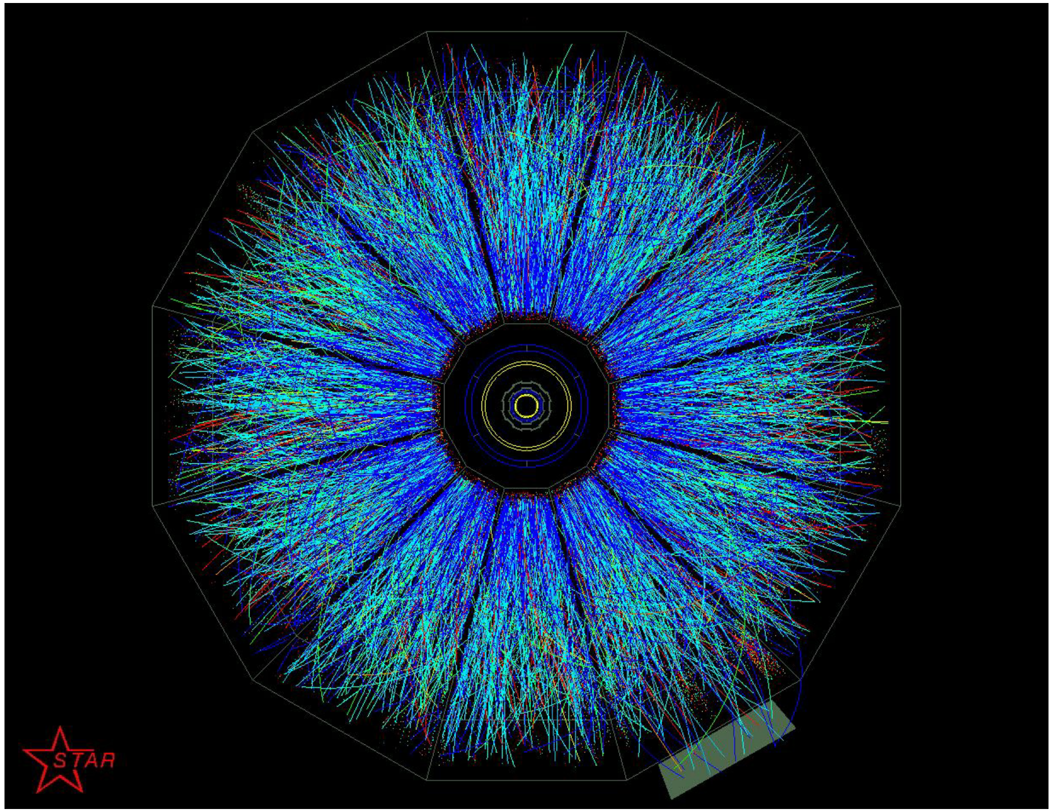
The critical point in QCD

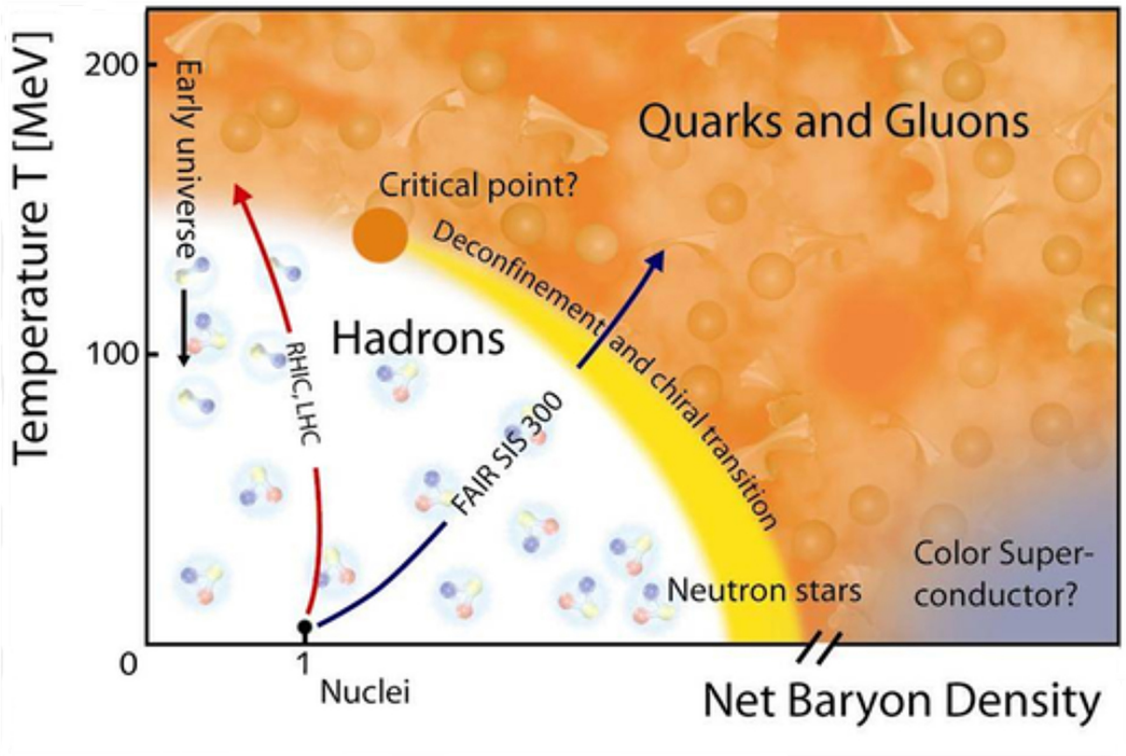
Thomas Schäfer

North Carolina State University

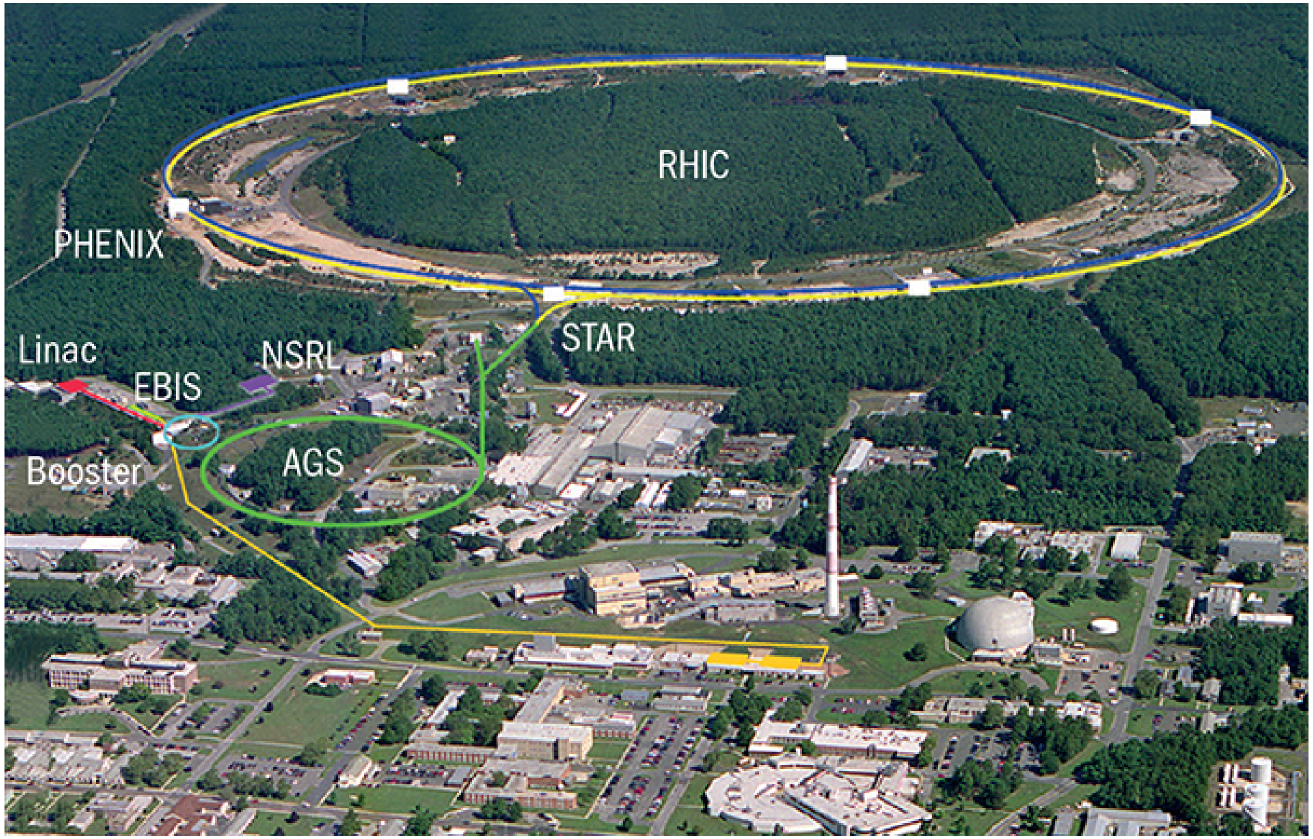


The phase diagram of QCD

$$\boxed{\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu}}$$



2000: Dawn of the collider era at RHIC

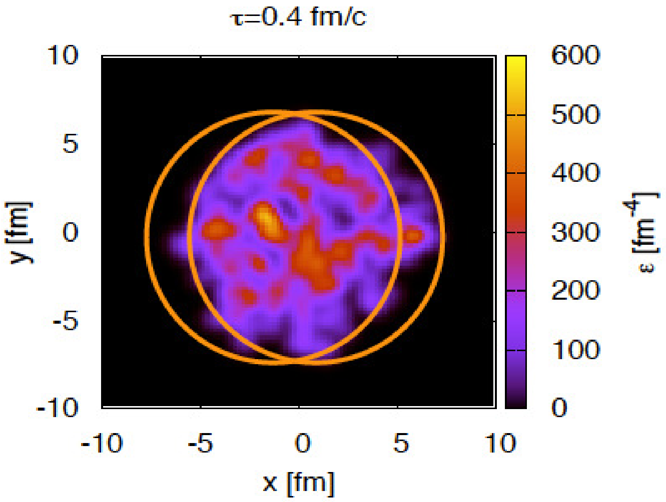


$Au + Au$ @200 $AGeV$

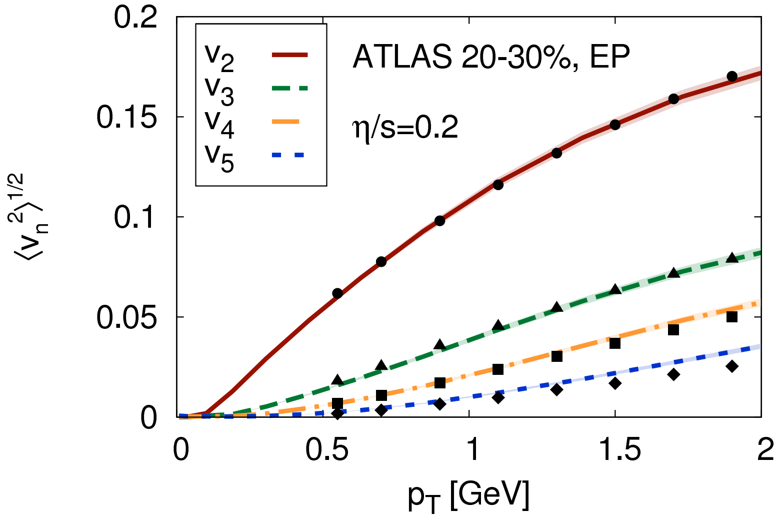
What did we find?

Heavy ion collisions at RHIC are described by a very simple theory:

$$\pi\alpha\nu\tau\alpha\ \rho\varepsilon\iota \quad (\textit{everything flows})$$



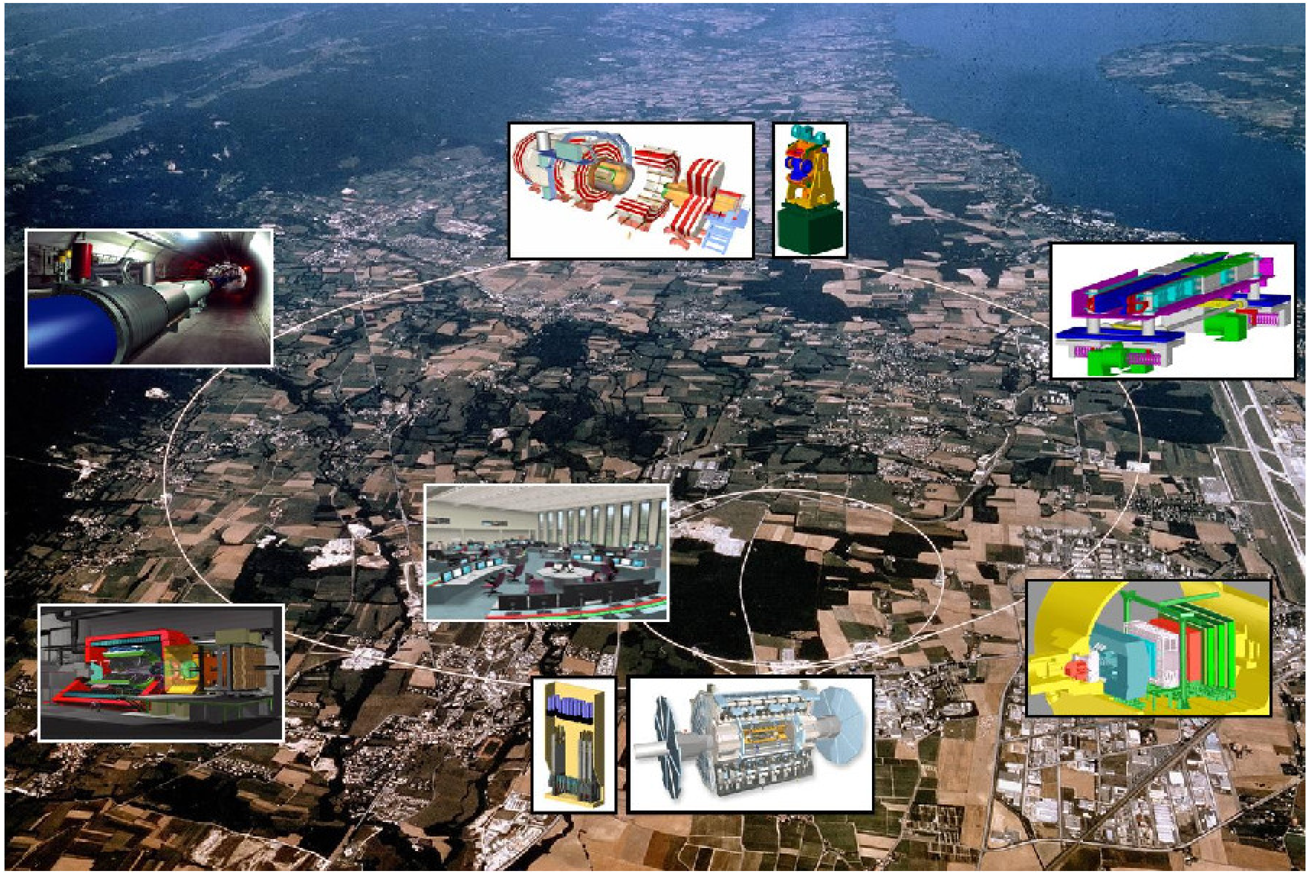
B. Schenke



C. Gale et al.

Hydro converts initial state fluctuations to flow fluctuations. Attenuation coefficient is small, $\eta/s \simeq 0.08\hbar/k_B$, indicating that the plasma is strongly coupled.

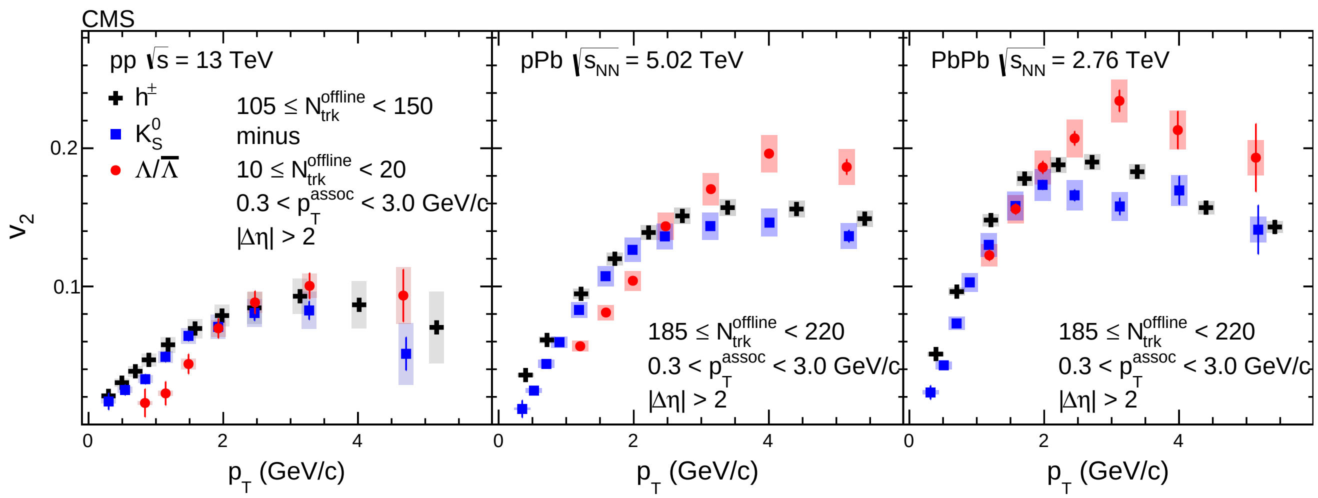
2010: The energy frontier at LHC



$Pb + Pb$ @$2.76\,ATeV$, now $5.5\,ATeV$

What did we find?

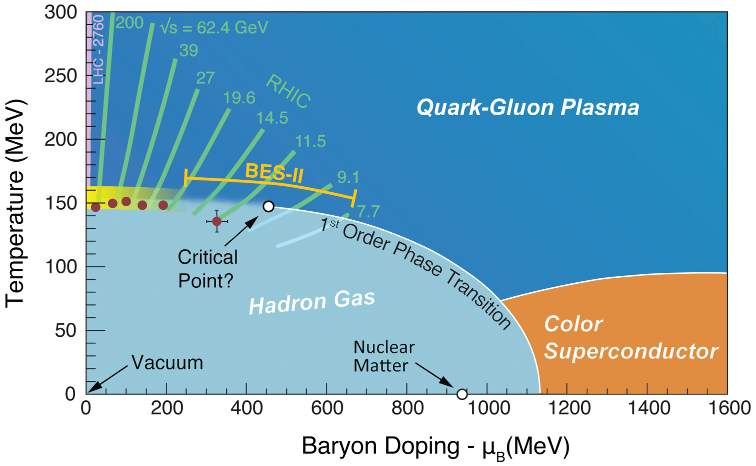
Even the smallest droplets of QGP fluid produced in (high multiplicty) pp and pA collisions exhibit collective flow.

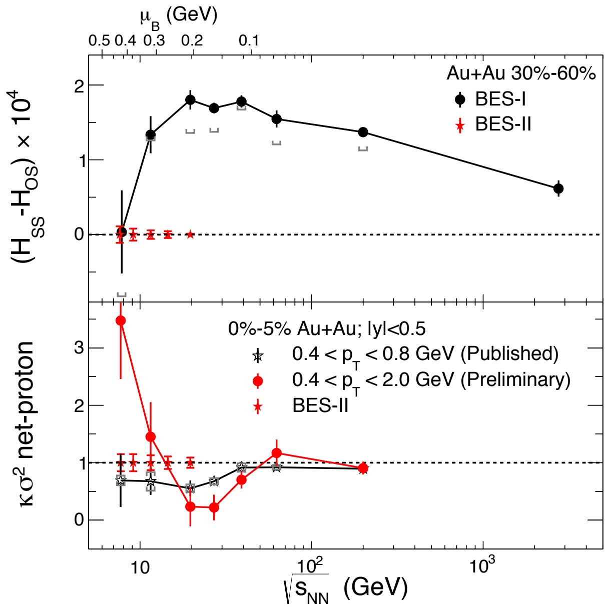


Small viscosity $\eta/s \simeq 0.08\hbar/k_B$ implies short mean free path and rapid hydrodynamization.

The next step: RHIC beam energy scan

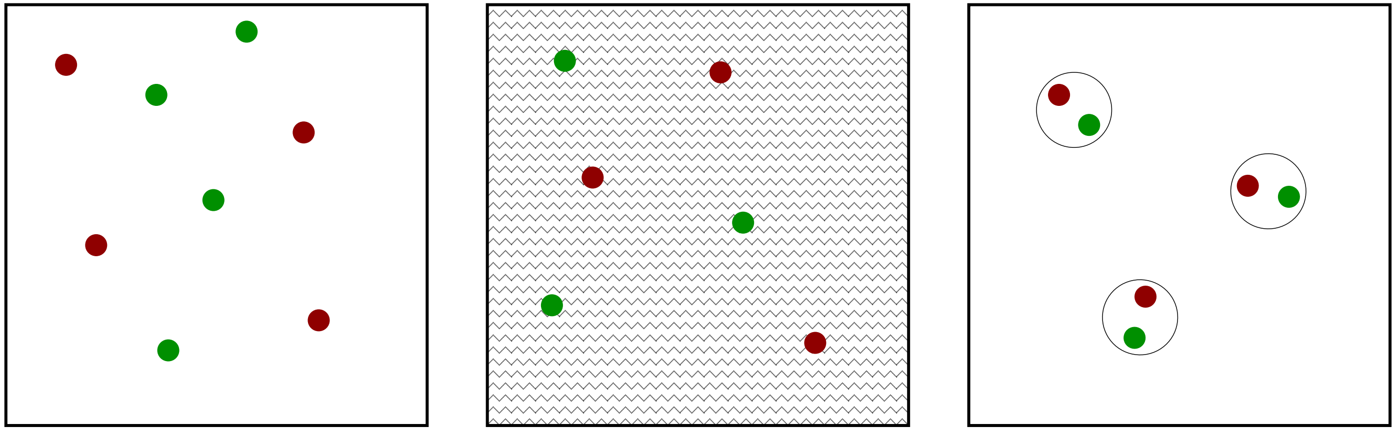
Can we locate the phase transition itself, either by locating a critical point, or identifying a first order transition?





What is a Phase of QCD? Phases of Gauge Theories

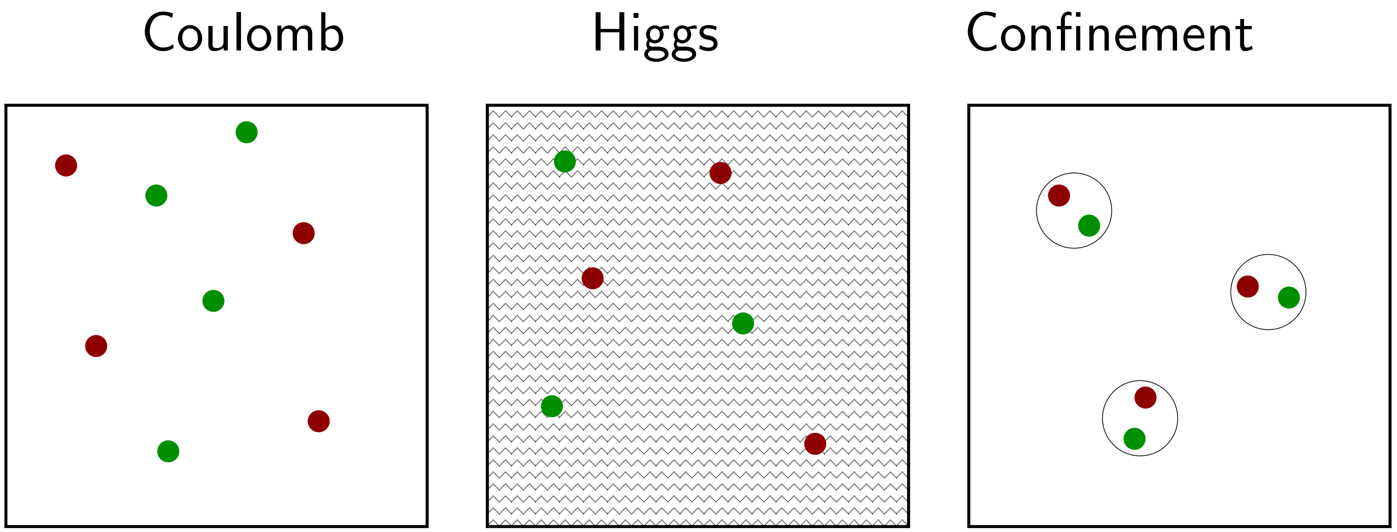
Coulomb Higgs Confinement



$$V(r) \sim -\frac{e^2}{r} \qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

Standard Model: $U(1) \times SU(2) \times SU(3)$

What is a Phase of QCD? Phases of Gauge Theories

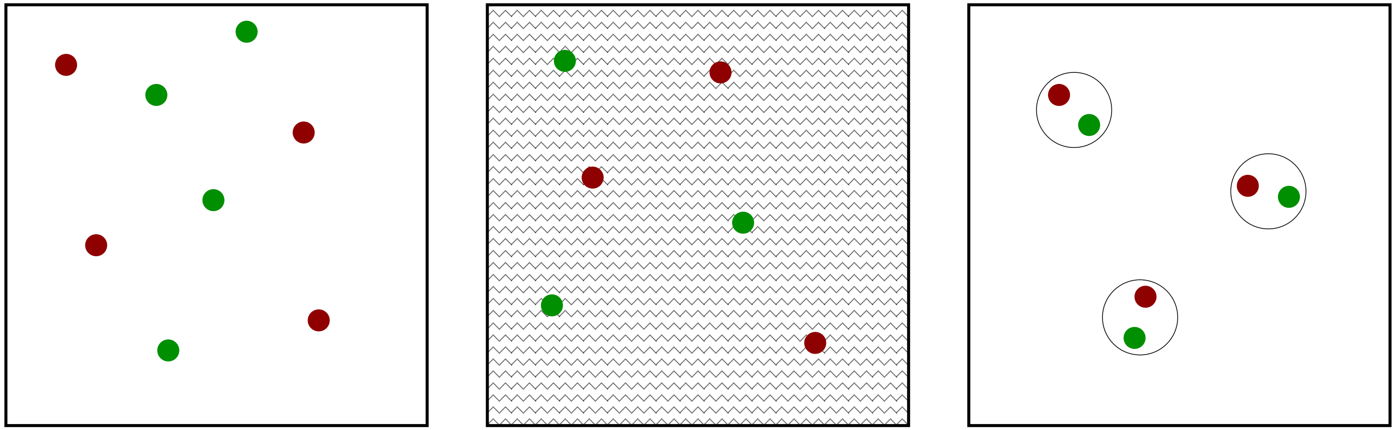


$$V(r) \sim -\frac{e^2}{r} \qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

QCD: High T phase — High μ phase — Low T, μ phase

What is a Phase of QCD? Phases of Gauge Theories

Coulomb | Higgs | Confinement



$$V(r) \sim -\frac{e^2}{r} \qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

No local order parameters: Phases can be continously connected.

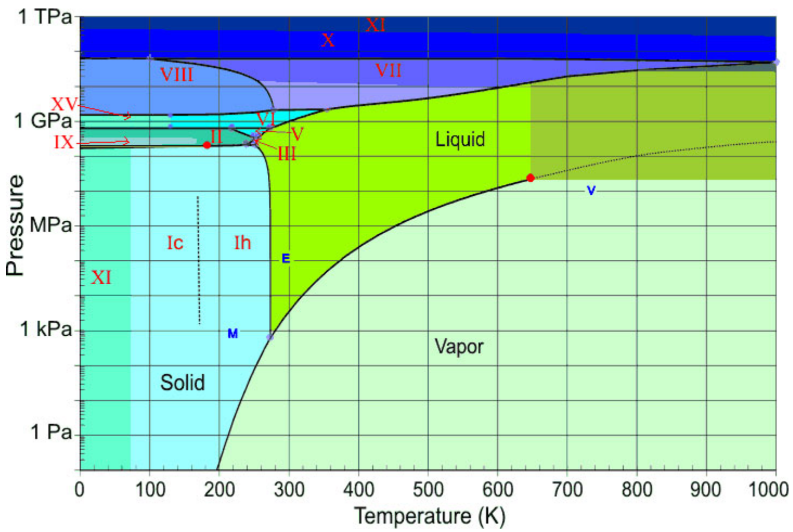
Phases of Matter: Symmetries

Local order parameters and change of symmetry: Sharp phase transitions.

phase	order param	broken symmetry	rigidity phenomenon	Goldstone boson
crystal	ρ_k	translations	rigid	phonon
magnet	$\vec{M}$	rotations	hysteresis	magnon
superfluid	$\langle\Phi\rangle$	particle number	supercurrent	phonon
supercond.	$\langle\psi\psi\rangle$	gauge symmetry	supercurrent	none (Higgs)
χsb	$\langle\bar{\psi}\psi\rangle$	chiral symmetry	axial current	pion

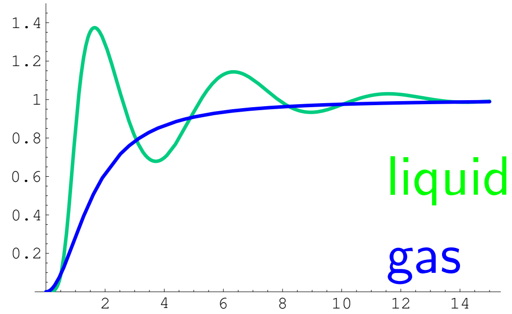
Transitions without change of symmetry: Liquid-Gas

Phase diagram of water

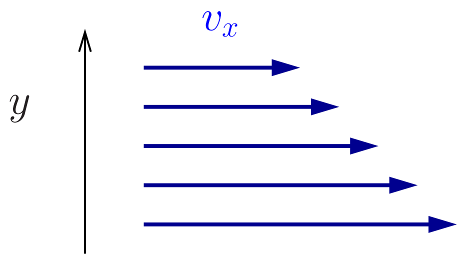


Characteristics of a liquid

Pair correlation function

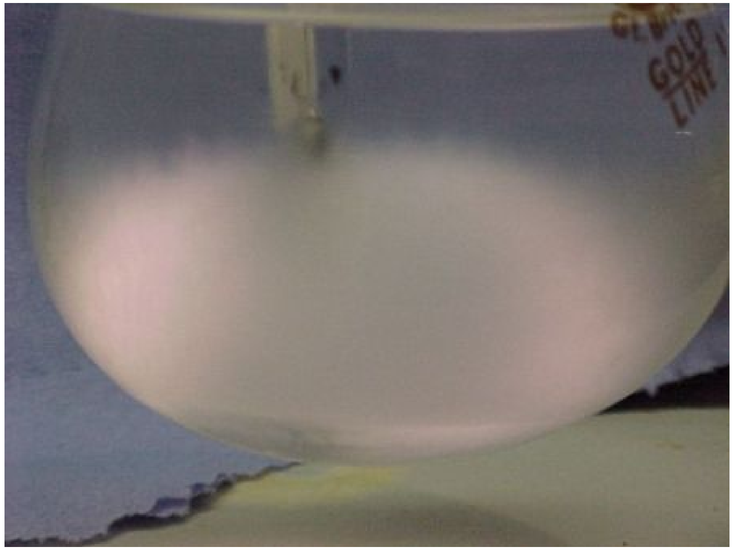


Good fluid: low viscosity



$$F_x = \eta A \frac{\partial v_x}{\partial y}$$

Signatures of the critical endpoint



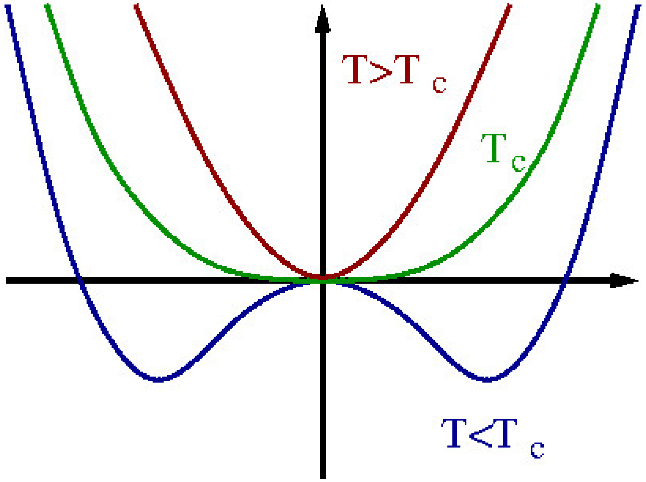
Correlation length diverges

Critical opalescence

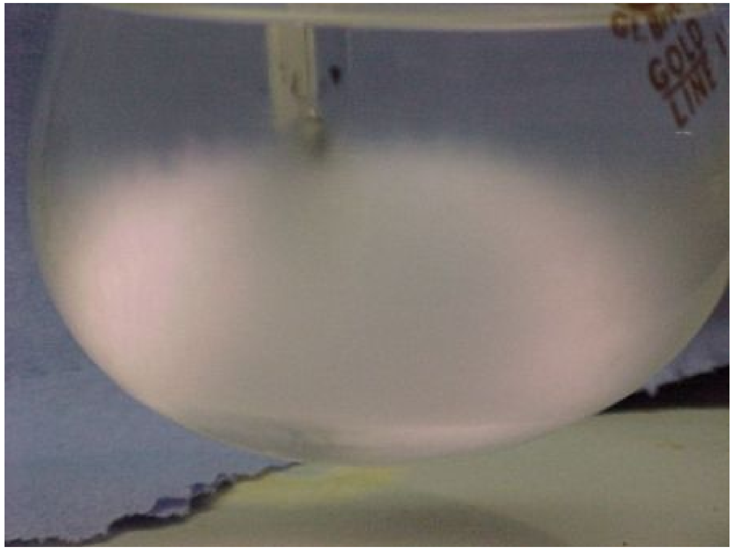
Scalar order parameter $\phi = \rho - \rho_0$

$$F[\phi] = \int d^3x \left\{ \kappa(\nabla\phi)^2 + r\phi^2 + \lambda\phi^4 + \ldots \right\}$$

Free energy functional:



Signatures of the critical endpoint



Correlation length diverges

Critical opalescence

Scalar order parameter $\phi = \rho - \rho_0$

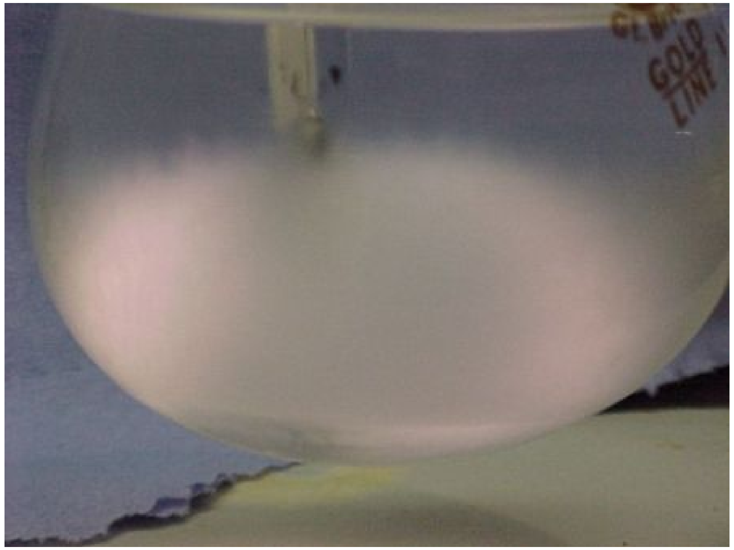
$$F[\phi] = \int d^3x \, \{\kappa(\nabla\phi)^2 + r\phi^2 + \lambda\phi^4 + \ldots\}$$

Predicts critical equation of state and correlation length

$$\xi \sim t^{-\nu} \qquad t = \frac{T - T_c}{T_c}$$

$$\nu = \frac{3}{4}\epsilon + O(\epsilon^2) \simeq 0.58$$

Signatures of the critical endpoint



Correlation length diverges

Critical opalescence

Scalar order parameter $\phi = \rho - \rho_0$

$$F[\phi] = \int d^3x \left\{ \kappa(\nabla\phi)^2 + r\phi^2 + \lambda\phi^4 + \ldots \right\}$$

$F[\phi]$ universal, ϕ could be the magnetization of a spin system.

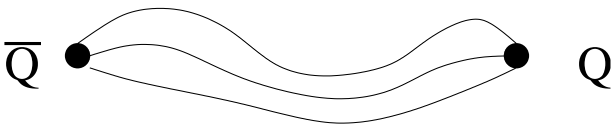
$$\xi \sim t^{-\nu} \qquad t = \frac{T - T_c}{T_c}$$

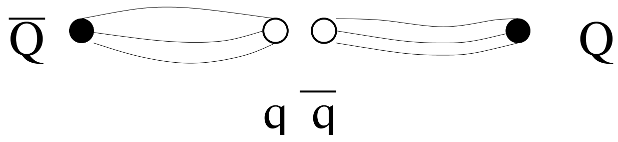
$$\nu = \frac{3}{4}\epsilon + O(\epsilon^2) \simeq 0.58$$

Results are said to belong to the $d = 3$ Ising universality class.

Critical endpoint in QCD?

Light fermions: Confinement is not a sharp phase transition





Massless fermions: Chiral symmetry breaking is a sharp transition

$$\langle \bar{\psi}_L^f \psi_R^g + \bar{\psi}_L^f \psi_R^g \rangle \simeq -(230\,\mathrm{MeV})^3\, \delta^{fg}$$

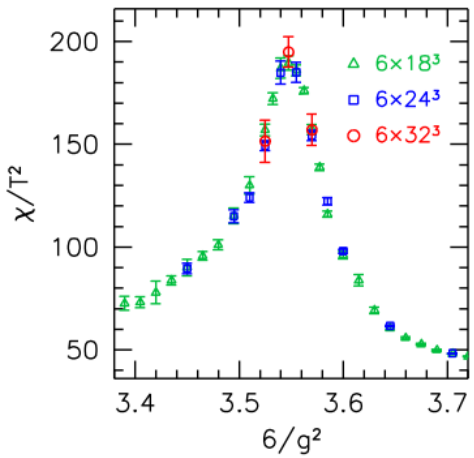
$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

$N_f = 2$: Second order.

$N_f = 3$: First order.

Real world, $m_s > m_{u,d} \neq 0$.

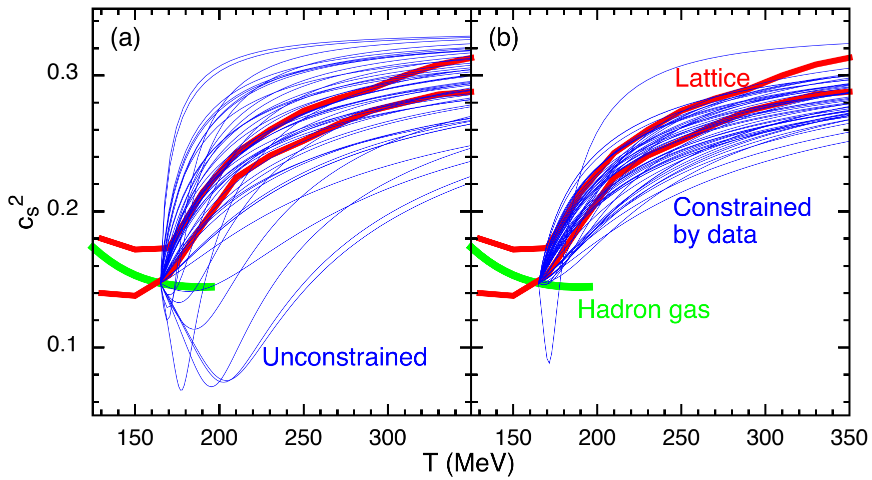
The $\mu = 0$ transition is a crossover.



Aoki et al., Nature (2006)

Crossover: Experimental indications

The speed of sound $c_s^2 = (\partial P)/(\partial \mathcal{E})$ determines the acceleration history of the fireball. Sharp phase transition: $c_s^2 = 0$. Crossover: Soft point $c_s^2(min) > 0$



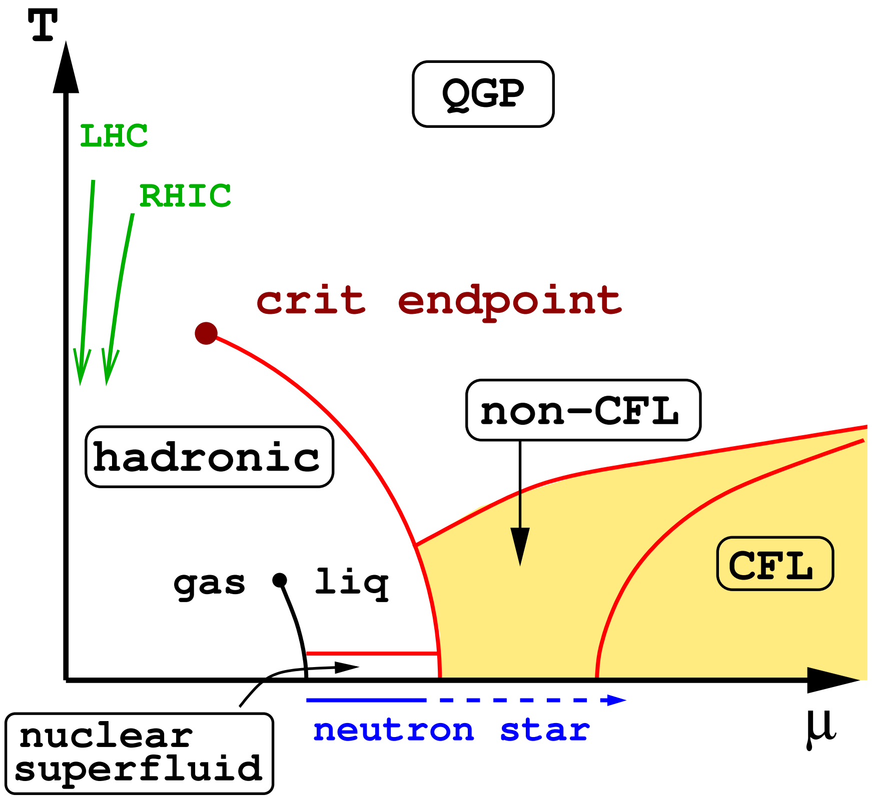
Pratt et al, PRL (2013)

Reconstruct sound speed from particle spectra, HBT source sizes and emission duration

Critical endpoint in QCD?

What happens for $\mu \neq 0$? Lattice calculations cannot tell (the QCD sign problem). Two options: The transition weakens, or it strengthens.

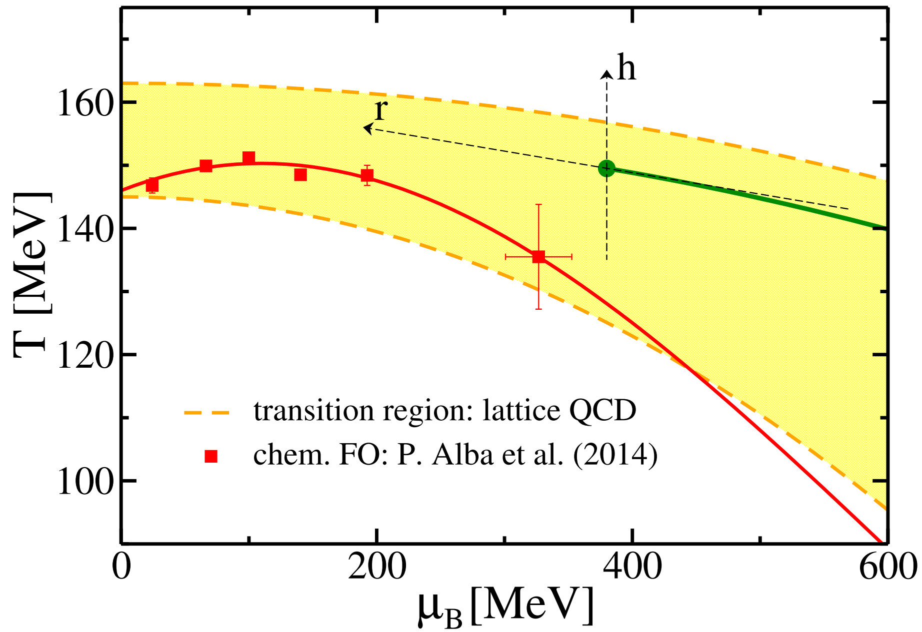
If the transition strengthen for $\mu > 0$ there is a critical endpoint.



Critical endpoint in QCD?

Several possible order parameters: $\langle\bar{\psi}\psi\rangle - \Sigma_0$, $\rho - \rho_0$, $s - s_0$.

All of them mix, obtain one critical mode. Free energy in $d = 3$ Ising universality class.



$$\mathcal{F} = \kappa(\nabla\phi)^2 + r\phi^2 + \phi h$$
$$+ g\phi^3 + \lambda\phi^4$$

External field h.

Reduced temperature r.

$$\xi \sim r^{-\nu}$$

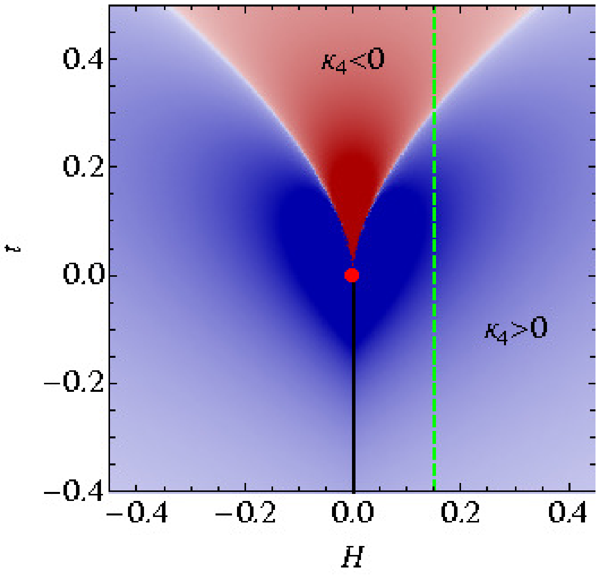
Freezout curve (exp). Transition regime (lattice). Critical line (model).

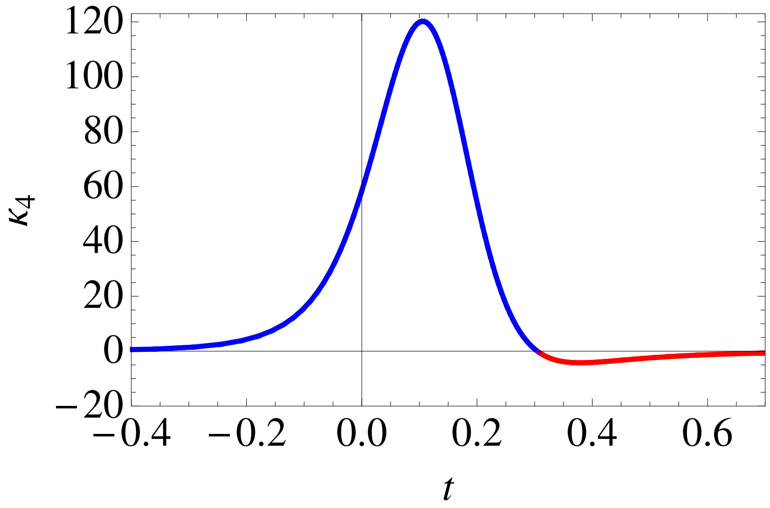
More sensitive observables: Higher order cumulants

Consider curtosis: $\kappa_4 = \langle \phi^4 \rangle - 3\langle \phi^2 \rangle^2$

Stronger divergence near critical point: $\kappa_4/\kappa_2^2 \sim \xi^3$

Non-trivial dependence on t ($\rightarrow$ beam energy)

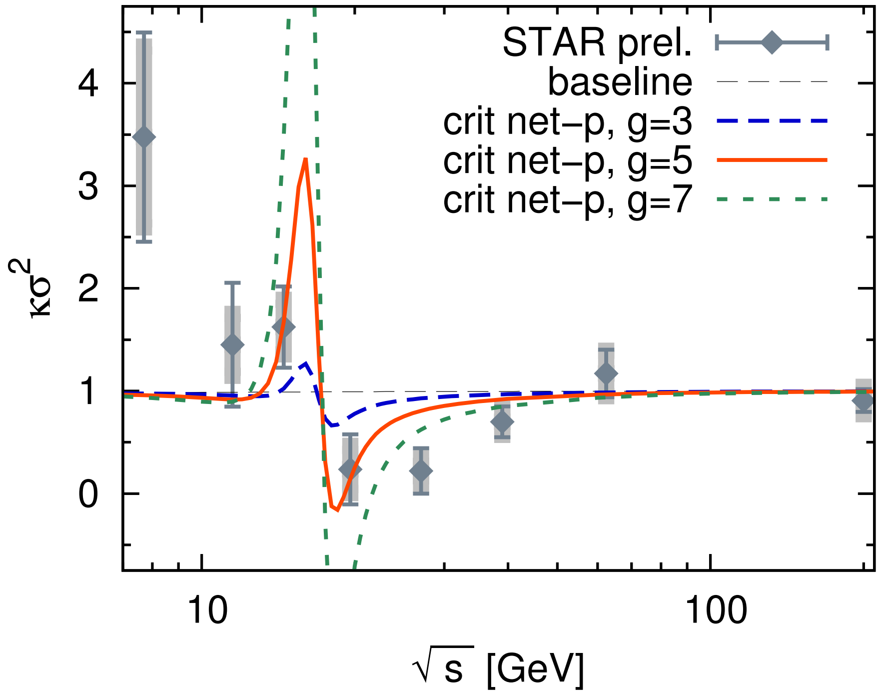




Stephanov, PRL (2011)

Compare to BES-I data

Many details: Couple fluctuations to particles $\delta N_p \sim m$, model freezeout curve, map Ising EOS to QCD phase diagram, include resonance decays.



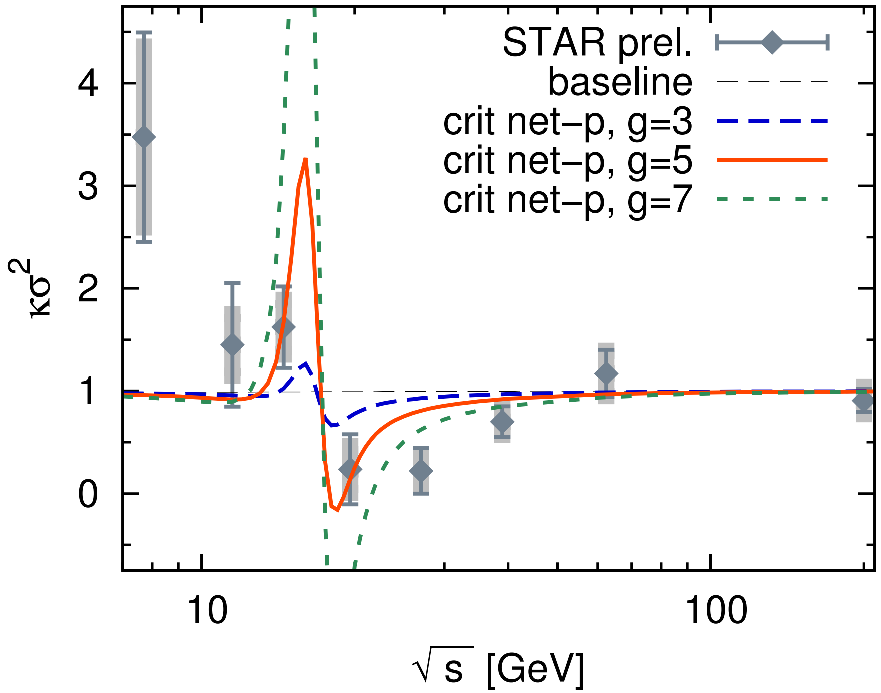
M. Bluhm et al. (2016)

High energy baseline, fluctuations are Gaussian.

Some indication of non-Gaussian behavior at lower energy.

Future improvements

Many details: Couple fluctuations to particles $\delta N_p \sim m$, model freezeout curve, map Ising EOS to QCD phase diagram, include resonance decays.



M. Bluhm et al. (2016)

Experiment: BES-II will provide smaller error bars, more energy bins. Also: Correlate with other observables.

Theory: Dynamical evolution of fluctuations.

Digression: Diffusion

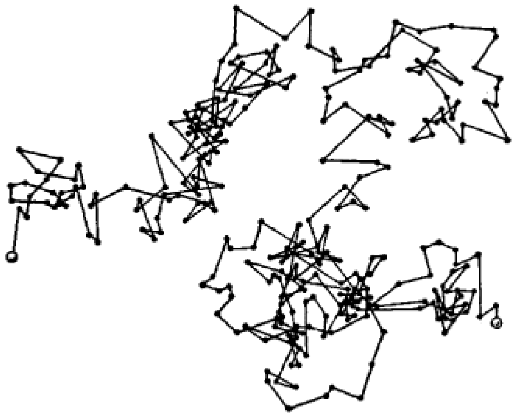
Consider a Brownian particle

$$\dot{p}(t) = -\gamma_D p(t) + \zeta(t) \qquad \langle \zeta(t)\zeta(t')\rangle = \kappa\delta(t-t')$$

drag (dissipation) white noise (fluctuations)

For the particle to eventually thermalize

$$\langle p^2 \rangle = 2mT$$



drag and noise must be related

$$\kappa = \frac{mT}{\gamma_D}$$

Einstein (Fluctuation-Dissipation)

Hydrodynamic equation for critical mode

Equation of motion for critical mode ϕ ("model H")

$$\frac{\partial\phi}{\partial t} = D_0\nabla^2\frac{\delta\mathcal{F}}{\delta\phi} - g_0\vec{\nabla}\phi\cdot\frac{\delta\mathcal{F}_T}{\delta\vec{\pi}} + \zeta_\phi$$

Diffusive Reactive White Noise

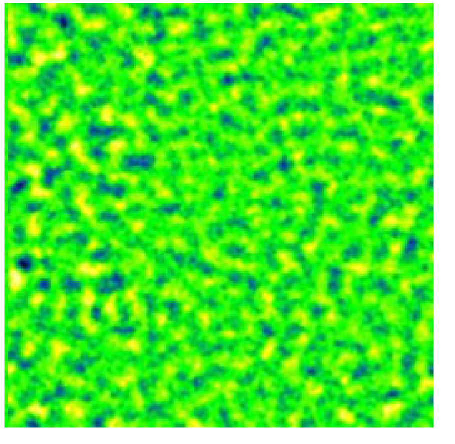
Free energy functional: Order parameter ϕ, momentum density $\vec{\pi} = \rho\vec{v}$

$$\mathcal{F} = \int d^dx\,\left[\frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{r_0}{2}\phi^2 + \lambda_0\phi^4 + \frac{1}{2}\vec{\pi}^2\right]$$

Fluctuation-Dissipation relation

$$\langle\zeta_\phi(x,t)\zeta_\phi(x',t')\rangle = 2D_0T\delta(x-x')\delta(t-t')$$

ensures $P[\phi] \sim \exp(-\mathcal{F}[\phi]/T)$

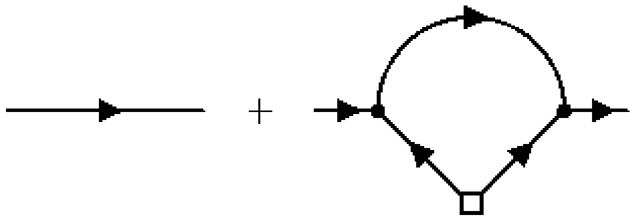


Linearized analysis (non-critical fluid)

Navier-Stokes equation: $\partial_0 \vec{v} + \nu \nabla^2 \vec{v} = mode\ couplings + noise$

Linearized propagator: $\langle \delta v_i^T \delta v_j^T \rangle_{\omega,k} = \frac{1}{\rho} \frac{-\nu k^2 P_{ij}^T}{-i\omega + \nu k^2}$ $\qquad \nu = \frac{\eta}{\rho}$

Fluctuation correction:



Renormalized viscosity:

$$\eta = \eta_0 + c_\eta \frac{T\rho\Lambda}{\eta_0} - c_\tau \sqrt{\omega} \frac{T\rho^{3/2}}{\eta_0^{3/2}}$$

Hydro is a renormalizable stochastic field theory

Linearized analysis (critical fluid)

Consider order parameter mode

$$\partial_0 \phi = -D\nabla^2 \frac{\delta \mathcal{F}}{\delta \phi} + \zeta_\phi$$

$$\mathcal{F} = \int d^3x \left\{ \frac{1}{2}(\nabla\phi)^2 + \frac{r}{2}\phi^2 + g\phi^3 + \lambda\phi^4 \right\}$$

Dispersion relation $i\omega = Dq^2(r_0 + q^2) + \ldots$

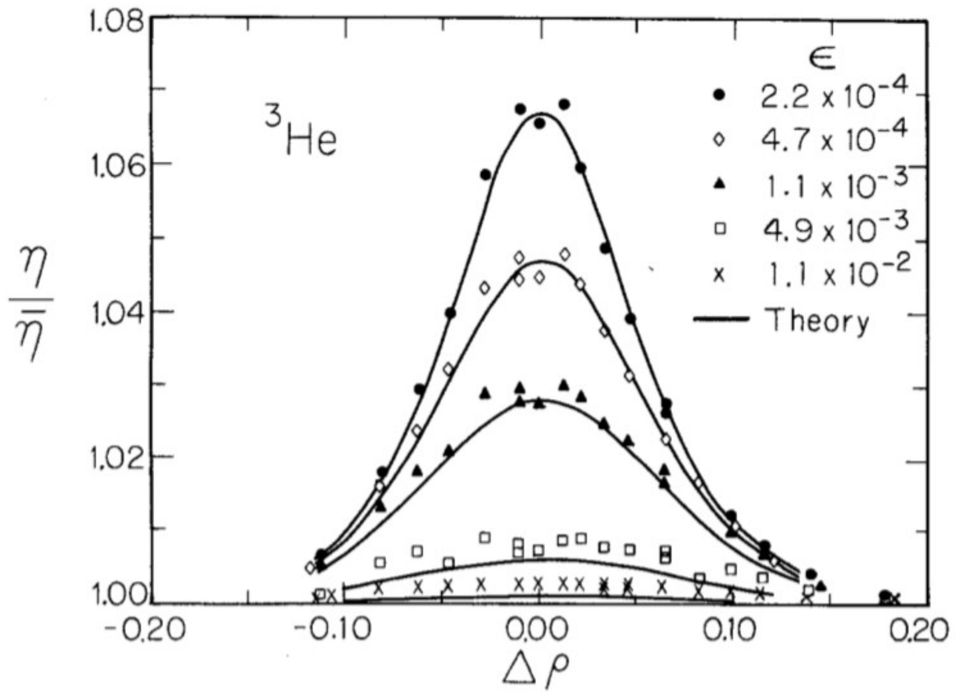
Use $r_0 \sim \xi^{-2}$. Relaxation time for modes $q \sim \xi^{-1}$:

$$\tau \sim \xi^z \quad (z = 4) \qquad \text{"Critical slowing down"}$$

A more sophisticated analysis gives $z \simeq 3$ and

$$\eta \sim \xi^{0.05} \qquad \kappa \sim \xi^{0.9} \qquad \zeta \sim \xi^{2.8}$$

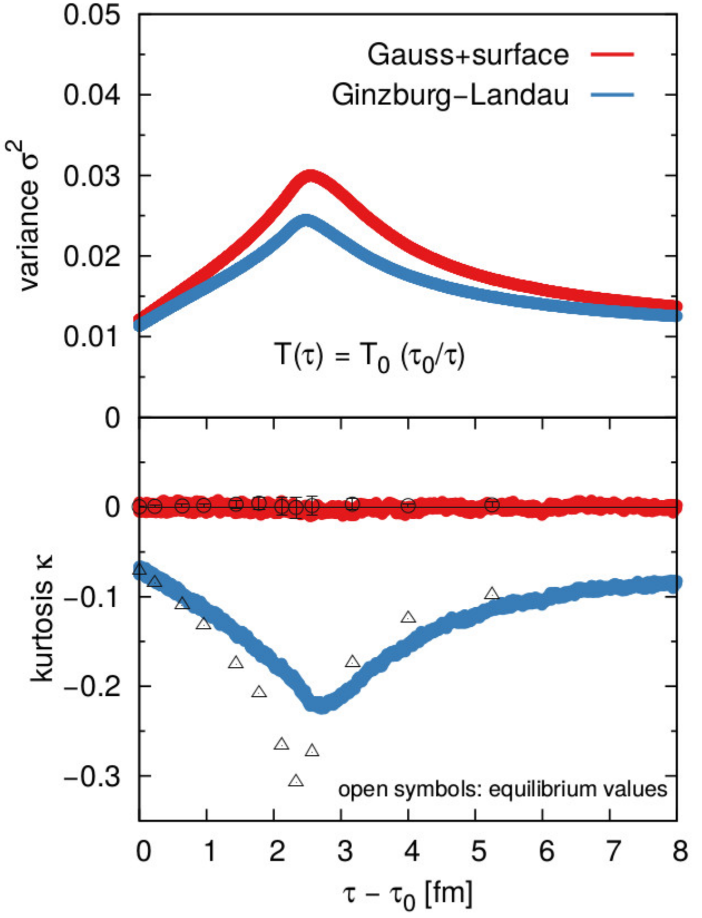
Critical transport (liquid gas phase transition in helium)



Agosta, Cohen, Wang, Meyer (1987)

Critical behavior of shear viscosity very weak, but bulk viscosity and critical slowing down expected to be much more important.

Numerical work (diffusion in expanding critical fluid)



Model B in a system with longitudinal expansion.

Note that non-Gaussian fluctuations (kurtosis) are generated from Gaussian white noise and non-linear mode couplings.

Observe critical slowing down (memory effect). Affects different observables differently.

M. Nahrgang et al. (2017)

Outlook

Opportunity: Discover QCD critical point by observing critical fluctuations in heavy ion collisions. Intriguing hints present in BES-I data.

Challenge: Propagate fluctuations of conserved charges in relativistic fluid dynamics. Describe initial state fluctuations and final state freezeout.

Experiment: BES-II scheduled for 2019/2020 (24wks)

Other opportunities: Anomalous transport.