Fluctuations in Cold Atomic Gases

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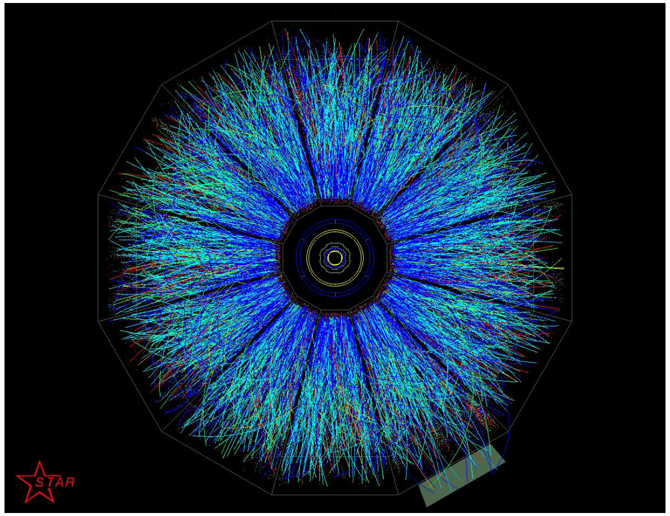
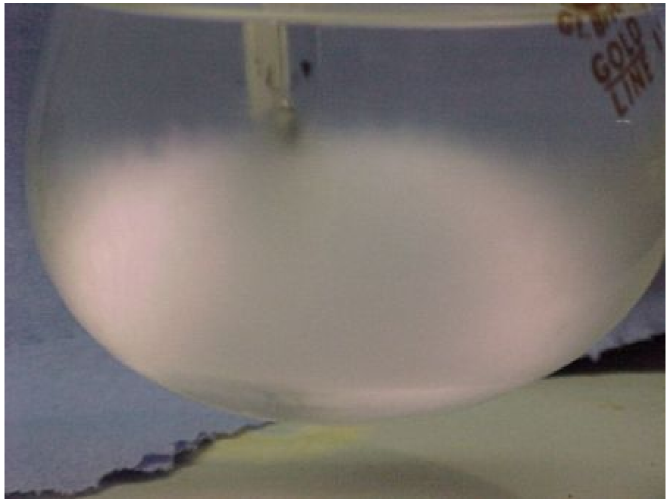
BEST COLLABORATION





Why consider fluctuations?

For consistency: Satisfy fluctuation-dissipation relations.

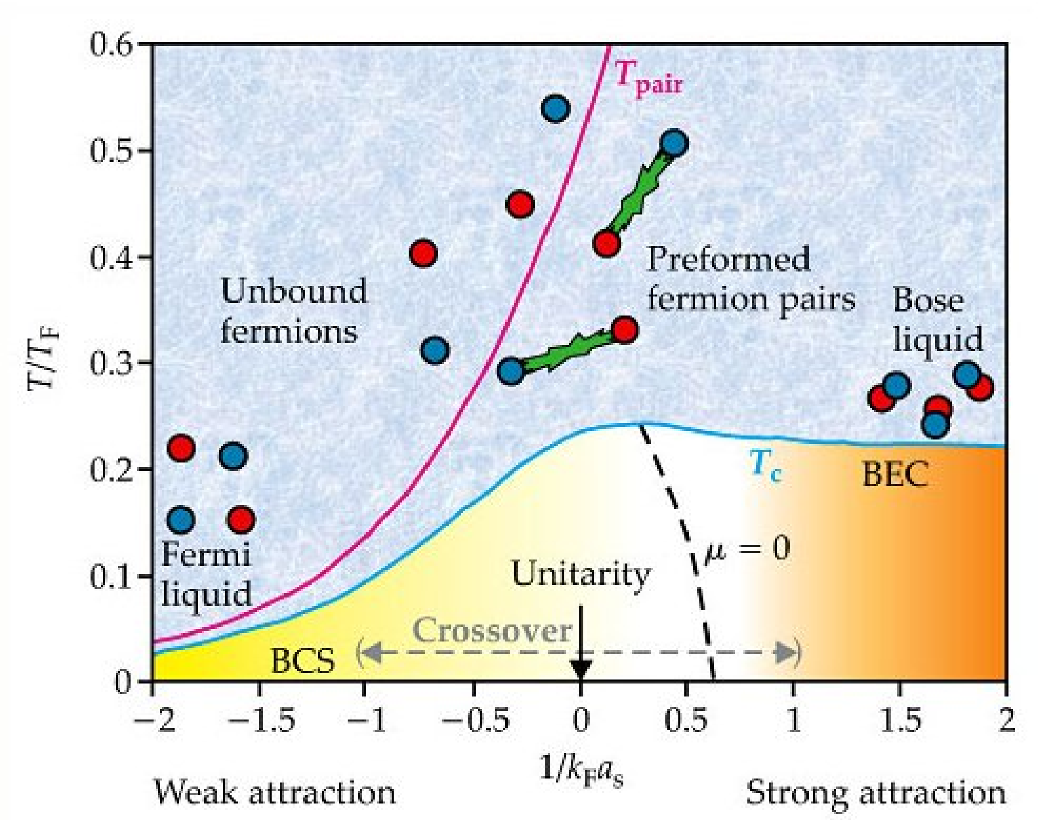
Fluid dynamics as an EFT: Fluctuations determine non-analyticities in (ω, k), and encode the resolution dependence of low energy parameters (such as transport coefficients).

Role of fluctuations enhanced in nearly perfect fluids ($\eta/s \lesssim 1$).

Fluctuations are dominant near critical points.

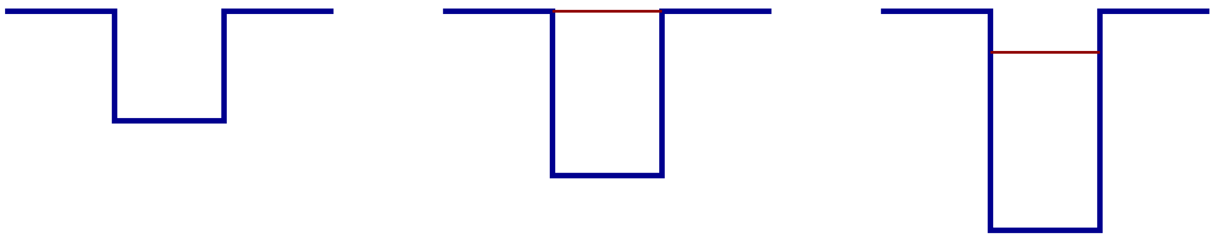
Introduction: Dilute Fermi gas, BCS-BEC crossover

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$



Unitarity limit

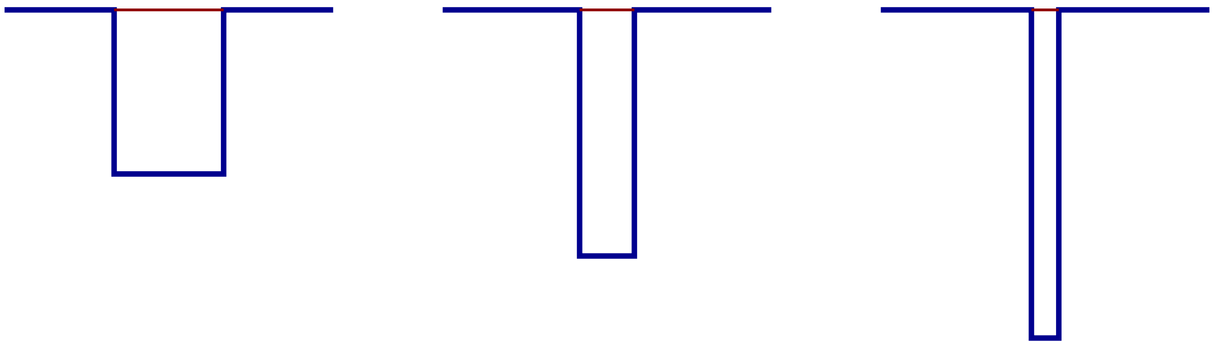
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Unitarity limit

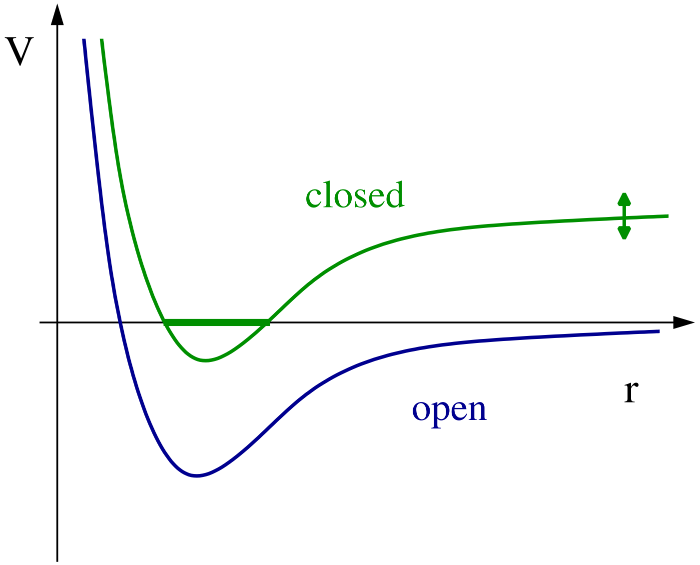
Now take the range to zero, keeping $\epsilon_B \simeq 0$

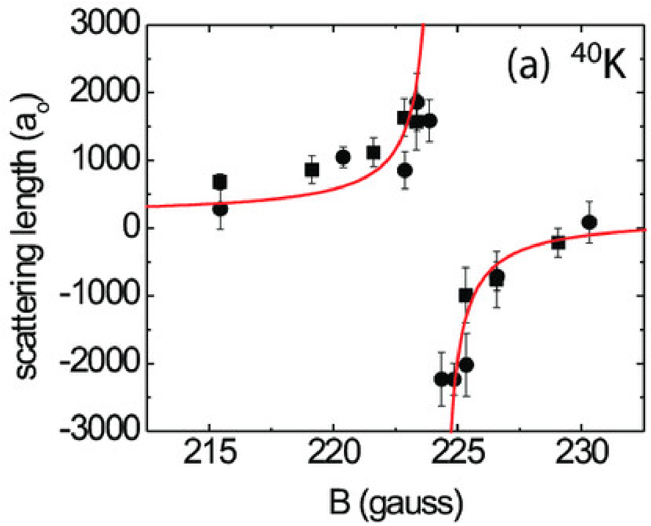


Universal scattering amplitude $\quad \mathcal{T} = \dfrac{1}{ik}$

Feshbach resonances

Atomic gas with two spin states: "↑" and "↓"

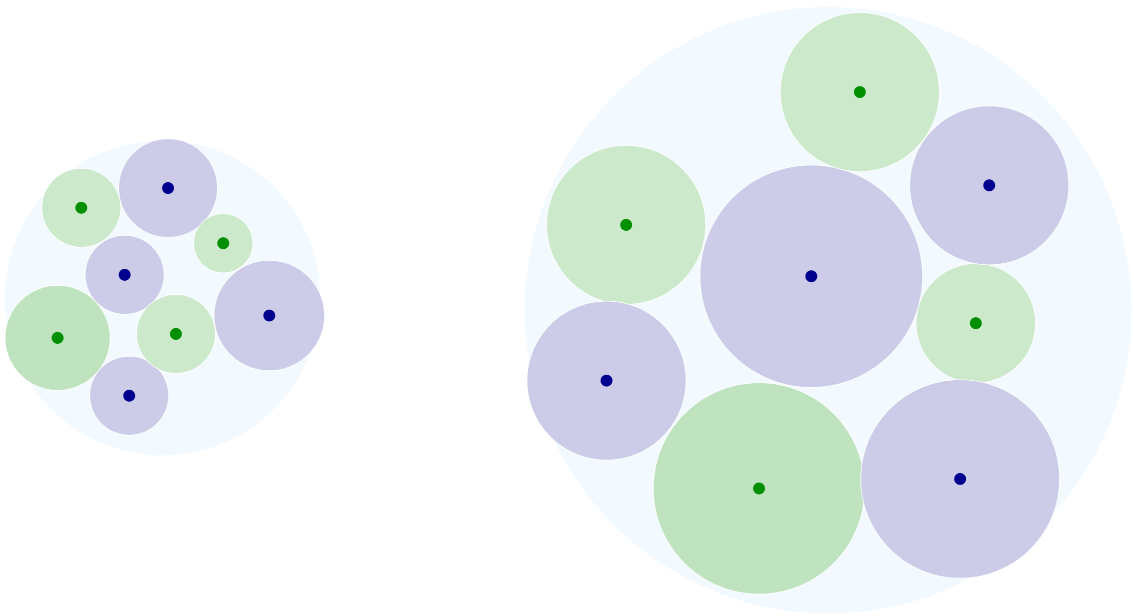




Feshbach resonance $$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

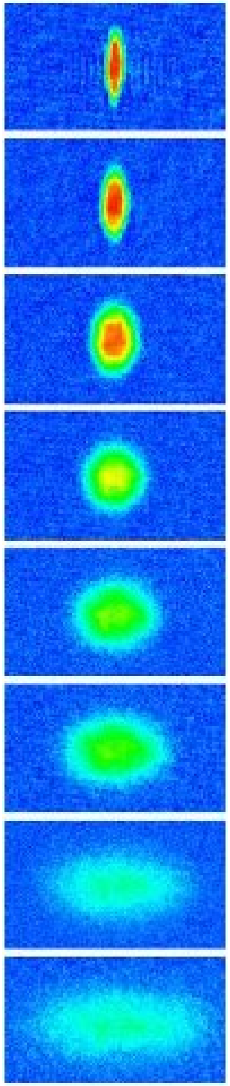
Universal fluid dynamics

Many body system: Effective cross section $\sigma_{tr} \sim n^{-2/3}$ (or $\sigma_{tr} \sim \lambda^2$)

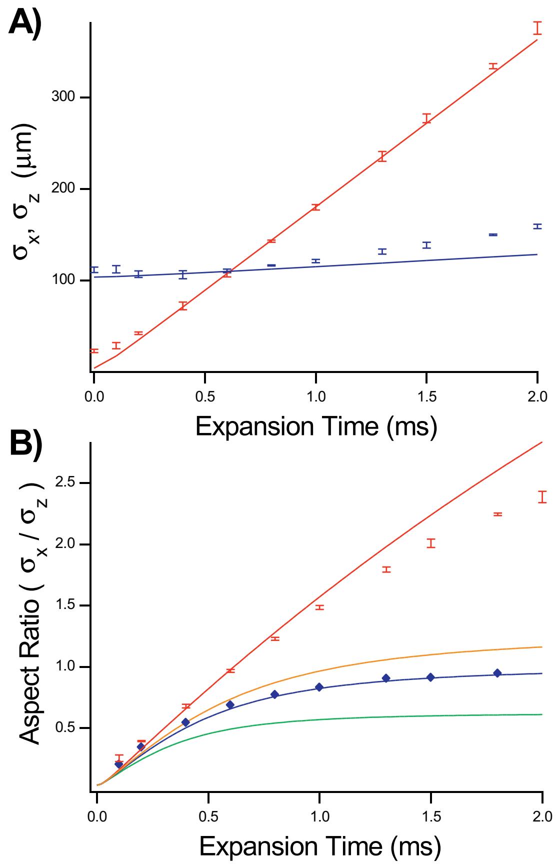


Systems remains hydrodynamic despite expansion

Almost ideal fluid dynamics



O'Hara et al. (2002)



Hydrodynamic expansion

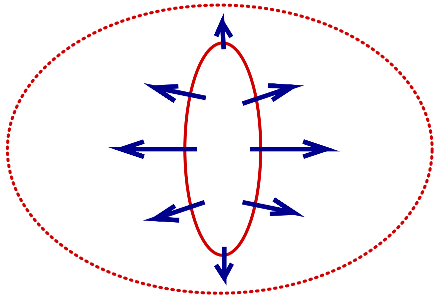
converts

coordinate space

anisotropy

to momentum space

anisotropy



Part I: Noncritical fluctuations

Hydrodynamic tails (non-analytic terms in the gradient expansion).

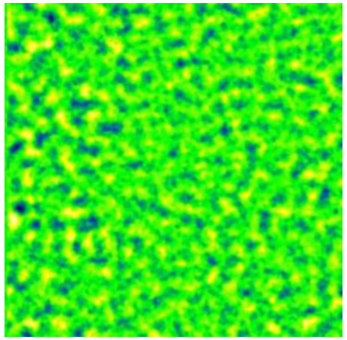
Fluctuations bounds on transport coefficients.

Fluid dynamics as a renormalizable effective theory.

Phenomenology?

Hydrodynamic fluctuations

Hydrodynamic variables fluctuate



$$\langle \delta v_i(x,t)\delta v_j(x',t)\rangle = \frac{T}{\rho}\delta_{ij}\delta(x-x')$$

Linearized hydrodynamics propagates fluctuations as shear or sound

$$\langle \delta v_i^T \delta v_j^T\rangle_{\omega,k} = \frac{2T}{\rho}(\delta_{ij}-\hat{k}_i\hat{k}_j)\frac{\nu k^2}{\omega^2+(\nu k^2)^2} \qquad \textit{shear}$$

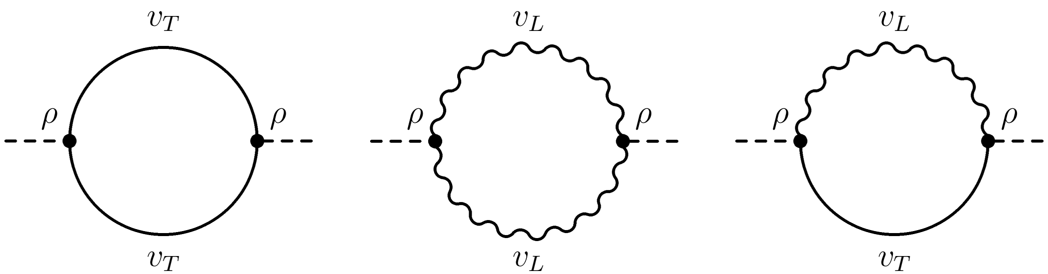
$$\langle \delta v_i^L \delta v_j^L\rangle_{\omega,k} = \frac{2T}{\rho}\hat{k}_i\hat{k}_j\frac{\omega k^2\Gamma}{(\omega^2-c_s^2k^2)^2+(\omega k^2\Gamma)^2} \qquad \textit{sound}$$

$v = v_T + v_L$: $\nabla\cdot v_T = 0$, $\nabla\times v_L = 0$ $\qquad$ $\nu = \eta/\rho$, $\Gamma = \frac{4}{3}\nu + \ldots$

Hydro Loops: "Breakdown" of second order hydro

Correlation function in hydrodynamics

$$G_S^{xyxy} = \langle\{\Pi^{xy}, \Pi^{xy}\}\rangle_{\omega,k} \simeq \rho_0^2 \langle\{v_x v_y, v_x v_y\}\rangle_{\omega,k}$$



Match to response function in $\omega \to 0$ (Kubo) limit

$$G_R^{xyxy} = P + \delta P - i\omega[\eta + \delta\eta] + \omega^2 \left[\eta\tau_\pi + \delta(\eta\tau_\pi)\right]$$

with

$$\delta P \sim T\Lambda^3 \qquad \delta\eta \sim \frac{T\rho\Lambda}{\eta} \qquad \delta(\eta\tau_\pi) \sim \frac{1}{\sqrt{\omega}} \frac{T\rho^{3/2}}{\eta^{3/2}}$$

Hydro Loops: RG and "breakdown" of 2nd order hydro

Cutoff dependence can be absorbed into bare parameters. Non-analytic terms are cutoff independent.

Fluid dynamics is a "renormalizable" effective theory.

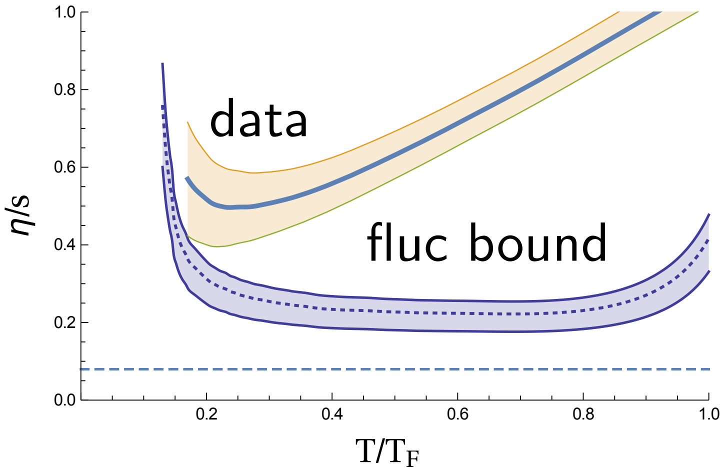
Small η enhances fluctuation corrections: $\delta\eta \sim T \left(\frac{\rho}{\eta}\right)^2 \left(\frac{P}{\rho}\right)^{1/2}$

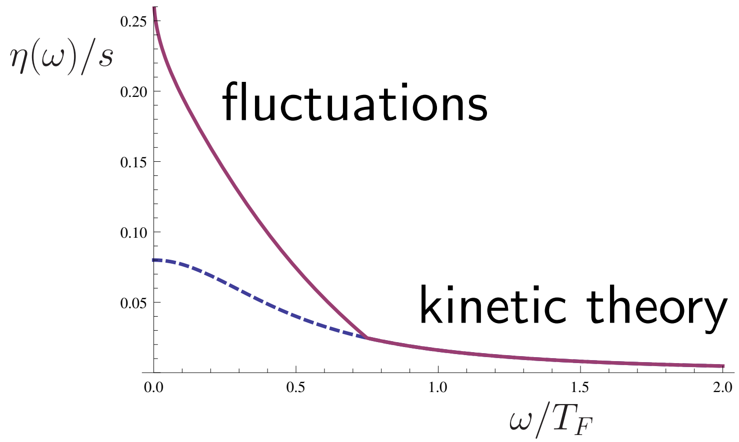
Small η leads to large $\delta\eta$: There must be a bound on η/n.

Relaxation time diverges: $\delta(\eta\tau_\pi) \sim \frac{1}{\sqrt{\omega}} \left(\frac{\rho}{\eta}\right)^{3/2}$

2nd order hydro without fluctuations inconsistent.

Fluctuation induced bound on η/s





$(\eta/s)_{min} \simeq 0.2$

spectral function

non-analytic $\sqrt{\omega}$ term

Schaefer, Chafin (2012), see also Kovtun, Moore, Romatschke (2011)

Fluctuation induced bulk stresses

Kubo relation for bulk viscosity

$$\zeta = - \lim_{\omega\to 0} \operatorname{Im} \frac{1}{9\omega} \int dt d^3x \, e^{-i\omega t} \, \langle [\Pi_{ii}(t,x), \Pi_{jj}(0)] \Theta(t) \rangle$$

Scale invariance not manifest

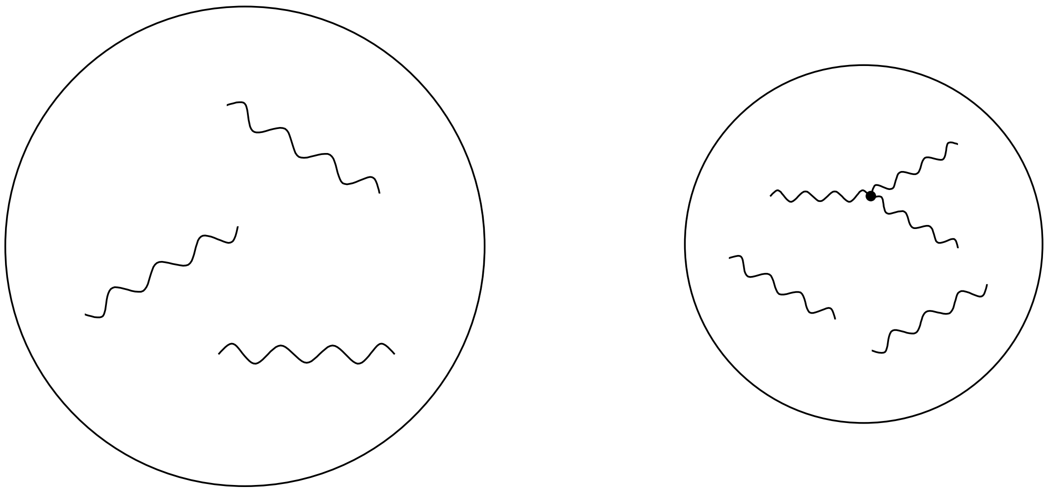
May use conservation of energy $\partial_t \mathcal{E} + \vec{\nabla} \cdot \vec{\jmath}^{\epsilon} = 0$ to rewrite Kubo formula

$$\zeta = - \lim_{\omega\to 0} \operatorname{Im} \frac{1}{\omega} \langle [\mathcal{O}(t,x), \mathcal{O}(0)] \rangle_{\omega k} \qquad \mathcal{O} = \frac{1}{3}\Pi_{ii} - \frac{2}{3}\mathcal{E}$$

and consider coupling to fluctuations of ρ and T

$$\mathcal{O} = \mathcal{O}_0 + a_{\rho\rho}(\Delta\rho)^2 + a_{\rho T}\Delta\rho\Delta T + a_{TT}(\Delta T)^2 + \ldots .$$

Fluctuation induced bulk stresses



Fluctuation contribution to bulk spectral function ($A_i \sim (P - \frac{2}{3}\mathcal{E})^2$):

$$\zeta(\omega) = \zeta(0) - \left(\frac{A_T}{(2D_T)^{3/2}} + \frac{A_\Gamma}{\Gamma^{3/2}} \right) \frac{\sqrt{\omega}}{36\sqrt{2}\pi} \, .$$

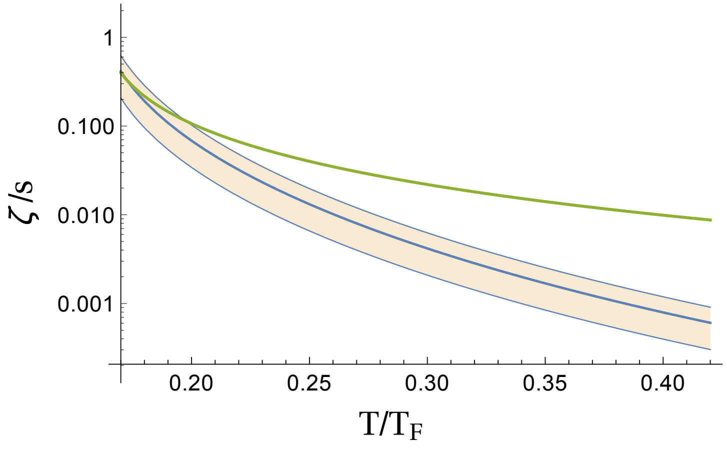
Fluctuation bound

$$\zeta_{min} = \left(\frac{A_T}{2D_T^2} + \frac{\sqrt{5}A_\Gamma}{\sqrt{3}\Gamma^2} \right) \sqrt{\frac{T}{m}} \, .$$

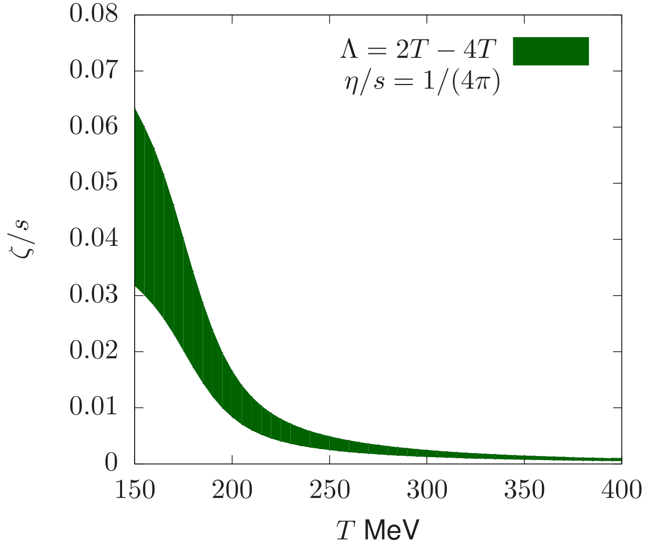
Consider $\lambda/a \sim 1$. Get $\zeta/s \gtrsim 0.1$

Fluctuation induced bound on ζ/s

(Detuned) Unitary Fermi Gas



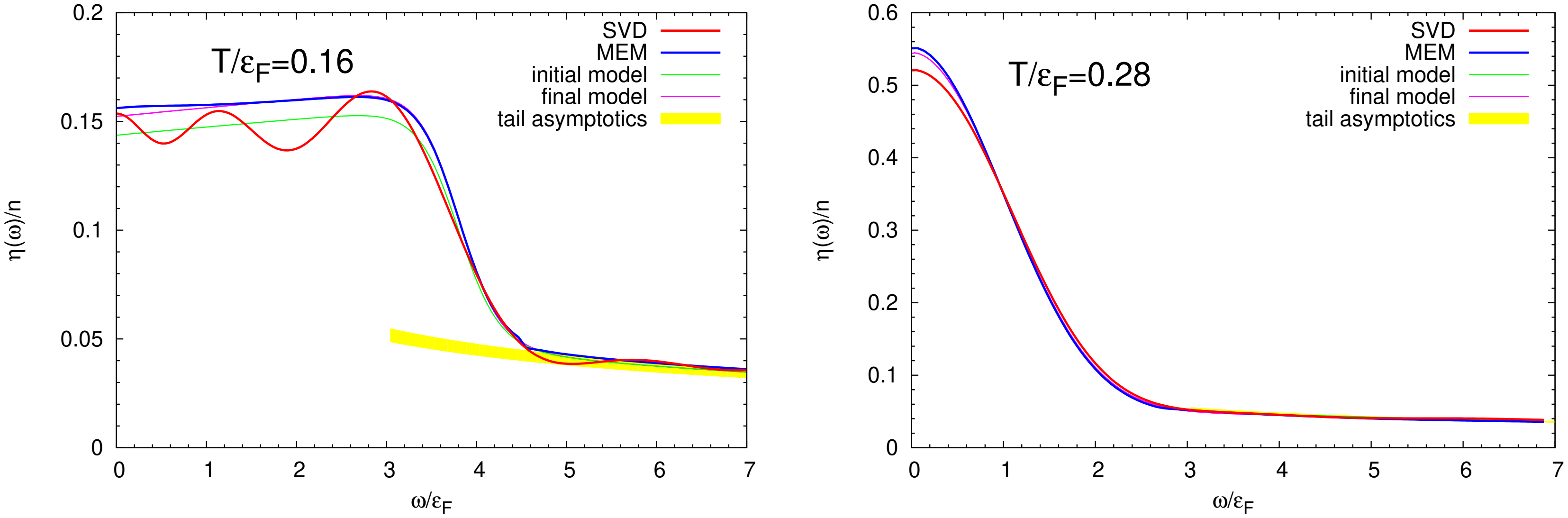
Quark Gluon Plasma



Non-relativistic fluid, M. Martinez, T. S. (2017); relativistic fluid, Akamatsu et al. (2018)

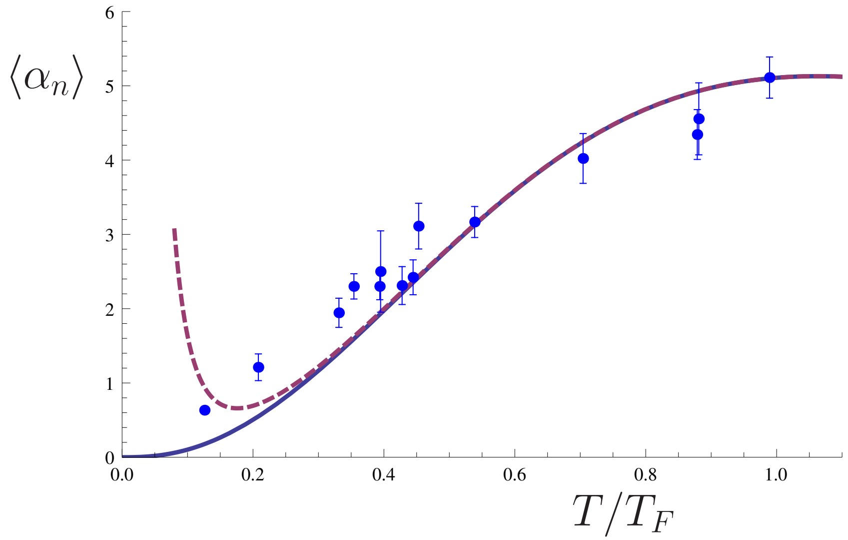
See also Kovtun, Yaffe (2003)

Can we detect tails (numerical simulation)?



Numerical results for spectral functions (Wlazlowski et al.)

Can we detect tails (experimentally)?



Damping of collective modes (3d transverse breathing mode)

2d breathing mode: $\Gamma/\omega \simeq \frac{1}{16\pi}\log(N)$

Part II: Critical Fluctuations

Expectation: If there is a critical endpoint in the QCD phase diagram, then the dynamical universality class is that of model H (liquid-gas).

Possible simplifications: Purely diffusive dynamics of order parameter mode (model B).

The superfluid transition in the unitary Fermi gas (and in liquid He) is described by model F.

Digression: Diffusion

Consider a Brownian particle

$$\dot{p}(t) = -\gamma_D p(t) + \zeta(t) \qquad \langle \zeta(t)\zeta(t') \rangle = \kappa \delta(t - t')$$

drag (dissipation) white noise (fluctuations)

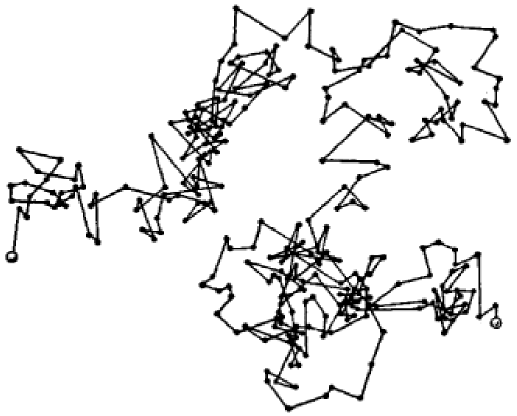
For the particle to eventually thermalize

$$\langle p^2 \rangle = 2mT$$

drag and noise must be related

$$\kappa = \frac{mT}{\gamma_D}$$

Einstein (Fluctuation-Dissipation)



Hydrodynamic equation for critical mode

Equation of motion for critical mode ϕ ("model H")

$$\frac{\partial \phi}{\partial t} = D\nabla^2 \frac{\delta \mathcal{F}}{\delta \phi} - g\vec{\nabla}\phi \cdot \frac{\delta \mathcal{F}_T}{\delta \vec{\pi}} + \zeta_\phi$$

Diffusive Reactive White Noise

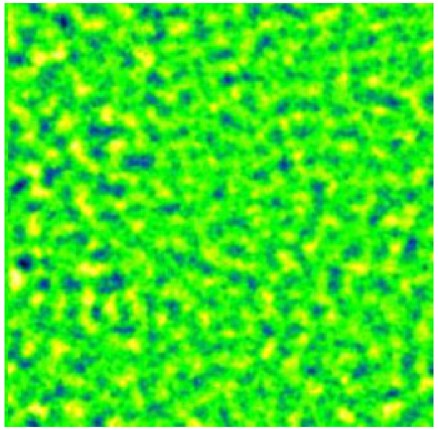
Free energy functional: Order parameter ϕ, momentum density $\vec{\pi} = \rho\vec{v}$

$$\mathcal{F} = \int d^d x \left[\frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{r}{2}\phi^2 + \lambda\phi^4 + \frac{1}{2}\vec{\pi}^2 \right]$$

Fluctuation-Dissipation relation

$$\langle \zeta_\phi(x,t)\zeta_\phi(x',t')\rangle = 2DT\delta(x-x')\delta(t-t')$$

ensures $P[\phi] \sim \exp(-\mathcal{F}[\phi]/T)$

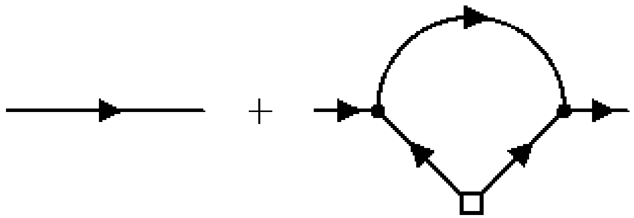


Linearized analysis (non-critical fluid)

Navier-Stokes equation: $\partial_0 \vec{v} + \nu \nabla^2 \vec{v} = mode\ couplings + noise$

Linearized propagator: $\langle \delta v_i^T \delta v_j^T \rangle_{\omega,k} = \frac{1}{\rho} \frac{-\nu k^2 P_{ij}^T}{-i\omega + \nu k^2}$ $\qquad \nu = \frac{\eta}{\rho}$

Fluctuation correction:



Renormalized viscosity:

$$\eta = \eta_0 + c_\eta \frac{T\rho\Lambda}{\eta_0} - c_\tau \sqrt{\omega} \frac{T\rho^{3/2}}{\eta_0^{3/2}}$$

Hydro is a renormalizable stochastic field theory

Linearized analysis (critical fluid)

Consider order parameter mode

$$\partial_0 \phi = -D\nabla^2 \frac{\delta \mathcal{F}}{\delta \phi} + \textit{mode couplings} + \zeta_\phi$$

$$\mathcal{F} = \int d^3x \left\{ \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + \lambda\phi^4 + \frac{1}{2}\vec{\pi}^2 \right\}$$

Dispersion relation $i\omega = Dq^2(r+q^2) + \ldots$

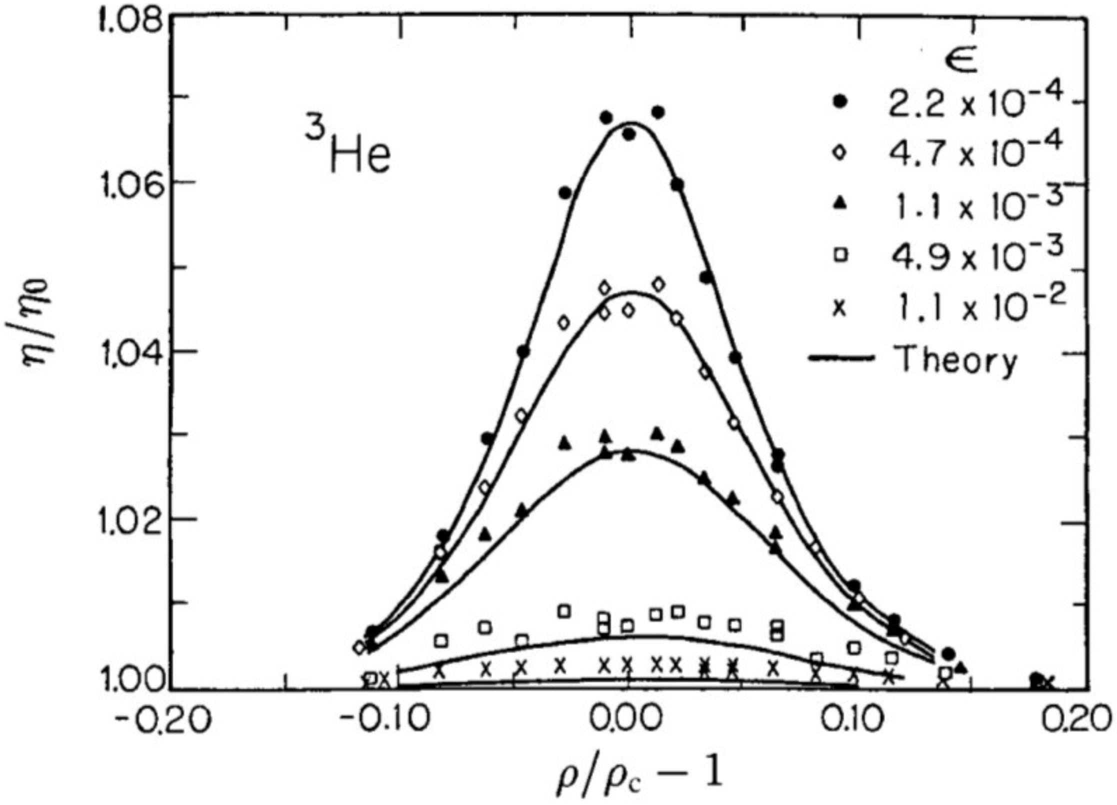
Use $r \sim \xi^{-2}$. Relaxation time for modes $q \sim \xi^{-1}$:

$$\tau \sim \xi^z \quad (z=4) \qquad \text{"Critical slowing down"}$$

A more sophisticated analysis gives $z \simeq 3$ and

$$\eta \sim \xi^{0.05} \qquad \kappa \sim \xi^{0.9} \qquad \zeta \sim \xi^{2.8}$$

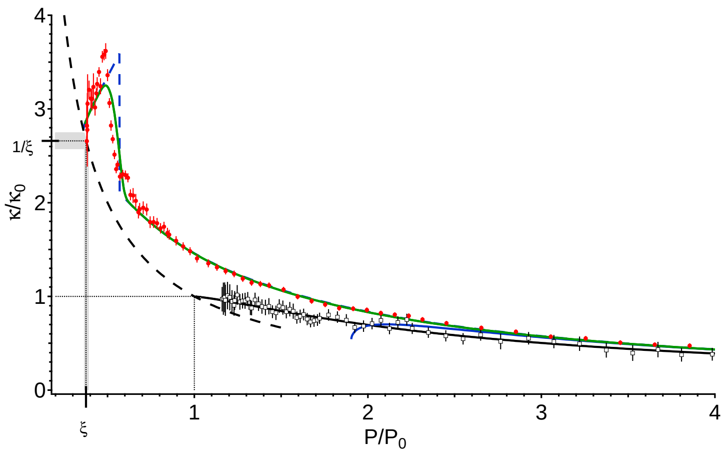
Critical transport (helium)



Agosta, Wang, Meyer (1987)

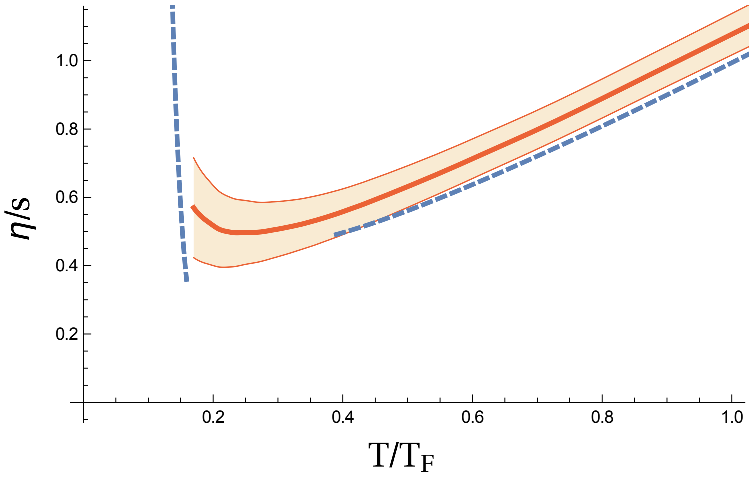
More dramatic enhancement in thermal conductivity and bulk viscosity (sound attenuation)

Critical behavior (unitary Fermi gas)



λ-transition in κ, c_V

Ku et al. (Science, 2012)



η/s looks smooth

Bluhm, J. H., T.S (2017)

Summary

Beautiful theory of hdrodynamic tails (RG, bounds on transport coefficients, non-analyticities), but difficult to observe in experiment (applies to both unitary gases and heavy ion collisions).

Equilibrium critical behavior (λ-transition) in cold gases has been observed, transport behavior more difficult. Recent developments (box potentials, local thermometers) should help.

Possible test bed for studying critical behavior in expanding systems.