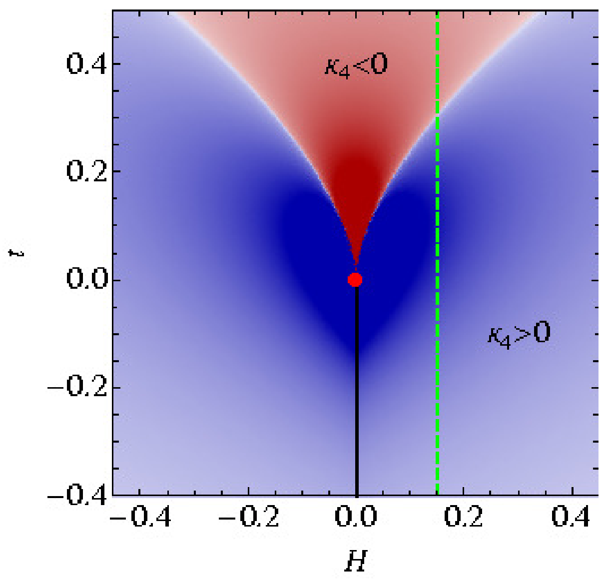
Look for enhancement/non-monotonic behavior.

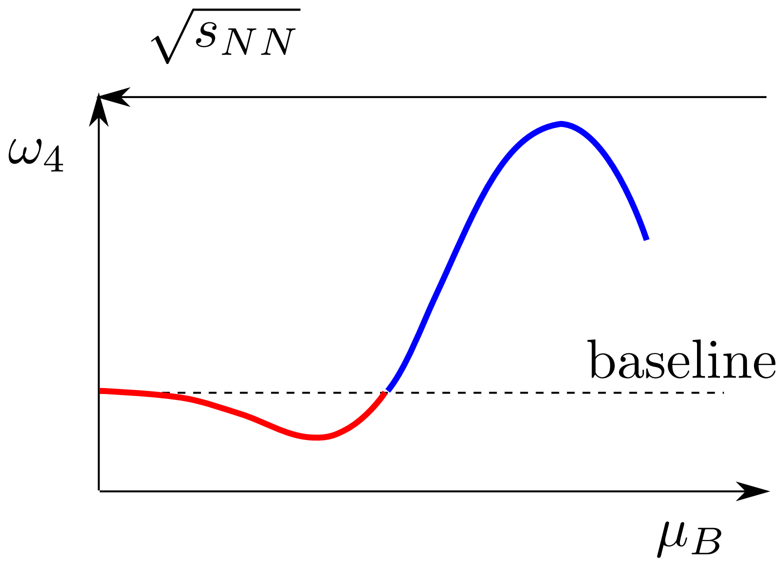
More sensitive observables: Higher order cumulants

Consider curtosis: $\kappa_4 = \langle\phi^4\rangle - 3\langle\phi^2\rangle^2$

Stronger divergence near critical point: $\kappa_4/\kappa_2^2 \sim \xi^3$

Non-trivial dependence on t ($\rightarrow$ beam energy)

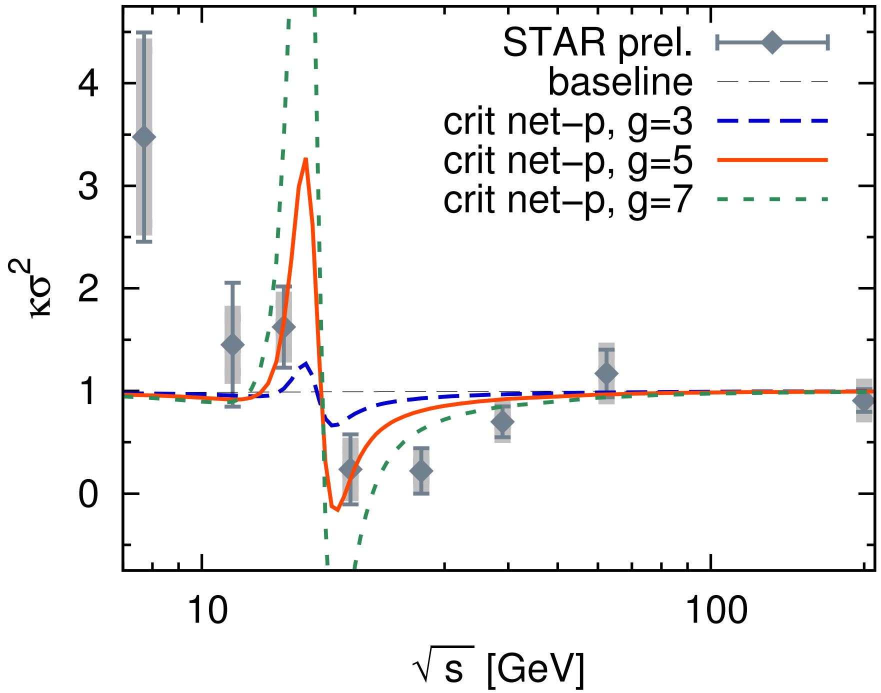




Stephanov, PRL (2011)

Compare to BES-I data

Many details: Couple fluctuations to particles $\delta N_p \sim \phi$, model freezeout curve, map Ising EOS to QCD phase diagram, include resonance decays.



M. Bluhm et al. (2016)

High energy baseline, fluctuations are Gaussian.

Some indication of non-Gaussian behavior at lower energy.

Dynamical Theory

What is the dynamical theory near the critical point?

The basic logic of fluid dynamics still applies. Important modifications:

- Critical equation of state.
- Possible Goldstone modes (chiral field in QCD?)
- Stochastic fluxes, fluctuation-dissipation relations.

Digression: Diffusion

Consider a Brownian particle

$$\dot{p}(t) = -\gamma_D p(t) + \zeta(t) \qquad \langle \zeta(t)\zeta(t') \rangle = \kappa \delta(t - t')$$

drag (dissipation) white noise (fluctuations)

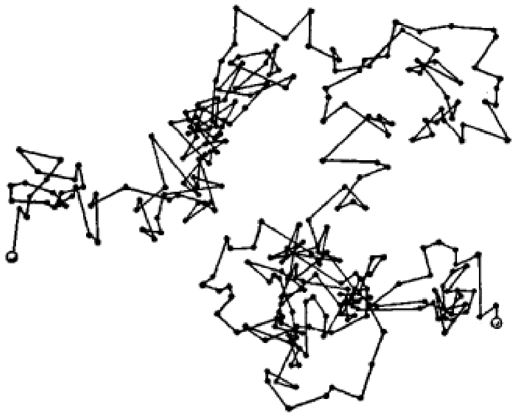
For the particle to eventually thermalize

$$\langle p^2 \rangle = 2mT$$

drag and noise must be related

$$\kappa = \frac{mT}{\gamma_D}$$

Einstein (Fluctuation-Dissipation)



Hydrodynamic equation for critical mode

Equation of motion for critical mode ϕ ("model H")

$$\frac{\partial \phi}{\partial t} = \kappa \nabla^2 \frac{\delta \mathcal{F}}{\delta \phi} - g \vec{\nabla}\phi \cdot \frac{\delta \mathcal{F}}{\delta \vec{\pi}} + \zeta_\phi$$

Diffusion Advection Noise

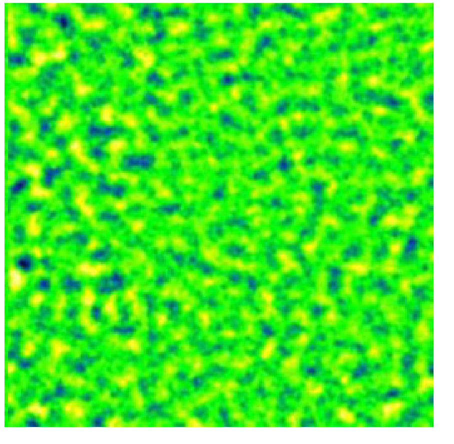
Free energy functional: Order parameter ϕ, momentum density $\vec{\pi} = w\vec{v}$

$$\mathcal{F} = \int d^d x \left[\frac{1}{2w}\vec{\pi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{m^2}{2}\phi^2 + \lambda\phi^4 \right] \qquad D = m^2\kappa$$

Fluctuation-Dissipation relation

$$\langle \zeta_\phi(x,t)\zeta_\phi(x',t')\rangle = -2\kappa T \nabla^2 \delta(x-x')\delta(t-t')$$

ensures $P[\phi] \sim \exp(-\mathcal{F}[\phi]/T)$



Numerical realization

Stochastic relaxation equation ("model A")

$$\partial_t \psi = -\Gamma \frac{\delta \mathcal{F}}{\delta \psi} + \zeta \qquad \langle \zeta(x,t)\zeta(x',t')\rangle = \Gamma T \delta(x-x')\delta(t-t')$$

Naive discretization

$$\psi(t+\Delta t) = \psi(t) + (\Delta t)\left[-\Gamma \frac{\delta \mathcal{F}}{\delta \psi} + \sqrt{\frac{\Gamma T}{(\Delta t) a^3}}\,\theta\right] \qquad \langle \theta^2 \rangle = 1$$

Noise dominates as $\Delta t \to 0$, leads to discretization ambiguities in the equilibrium distribution.

Idea: Use Metropolis update

$$\psi(t+\Delta t) = \psi(t) + \sqrt{2\Gamma(\Delta t)}\,\theta \qquad p = min(1, e^{-\beta \Delta \mathcal{F}})$$

Numerical realization

Central observation

$$\begin{aligned}
\langle \psi(t+\Delta t, \vec{x}) - \psi(t, \vec{x}) \rangle &= -(\Delta t)\,\Gamma\,\frac{\delta \mathcal{F}}{\delta \psi} + O\left((\Delta t)^2\right) \\
\langle [\psi(t+\Delta t, \vec{x}) - \psi(t, \vec{x})]^2 \rangle &= 2(\Delta t)\,\Gamma\, T + O\left((\Delta t)^2\right) \,.
\end{aligned}$$

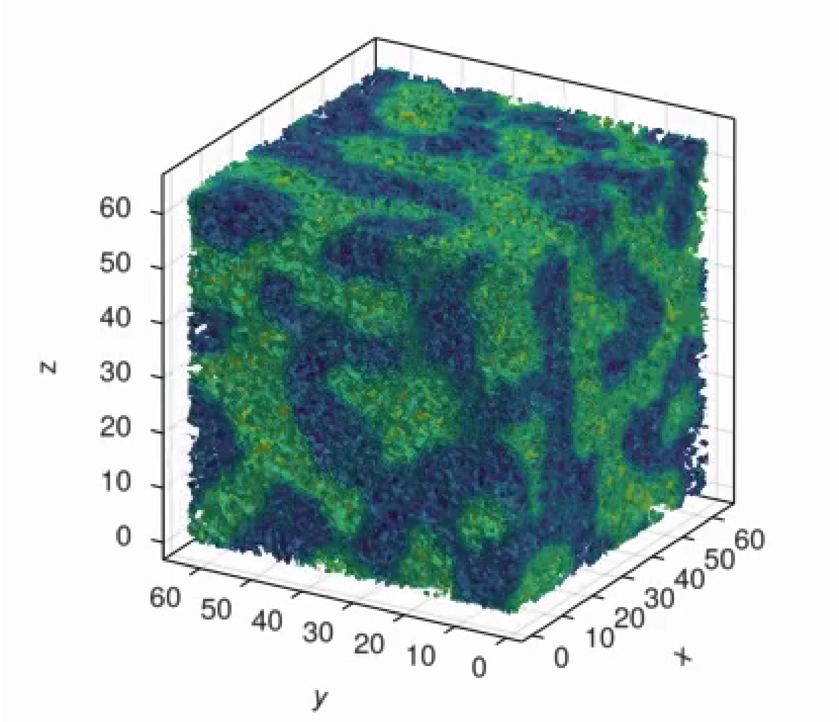
Metropolis realizes both diffusive and stochastic step. Also

$$P[\psi] \sim \exp(-\beta \mathcal{F}[\psi])$$

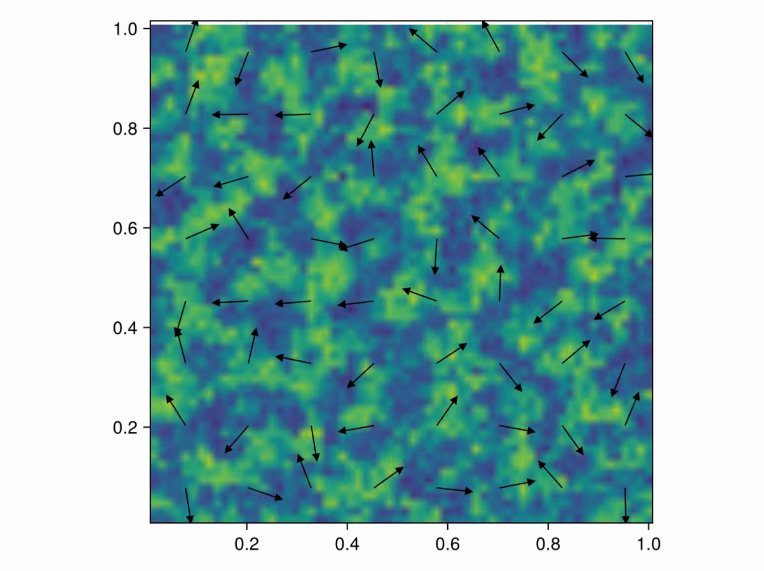
Note: Still have short distance noise; need to adjust bare parameters such as Γ, m^2, λ to reproduce physical quantities.

Numerical results (critical Navier-Stokes)

Order parameter (3d)

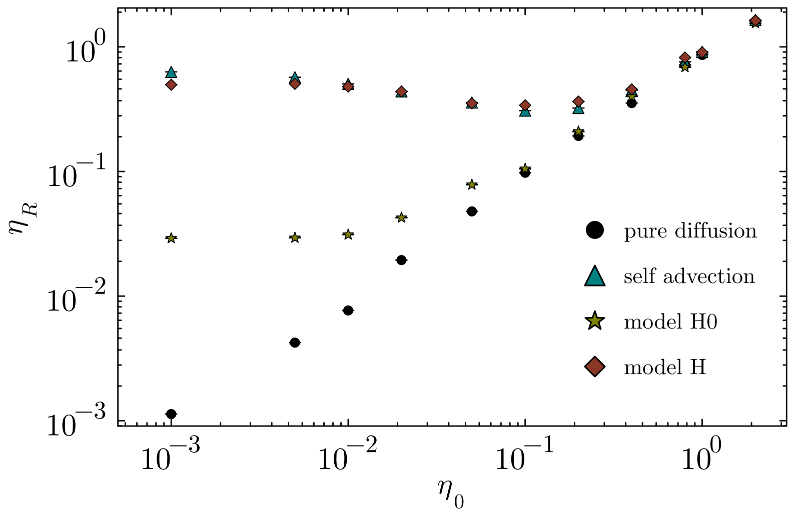


Order parameter/velocity field (2d)

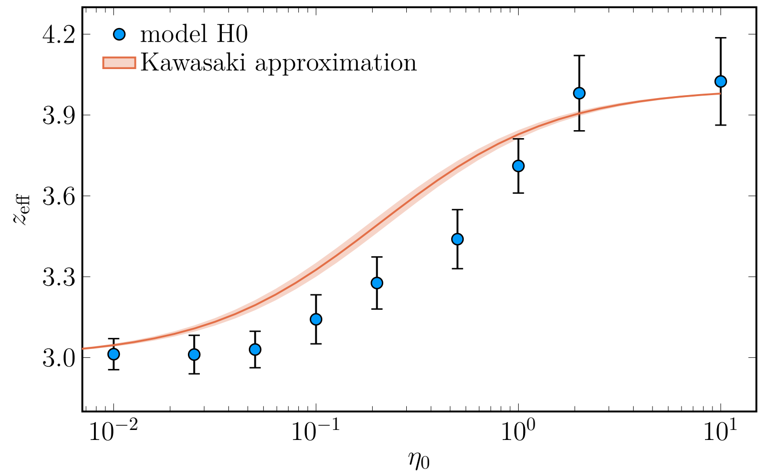


Critical Navier-Stokes (model H)

Renormalized viscosity



Dynamic exponent $\tau \sim \xi^z$



Top: Model H

Middle: No self-advection

Bottom: No advection

small η/large ξ $\leftrightarrow$ large η/small ξ

Stickiness of shear waves

Shear waves speed up relaxation

What's next?

Couple to realistic fluid background,

convert fluid elements to particles.

Outlook

Opportunity: Discover QCD critical point by observing critical fluctuations in heavy ion collisions. Intriguing hints present in BES-I data.

Challenge: Propagate fluctuations of conserved charges in relativistic fluid dynamics. Describe initial state fluctuations and final state freezeout.

Experiment: BES-II is being analyzed.

Other opportunities: Chiral dynamics, small systems.

Learned many things about fluid dynamics along the way.