# Perfect Fluid Olympics

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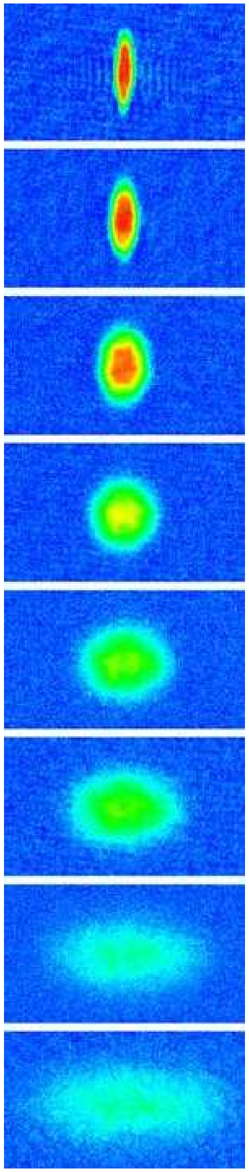
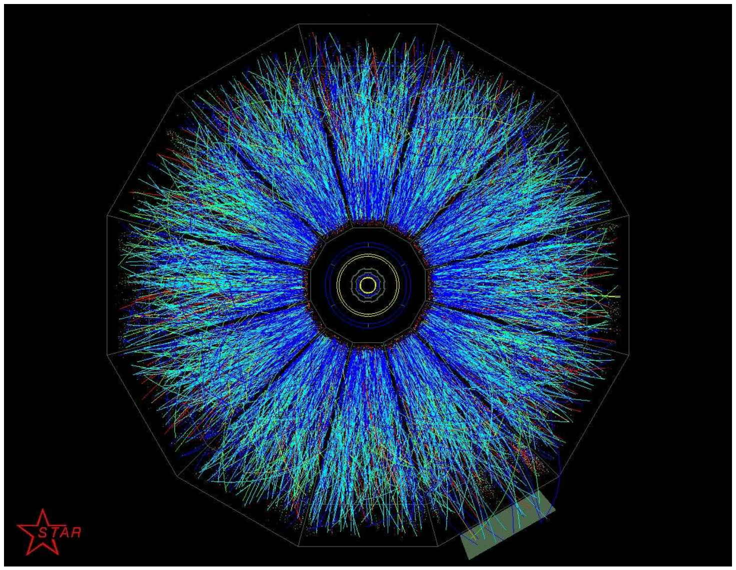
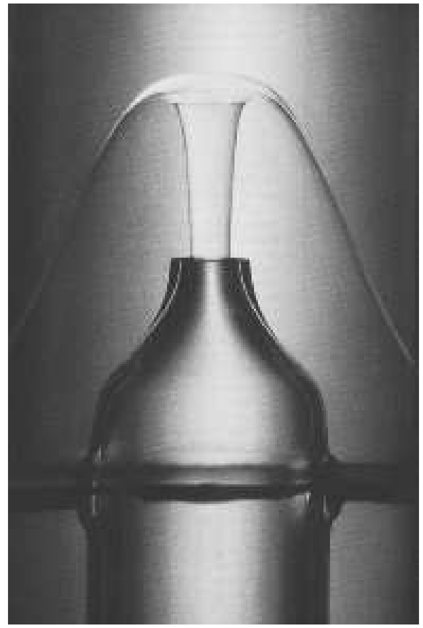
# Perfect Fluids: The contenders





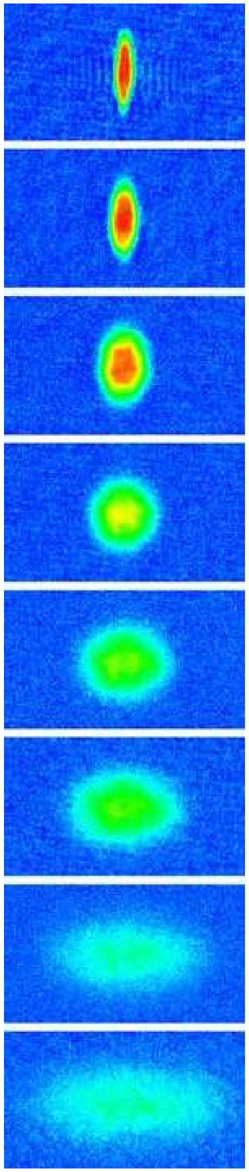

QGP (T=180 MeV)

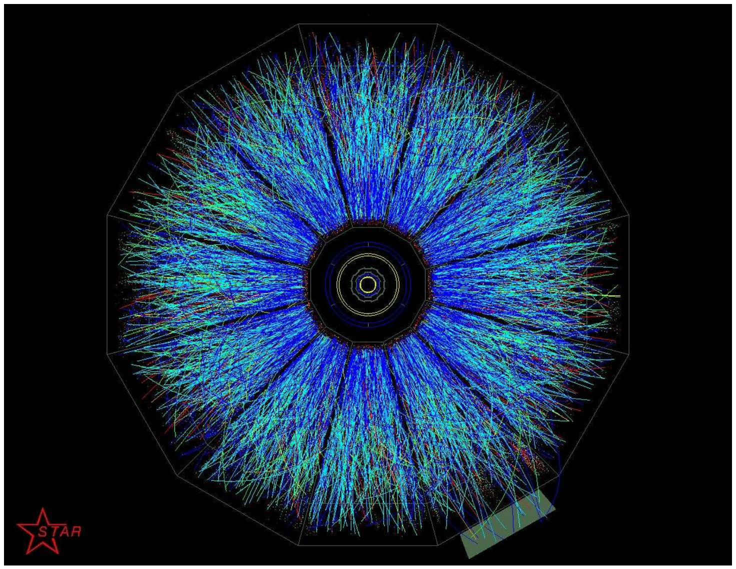


Liquid Helium
(T=0.1 meV)

Trapped Atoms
(T=0.1 neV)

# Perfect Fluids: The contenders






QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$

Liquid Helium

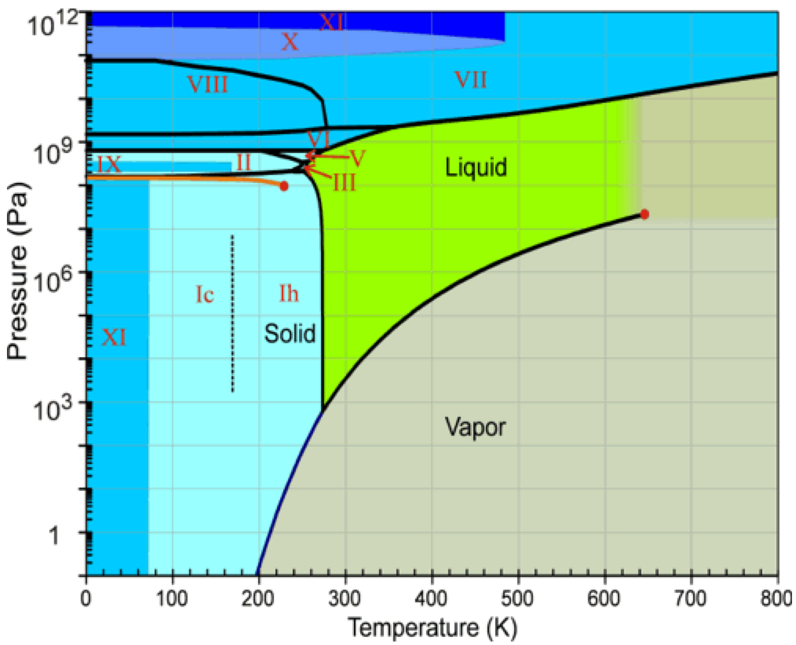
$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

$\eta / s$

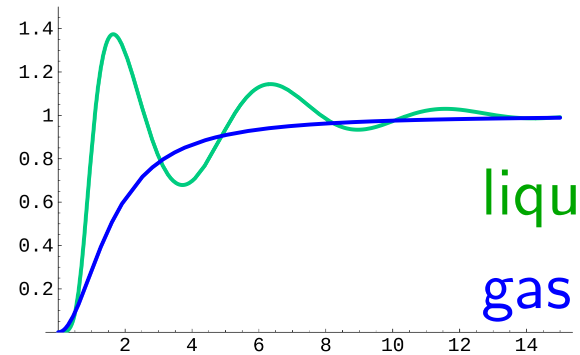
# Transitions without change of symmetry: Liquid-Gas

## Phase diagram of water

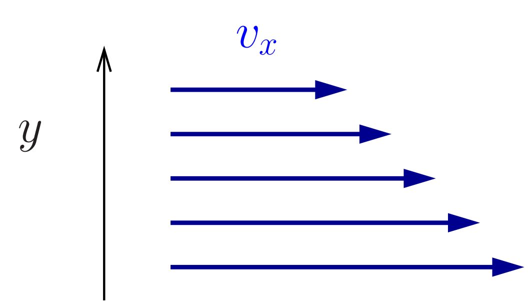



## Characteristics of a liquid

Pair correlation function



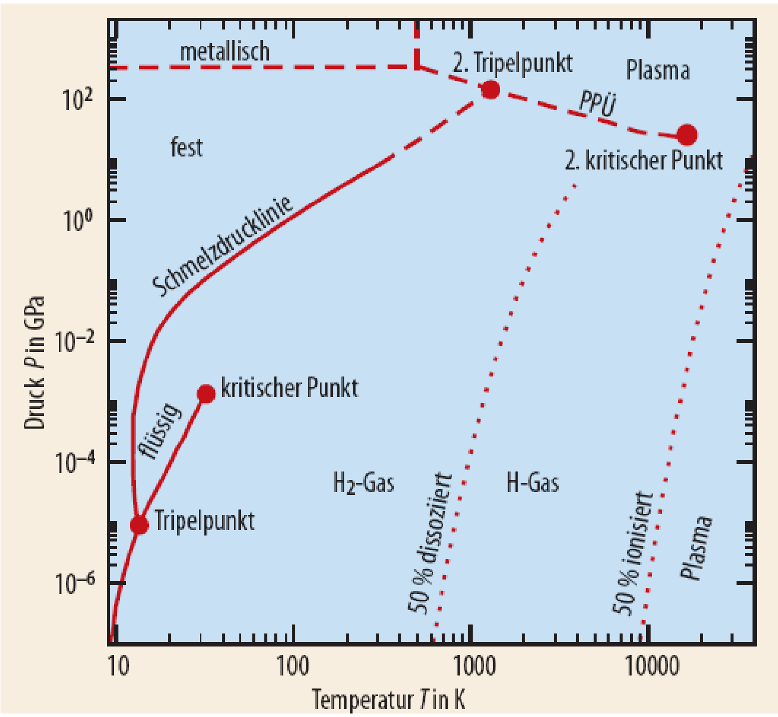

Good fluid: low viscosity




$$F_x = \eta A \frac{\partial v_x}{\partial y}$$

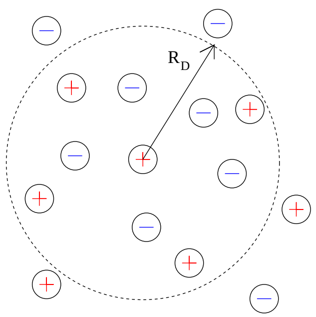
# Transitions without change of symmetry: Gas-Plasma

## Phase diagram of hydrogen




## Plasma Effects

Debye screening




$$V(r) = -\frac{e}{r} e^{-m_D r}$$

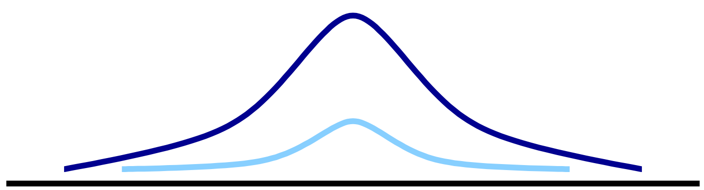
$$m_D^2 = \frac{4\pi e^2 n}{kT}$$

Plasma oscillations

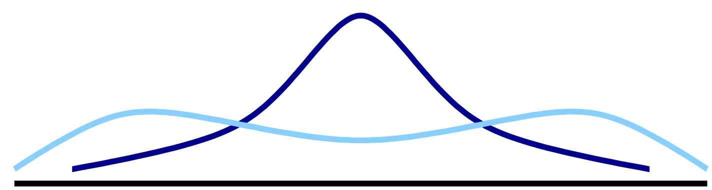
$$\omega_{pl} = \frac{4\pi e^2 n}{m}$$

# Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



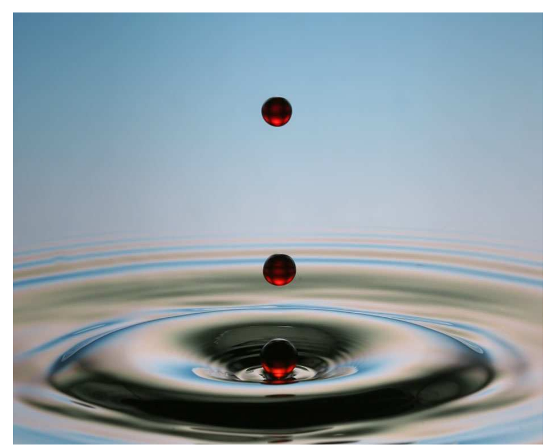
$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



# Example: Simple Fluid

Continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

Euler (Navier-Stokes) equation

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0$$

Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right) + \ldots$$

reactive　　dissipative

# Kinetic Theory

Quasi-Particles ($\gamma \ll \omega$): introduce distribution function $f_p(x,t)$

$$N = \int d^3p\, f_p \qquad T_{ij} = \int d^3p\, \frac{p_i p_j}{E_p} f_p,$$

Boltzmann equation

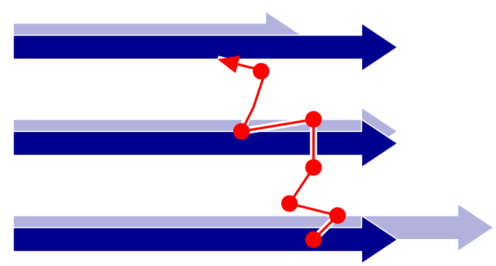
$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

Collision term $C[f_p] = C_{gain} - C_{loss}$

Linearized theory (Chapman-Enskog): $f_p = f_p^0(1 + \chi_p/T)$

suitable for transport coefficients

shear viscosity $\chi_p = g_p p_i p_j v_{ij}$



# Viscosity Bound: Rough Argument

Kinetic theory estimate of shear viscosity

$$\eta \sim \frac{1}{3} n\, \bar{p}\, l_{mfp} \qquad\qquad (\mathrm{Note}: l_{mfp} \sim 1/(n\sigma))$$

Normalize to density. Uncertainty relation implies

$$\frac{\eta}{n} \sim \frac{n\, \bar{p}\, l_{mft}}{n} \geq \hbar$$

Also: $s \sim k_B n$ and $\eta/s \geq \hbar/k_B$

Validity of kinetic theory as $\bar{p}\, l_{mfp} \sim \hbar$?

What is the "right" ratio, $\eta/s$ or $\eta/n$?

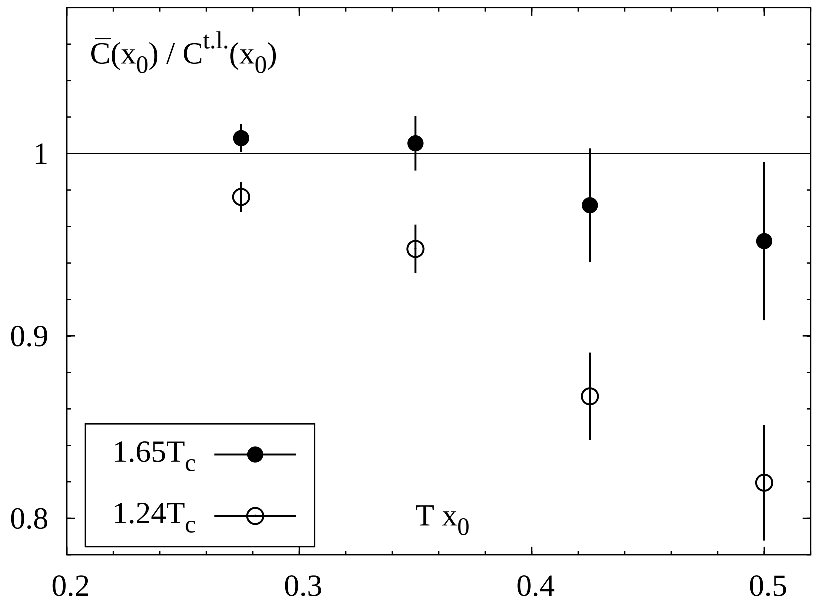
# Kubo Formula

Linear response theory provides relation to Green functions

$$\eta = - \lim_{\omega \to 0} \frac{1}{\omega} G_R(\omega, 0)$$

$$G_R(\omega, 0) = \int dt\, d^3x\, e^{i\omega t} \Theta(t) \langle [T_{xy}(t, x), T_{xy}(0, 0)] \rangle$$

Can be used in field theory (hard), lattice calculations (also hard)




H. Meyer (2007)

# Dynamic Universality

Continuous phase transition: Dynamics of low energy modes universal

Universality for transport coefficients

Universal theory: Hydro (diffusive modes), order parameters (time dependent LG), stochastic forces (Langevin)

$$\frac{\partial}{\partial t}(\rho v_i) = P_{ij}^{\perp}\left[\eta_0 \nabla^2 \frac{\delta \mathcal{H}}{\delta(\rho v_j)} + w_0 (\nabla_j \phi) \frac{\delta \mathcal{H}}{\delta \phi} + \zeta_j\right]$$

Model H of Hohenberg and Halperin

$$\eta \sim \xi^{x_\eta} \; (x_\eta \simeq 0.06) \qquad\qquad \zeta \sim \xi^{x_\zeta} \; (x_\zeta \simeq 2.8)$$

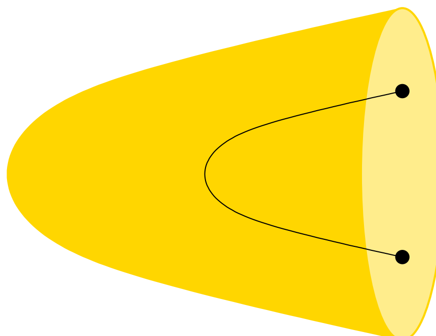
# Gauge Theory at Strong Coupling: Holographic Duals

The AdS/CFT duality relates

| | | |
|---|---|---|
| large $N_c$ (Conformal) gauge theory in 4 dimensions | $\Leftrightarrow$ | string theory on 5 dimensional Anti-de Sitter space $\times S_5$ |
| correlation fcts of gauge invariant operators | $\Leftrightarrow$ | boundary correlation fcts of AdS fields |

$$\langle \exp \int dx \; \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



The correspondence is simplest at strong coupling $g^2 N_c$

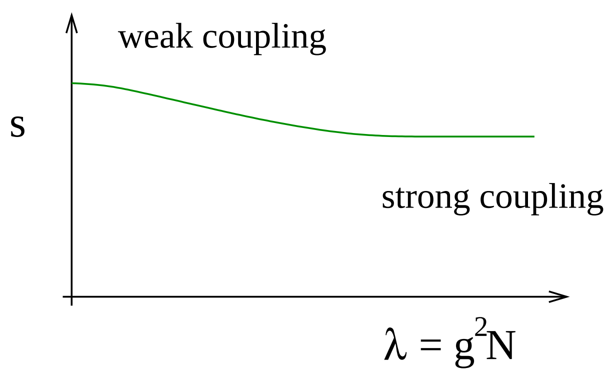
strongly coupled gauge theory $\Leftrightarrow$ classical string theory

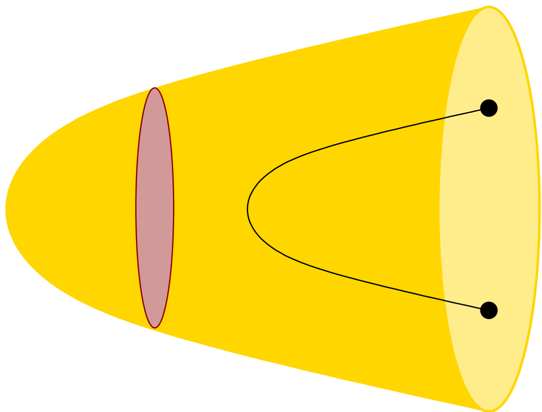
# Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature $\Leftrightarrow$ Hawking temperature of black hole

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy $\sim$ area of event horizon



$$s(\lambda \to \infty) = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s(\lambda = 0)$$

Gubser and Klebanov

# Holographic Duals: Transport Properties

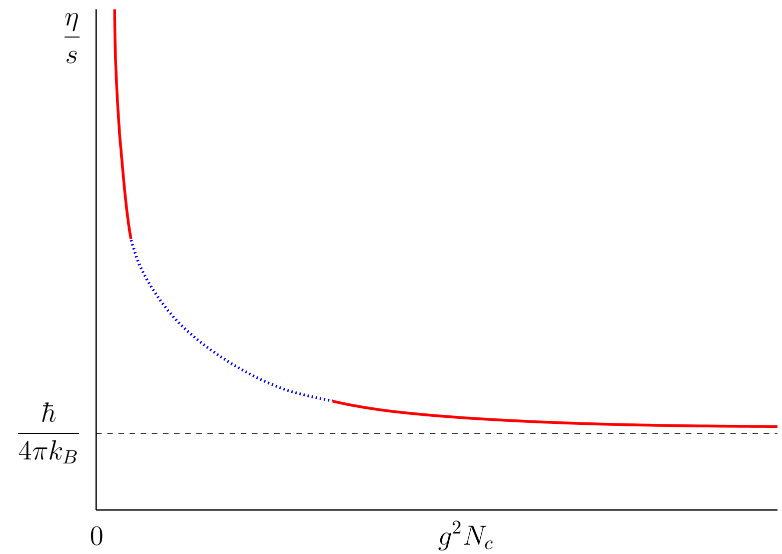
Thermal (conformal) field theory $\equiv AdS_5$ black hole

| | | |
|---|---|---|
| CFT entropy | $\Leftrightarrow$ | Hawking-Bekenstein entropy<br>$\sim$ area of event horizon |
| shear viscosity | $\Leftrightarrow$ | Graviton absorption cross section<br>$\sim$ area of event horizon |

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets




Strong coupling limit universal? Provides lower bound for all theories?

# Holographic Duals: Recent Work

Can we get $AdS/NRCFT$ using an embedding of non-relativistic conformal group?

$$Schr(d) \subset SO(d+1,2) \;\; (= Iso(AdS_{d+2}))$$

Not quite, but we can use $Schr(d) \subset SO(d+2,2)$

$$\text{New } 2+1 \text{ NRCFTs with } \frac{\eta}{s} = \frac{1}{4\pi}$$

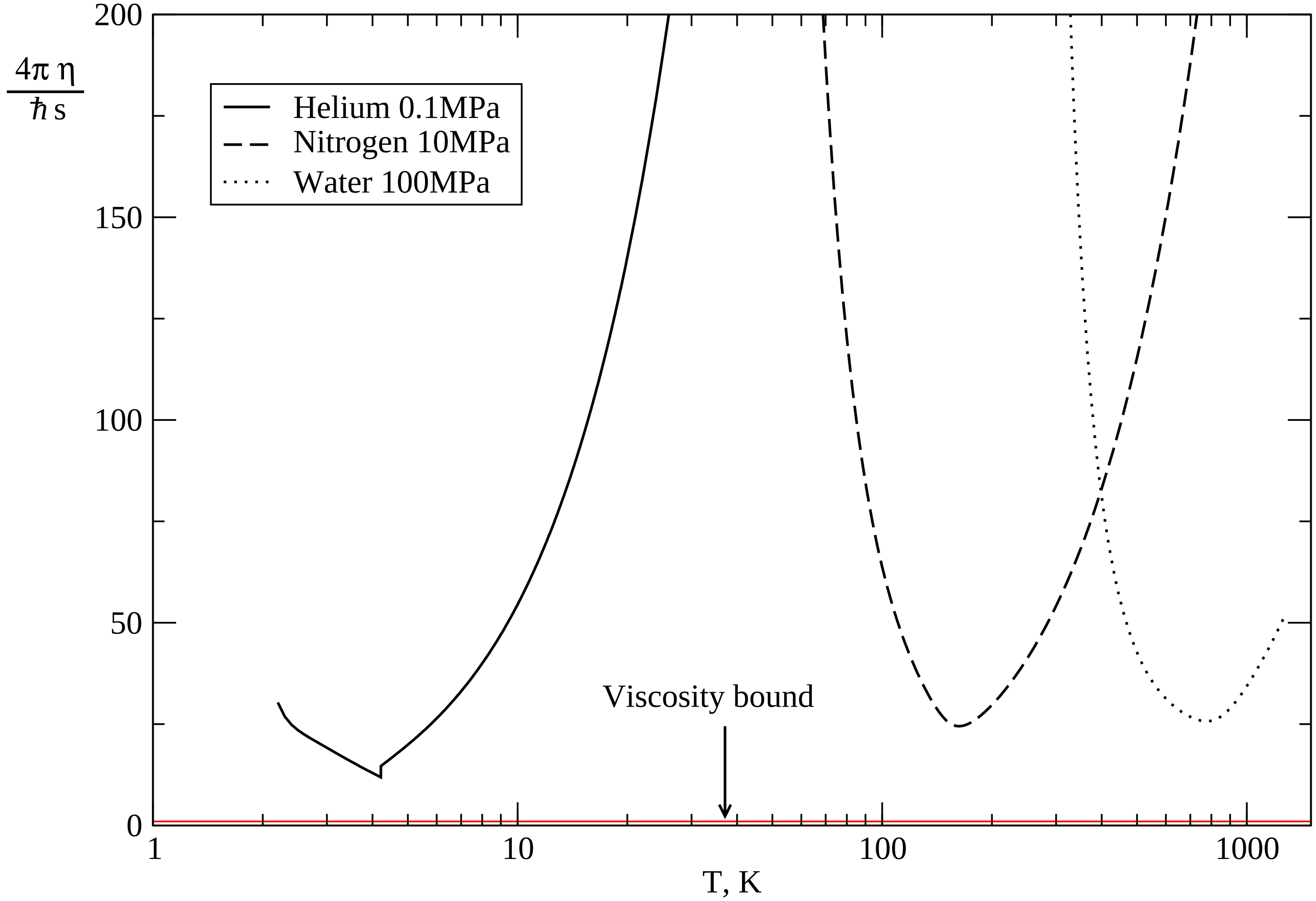
Son, McGreevy, Balasubramanian, . . .

Can we prove viscosity bound? Proven in certain classes of backgrounds, but some possible counterexamples exist

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - 4\lambda_{GB}) \qquad \lambda_{GB} < \frac{9}{100}$$

Myers, Shenker, Kats, Petrov, . . .

# Viscosity Bound: Common Fluids



# Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

(Almost) scale invariant systems

# I. Fermi gas at unitarity

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \Big( i\partial_0 + \frac{\nabla^2}{2M} \Big) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ ($C_0 \to \infty$)

This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$

$$\mathcal{L} = \Psi^\dagger \left[ i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left( \Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \; ,$$

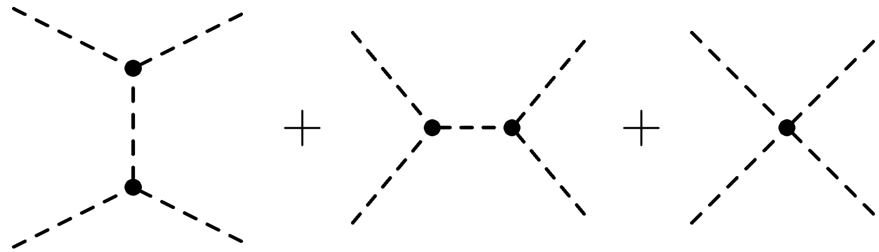
Low $T$ ($T < T_c \sim \mu$): Pairing and superfluidity

Low T: Phonons Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

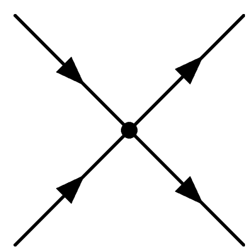
Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$




High T: Atoms Cross section regularized by thermal momentum

$$\eta = \frac{15}{32\sqrt{2}}(mT)^{3/2}$$



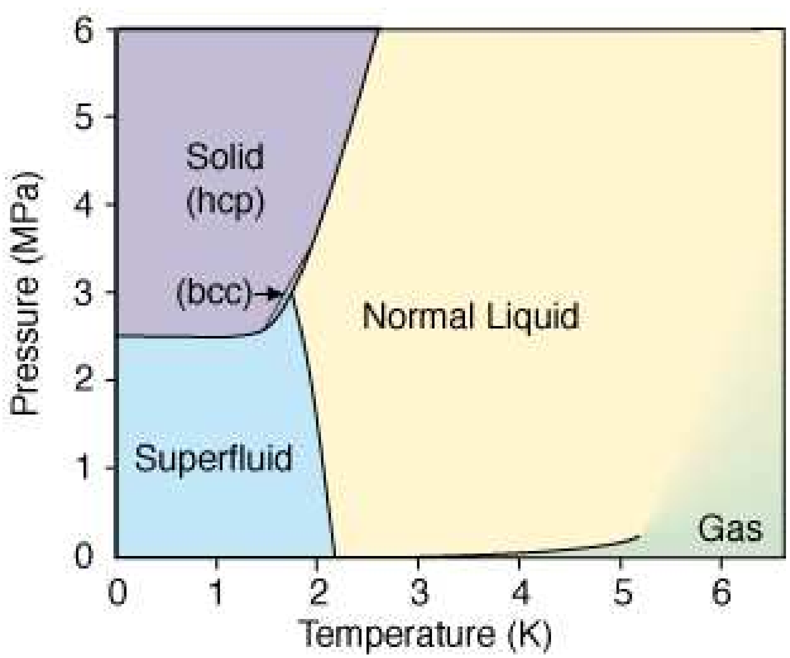
# II. Liquid Helium

Bosons, van der Waals + short range repulsion

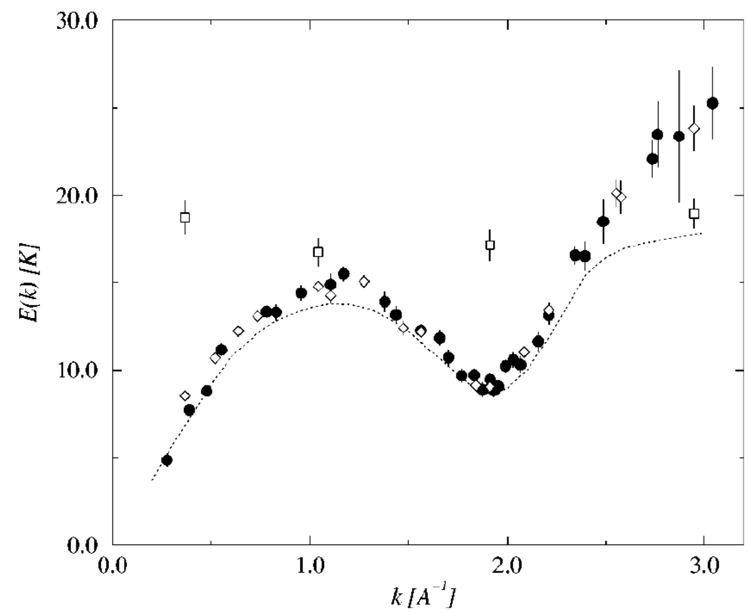
$$S = \int \Phi^\dagger \left( i\partial_0 + \frac{\nabla^2}{2M} \right) \Phi + \int \int \left( \Phi^\dagger \Phi \right) V(x-y) \left( \Phi^\dagger \Phi \right)$$

with $V(x) = V_{sr}(x) - c_6/x^6$. Note: $a = 189 a_0 \gg a_0$

## Phase Diagram




## Excitations



Low T: Phonons and Rotons Effective lagrangian

$$\mathcal{L} = \varphi^*(\partial_0^2 - v^2)\varphi + i\lambda\dot{\varphi}(\vec{\nabla}\varphi)^2 + \ldots$$

$$+ \varphi_{R,v}^*(i\partial_0 - \Delta)\varphi_{R,v} + c_0(\varphi_{R,v}^*\varphi_{R,v})^2 + \ldots$$

Shear viscosity

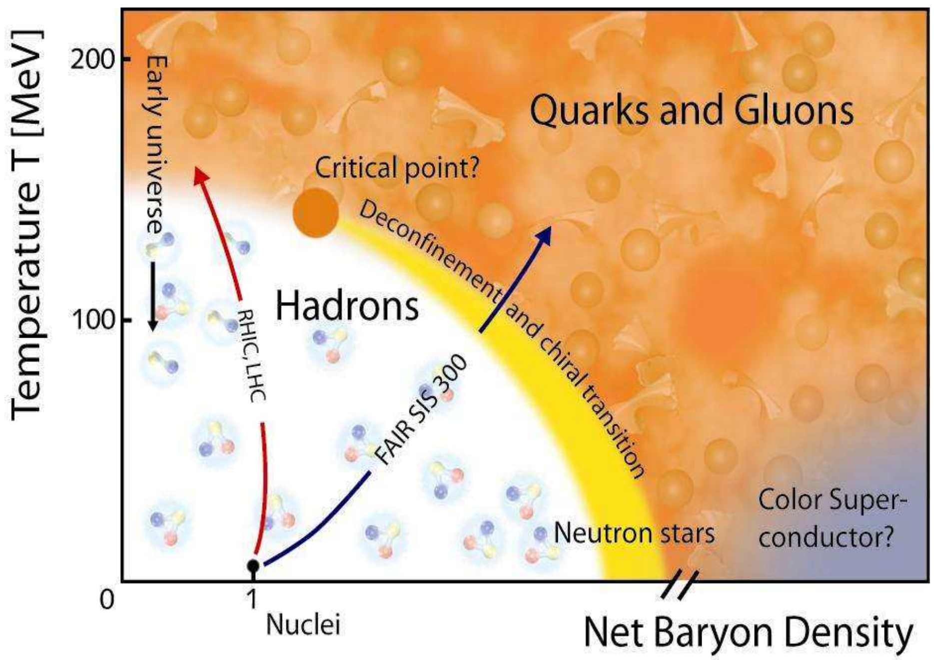
$$\eta = \eta_R + \frac{c_{Rph}}{\sqrt{T}}\, e^{\Delta/T} + \frac{c_{ph}}{T^5} + \ldots$$

High T: Atoms Viscosity governed by hard core ($V \sim 1/r^{12}$)

$$\eta = \eta_0(T/T_0)^{2/3}$$

# III. Quark Gluon Plasma

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

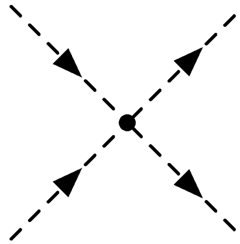


Low T: Pions Chiral perturbation theory

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + (B\mathrm{Tr}[MU] + h.c.) + \ldots$$

Viscosity dominated by $\pi\pi$ scattering

$$\eta = A\frac{f_\pi^4}{T}$$



High T: Quasi-Particles HTL theory (screening, damping, ...)

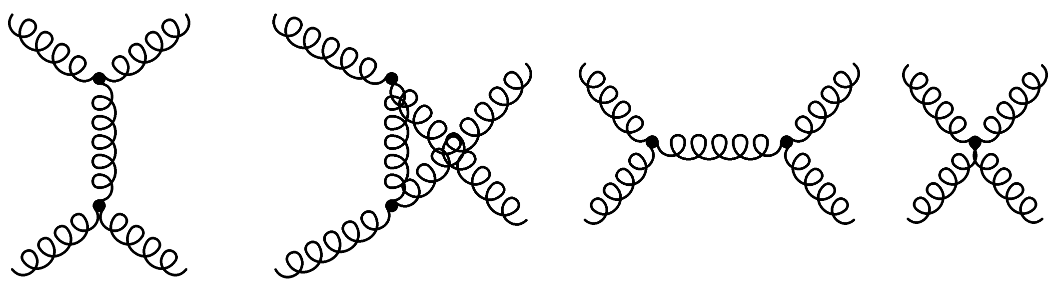
$$\mathcal{L}_{HTL} = \int d\Omega\, G^a_{\mu\alpha}\frac{v^\alpha v_\beta}{(v\cdot D)^2}G^{a,\mu\beta}$$

quasi-particle width

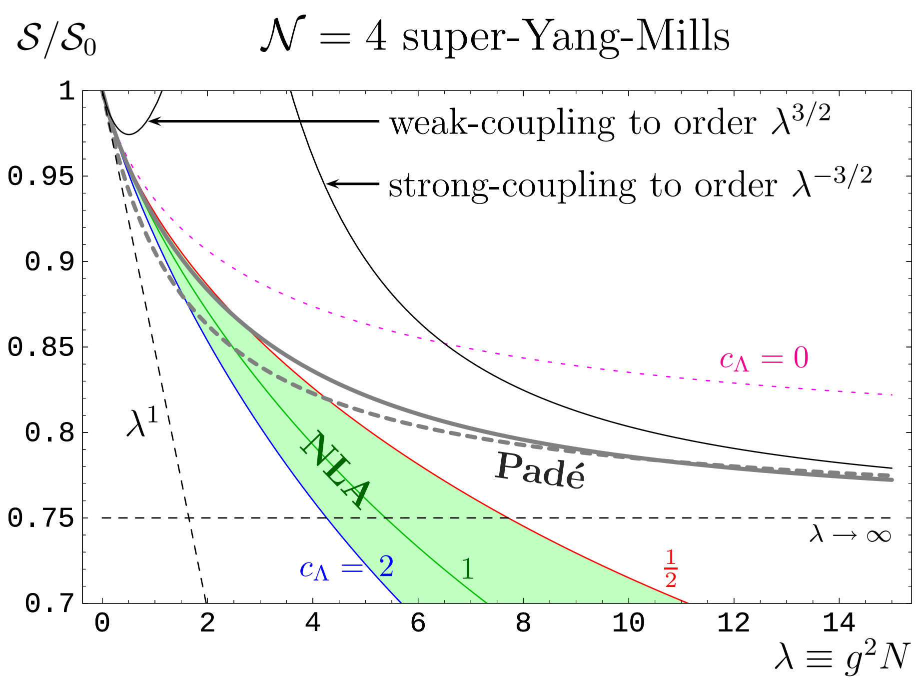
$$\gamma \sim g^2 T$$

Viscosity dominated by t-channel gluon exchange

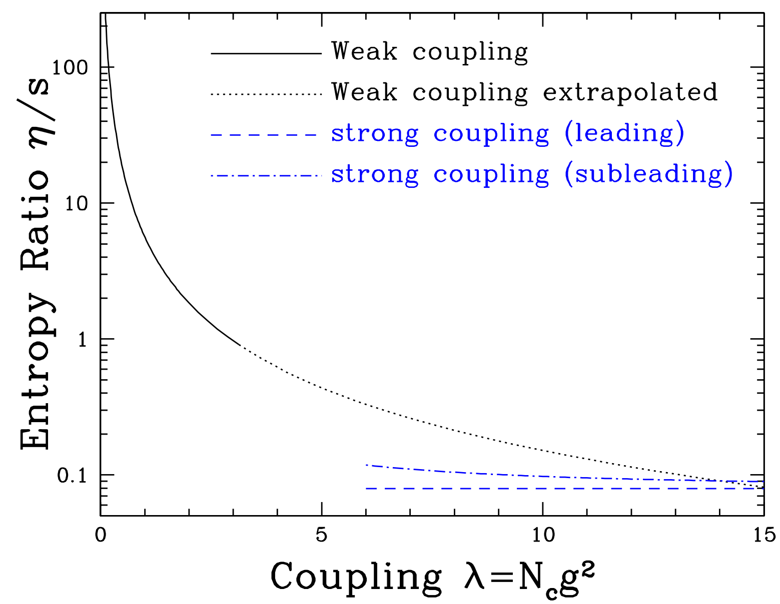
$$\eta = \frac{27.13T^3}{g^4\log(2.7/g)}$$



# What is the value of the coupling?




Blaizot, Iancu, Rebhan, Kraeemmer, Rebhan (2006)




Huot, Jeon, Moore (2006)

# I. Experiment (Liquid Helium)

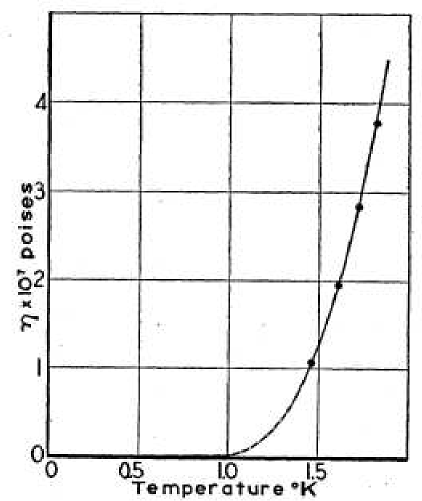



FIG. 1. The viscosity of liquid helium II measured by flow through a $10^{-4}$ cm channel.

2, § 3 LIQUID HELIUM II 23

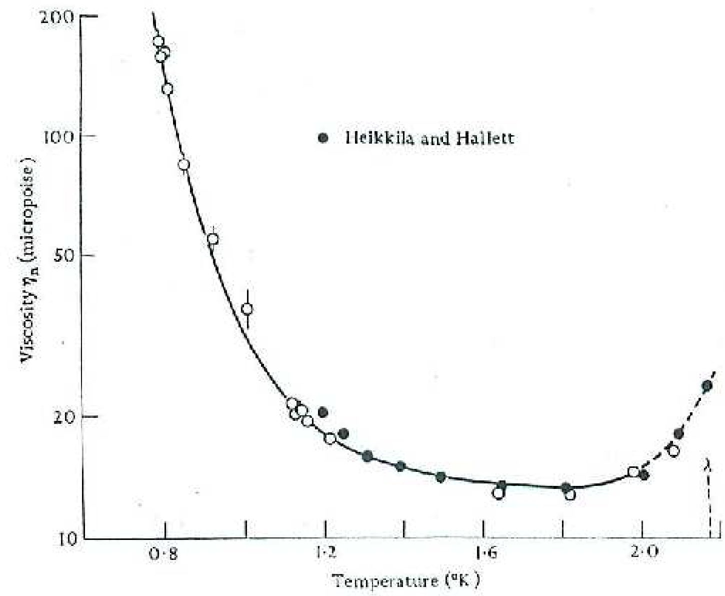



FIG. 11. The viscosity ($\eta_n$) of helium II as measured in a rotation viscometer (Woods and Hollis Hallett [50]). The full points show the earlier results of Heikkila and Hollis Hallett [51]).

Kapitza (1938)

viscosity vanishes below $T_c$

capillary flow viscometer

Hollis-Hallett (1955)

roton minimum, phonon rise

rotation viscometer

$$\eta/s \simeq 0.8\, \hbar/k_B$$

# Viscosity near Liquid-Gas Transition

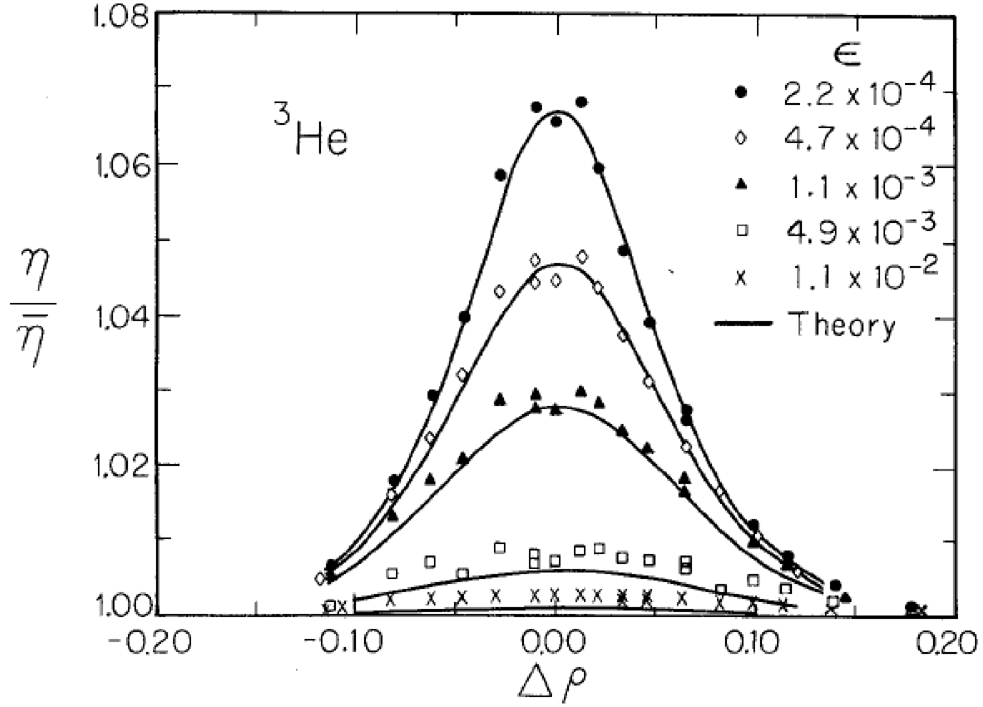
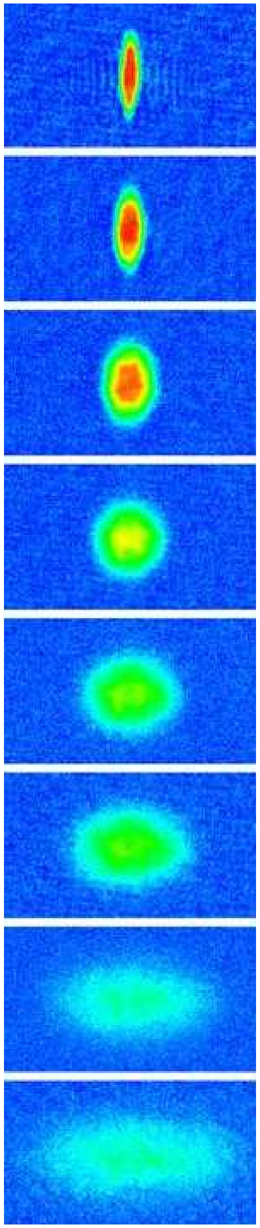
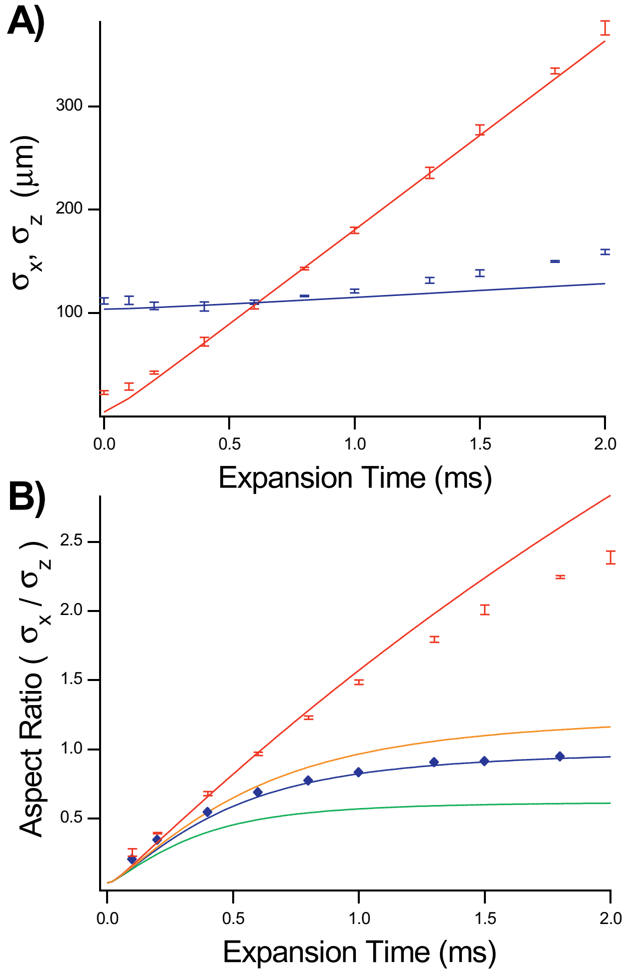



Fig. 14. The normalized viscosity $\tilde{\eta}/\bar{\eta}$ for $^3$He versus $\Delta\rho_c$ along several isotherms. (—) Fits of the MC theory with $q_D = 3\times10^6$ cm$^{-1}$.

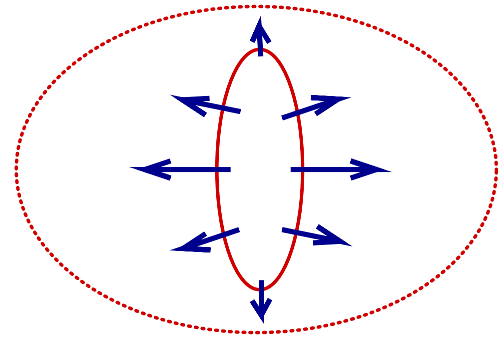
Agosta, Wang, Cohen, Meyer (1987)

# II. Elliptic Flow (Fermions)



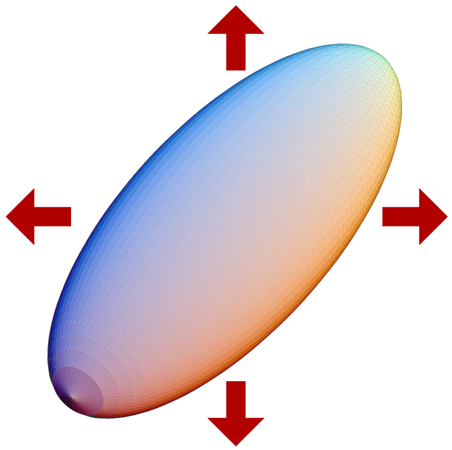



Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

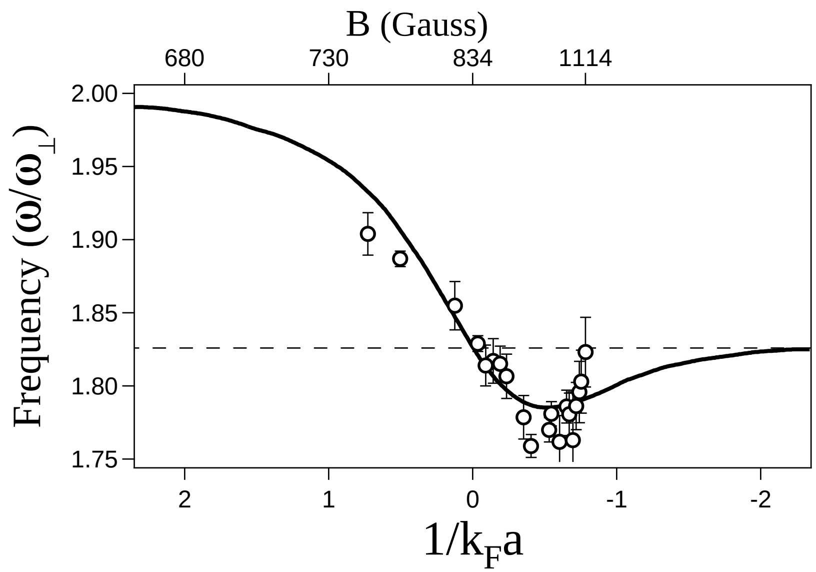


# Collective Modes

Radial breathing mode



Kinast et al. (2005)

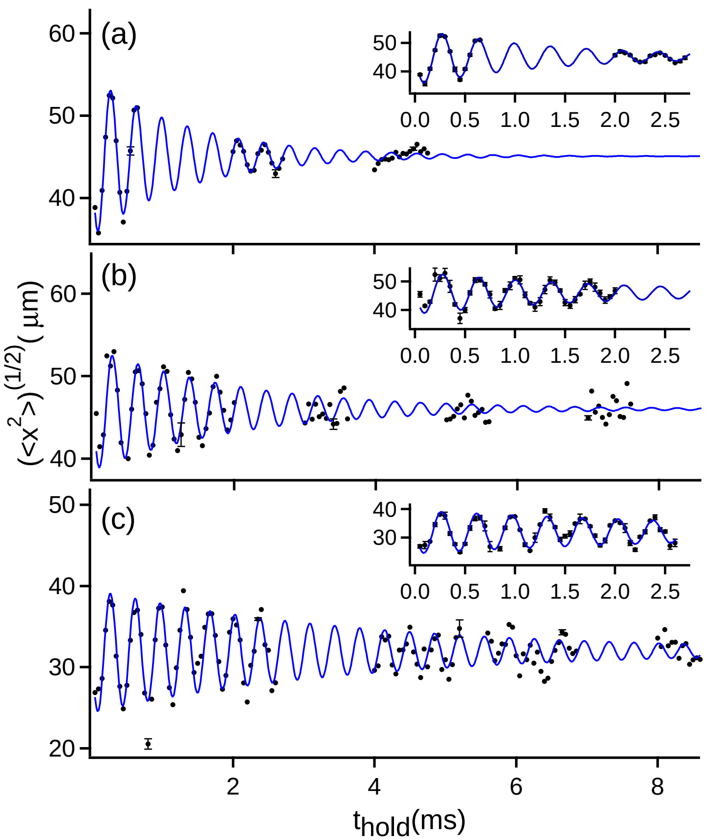


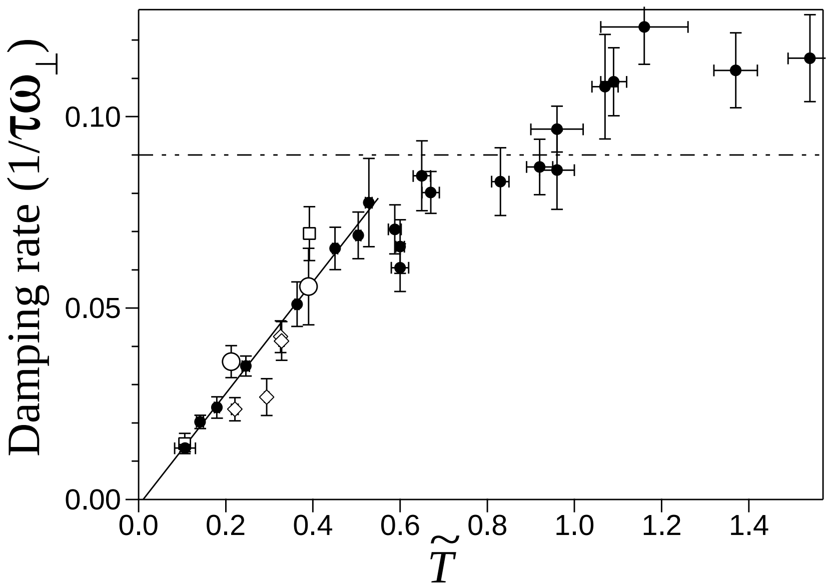

Ideal fluid hydrodynamics, equation of state $P \sim n^{5/3}$

$$\begin{aligned}\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) &= 0 \\ \frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} &= -\frac{1}{mn}\vec{\nabla}P - \frac{1}{m}\vec{\nabla}V\end{aligned}$$

$$\omega = \sqrt{\frac{10}{3}}\omega_\perp$$

# Damping of Collective Excitations




$$T/T_F = (0.5, 0.33, 0.17)$$

$\tau\omega$: decay time $\times$ trap frequency

Kinast et al. (2005)

# Viscous Hydrodynamics

Energy dissipation ($\eta, \zeta, \kappa$: shear, bulk viscosity, heat conductivity)

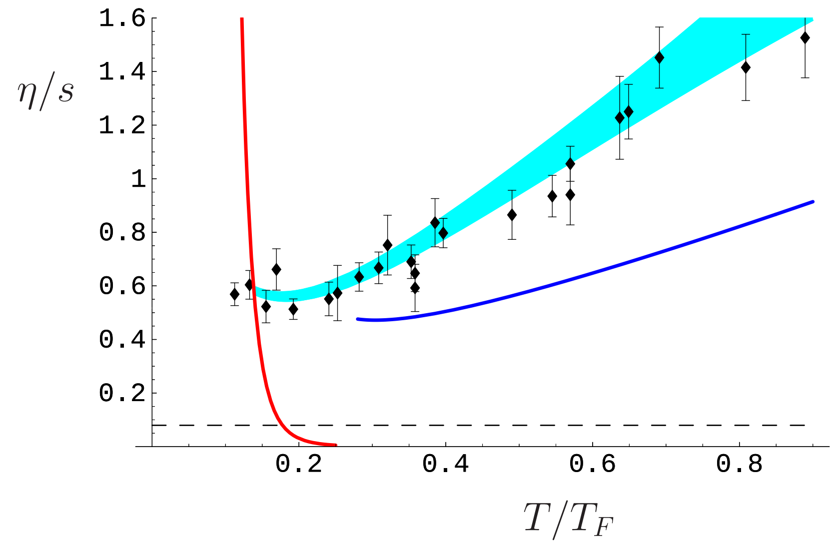
$$\dot{E} = -\frac{\eta}{2}\int d^3x \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \zeta\int d^3x\,(\partial_i v_i)^2 - \frac{\kappa}{T}\int d^3x\,(\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

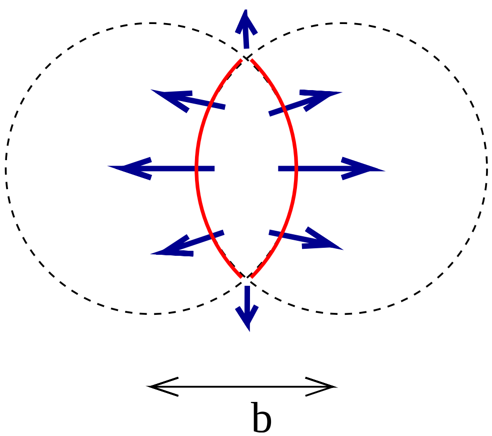
$$\frac{\eta}{s} = \frac{3}{4}\xi^{\frac{1}{2}}(3N)^{\frac{1}{3}}\frac{\Gamma}{\omega_\perp}\frac{\bar{\omega}}{\omega_\perp}\frac{N}{S}$$

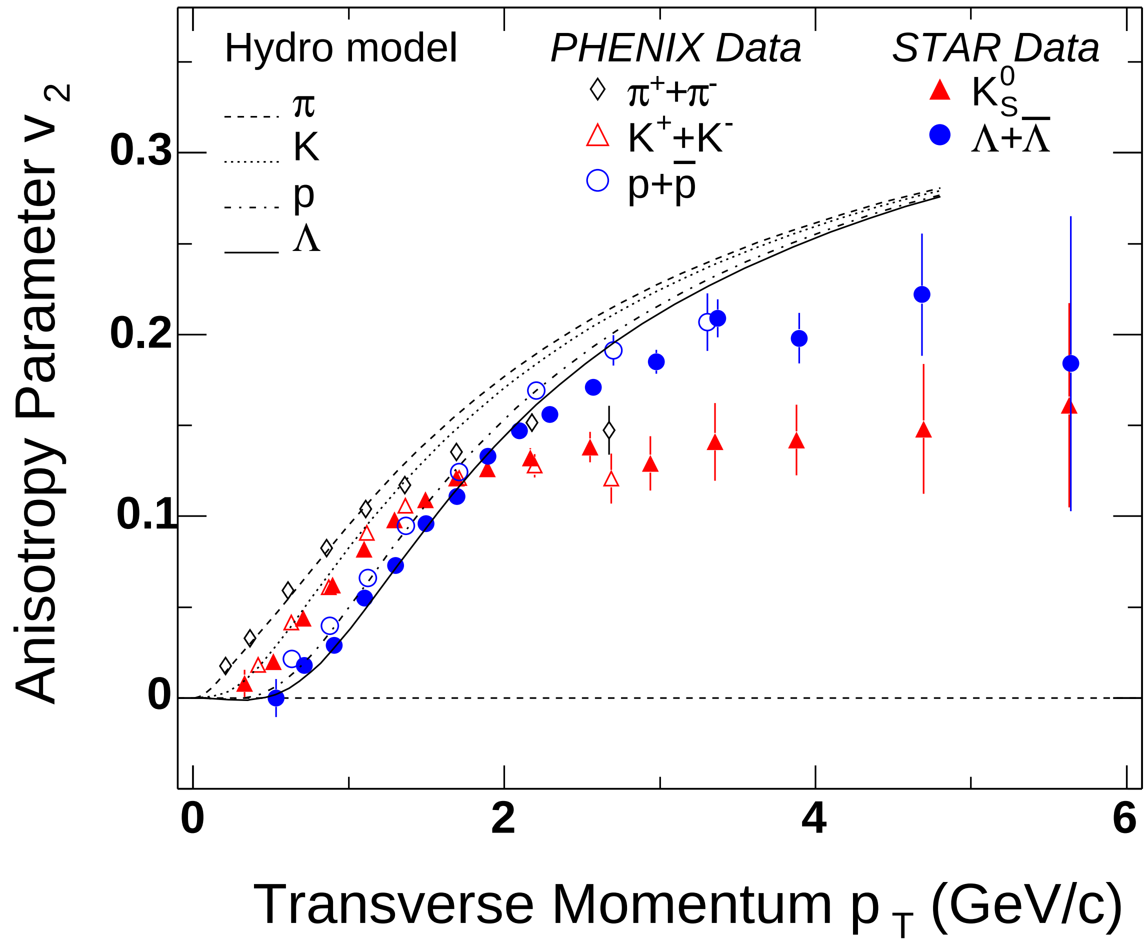
see also Bruun, Smith, Gelman et al.



# III. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy




source: U. Heinz (2005)

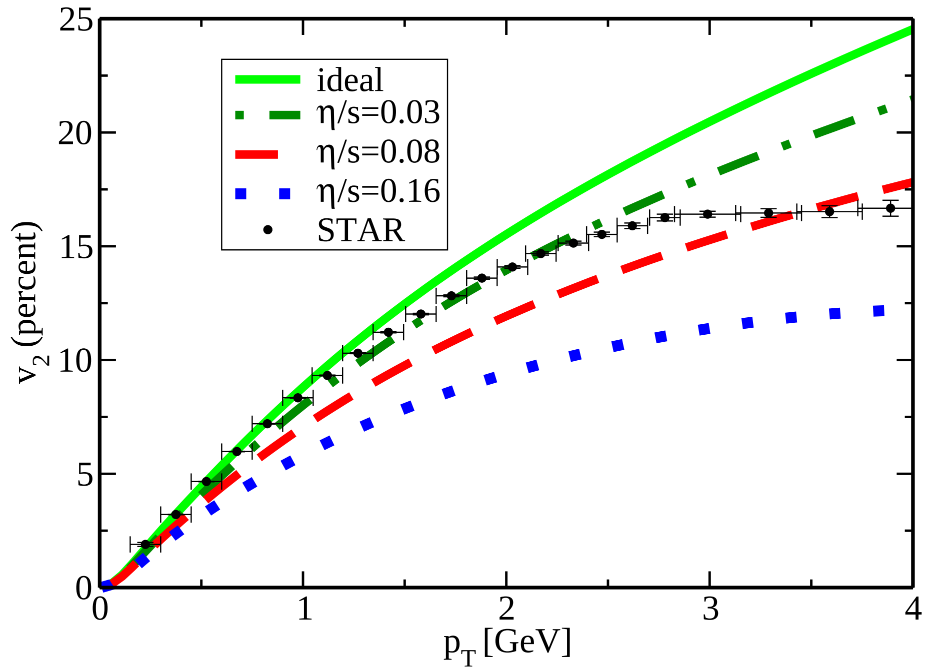
# Viscosity and Elliptic Flow

Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$ (applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm




Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

# Outlook

Too early to declare a winner.

$$\eta/s \simeq 0.8 \text{ (He)}, \qquad \eta/s \leq 0.5 \text{ (CA)}, \qquad \eta/s \leq 0.5 \text{ (QGP)}$$

Other experimental constraints, more analysis needed.

Existing theory tools: Kinetic theory in low/high $T$ limit, Kubo+field theory tools (lattice, etc), dynamic universality, ...

New theory tools: AdS/Cold Atom correspondence? Field theory approaches in cross over regime (large N, epsilon expansions, ...)