Hydrodynamics and Transport in Ultracold Fermi Gases

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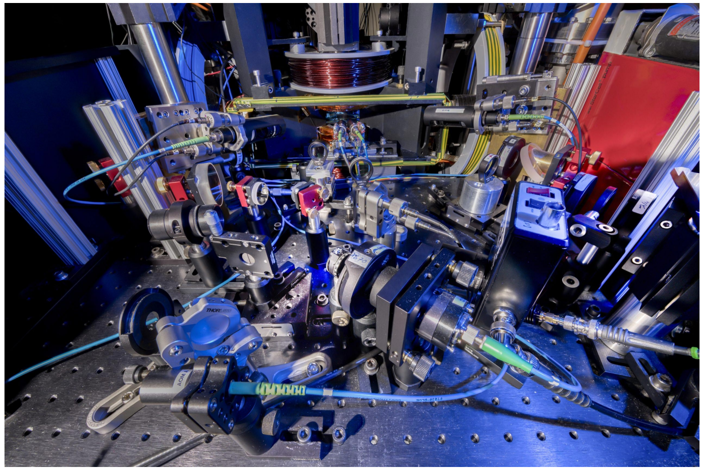
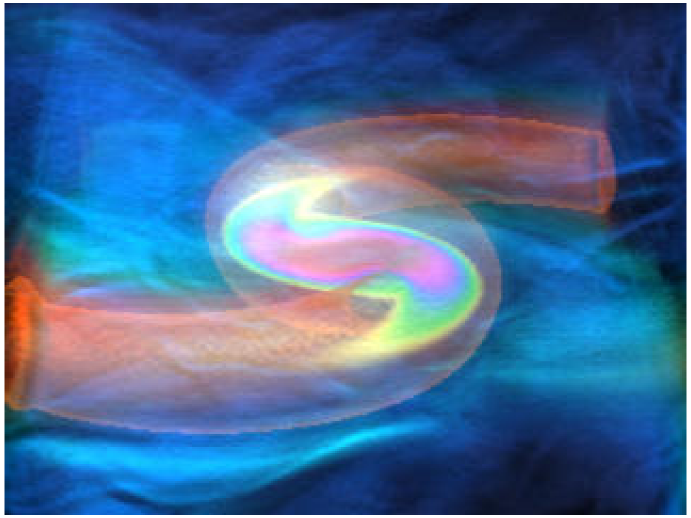




DOE Quantum Horizons, T.S., S. König (NCSU), M. Zwierlein (MIT).

Fermi Gas at Unitarity: Field Theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$ (DR: $C_0 \to \infty$)

This limit is smooth (HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$)

$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$

$\phi \sim \psi_\uparrow \psi_\downarrow$ auxiliary "pair" or "dimer" field.

Fermi Gas at Unitarity: Questions

Validity of hydrodynamics and kinetic theory: From large to small systems, from small to large (ω, k).

Transport coefficients: η, κ, spin diffusion, first and second sound diffusivity, ζ (away from unitarity).

Spectral functions $\eta(\omega)$ etc. Quasi-particles? Validity of (resummed) many-body perturbation theory.

New topics: OTOCs, fluctuations, etc.

Outline

1. Transport coefficients: Theory

2. Transport: Viscosity from elliptic flow.

3. Transport: Linear response.

4. Outlook: External fields, OTOCs, etc.

1. Fluid dynamics

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\vec{j}^{\rho} = 0 \qquad \frac{\partial \mathcal{E}}{\partial t} + \vec{\nabla}\vec{j}^{\epsilon} = 0$$

$$\frac{\partial \pi_i}{\partial t} + \nabla_j \Pi_{ij} = 0 \qquad \vec{j}^{\rho} \equiv \rho\vec{v} = \vec{\pi}$$

Scale invariance: Ideal fluid dynamics

$$\Pi^0_{ij} = P g_{ij} + \rho v_i v_j, \qquad P = \frac{2}{3}\mathcal{E}$$

First order viscous hydrodynamics

$$\delta^{(1)}\Pi_{ij} = -\eta\sigma_{ij} - \zeta g_{ij}\langle\sigma\rangle \qquad \zeta = 0$$

$$\sigma_{ij} = (\nabla_i v_j + \nabla_j v_i - 2/3\delta_{ij}\nabla \cdot v) \qquad \langle\sigma\rangle = \sigma_{ii}$$

Microscopic Theory

$P = P(\mathcal{E})$ fixed by conformal symmetry. $P(\mu, T)$ can be computed from euclidean data

$$P = \log Z(\mu, T) \qquad Z = \int D\psi D\psi^\dagger e^{-S_E}$$

But: Transport coefficients determined by Kubo relations

$$\eta = -\lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im} \int dt d^3x \, e^{-i(\omega t - kx)} \, \Theta(t) \langle [\Pi_{xy}(0), \Pi_{xy}(t, x)] \rangle$$

Requires real time data.

Kinetic theory

Microscopic picture: Quasi-particle distribution function $f_p(x,t)$

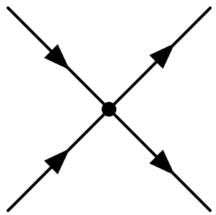
$$\rho(x,t) = \int d\Gamma_p \, \sqrt{g} m f_p(x,t) \qquad \pi_i(x,t) = \int d\Gamma_p \, \sqrt{g} p_i f_p(x,t)$$

$$\Pi_{ij}(x,t) = \int d\Gamma_p \, \sqrt{g} p_i v_j f_p(x,t)$$

Boltzmann equation

$$\left(\frac{\partial}{\partial t} + \frac{p^i}{m} \frac{\partial}{\partial x^i} - \left(g^{il} \dot{g}_{lj} p^j + \Gamma^i_{jk} \frac{p^j p^k}{m} \right) \frac{\partial}{\partial p^i} \right) f_p(t,x,) = C[f]$$

$$C[f] =$$



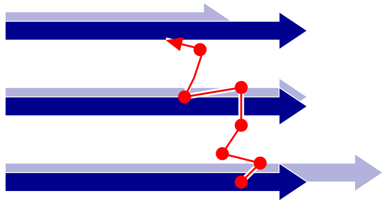
Solve order-by-order in Knudsen number $Kn = l_{mfp}/L$

Kinetic theory: Knudsen expansion

Chapman-Enskog expansion $f = f_0 + \delta f_1 + \delta f_2 + \ldots$

Gradient exp. $\delta f_n = O(\nabla^n)$

$\equiv$ Knudsen exp. $\delta f_n = O(Kn^n)$



First order result

Bruun, Smith (2005)

$$\delta^{(1)}\Pi_{ij} = -\eta\sigma_{ij} \qquad \delta^{(1)} j_i^{\epsilon} = -\kappa\nabla_i T \qquad \eta = \frac{15}{32\sqrt{\pi}}(mT)^{3/2} \qquad \kappa = \frac{2}{3}c_P\eta$$

Second order result

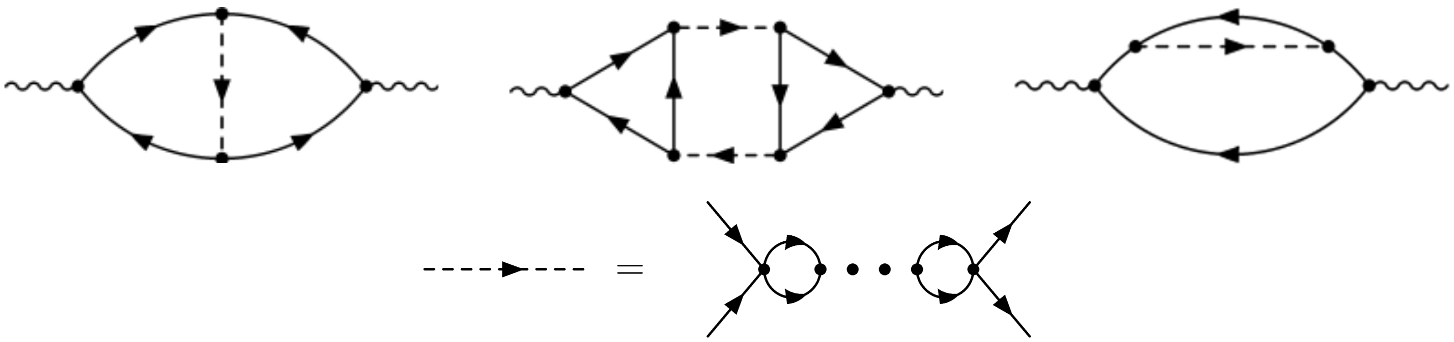
Chao, Schaefer (2012), Schaefer (2014)

$$\delta^{(2)}\Pi^{ij} = \frac{\eta^2}{P}\left[\langle D\sigma^{ij}\rangle + \frac{2}{3}\sigma^{ij}(\nabla\cdot v)\right] + \frac{\eta^2}{P}\left[\frac{15}{14}\sigma^{\langle i}{}_k\sigma^{j\rangle k} - \sigma^{\langle i}{}_k\Omega^{j\rangle k}\right] + O(\kappa\eta\nabla^i\nabla^j T)$$

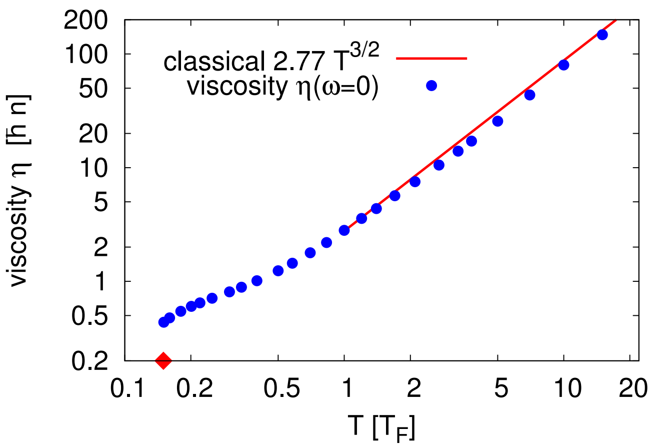
relaxation time $\tau_\pi = \eta/P$

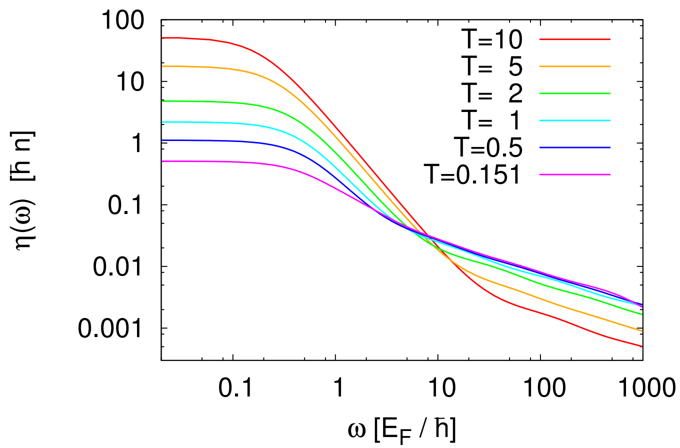
Quantum Field Theory

The diagrammatic content of the Boltzmann equation is known: Kubo formula with "Maki-Thompson" + "Azlamov-Larkin" +"Self-energy"



Limits subtle ($\omega \to 0$ and $n\lambda^3 \to 0$ don't commute). Can be used to extrapolate Boltzmann result to $T \sim T_F$





Enss, Zwerger (2011), see also Levin (2014)

Short time behavior: OPE

Operator product expansion (OPE)

$$\eta(\omega) = \sum_n \frac{\langle \mathcal{O}_n \rangle}{\omega^{(\Delta_n - d)/2}} \qquad \mathcal{O}_n(\lambda^2 t, \lambda x) = \lambda^{-\Delta_n} \mathcal{O}_n(t, x)$$

Leading operator: Contact density (Tan)

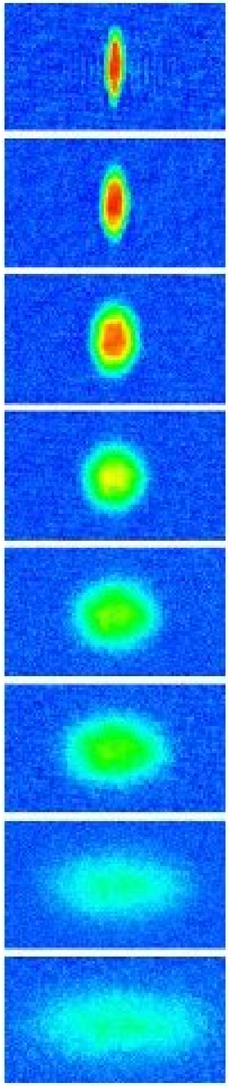
$$\mathcal{O}_{\mathcal{C}} = C_0^2 \psi\psi\psi^\dagger\psi^\dagger = \Phi\Phi^\dagger \qquad \Delta_{\mathcal{C}} = 4$$

$\eta(\omega) \sim \langle \mathcal{O}_{\mathcal{C}} \rangle / \sqrt{\omega}$. Asymptotic behavior + analyticity gives sum rule

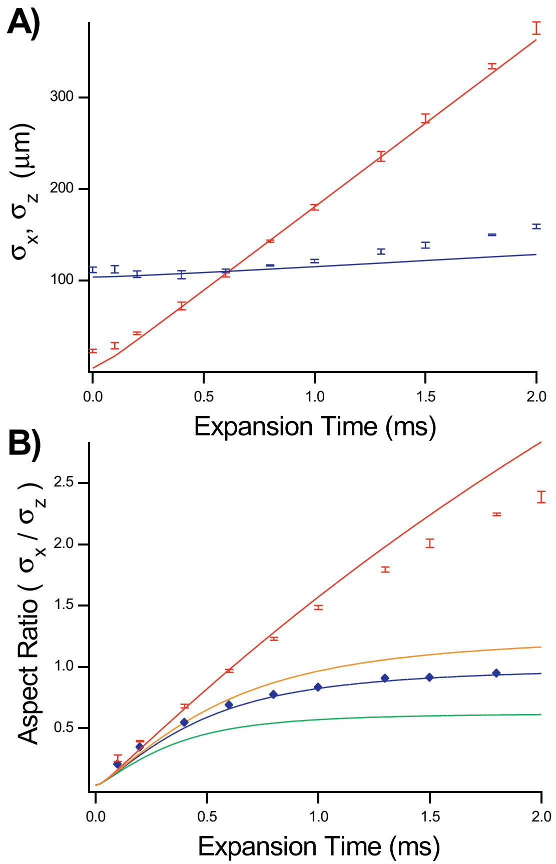
$$\frac{1}{\pi} \int dw \left[\eta(\omega) - \frac{\langle \mathcal{O}_{\mathcal{C}} \rangle}{15\pi\sqrt{m\omega}} \right] = \frac{\mathcal{E}}{3}$$

Randeria, Taylor (2010), Enss, Zwerger (2011), Hoffman (2013)

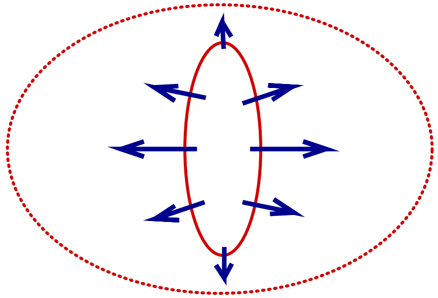
2. Elliptic flow in the unitary Fermi gas



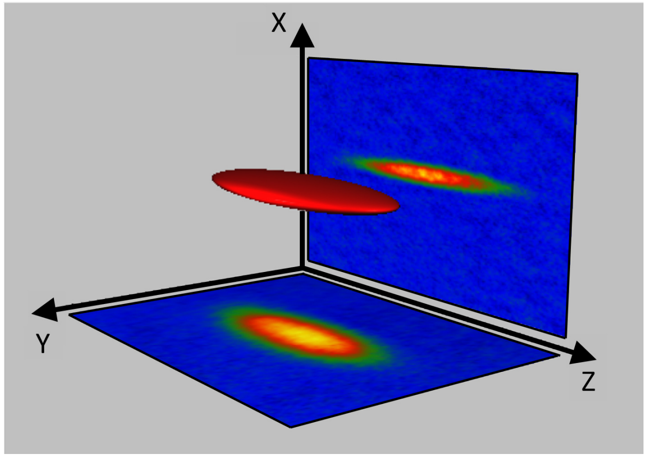
O'Hara et al. (2002)

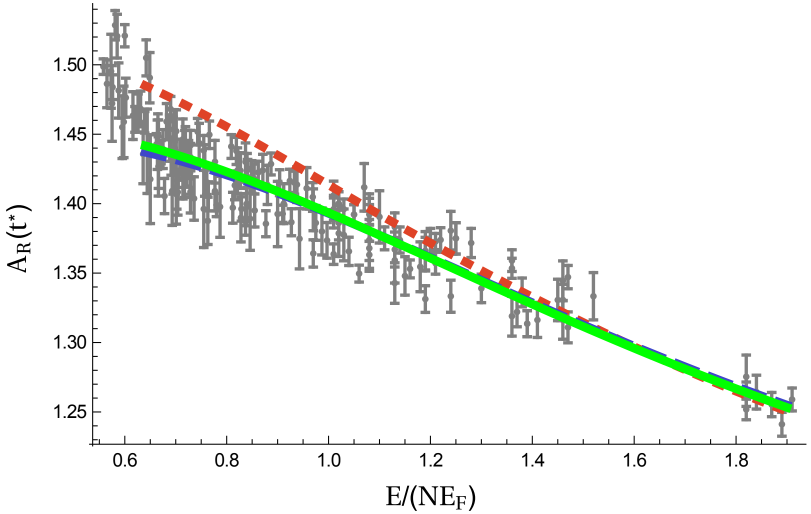


Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Fluid dynamics analysis



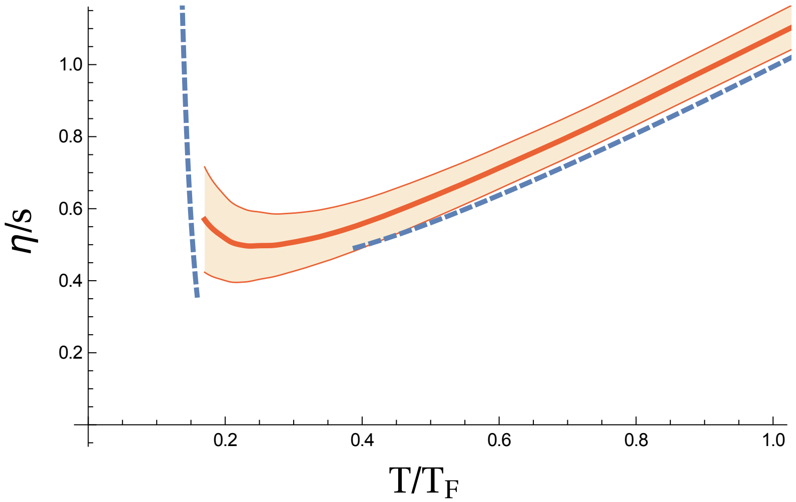


$A_R = \sigma_x/\sigma_y$ as function of total energy. Data: Joseph et al (2016). $E/(NE_F) \sim 0.6$ is the superfluid transition.

Grey, Blue, Green: LO, NLO, NNLO fit.

$$\eta = \eta_0 (mT)^{3/2} \left\{ 1 + \eta_2 n\lambda^3 + \eta_3 (n\lambda^3)^2 + \ldots \right\}$$

Reconstruct η/s (normal fluid)



$T_c \sim 0.17 T_F$. Kinetic theory at low and high T (blue dashed)

Consistency check: $T \gg T_c$

$$\eta|_{T\gg T_c} = (0.265 \pm 0.02)(mT)^{3/2}$$

$$\eta_0(th) = \frac{15}{32\sqrt{\pi}} = 0.269$$

Phenomenology (normal phase): Two-term virial expansion works well,
$\eta \sim \eta_0 (mT)^{3/2} + \eta_1 \hbar n$

$$\eta/s|_{T_c} = 0.56 \pm 0.20$$

Superfluid hydrodynamics

Spontaneous symmetry breaking: $\langle \Psi \rangle = v_0 e^{i\theta}$.

Goldstone boson is a new hydro mode: $\vec{v}_s = \frac{\hbar}{m} \vec{\nabla}\theta$

$$\partial_t \vec{v}_s + \frac{1}{2}\vec{\nabla}(v_s^2) = -\vec{\nabla}\mu_s$$

Momentum density: $\vec{\pi} = \rho_n \vec{v}_n + \rho_s \vec{v}_s$

$$\rho = \rho_n + \rho_s \qquad \rho_n = 2\, \frac{\partial P}{\partial w^2}\bigg|_{\mu_s, T} \qquad \vec{w} = \vec{v}_n - \vec{v}_s$$

Stress tensor and energy current

$$\begin{aligned} \Pi_{ij} &= P\delta_{ij} + \rho_n v_{n,i} v_{n,j} + \rho_s v_{s,i} v_{s,j} \\ \vec{j}^{\epsilon} &= sT\vec{v}_n + \left(\mu_s + \frac{1}{2} v_s^2\right)\vec{\pi} + \rho_n \vec{v}_n \vec{v}_n \cdot \vec{w} \end{aligned}$$

Superfluid hydrodynamics

Dissipative stresses

$$\delta\Pi_{ij} = -\eta\left(\nabla_i v_{n,j} + \nabla_j v_{n,i} - \frac{2}{3}\delta_{ij}\vec{\nabla}\cdot\vec{v}_n\right)$$

$$-\,\delta_{ij}\Big(\zeta_1\vec{\nabla}\left(\rho_s\left(\vec{v}_s - \vec{v}_n\right)\right) + \zeta_2\left(\vec{\nabla}\cdot\vec{v}_n\right)\Big)$$

Equation of motions for v_s: $\dot{v}_s + \frac{1}{2}\nabla(v_s^{\,2}) = -\nabla(\mu_s + H)$ with

$$H = -\zeta_3\vec{\nabla}\left(\rho_s\left(\vec{v}_s - \vec{v}_n\right)\right) - \zeta_4\vec{\nabla}\cdot\vec{v}_n$$

Conformal symmetry: $\zeta_1 = \zeta_2 = \zeta_4 = 0$

Son (2007)

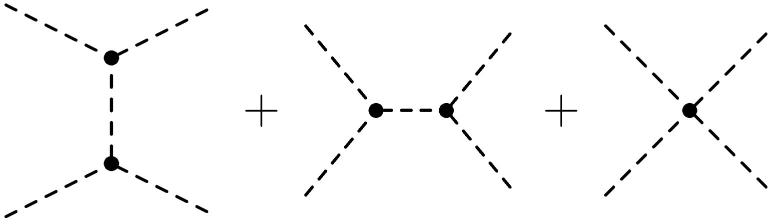
Low T: Phonons

Goldstone boson $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$. Effective Lagrangian

$$\mathcal{L} = c_0 m^{3/2}\left(\mu - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}\right)^{5/2} + \ldots$$

Viscosity dominated by $\varphi + \varphi \to \varphi + \varphi$

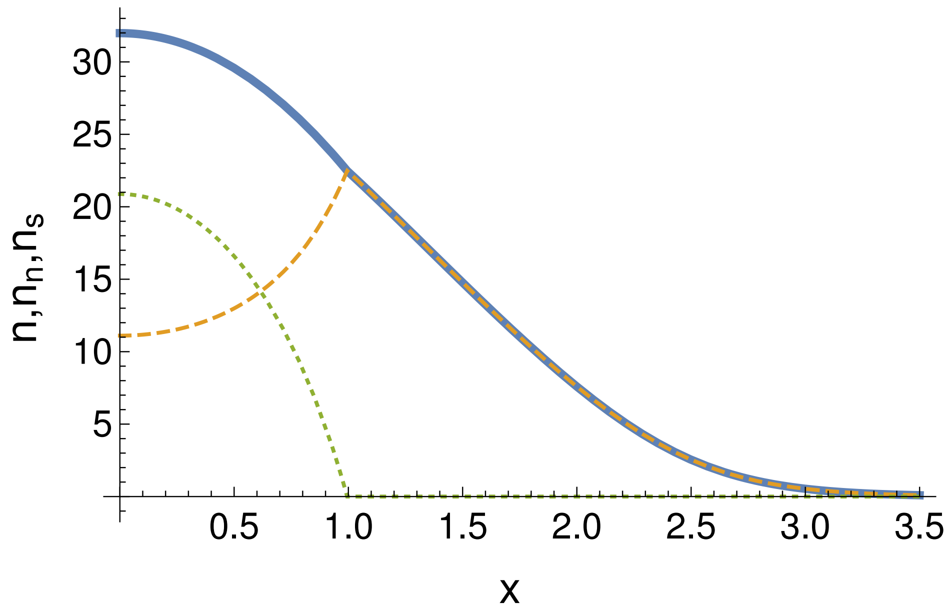
$$\eta = A\frac{\xi^5}{c_s^3}\frac{T_F^8}{T^5}$$



Thermal conductivity is subtle, because quasi-particles with $E_p \sim c_s p$ do not contribute. The dominant process is phonon splitting, made possible by non-linear terms in the dispersion relation.

$$\kappa = \frac{128}{3\pi}\frac{\gamma^2}{g_3^2}\frac{T^2}{c_s^2}D_H = \frac{256\sqrt{2}}{25\pi^3\xi^2 m}(mT)^{3/2}\left(\frac{T}{T_F}\right)^2 D_H$$

Two-fluid hydro for an expanding cloud

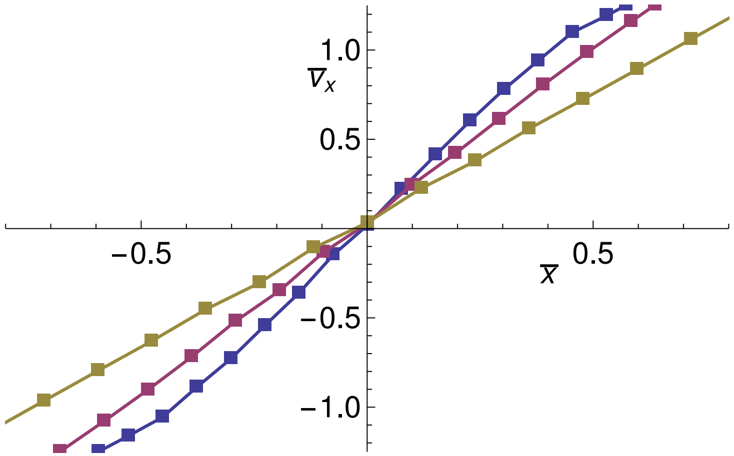


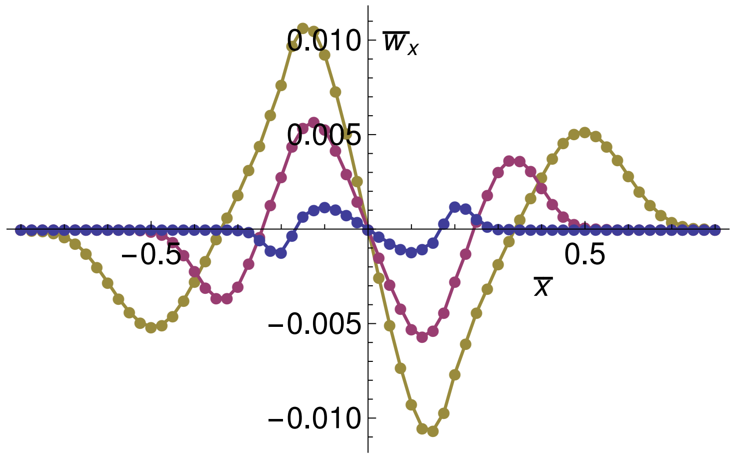
$\rho = \rho_s + \rho_n$ (solid), ρ_n (dashed), ρ_s (dotted)

Gibbs-Duhem relation

$$dP = n d\mu_s + s dT + \frac{\rho_n}{2} dw^2$$

Two-fluid hydro for an expanding cloud



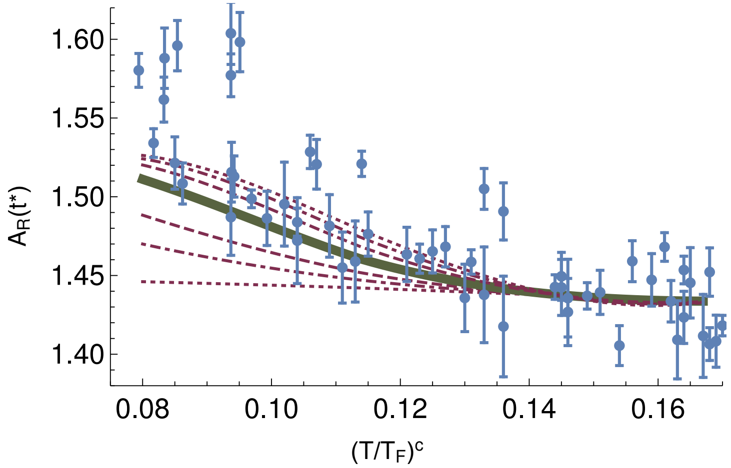


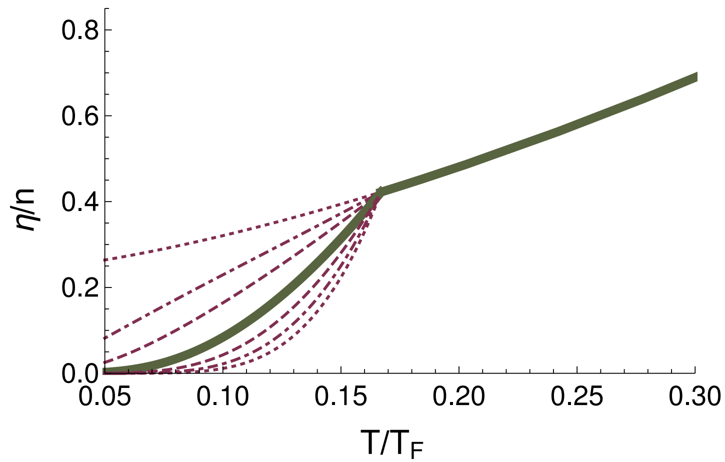
Average fluid velocity $v_x(x,t)$. Superfluid $w_x(x,t) = v_x^n(x,t) - v_x^s(x,t)$

Superfluid $\vec{w} = \vec{v}^n - \vec{v}^s$ can be computed perturbatively.

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}\right)\vec{w} = -\frac{s}{\rho_n}\vec{\nabla}T + O(w^2)\,.$$

Two-fluid hydro analysis of expanding cloud





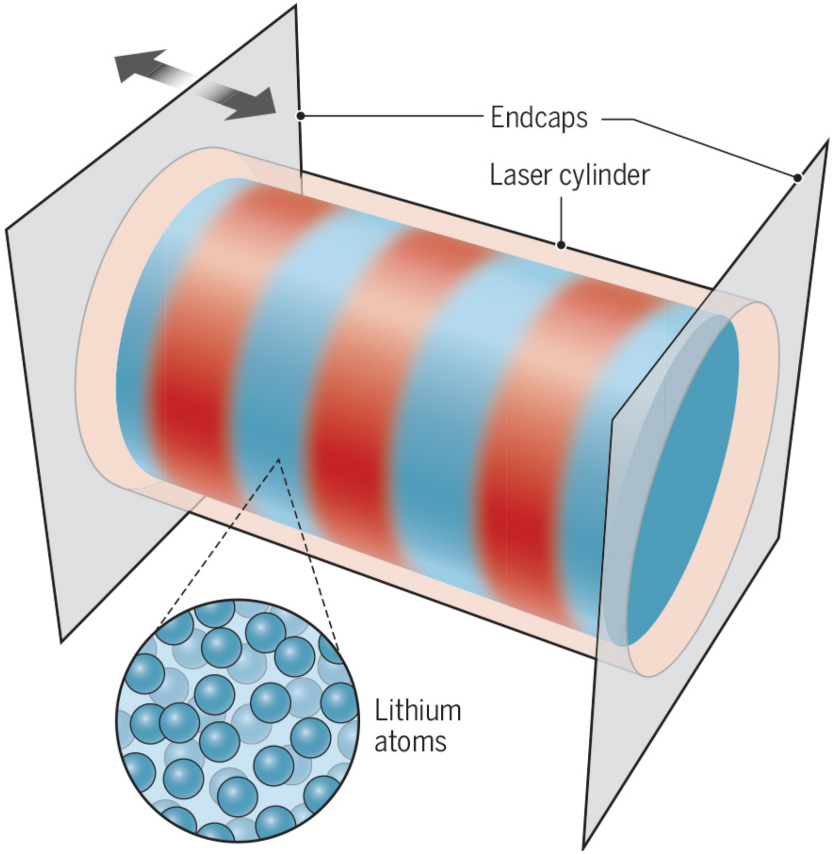
A_R in low T regime.

Small η corresponds to large A_R.

Fits for $\eta(T < T_c)$:

$$\eta \simeq \eta_0 \exp\left[-2\frac{T_c - T}{T}\right]$$

3. Linear Response: Sound attenuation



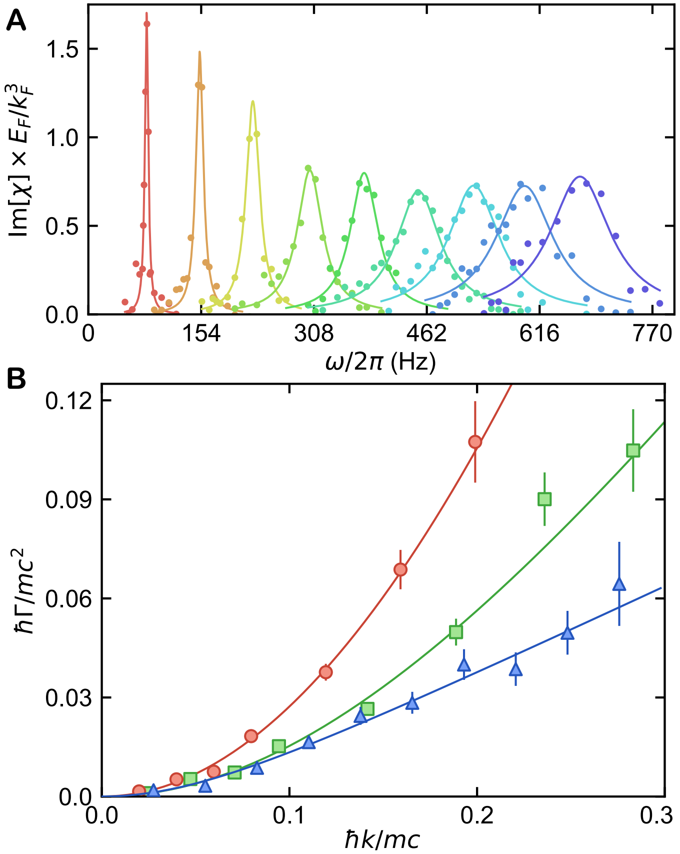
Cylindrical box, response to small harmonic drive.

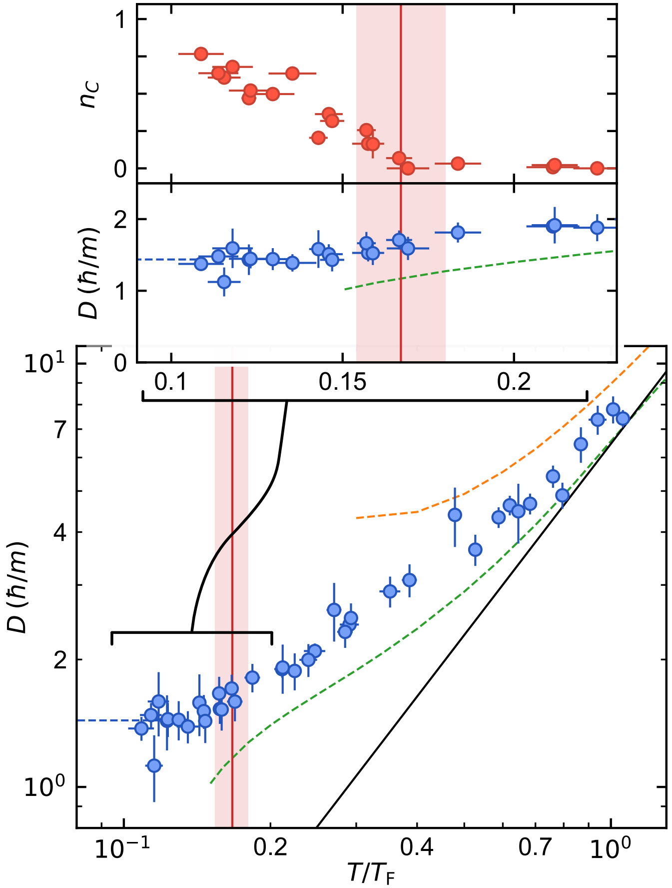
Patel et al., Science (2021)

Sound attenuation (MIT)

Spectral response $\rho_k(\omega)$.

Sound diffusivity $D_s(T)$





Damping rate $\Gamma(k)$

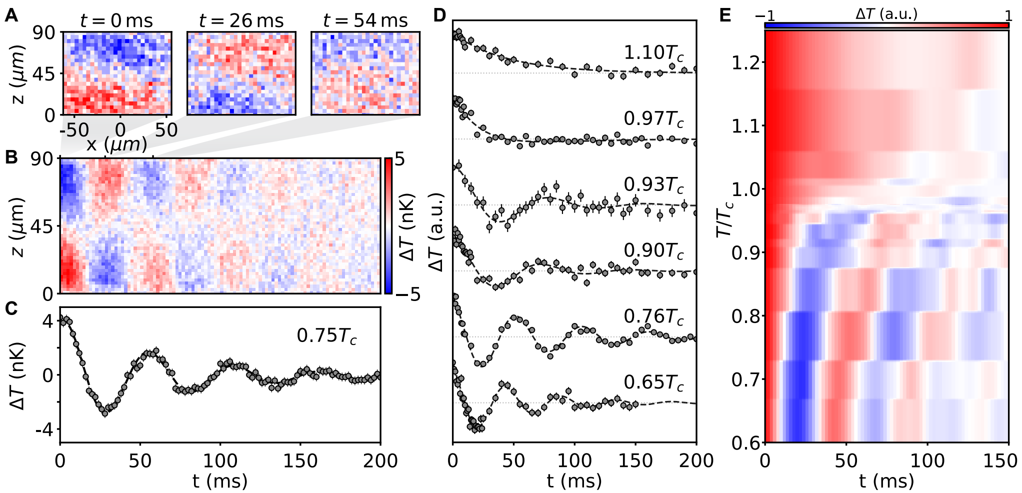
$$D_s = \frac{4\eta}{3\rho} + \frac{4\kappa T}{15P}.$$

$(T/T_F = 0.36, 0.21, 0.13)$.

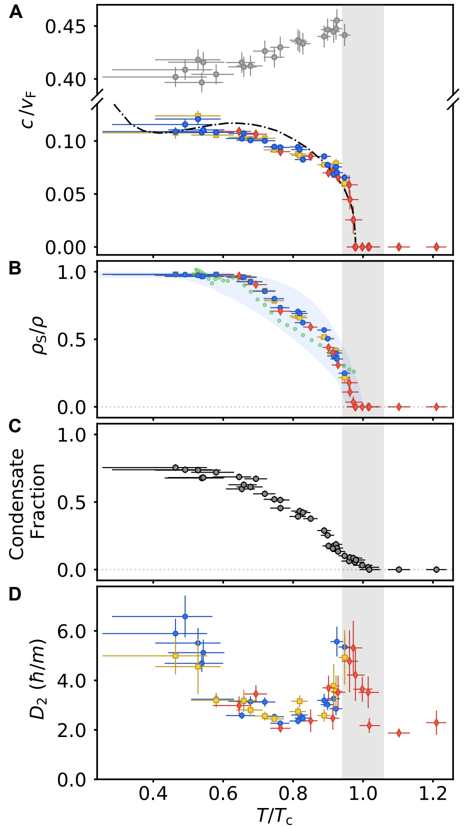
Patel et al., Science (2021)

MIT: Thermography and second sound

Heat propagation above and below T_c



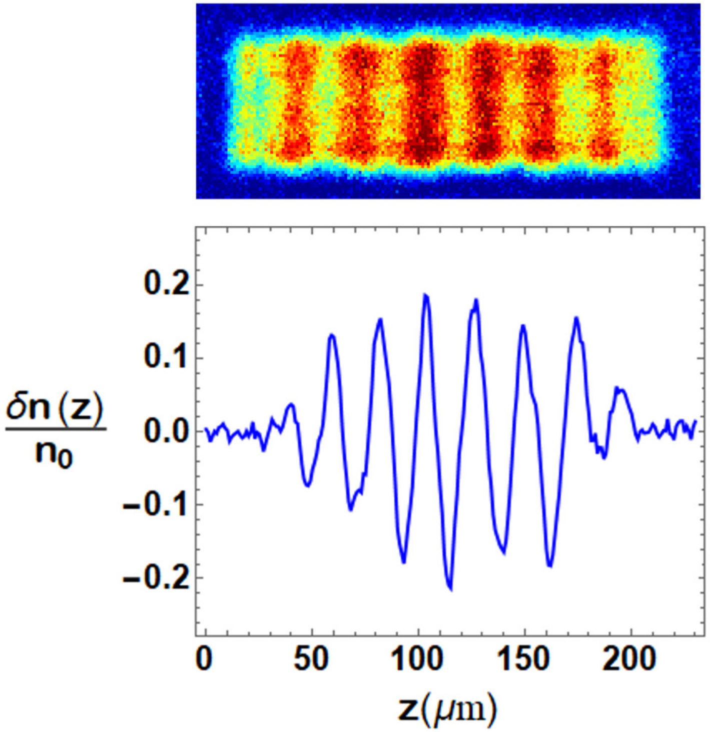
2^{nd} sound diffusivity

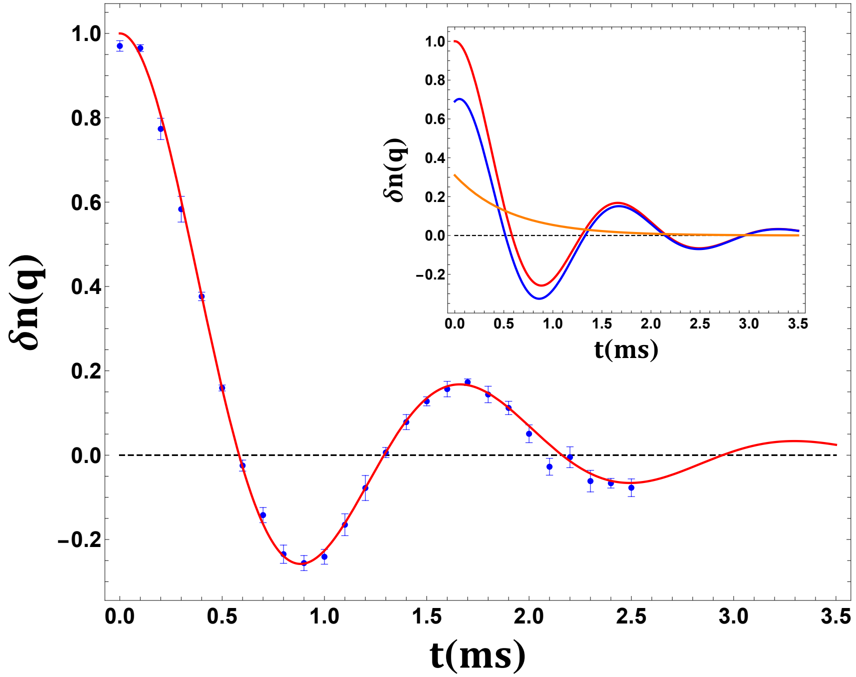


Yan et al., Science (2024)

Linear Response (NC State)

Baird et al., PRL 2019; Wang et al, PRL 2022.





$$\left.\frac{\kappa}{\eta}\right|_{T\gg T_c} = 0.93(14)\,\frac{15k_B}{4m}$$

4. Outlook: External fields, OTOCs, etc.

Can realize response to $A_0(x,t)$, as well as spatial/time variation of scattering length

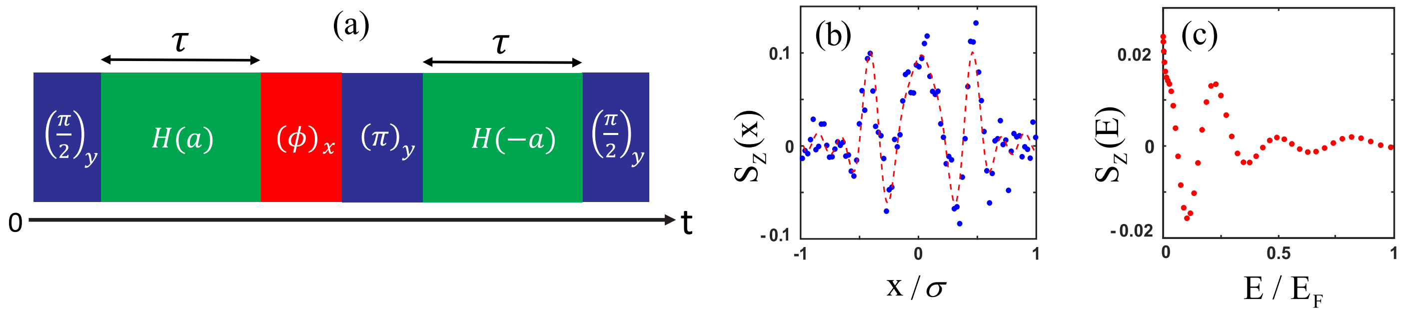
$$H' = \psi^\dagger \psi A_0(x,t), \qquad H' = C_0(x,t)(\psi^\dagger\psi)^2$$

We would like to realize non-trivial metric perturbations

$$H' = \frac{g_{xy}(x,t)}{m} \psi^\dagger \nabla_x \nabla_y \psi$$

We would also like to realize out-of-time-order correlators

$$C(t) = \langle [V(t), W(0)]^2 \rangle$$



Pegahan et al., PRL 2021.