(Nearly) Scale invariant fluid dynamics for the dilute Fermi gas in two and three dimensions

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Outline

I. Conformal hydrodynamics

II. Observations (3d)

III. Scale breaking (3d)

IV. Two dimensional fluids

V. Fluctuations

Two body scattering in 2d and 3d

Consider zero range interaction

$$\mathcal{L} = \psi^\dagger \Big(i\partial_0 + \frac{\nabla^2}{2M} \Big) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

3d: Tune C_0 to unitarity. Two body scattering matrix

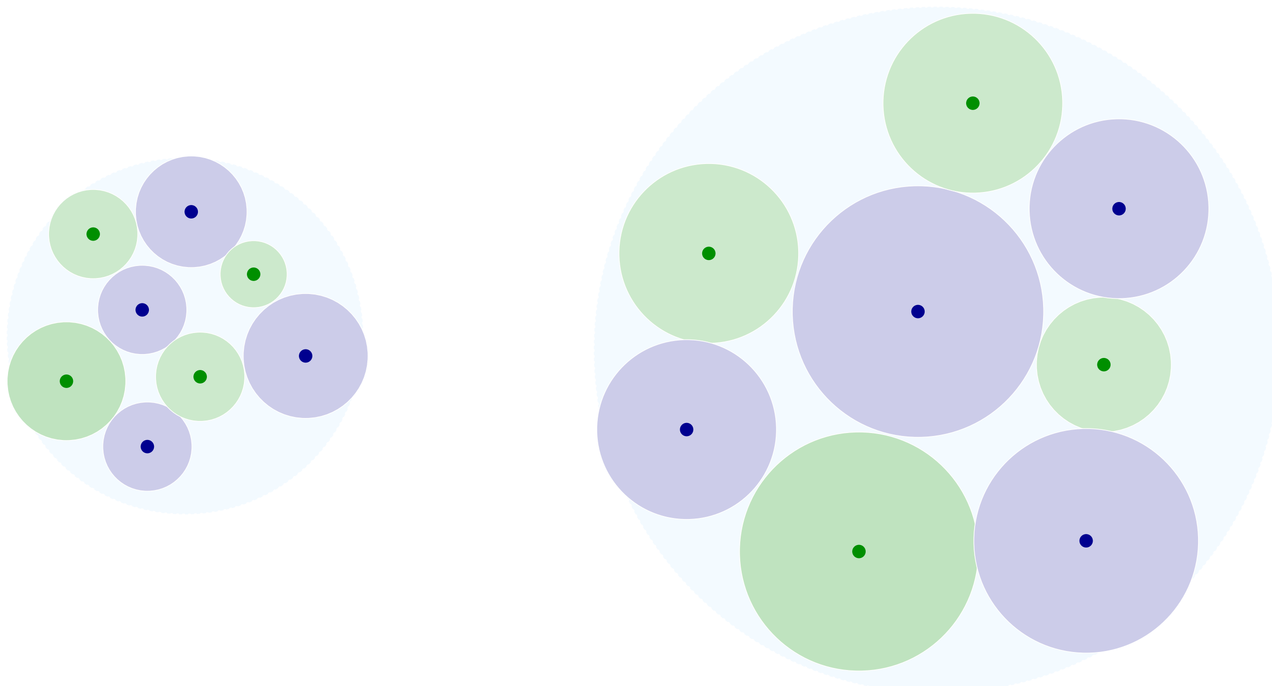
$$\mathcal{T} = \frac{4\pi}{m} \frac{1}{-1/a - ik} \qquad \Rightarrow \qquad \mathcal{T} = \frac{4\pi}{m} \frac{1}{-ik}$$

2d: Classical scale invariance, broken by quantum effects

$$\mathcal{T} = \frac{4\pi}{m} \frac{1}{\log[(ka_{2d})^2] - i\pi}$$

I. Scale invariant fluid dynamics in 3d

Many body system: Effective cross section $\sigma_{tr} \sim n^{-2/3}$ (or $\sigma_{tr} \sim \lambda^2$)



Systems remains hydrodynamic despite expansion

Scale and conformal symmetry

Gallilean boosts	$\vec{x}' = \vec{x} + \vec{v}t$	$t' = t$
scale trafo	$\vec{x}' = e^s \vec{x}$	$t' = e^{2s} t$
conformal trafo	$\vec{x}' = \vec{x}/(1+ct)$	$1/t' = 1/t + c$

Ideal fluid dynamics

$$\Pi^0_{ij} = P\delta_{ij} + \rho v_i v_j, \qquad P = \frac{2}{3}\mathcal{E}$$

First order viscous hydrodynamics

$$\delta^{(1)}\Pi_{ij} = -\eta\sigma_{ij}, \quad \sigma_{ij} = \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3}\delta_{ij}(\nabla\cdot v)\right), \qquad \zeta = 0$$

Second order conformal hydrodynamics

Relaxation of shear stress is a second order hydro term. Complete list

$$\begin{aligned}\delta^{(2)}\Pi^{ij} \;=\; & \eta\tau_\pi\left[{}^{\langle}D\sigma^{ij\rangle} + \frac{2}{3}\sigma^{ij}(\nabla\cdot v)\right] \\ & + \lambda_1\sigma^{\langle i}{}_k\sigma^{j\rangle k} + \lambda_2\sigma^{\langle i}{}_k\Omega^{j\rangle k} + \lambda_3\Omega^{\langle i}{}_k\Omega^{j\rangle k} \\ & + \gamma_1\nabla^{\langle i}T\nabla^{j\rangle}T + \gamma_2\nabla^{\langle i}P\nabla^{j\rangle}P + \gamma_3\nabla^{\langle i}\nabla^{j\rangle}T + \ldots\end{aligned}$$

$$D = \partial_0 + v\cdot\nabla \quad A^{\langle ij\rangle} = \frac{1}{2}\left(A_{ij} + A_{ji} - \frac{2}{3}g_{ij}A^k{}_k\right) \quad \Omega^{ij} = \left(\nabla_i v_j - \nabla_j v_i\right)$$

New transport coefficients $\tau_\pi, \lambda_i, \gamma_i$

Can be written as a relaxation equation for $\pi^{ij} \equiv \delta\Pi^{ij}$

$$\left[{}^{\langle}D\pi^{ij\rangle} + \frac{5}{3}(\nabla\cdot v)\pi^{ij}\right] = -\tau_\pi\left(\pi^{ij} + \eta\sigma^{ij}\right) + \ldots$$

Chao, Schaefer (2011)

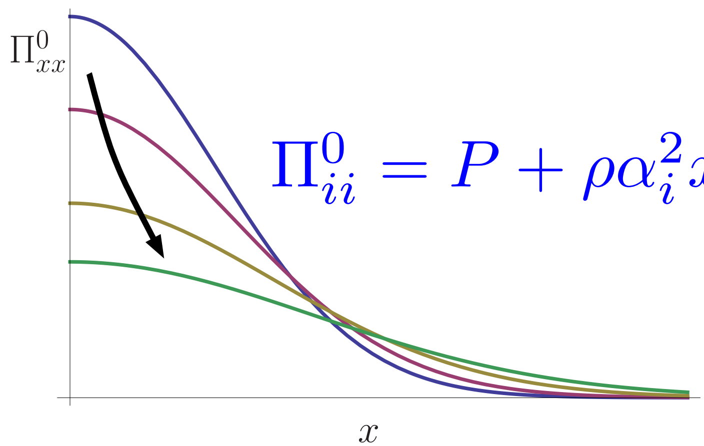
Why second order fluid dynamics?

Scaling ("Hubble") expansion

$$\rho(x_i,t) = \rho_0(b_i(t)x_i), \qquad v_i(x_j,t) = \alpha_i(t)x_i$$

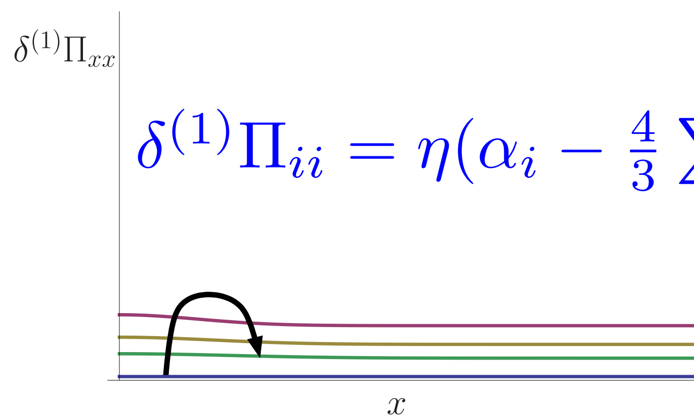
ideal stresses

$$\Pi^0_{ii} = P + \rho\alpha_i^2 x_i^2$$



dissipative stresses

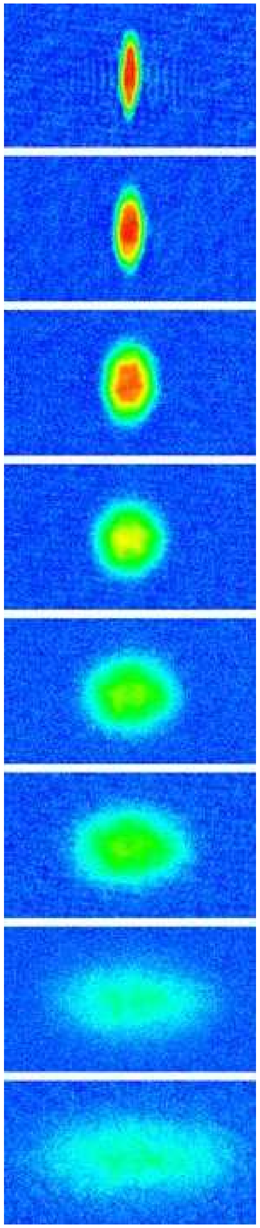
$$\delta^{(1)}\Pi_{ii} = \eta(\alpha_i - \tfrac{4}{3}\textstyle\sum_j \alpha_j)$$



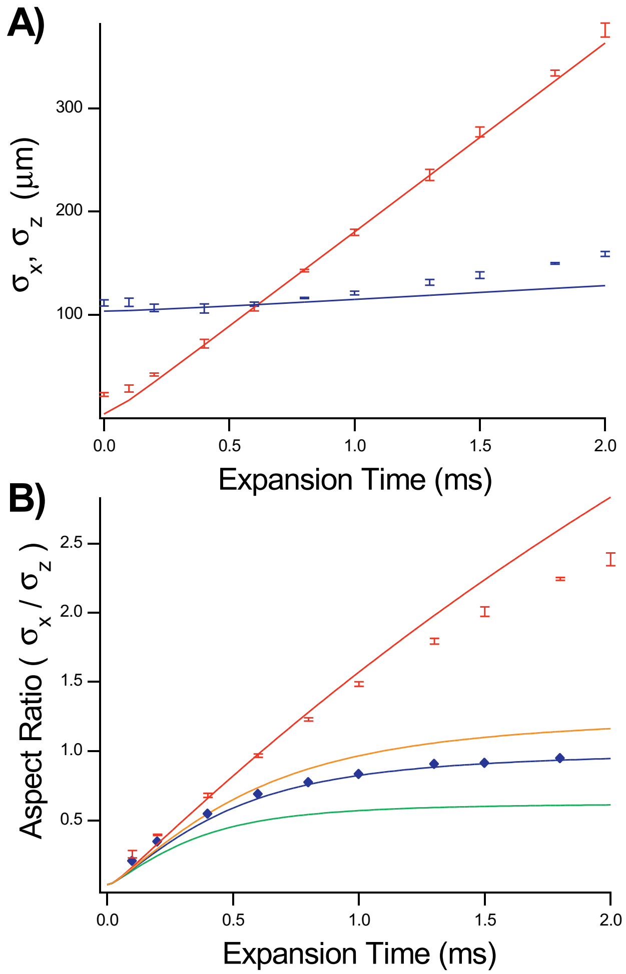
Ideal stresses propagate with speed $\sim c_s$, dissipative stresses propagate with infinite speed. Hydro always breaks down in the dilute corona.

Solved by relaxation time $\tau_\pi \sim \frac{\eta}{P}$.

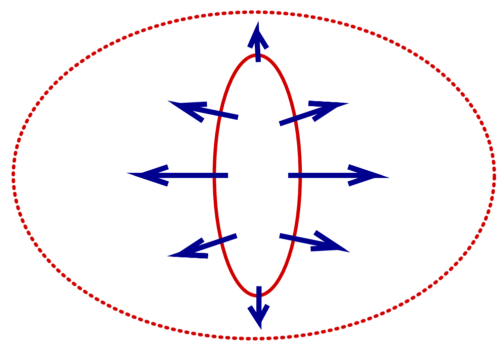
II. Scale invariant fluid dynamics: Observations



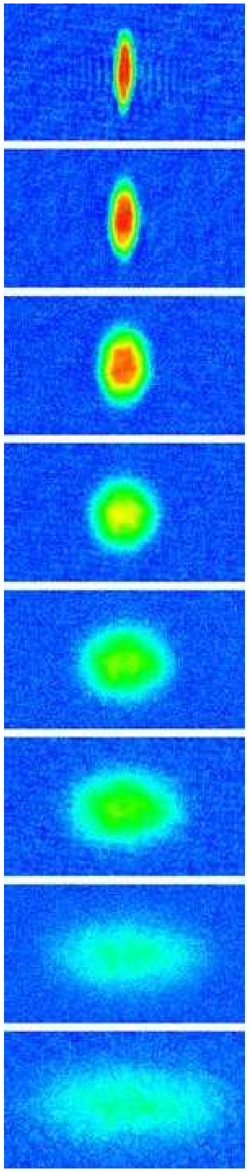
O'Hara et al. (2002)



Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

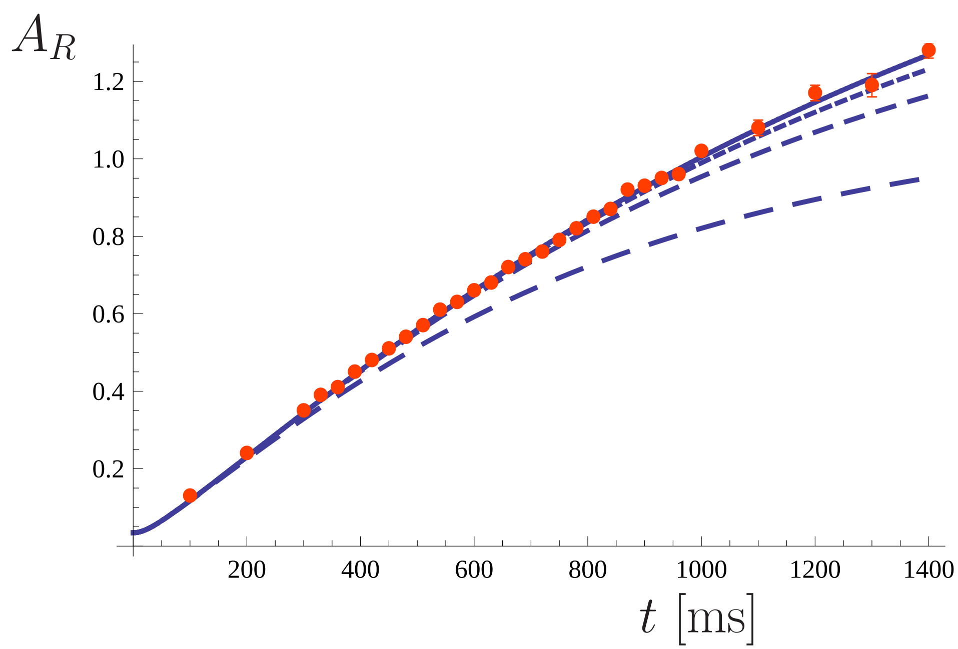


Elliptic flow: Breakdown of scale invariant hydrodynamics?



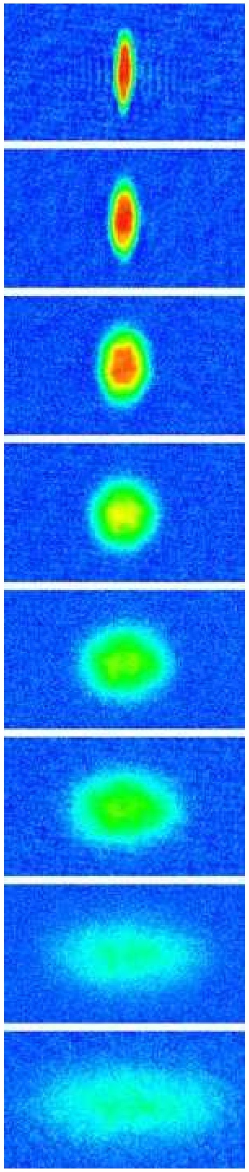
switch from conformal hydro to scale breaking kinetics

at scale factor $b_\perp^{fr} = 1, 5, 10, 20$

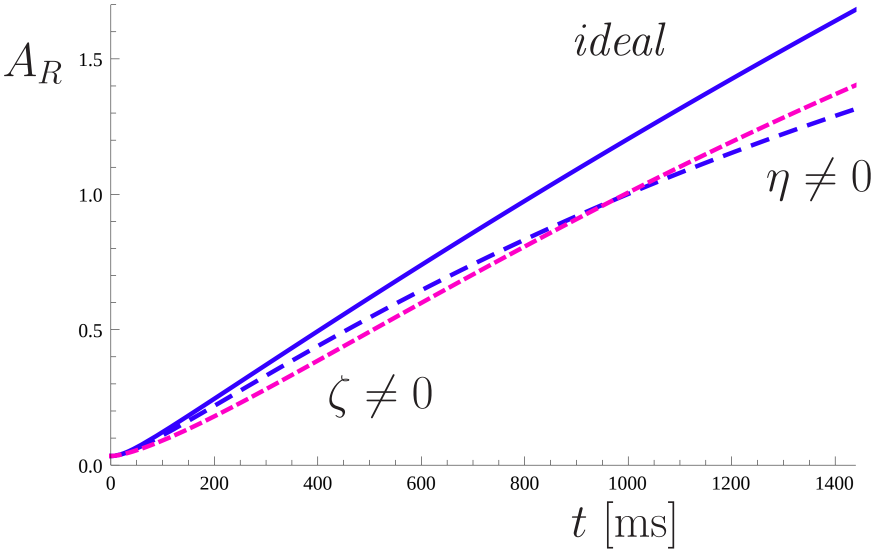


no breakdown seen in the data

Elliptic flow: Shear vs bulk viscosity



Dissipative hydro with both η, ζ



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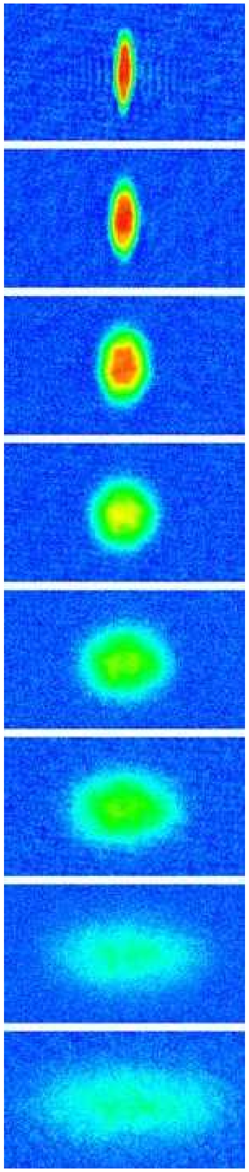
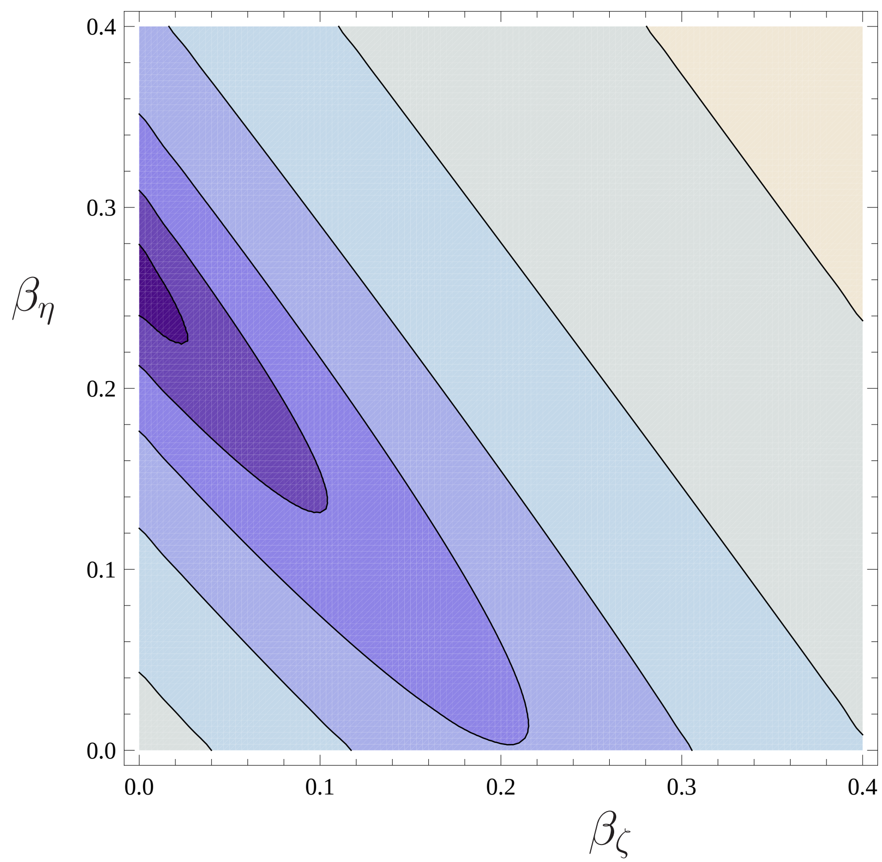


Dissipative hydro with both η, ζ

$$\beta_{\eta,\zeta} = \frac{[\eta,\zeta]}{n}\,\frac{E_F}{E}\,\frac{1}{(3\lambda N)^{1/3}}$$



$\eta \gg \zeta$

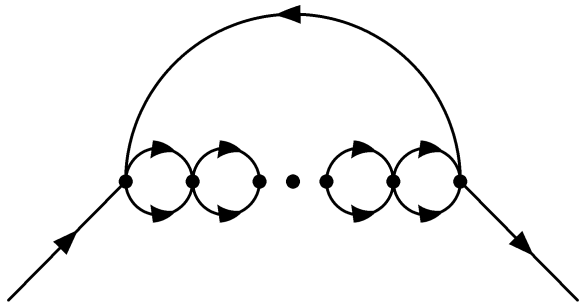
Dusling, Schaefer (2010)

III. Bulk viscosity and conformal symmetry breaking

Conformal symmetry breaking (thermodynamics)

$$1 - \frac{2\mathcal{E}}{3P} = \frac{C}{12\pi maP} \sim \frac{1}{6\pi}\, n\lambda^3\, \frac{\lambda}{a}$$

Quasi-particles:



$$Im\,\Sigma(k) \sim zT\sqrt{\frac{T}{\epsilon_k}} Erf\left(\sqrt{\frac{\epsilon_k}{T}}\right) \ll T$$

$$Re\,\Sigma(k) \sim zT\frac{\lambda}{a}\sqrt{\frac{T}{\epsilon_k}} F_D\left(\sqrt{\frac{\epsilon_k}{T}}\right)$$

Bulk viscosity

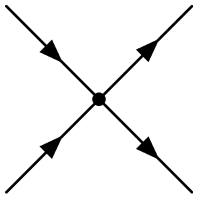
$$\zeta = \frac{1}{24\sqrt{2}\pi}\lambda^{-3}\left(\frac{z\lambda}{a}\right)^2$$

$$\zeta \sim \left(1 - \frac{2\mathcal{E}}{3P}\right)^2 \eta$$

IV. Viscosity in two dimensions

Kinetic theory with zero range interaction

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p = C[f_p] \qquad\qquad C[f] =$$



Shear viscosity in 2d and 3d

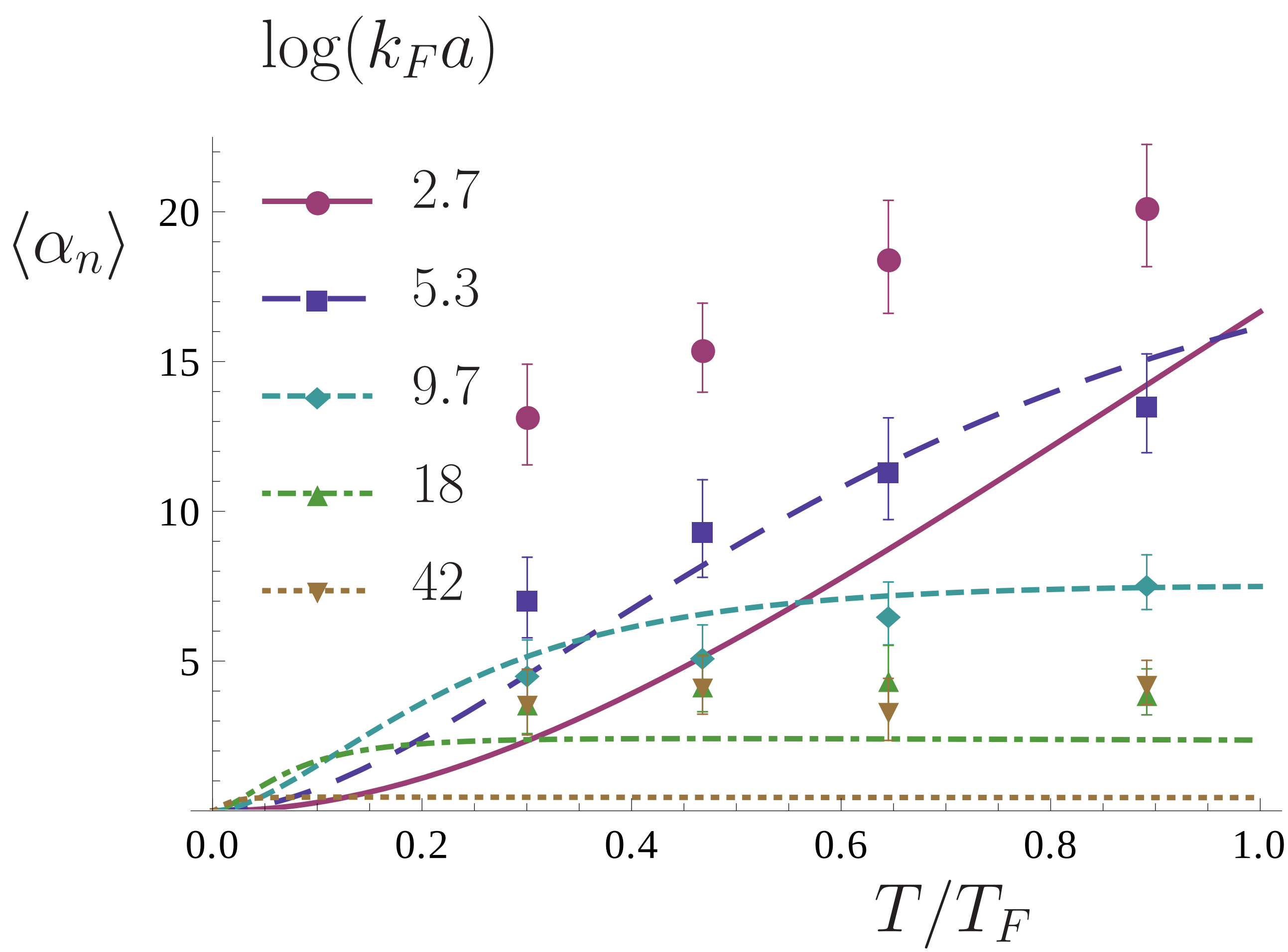
$$\eta_{2d} = \frac{\lambda^{-2}}{\pi}\left(\left[\log\left(5\pi\frac{a^2}{\lambda^2}\right)\right]^2 + \pi^2\right)$$

$$\eta_{3d} = \frac{15\sqrt{2}\pi\lambda^{-3}}{16}\left(1 + \frac{1}{4\pi}\frac{\lambda^2}{a^2} + \ldots\right)$$

Viscous damping

$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\,(\sigma_{ij})^2 = -\frac{N}{2}\langle\alpha_n\rangle\,(\sigma_{ij})^2$$

Comparison to collective mode data

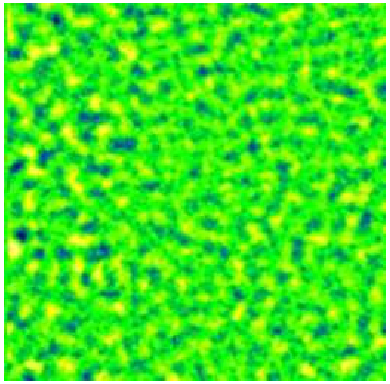


See arXiv:1111.7242, data from Vogt et al. arXiv:1111.1173.

Also: Bruun arXiv:1112.2395, Baur et al. arXiv:1301.0358.

V. Thermal fluctuations

Hydrodynamic variables fluctuate



$$\langle \delta v_i(x,t)\delta v_j(x',t)\rangle = \frac{T}{\rho}\delta_{ij}\delta(x-x')$$

Linearized hydrodynamics propagates fluctuations as shear or sound

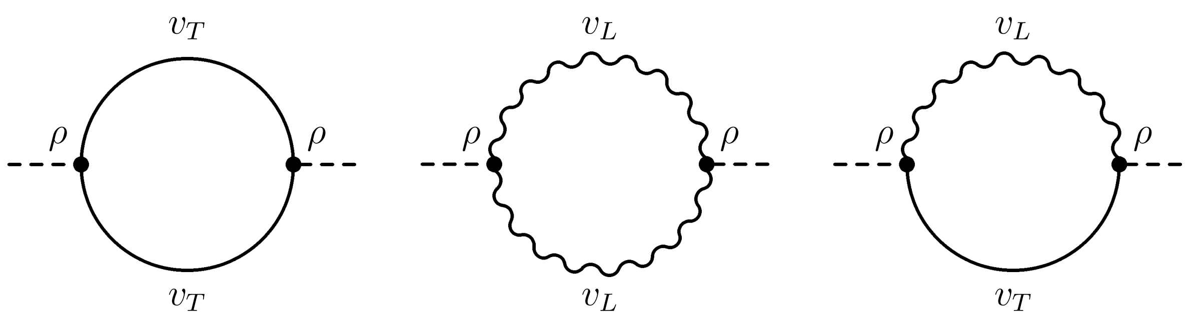
$$\langle \delta v_i^T \delta v_j^T \rangle_{\omega,k} = \frac{2T}{\rho}(\delta_{ij} - \hat{k}_i\hat{k}_j)\frac{\nu k^2}{\omega^2 + (\nu k^2)^2} \qquad \textit{shear}$$

$$\langle \delta v_i^L \delta v_j^L \rangle_{\omega,k} = \frac{2T}{\rho}\hat{k}_i\hat{k}_j\frac{\omega k^2\Gamma}{(\omega^2 - c_s^2k^2)^2 + (\omega k^2\Gamma)^2} \qquad \textit{sound}$$

$v = v_T + v_L$: $\nabla \cdot v_T = 0$, $\nabla \times v_L = 0$ $\qquad$ $\nu = \eta/\rho$, $\Gamma = \frac{4}{3}\nu + \ldots$

Hydro Loops: “Breakdown” of second order hydro

Response function $G_R^{xyxy} = \langle \theta(t)[\Pi^{xy}, \Pi^{xy}] \rangle_{\omega,k}$ $\Pi_{xy} = \rho v_x v_y$



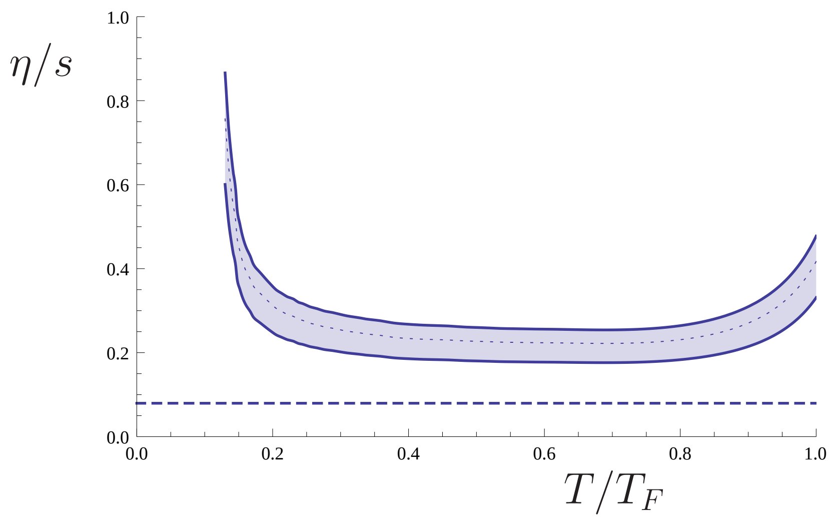
$$G_R^{xyxy} = P + \delta P + i\omega[\eta + \delta\eta] + \omega^2 \left[\eta\tau_\pi + \delta(\eta\tau_\pi)\right]$$

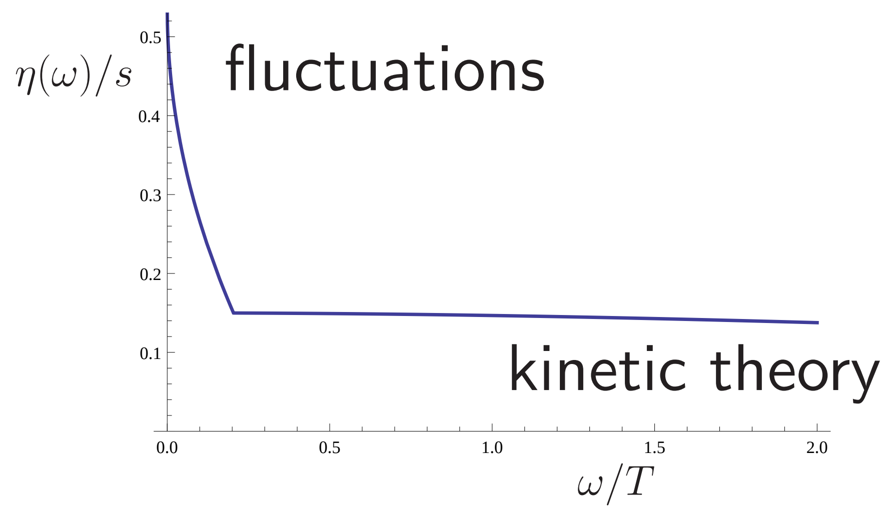
3d: Enhanced shear viscosity, divergent relaxation time

$$\delta\eta \sim T \left(\frac{\rho}{\eta}\right)^2 \left(\frac{P}{\rho}\right)^{1/2} \qquad \delta(\eta\tau_\pi) \sim \frac{1}{\sqrt{\omega}} \left(\frac{\rho}{\eta}\right)^{3/2}$$

Fluctuations large if bare viscosity is small

Fluctuation induced bound on η/s





$(\eta/s)_{min} \simeq 0.2$

spectral function

non-analytic $\sqrt{\omega}$ term

See arXiv:1209.1006, also Kovtun, Moore, Romatschke (2011).

Fluctuations in 2d

2d: Logarithmic divergence in shear viscosity

$$\delta\eta \sim \frac{1}{16\pi}\frac{mnT}{\eta}\log\left(\frac{2nT}{\eta\omega}\right)$$

Collective modes: Logarithmic dependence on number of particles

$$\delta\left(\frac{\eta}{n}\right) = \frac{log(N)}{16\pi} \qquad \text{hard to observe}$$

Power divergence in relaxation time

$$\delta(\eta\tau_\pi) \sim \frac{1}{\omega}\frac{mnT}{\eta^2}$$

Outlook

Can we observe bulk viscosity away from unitarity in 3d, or near the crossover in 2d? What about bulk viscosity in the superfluid phase?

Need local measurements of η/s in 2d and 3d. Requires second order hydrodynamics or hydro+kinetics calculations.

Measurements of the viscous relaxation time (based on collective modes and elliptic flow?).

QMC measurements of the viscosity spectral function. Can we see fluctuation effects? Quasi-particle behavior?

Scale breaking in 2d monopole frequency

Scale invariance implies undamped monopole mode $\omega = 2\omega_0$.

Frequency shift due to scale breaking

Randeria, Taylor (2012)

$$\frac{\omega^2}{4\omega_0^2} = 1 - \frac{d^2}{8}\frac{\int d^d r\,\gamma_d}{\int d^d r\, nV(r)} \qquad \gamma_d = \frac{2+d}{d}P - \rho\frac{\partial P}{\partial\rho}\bigg|_s$$

High temperature limit: Virial expansion

$$\gamma_d = \frac{mz^2T^2}{\pi}\left(2Tb_2' + T^2 b_2''\right)$$

$$b_2(T) = \nu\left(\frac{E_B}{T}\right) \qquad E_B = \frac{1}{ma^2}$$