QCD and Instantons: 12 Years Later

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ESQGP: A man ahead of his time

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QUARK–GLUON PLASMA AND HADRONIC PRODUCTION OF LEPTONS, PHOTONS AND PSIONS

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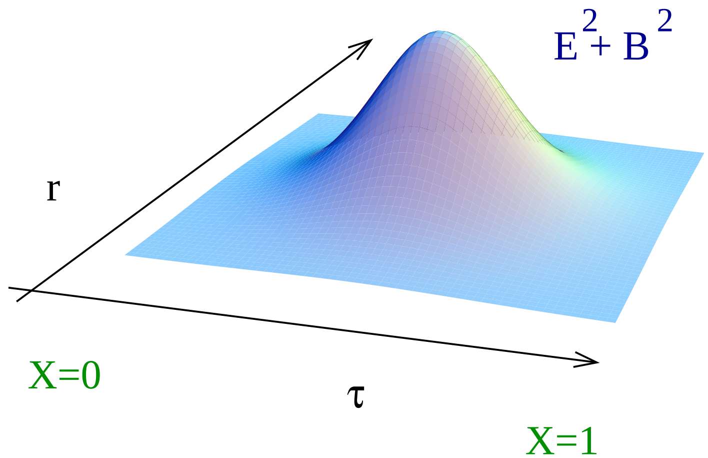
Received 16 March 1978

Instanton Liquid: Pre-History

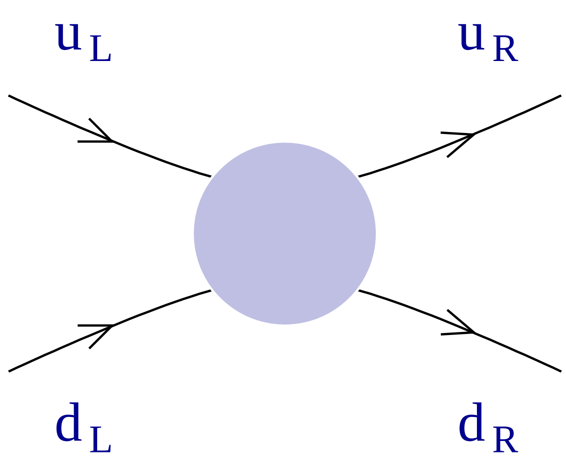
1975 (Polyakov): The instanton solution



$$A^a_\mu(x) = 2\frac{\eta_{a\mu\nu}x_\nu}{x^2+\rho^2},$$

$$G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} = \frac{192\rho^4}{(x^2+\rho^2)^4}.$$

1976 ('t Hooft): Fermion zero modes



$$\mathcal{L} = G \det_f(\bar\psi_{L,f}\psi_{R,g})$$

$$G = \int d\rho\, n(\rho)$$

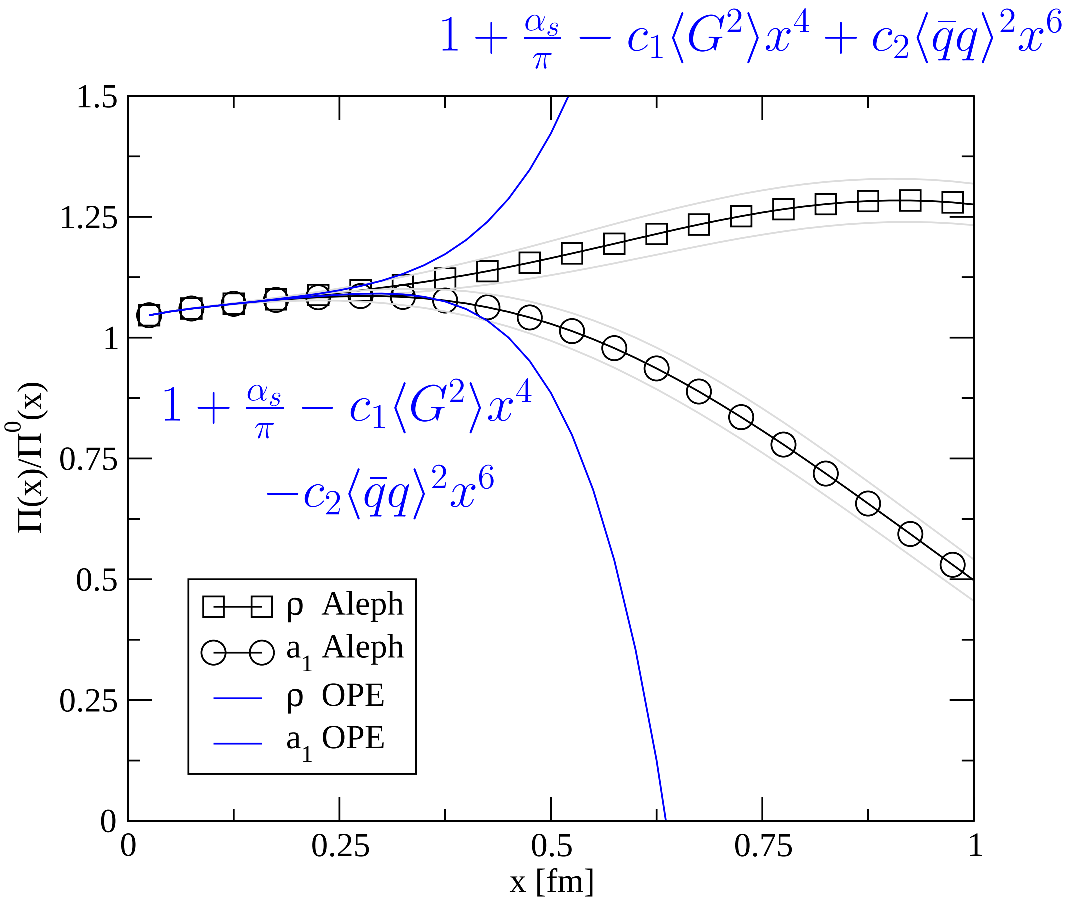
violates $U(1)_A$ but

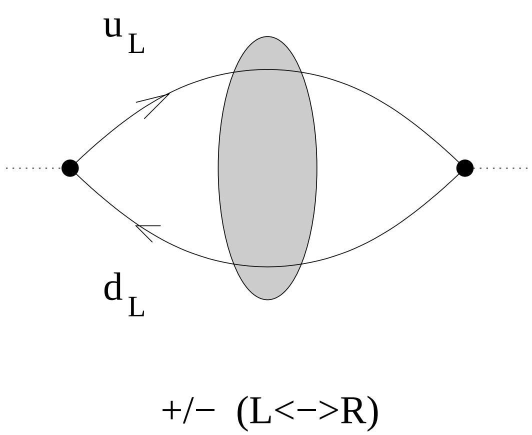
preserves $SU(2)_{L,R}$

. . . and contributes to

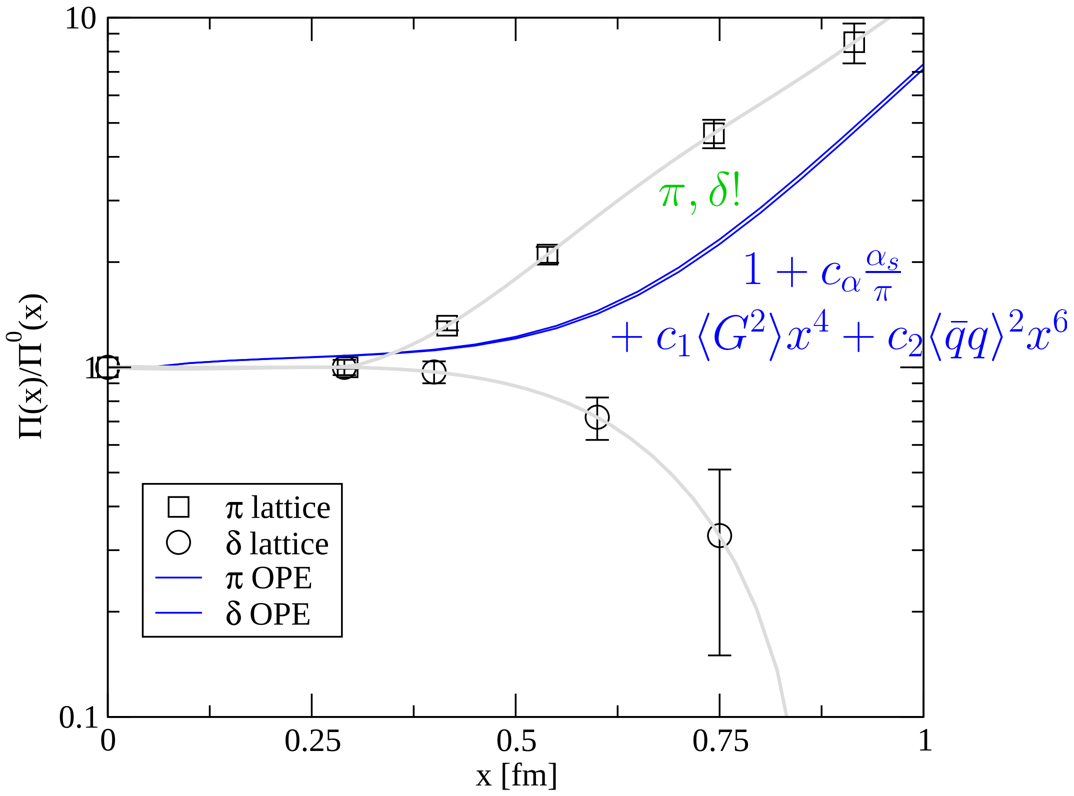
the η' mass

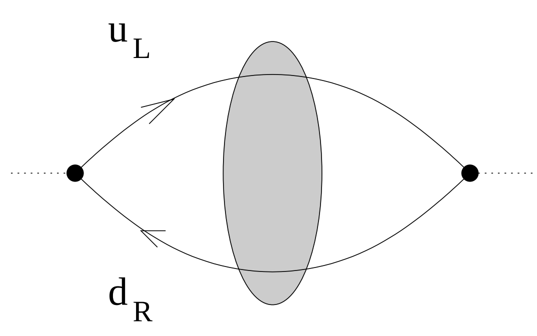
Phenomenology: Vector Channels (ρ and a_1)



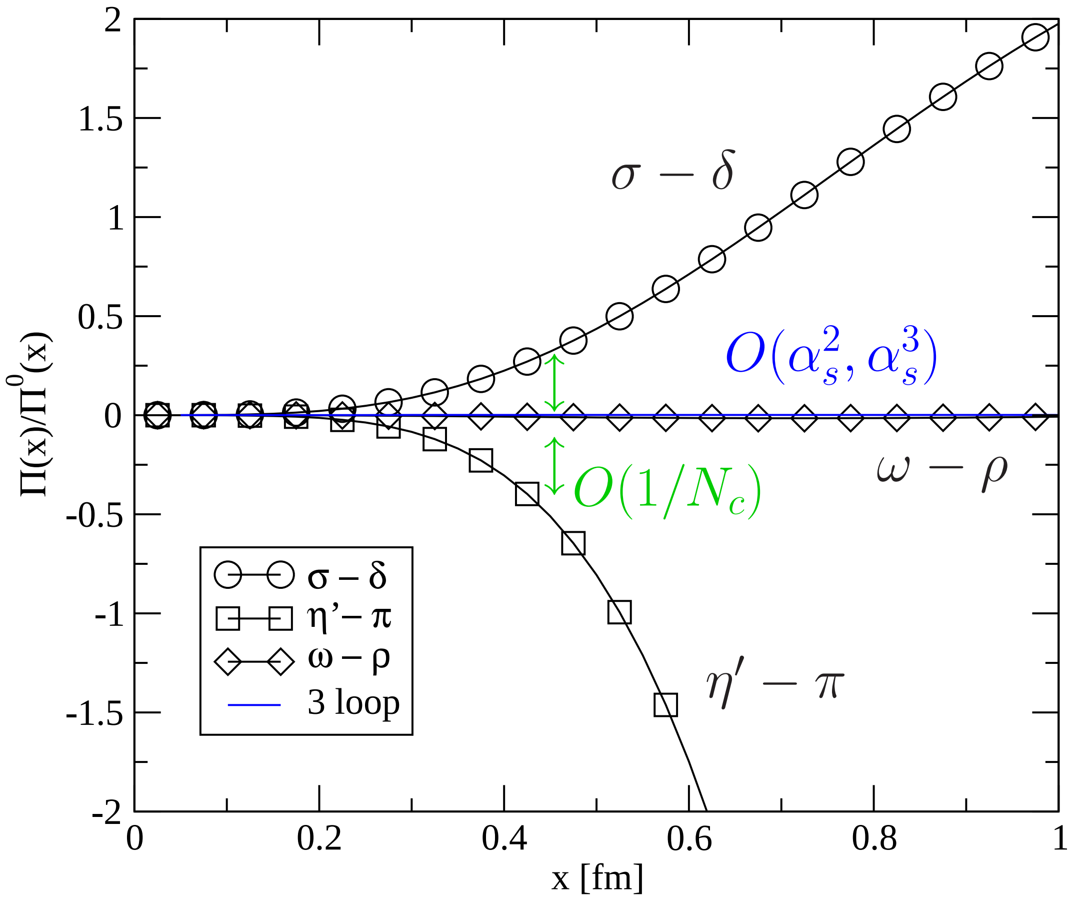


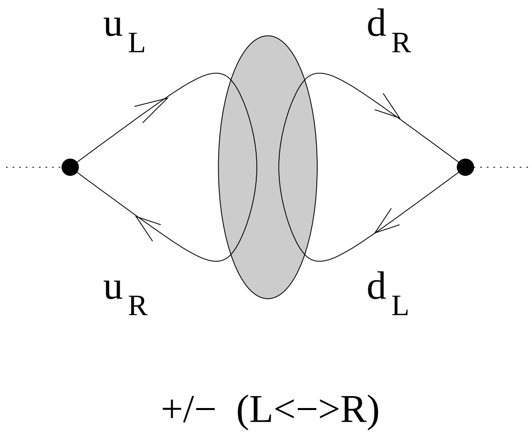
Phenomenology: Scalar Channels (π and δ)





Phenomenology: OZI Violation





Phenomenology: Summary

Only small effects in $(\bar{L}L \pm \bar{R}R)^2$.

Sign changes for $(\bar{L}R + \bar{R}L) \leftrightarrow (\bar{L}R - \bar{R}L)$.

Sign changes for $(\bar{u}d)(\bar{u}d) \leftrightarrow (\bar{u}u)(\bar{d}d)$.

$$\mathcal{L} = G \det_f (\bar{\psi}_L \psi_R) + (L \leftrightarrow R)$$

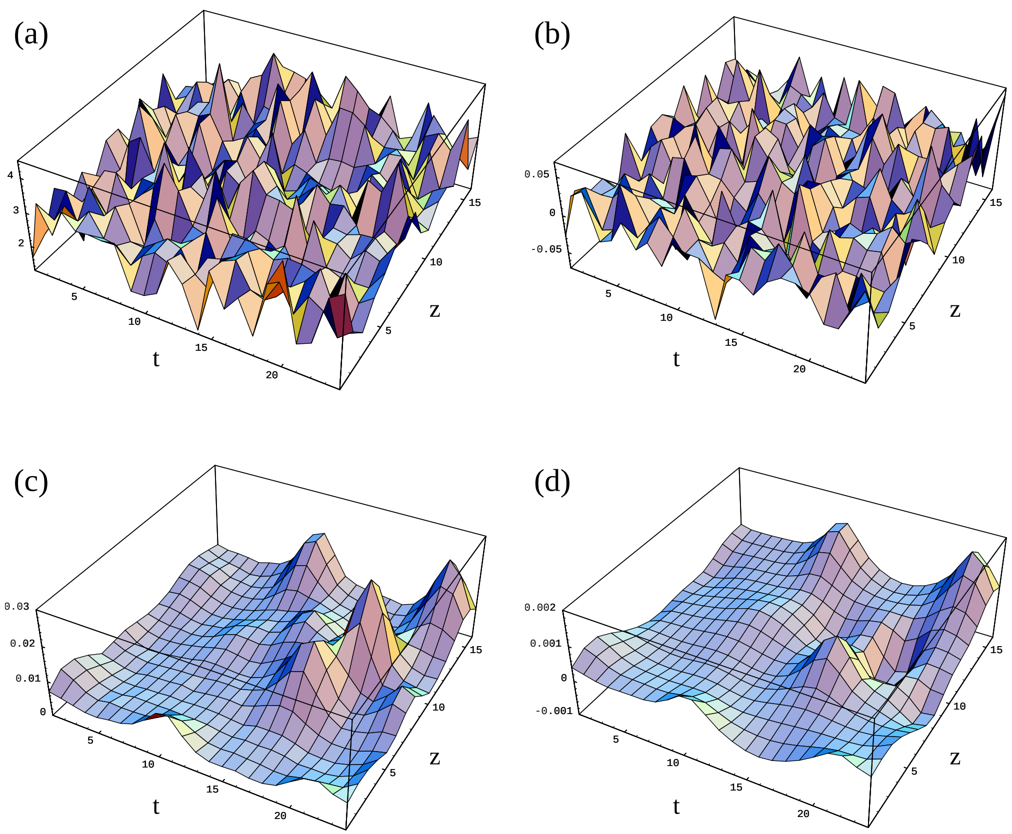
The Instanton Liquid

ES (1982): Instantons provide a quantitative description of QCD correlations functions

$$\rho = 0.3 \text{ fm} \quad \frac{N}{V} = 1 \text{ fm}^{-4}$$

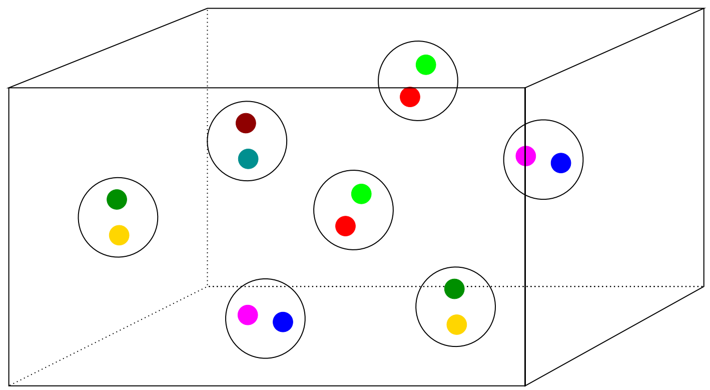
$$S \sim 10 \gg 1$$

$$\delta S \sim 1 \ll S$$



The Instanton Ensemble

Instanton liquid described by partition function (one parameter)



$$Z = \frac{1}{N_I! N_A!} \prod_I^{N_I+N_A} \int [d\Omega_I \, n(\rho_I)]$$

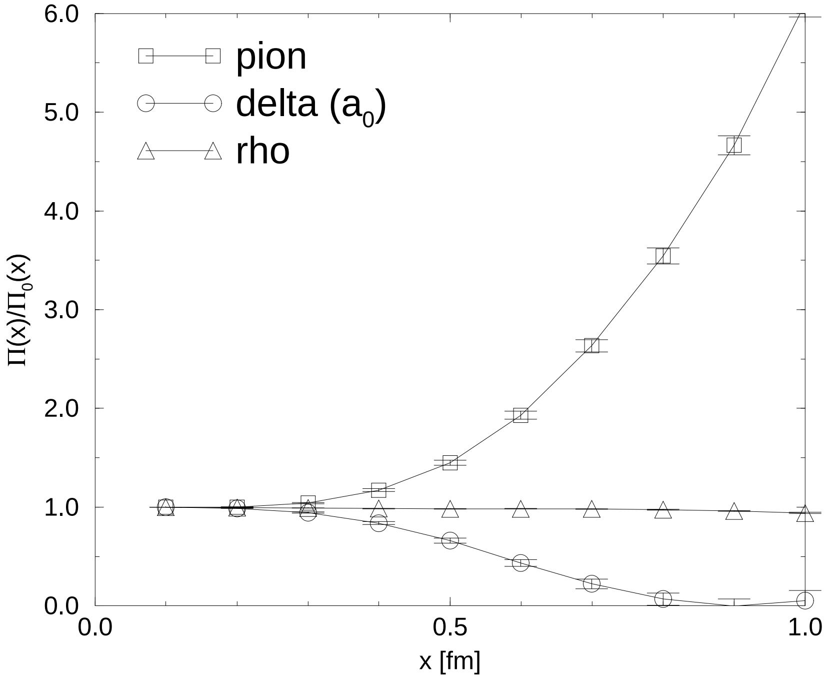
$$\times \det(\not{D}) \exp(-S_{int})$$

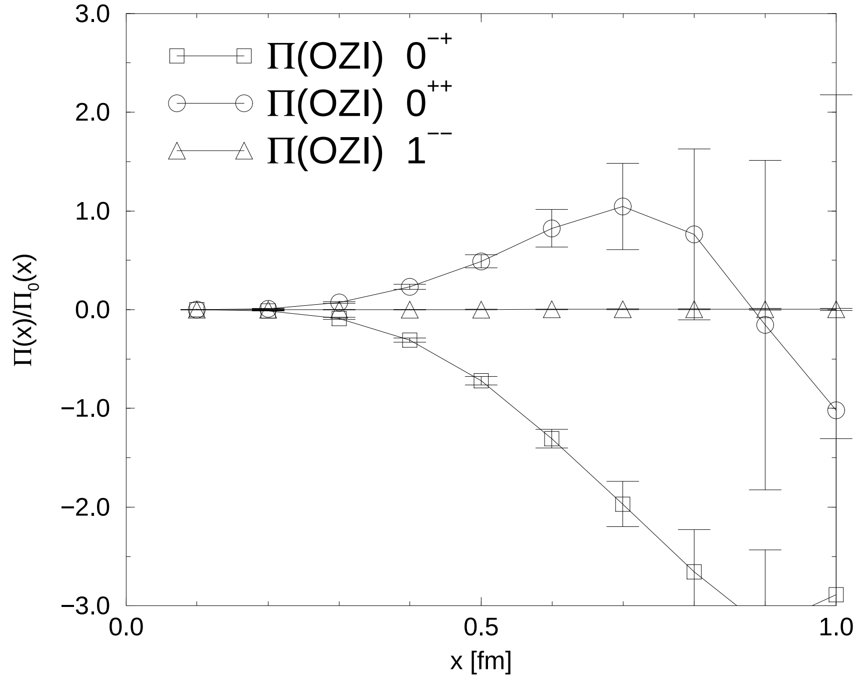
Quark propagator

$$S(x,y) = \sum_{IJ} \psi_I(x) \left(\frac{1}{T+im} \right)_{IJ} \psi_J^\dagger(y) + S_{NZM}(x,y)$$

"Instantons in QCD", Rev. Mod. Phys (1998)

Meson Correlation Functions





$$m_\pi = 140^* \text{ MeV} \quad (f_\pi = 71\,\text{MeV})$$

$$m_\rho \simeq m_\omega$$

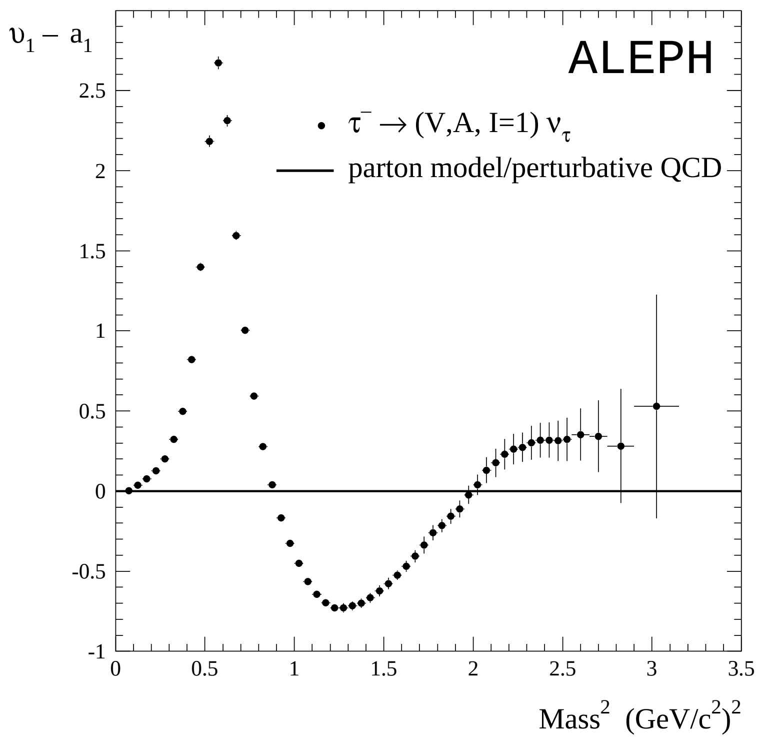
$$m_\rho = 795\,\text{MeV}$$

$$m_\sigma \simeq 580\,\text{MeV}$$

$$m_{a_0} \simeq 1\,\text{GeV}$$

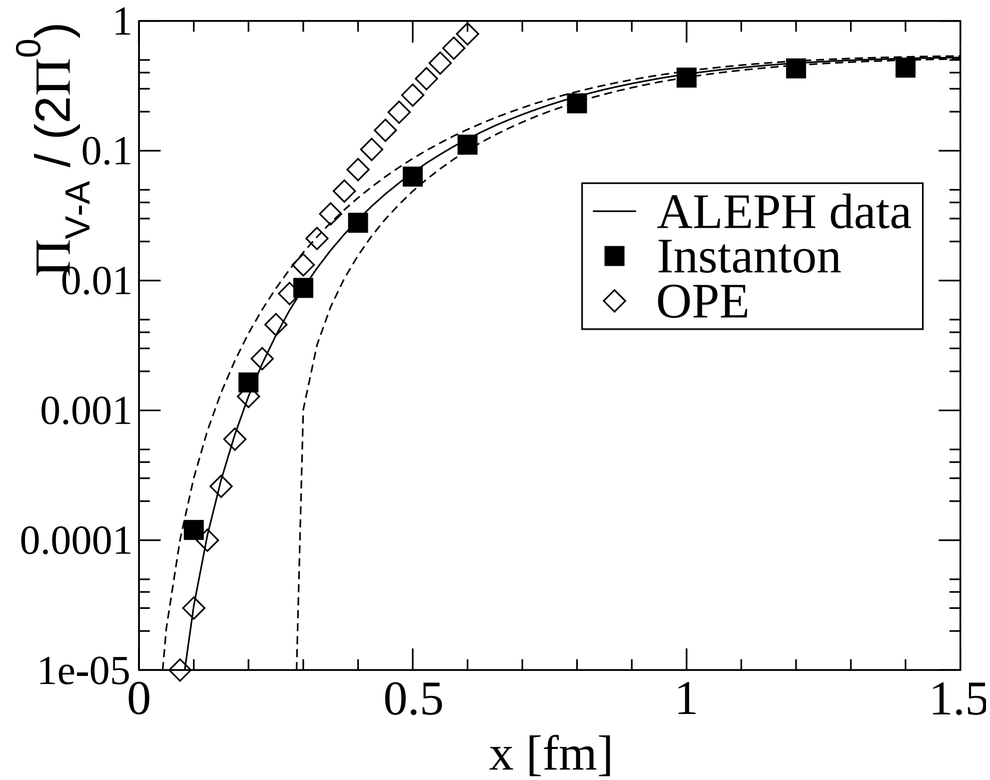
$$m_{\eta'} \simeq 1\,\text{GeV}$$

V−A Correlation Functions



Aleph spectral function

$\tau \to (V, A, I = 1)\nu_\tau$



coordinate space correlator

OPE, instanton liquid, data

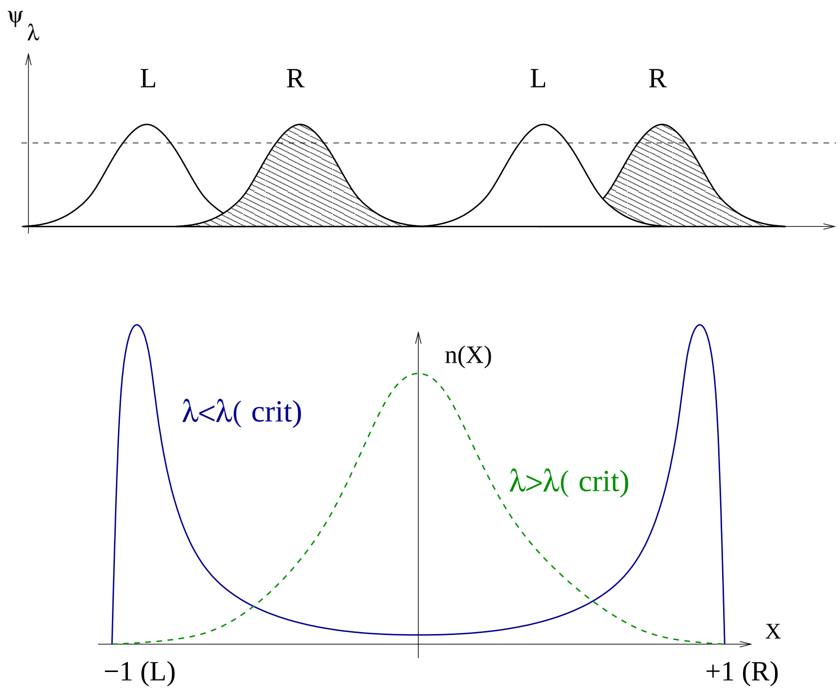
Instantons in QCD: 12 Years Later

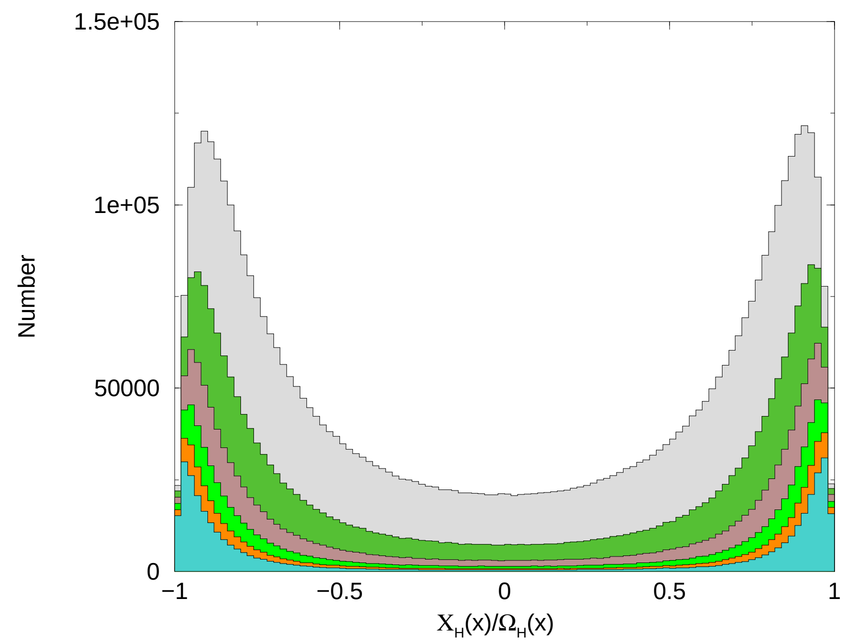
Chirality and zero modes on the lattice

High density QCD

SUSY, large N_c, AdS/CFT, AdS/QCD

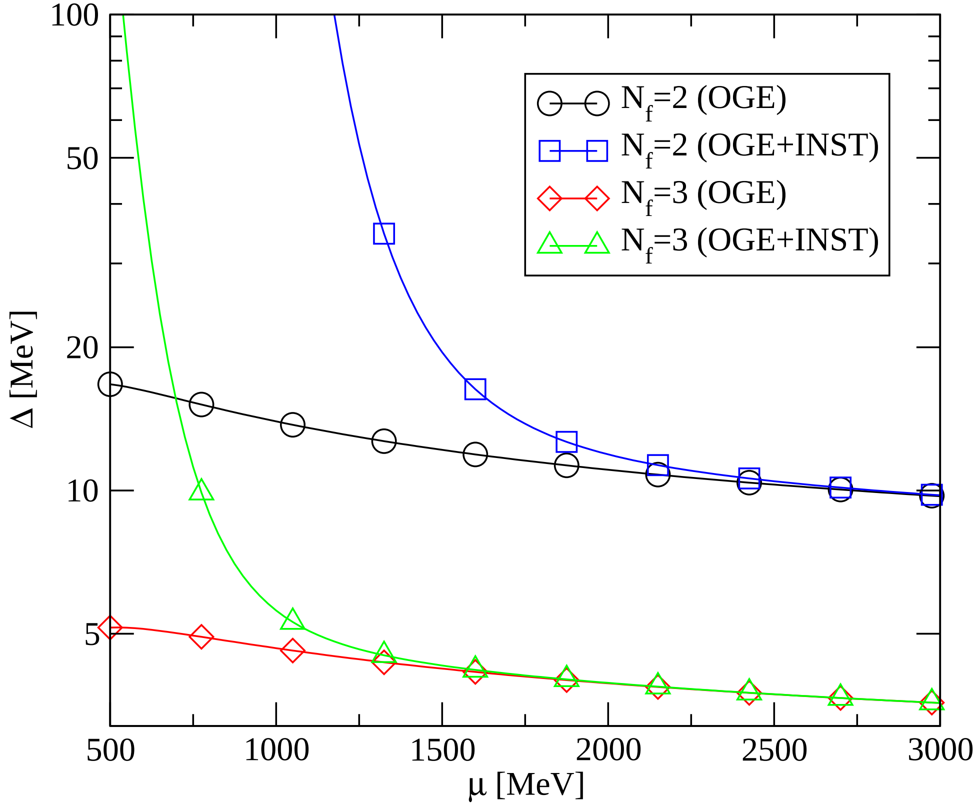
Chiral Symmetry Breaking on the Lattice

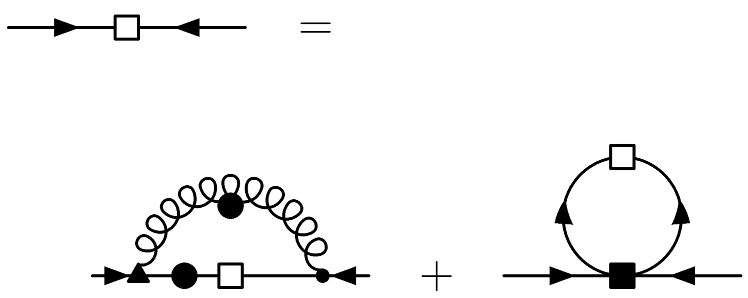




chirality distribution from T. Blum et al., [hep-lat/0105006]

Instantons and Color Superconductivity



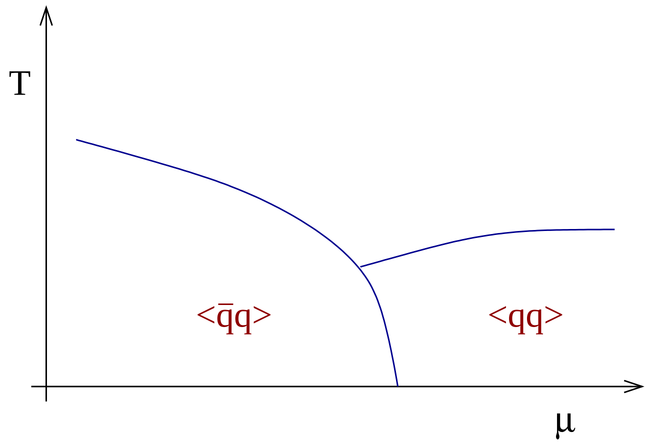


$\Delta \simeq 100$ MeV, $T_c \simeq 60$ MeV

RSSV (1998), ARW (1998)

A pQCD Instanton Plasma ($\mu \gg \Lambda_{QCD}$)

Schematic phase diagram (Here: $N_f = N_c = 2$)

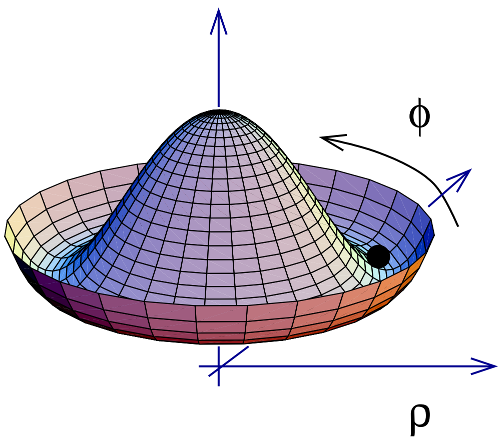


diquark condensate breaks $U(1)_B$ and $U(1)_A$

$$\langle q_L q_L \rangle = \rho\, e^{i(\chi+\phi)/2}$$
$$\langle q_R q_R \rangle = \rho\, e^{i(\chi-\phi)/2}$$

Effective lagrangian for $U(1)_A$ Goldstone boson

Son, Stephanov, Zhitnitsky

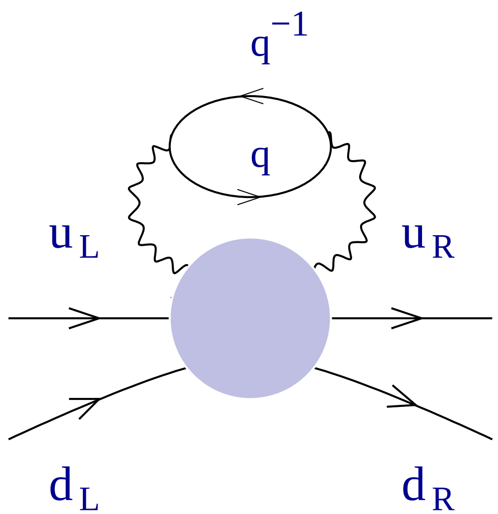


$$\mathcal{L} = \frac{f^2}{2}\left[(\partial_0\phi)^2 - v^2(\partial_i\phi)^2\right] - V(\phi+\theta) \;+\; \mathcal{L}(\rho,\chi)$$

$V(\phi+\theta)$ vanishes in perturbation theory

η' Mass at Large Baryon Density

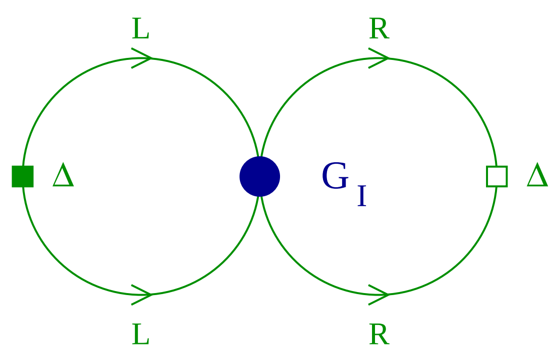
Instanton induced effective interaction for quarks with $p \sim p_F$



$$n(\rho,\mu) = n(\rho,0)\exp\left[-N_f\rho^2\mu^2\right]$$

$$\rho \sim \mu^{-1} \ll \Lambda_{QCD}^{-1}$$

Instanton contribution to vacuum energy

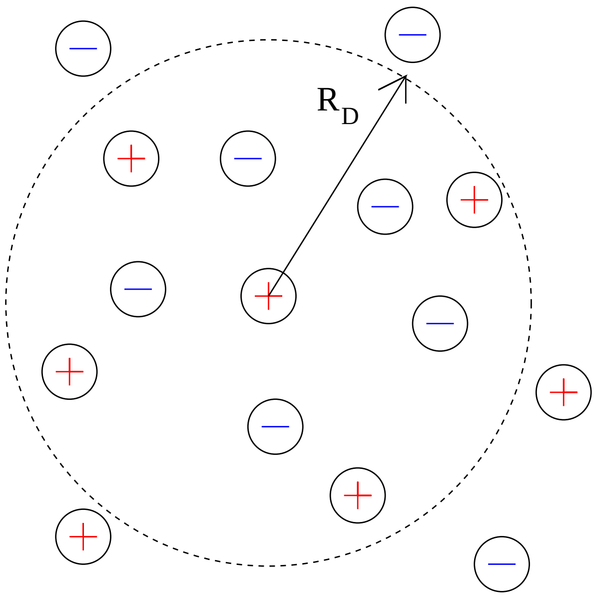


$$\langle \mathcal{L} \rangle = A\cos(\phi+\theta)$$

$$A = C_N \Phi^2 \left[\log\left(\frac{\mu}{\Lambda}\right)\right]^4 \left(\frac{\Lambda}{\mu}\right)^8 \Lambda^{-2}$$

η' mass satisfies “Witten-Veneziano” relation $f^2 m_\phi^2 = A$

Very dilute instanton gas



$$\rho \ll r_{IA} \ll R_D$$
$$\rho \sim \mu^{-1}$$
$$r_{IA} = A^{1/4}$$
$$R_D = m_\phi^{-1}$$

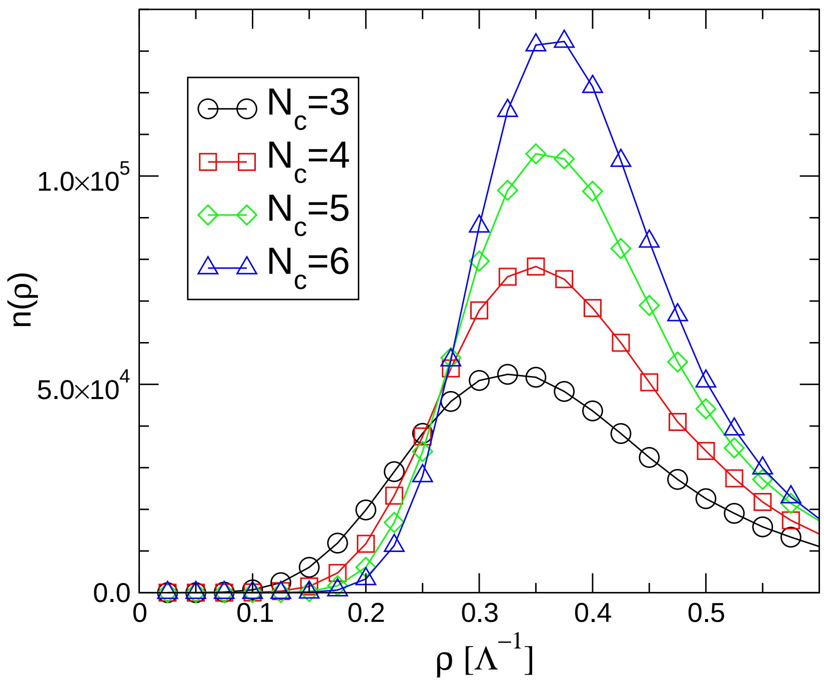
A is the local topological susceptibility

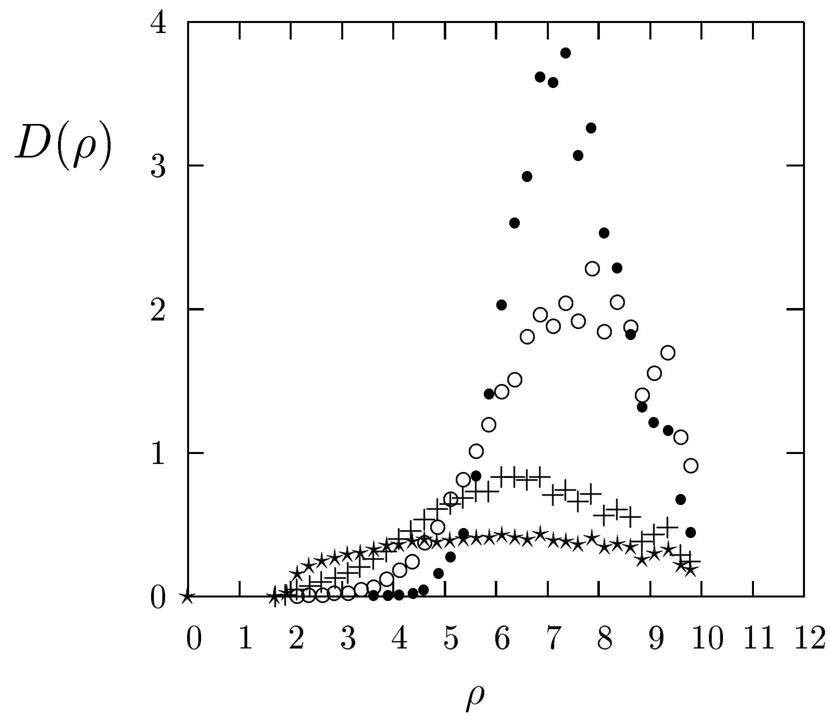
$$A = \chi_{top}(V) = \frac{\langle Q_{top}^2 \rangle_V}{V} \qquad r_{IA}^4 \ll V \ll R_D^4$$

Global topological susceptibility vanishes

$$\chi_{top} = \lim_{V\to\infty} \frac{\langle Q_{top}^2 \rangle_V}{V} = 0 \qquad (m = 0)$$

Instantons and Large N_c



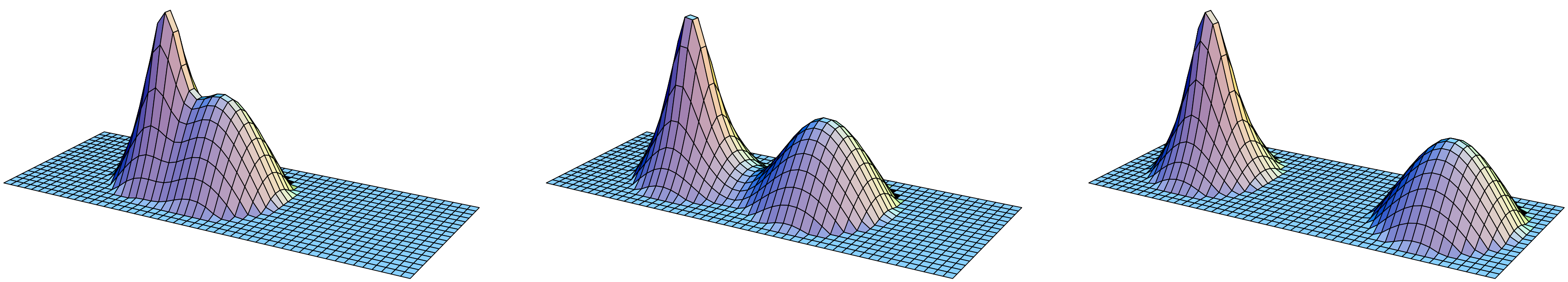


B. Lucini, M. Teper

$$\langle \bar{q} q \rangle \sim N_c \qquad \chi_{top} \sim 1 \qquad m^2_{\eta'} \sim 1/N_c$$

From Instanton to Monopoles

Kraan, van Baal: Instantons with non-zero holonomy



Monopole constituents with fractional top charge (⇒ confinement?)

New WCI calculation of gluino condensate

$$\frac{1}{16\pi^2}\langle \mathrm{Tr}[\bar{\lambda}\lambda]\rangle = \Lambda^3 \exp(2\pi i k/N_c)$$

AdS/CFT: $\mathcal{N}=4$ SUSY Yang Mills

String/field theory duality (Maldacena)

$$\begin{array}{ccc} \mathcal{N}=4\ \text{SUSY YM} & \Leftrightarrow & \text{IIB strings on } AdS_5\times S_5 \\ \lambda=g^2N\to\infty & \Leftrightarrow & (l_s/R)^4\to 0 \\ (g^2\to 0) & & (g_s\to 0) \end{array}$$

String theory contains D-instantons characterized by location on $AdS_5 \times S_5 \Leftrightarrow$ field theory instantons

$$\int d^4x \int \frac{d\rho}{\rho^5} \ \int d\Lambda^{ab}$$

$$AdS_5 \times S_5$$

Charge k instanton amplitudes

$$(AdS_5\times S_5)^k \quad \rightarrow \quad (AdS_5\times S_5)$$

$$\text{k instantons in commuting } SU(2)'s$$

$$\text{(bound by fermions)}$$

Instantons and AdS/QCD

Add singlet field $Y = \langle Y \rangle e^{ia}$ to AdS/QCD ("axion")

$$S = \int d^5x \sqrt{g} \left\{ \frac{1}{2}|DY|^2 + \frac{\kappa_0}{2} \left(Y^{N_f} \det(X) + h.c. \right) \right\}$$

Katz & Schwartz (2007)

Topological charge correlator: Treat κa^2 as a perturbation

$$\Pi_P(Q) = -\frac{1}{2N_f} \int_0^{z_m} \frac{dz}{z^5} \bar{\kappa} \left[\frac{1}{2}(Qz)^2 K_2(Qz) \right]^2 ,$$

AdS_5 measure $\times$ (Bulk-to-boundary prop)^2

Compare to instanton result

$$\Pi_P(Q) = -2 \int \frac{d\rho}{\rho^5} d(\rho) \left[\frac{1}{2}(Q\rho)^2 K_2(\rho Q) \right]^2 ,$$

instanton measure $\times$ (F-trafo of $G\tilde{G}_I$)^2

Happy Birthday Edward!!