Phase Structure and Transport Properties of (Very) Dense QCD Matter

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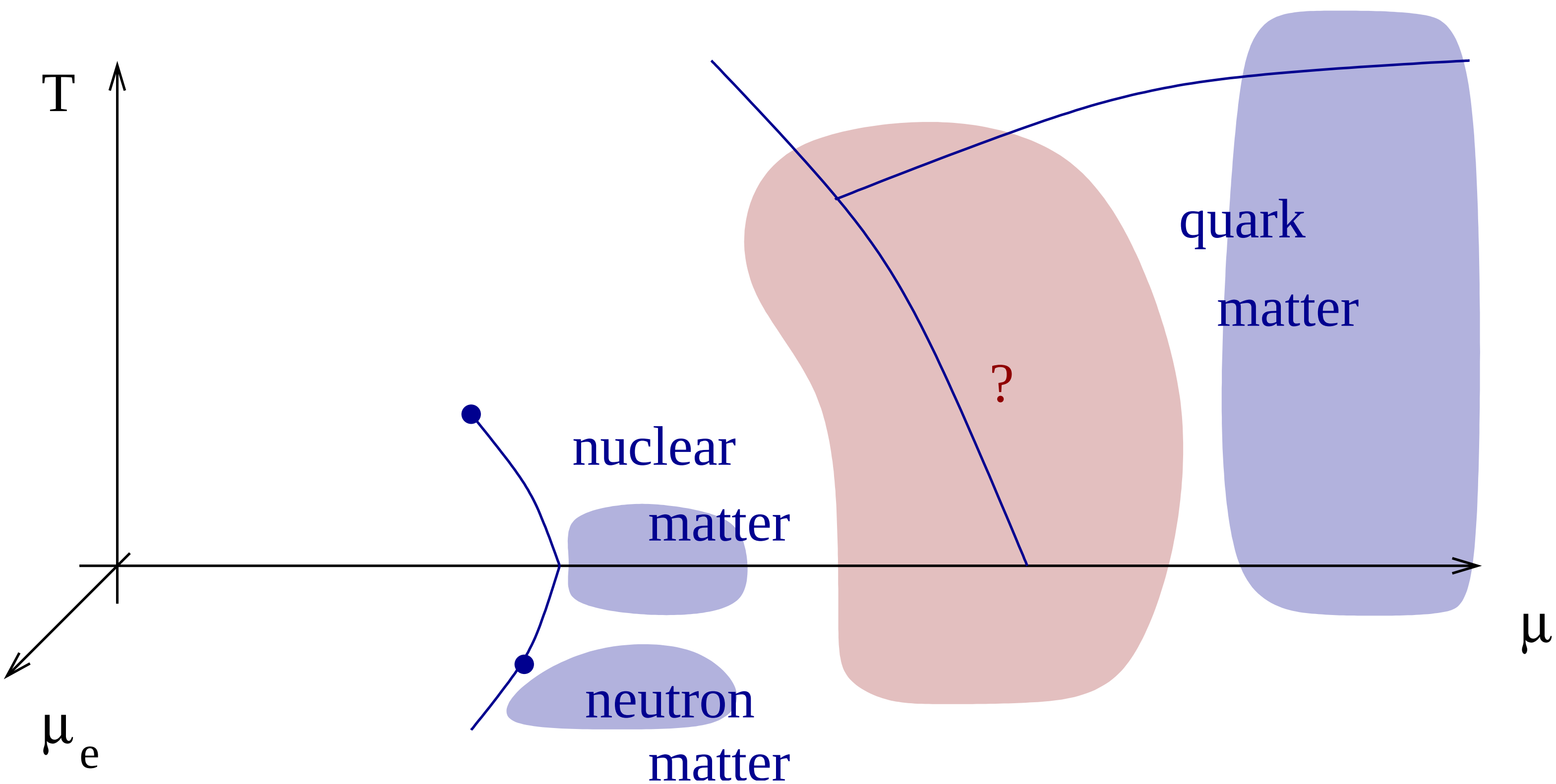
Schematic Phase Diagram



High Density Quark Matter

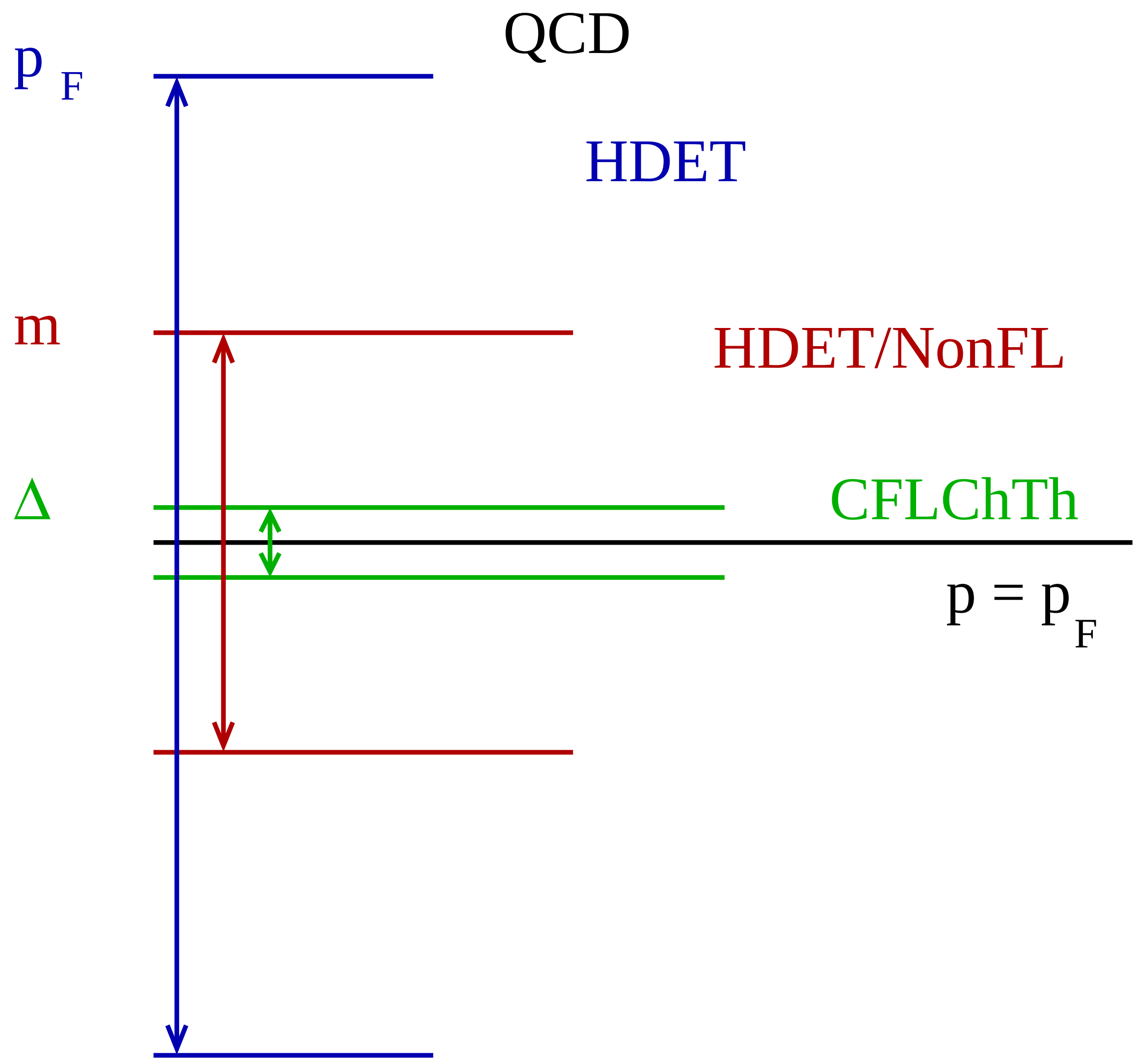
Goal: What is the densest (2nd densest, 3rd densest, ...) phase of (three flavor) quark matter?

What are the properties (thermodynamics, transport, ...) of these phases?

Are these properties consistent with observational constraints? Do they provide unique signatures?

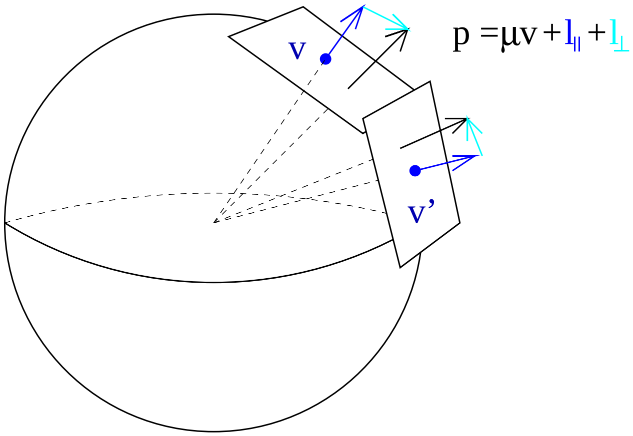
Strategy: Weak coupling/effective field theory methods.

Very Dense Matter: Effective Field Theories



High Density Effective Theory

Effective field theory on v-patches



$$\psi_{v\pm} = e^{-i\mu v\cdot x}\left(\frac{1\pm\vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

Effective lagrangian for $p_0 < m$

$$\mathcal{L} = \psi_v^\dagger\left(iv\cdot D - \frac{D_\perp^2}{2\mu}\right)\psi_v + \mathcal{L}_{4f} - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

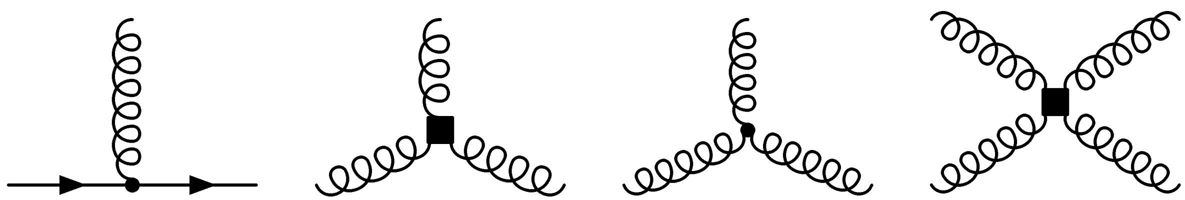
$$\mathcal{L}_{HDL} = -\frac{m^2}{2}\sum_v G^a_{\mu\alpha}\frac{v^\alpha v^\beta}{(v\cdot D)^2}G^b_{\mu\beta}$$

Non-Fermi Liquid Expansion

Scale momenta $(k_0, k_{||}, k_\perp) \to (sk_0, s^{2/3}k_{||}, s^{1/3}k_\perp)$

$$[\psi] = 5/6 \qquad [A_i] = 5/6 \qquad [S_0] = [S_{HDL}] = 0$$

Scaling behavior of vertices

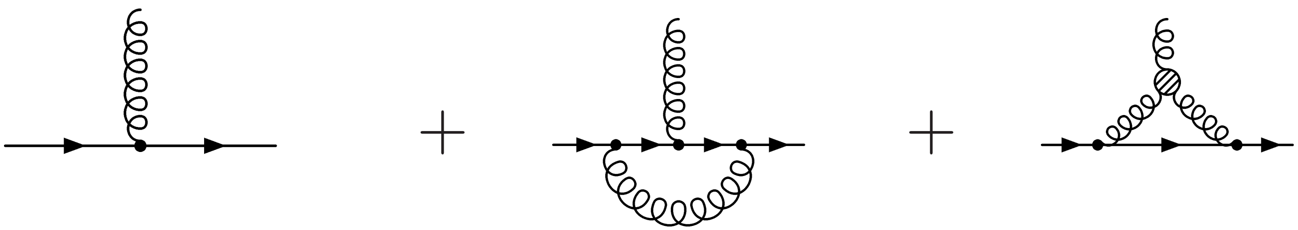


$s^{1/6}$ $\quad$ $s^{1/2}$ $\quad$ $s^{5/6}$ $\quad$ s

Systematic expansion in $\epsilon^{1/3} \equiv (\omega/m)^{1/3}$

Vertex Corrections, Migdal's Theorem

Corrections to quark gluon vertex

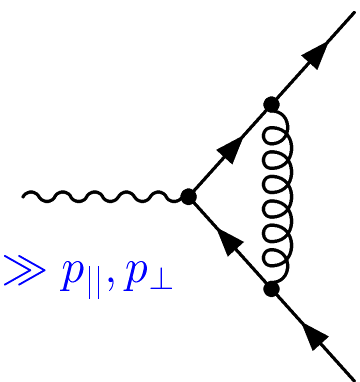


$$\sim \quad gv(1 + O(\epsilon^{1/3}))$$

Analogous to electron-phonon coupling

Can this fail? Yes, if external momenta fail to satisfy $p_\perp \gg p_0$

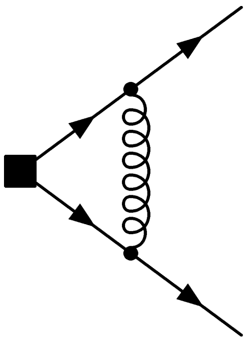
$p_0 \gg p_{||}, p_\perp$



$$= \quad \frac{eg^2}{9\pi^2} v_\mu \log(\epsilon)$$

Superconductivity

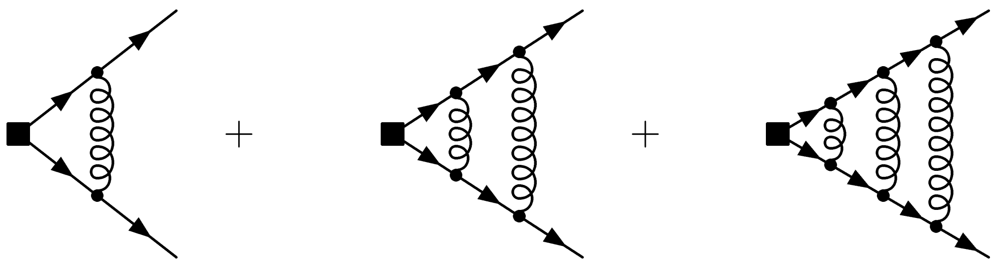
Same phenomenon occurs in anomalous self energy



$$= \frac{g^2}{18\pi^2} \int dq_0 \log\left(\frac{\Lambda_{BCS}}{|p_0 - q_0|}\right) \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta(q_0)^2}}$$

$\Lambda_{BCS} = 256\pi^4 g^{-5}\mu$ determined by electric exchanges

Have to sum all planar diagrams, non-planar suppressed by $\epsilon^{1/3}$



Solution at next-to-leading order (includes normal self energy)

$$\Delta_0 = 2\Lambda_{BCS} \exp\left(-\frac{\pi^2+4}{8}\right) \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) \qquad \Delta_0 \sim 50\,\text{MeV}$$

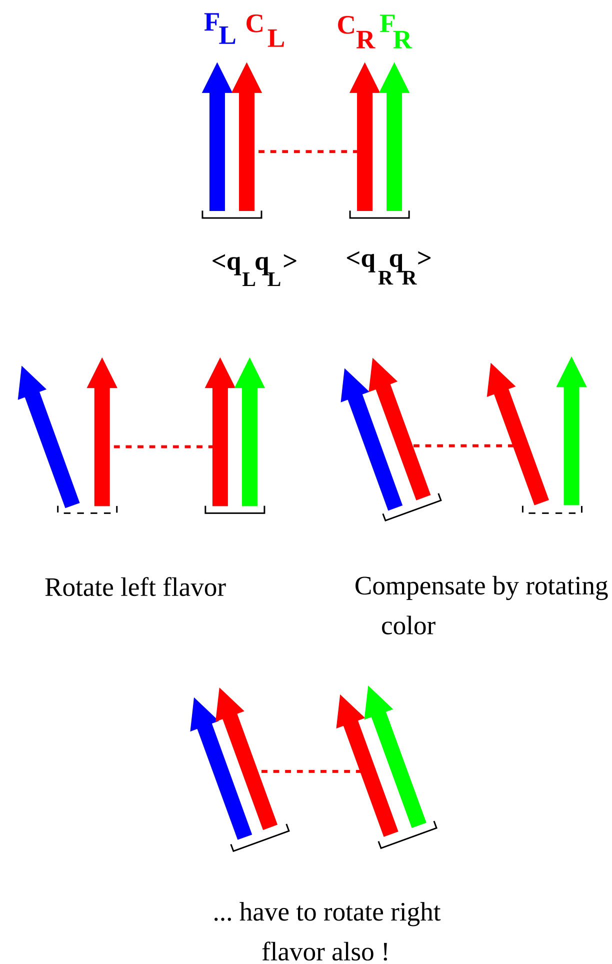
$N_f = 3$: CFL Phase

Consider $N_f = 3$ $(m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi\, \epsilon^{abI} \epsilon_{ijI}$$

$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$



Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \to SU(3)_{C+F}$$

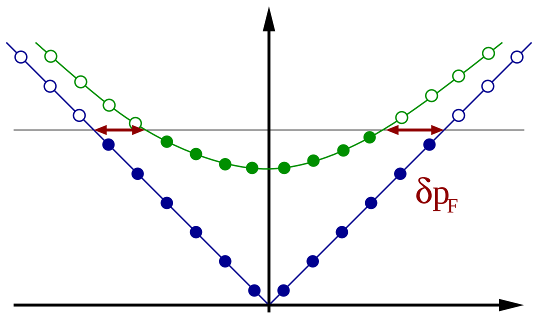
$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

All quarks and gluons acquire a gap

$[8] + [1]$ fermions, Q integer

Towards the real world: Non-zero strange quark mass

Have $m_s > m_u, m_d$: Unequal Fermi surfaces



$$\delta p_F \simeq \frac{m_s^2}{2p_F}$$

Also: If $p_F^s < p_F^{u,d}$ have unequal densities

Charge neutrality not automatic

Strategy

Consider $N_f = 3$ at $\mu \gg \Lambda_{QCD}$ (CFL phase)

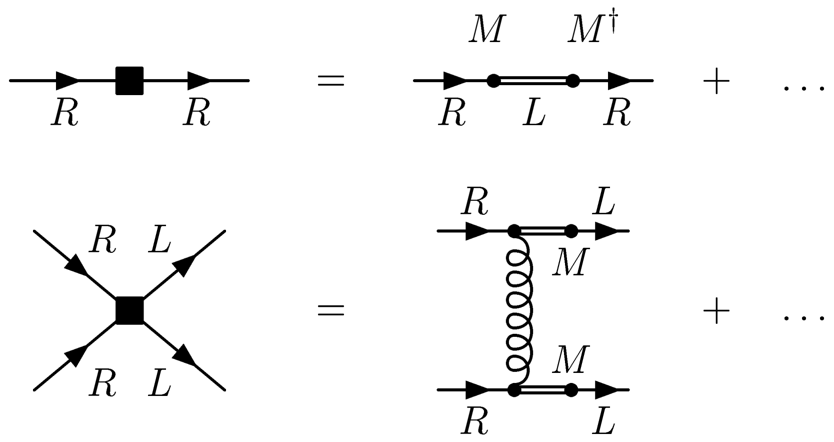
Study response to $m_s \neq 0$

Constrained by chiral symmetry

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu}\psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu}\psi_L$$

$$+ \ \frac{V_M^0}{\mu^2}(\psi_R^\dagger M\lambda^a\psi_L)(\psi_R^\dagger M\lambda^a\psi_L)$$



mass corrections to FL parameters $\hat{\mu}_{L,R}$ and $V^0(RR \to LL)$

EFT in the CFL Phase

Consider HDET with a CFL gap term

$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger (iv\cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger\psi_L X^\dagger\psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger\psi_L\right)\right]^2\right\} + (L\leftrightarrow R, X\leftrightarrow Y)$$

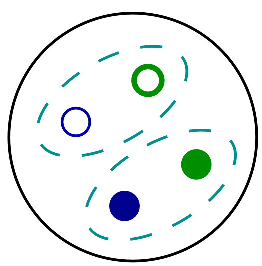
$$\psi_L \to L\psi_L C^T,\; X \to L X C^T, \qquad \langle X\rangle = \langle Y\rangle = 1\!\!1$$

Quark loops generate a kinetic term for X,Y

Integrate out gluons, identify low energy fields ($\xi = \Sigma^{1/2}$)

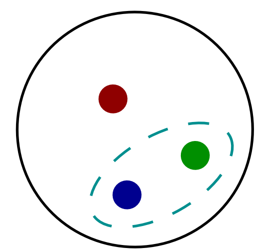
$$\Sigma = XY^\dagger$$

[8]+[1] GBs



$$N_L = \xi(\psi_L X^\dagger)\xi^\dagger$$

[8]+[1] Baryons



Effective theory: (CFL) baryon chiral perturbation theory

$$
\begin{aligned}
\mathcal{L} \;=\;& \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\nabla_0\Sigma\nabla_0\Sigma^\dagger\right) - v_\pi^2\,\mathrm{Tr}\left(\nabla_i\Sigma\nabla_i\Sigma^\dagger\right)\right\} \\
& + A\left\{\left[\mathrm{Tr}\left(M\Sigma\right)\right]^2 - \mathrm{Tr}\left(M\Sigma M\Sigma\right) + h.c.\right\} \\
& + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\,\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5\left\{\mathcal{A}_\mu, N\right\}\right) \\
& - F\,\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5\left[\mathcal{A}_\mu, N\right]\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\}
\end{aligned}
$$

$$\nabla_0\Sigma = \partial_0\Sigma + i\hat{\mu}_L\Sigma - i\Sigma\hat{\mu}_R$$

$$D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$$

$$f_\pi^2 = \frac{21 - 8\log 2}{18}\,\frac{\mu^2}{2\pi^2} \qquad v_\pi^2 = \frac{1}{3} \qquad A = \frac{3\Delta^2}{4\pi^2} \qquad D = F = \frac{1}{2}$$

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2} \mathrm{Tr}\left(\hat{\mu}_L \Sigma \hat{\mu}_R \Sigma^\dagger\right) - A\mathrm{Tr}(M\Sigma^\dagger) - B_1 \left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

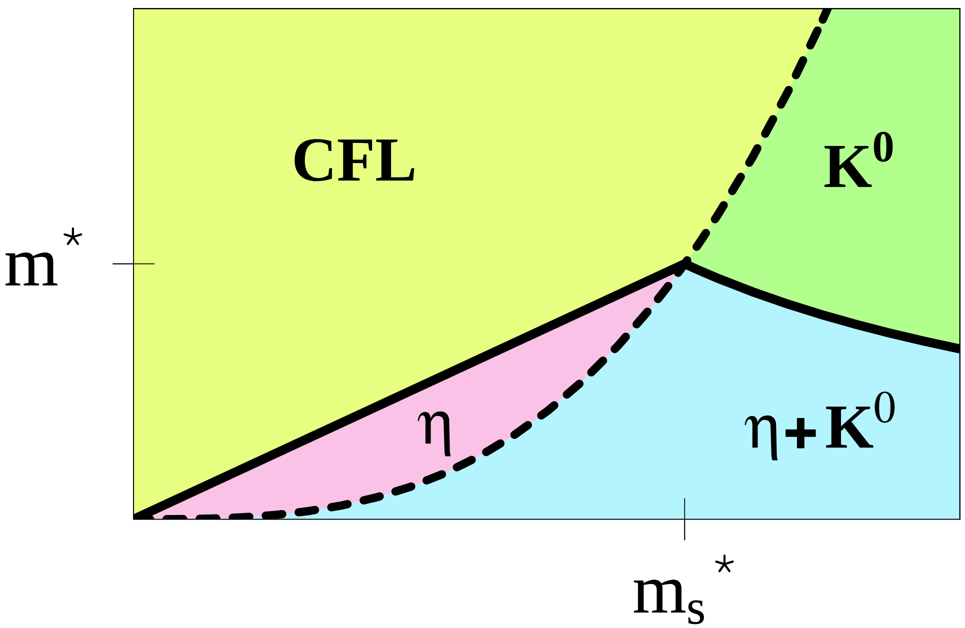
$$V(\Sigma_0) \equiv min$$

Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger \gamma_5 \rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$

$$\rho_{V,A} = \frac{1}{2}\left\{\xi \hat{\mu}_L \xi^\dagger \pm \xi^\dagger \hat{\mu}_R \xi\right\} \qquad\qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure of CFL Phase



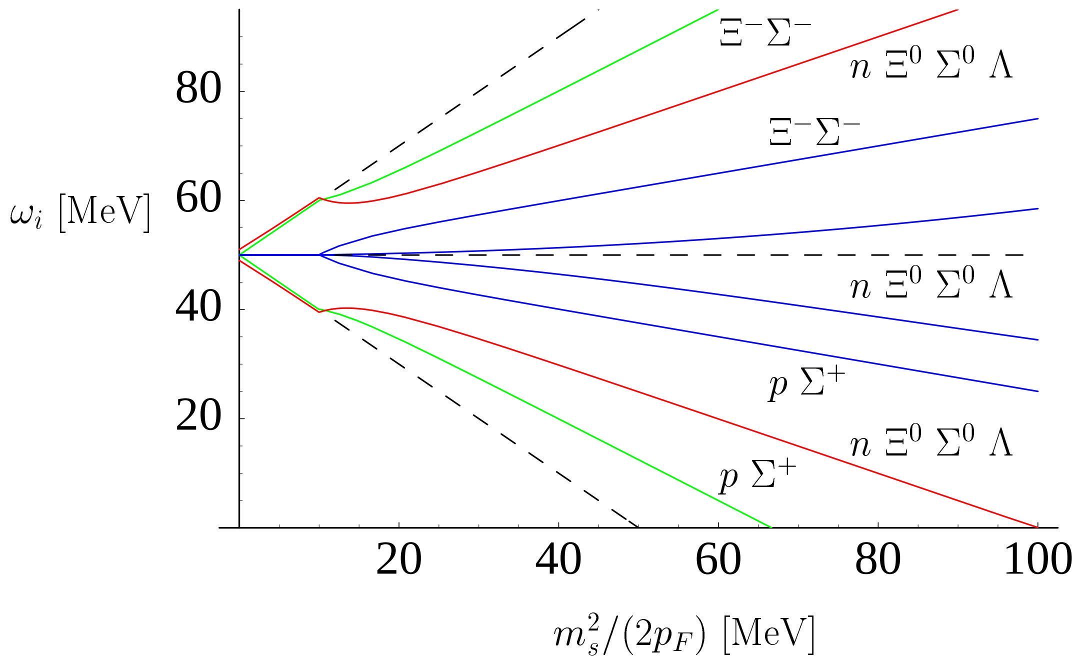
$$m_s^{crit} \sim 3.03\, m_d^{1/3} \Delta^{2/3}$$

$$m^* \sim 0.017\, \alpha_s^{4/3} \Delta$$

QCD realization of s-wave meson condensation

Driven by strangeness oversaturation of CFL state

Fermion Spectrum



$$m_s^{crit} \sim (8\mu\Delta/3)^{1/2}$$

gapless fermion modes (gCFLK)

(chromomagnetic) instabilities ?

Instabilities

Consider meson current

$$\Sigma(x) = U_Y(x)\Sigma_K U_Y(x)^\dagger \qquad U_Y(x) = \exp(i\phi_K(x)\lambda_8)$$

$$\vec{\mathcal{V}}(x) = \frac{\vec{\nabla}\phi_K}{4}(-2\hat{I}_3 + 3\hat{Y}) \qquad \vec{\mathcal{A}}(x) = \vec{\nabla}\phi_K(e^{i\phi_K}\hat{u}^+ + e^{-i\phi_K}\hat{u}^-)$$

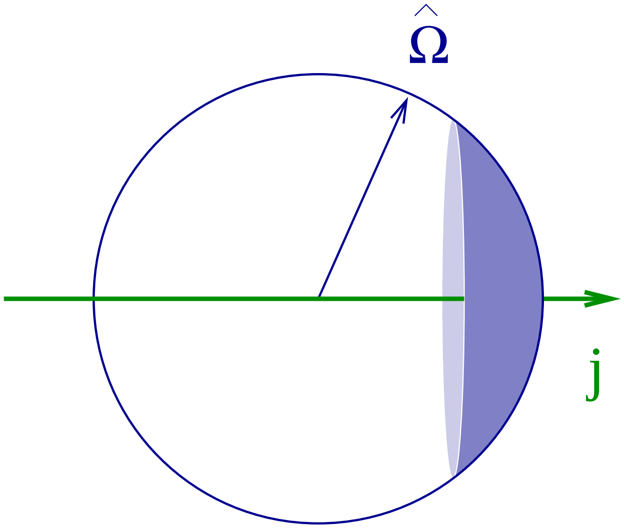
Gradient energy

$$\mathcal{E} = \frac{f_\pi^2}{2}v_\pi^2 \jmath_K^2 \qquad \vec{\jmath}_k = \vec{\nabla}\phi_K$$

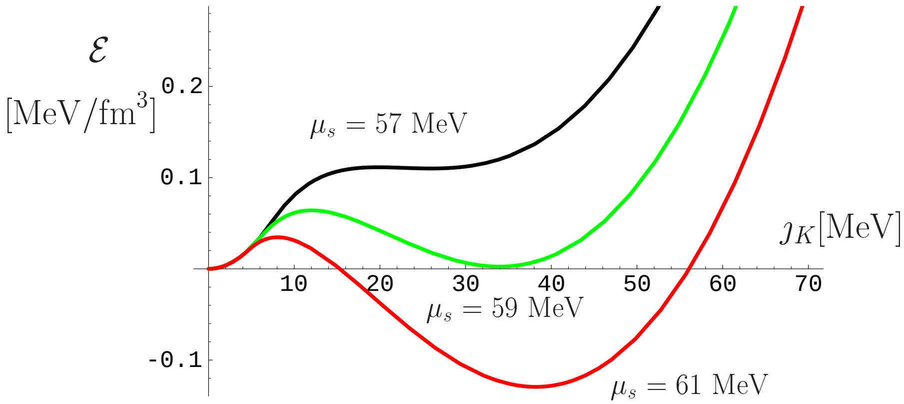
Fermion spectrum

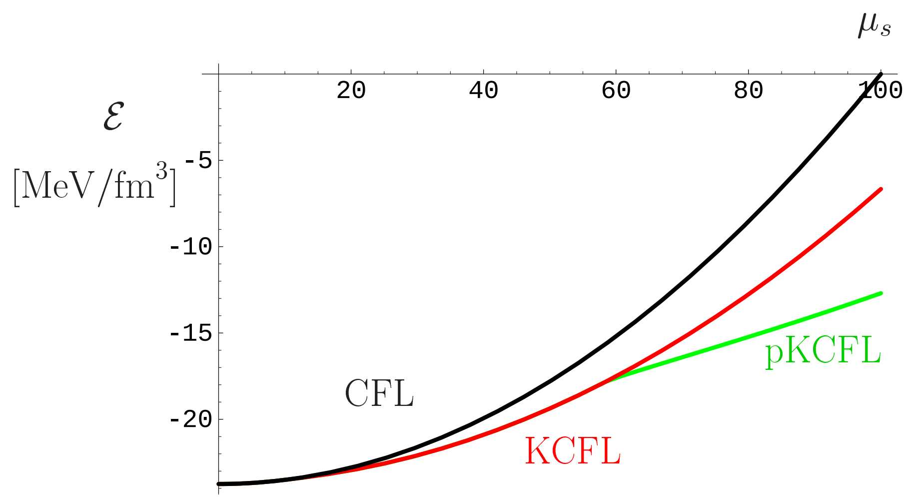
$$\omega_l = \Delta + \frac{l^2}{2\Delta} - \frac{4\mu_s}{3} - \frac{1}{4}\vec{v}\cdot\vec{\jmath}_K$$

$$\mathcal{E} = \frac{\mu^2}{2\pi^2}\int dl \int d\hat{\Omega}\ \omega_l \Theta(-\omega_l)$$



Energy Functional





$$\left.\frac{3\mu_s - 4\Delta}{\Delta}\right|_{crit} = a_{crit} \qquad \frac{\jmath_K}{\Delta} = c_{crit}$$

current strongly suppressed by electric charge neutrality

$m_s^2 \sim 2\mu\Delta$: multiple currents? crystalline state?

Transport Properties

Dissipative Terms

$$\dot{E} = -\frac{\eta}{2}\int d^3x \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2$$

$$-\zeta \int d^3x \left(\partial_i v_i\right)^2 - \frac{\kappa}{T}\int d^3x \left(\partial_i T\right)^2$$

Relevant to r-mode damping

Neutrino emissivity

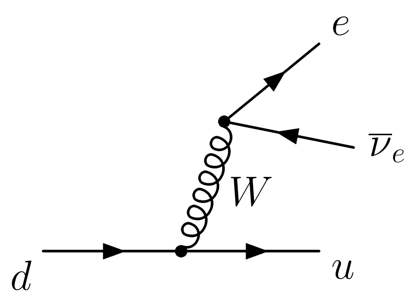
$$\epsilon_\nu = \frac{dE_\nu}{dt d^3x}$$

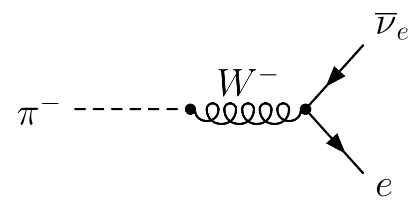
Relevant to cooling (together with κ, c_v)

Kinetics: Quasi-particles

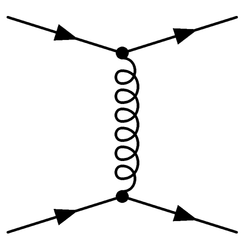
Quasi-particles control transport of flavor, energy, and momentum.

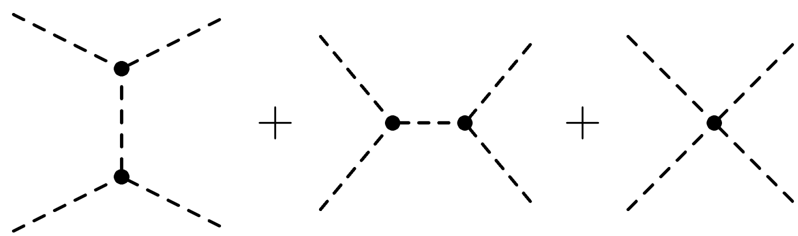
Neutrino emissivity, neutrino mean free path:



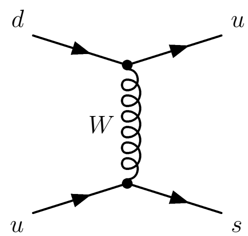


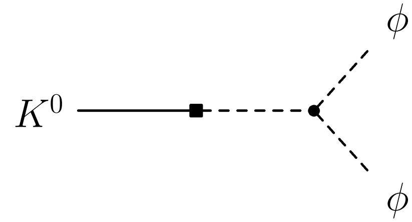
Shear viscosity, thermal conductivity



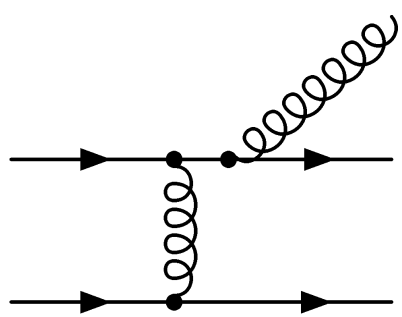


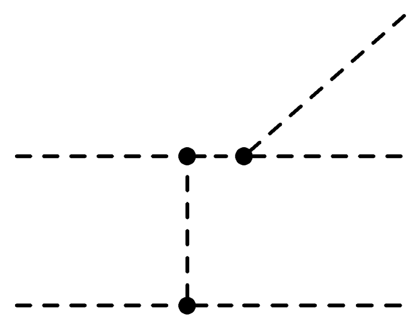
Bulk viscosity: Weak interaction





Bulk viscosity: Strong interaction





Umpaired Quark Matter

$$\kappa \simeq 0.5 \frac{m_D^2}{\alpha_s^2}$$ Non-FLT

$$\eta \simeq 4.4 \times 10^{-3} \, \frac{\mu^4 m_D^{2/3}}{\alpha_s^2 T^{5/3}}$$ Non-FLT

$$\zeta \simeq \frac{\alpha T^2}{\omega^2 + \beta T^4} \quad (\alpha, \beta^{1/2} \sim G_F^2)$$ QURCA

$$\epsilon_\nu \simeq \frac{457}{630} \alpha_s G_F^2 T^6 \mu_e \mu_u \mu_d$$ FLT, Non-FLT

CFL Quark Matter

$$\eta = 1.3 \times 10^{-4}\,\frac{\mu^8}{T^5}$$	phonons
$$\zeta = 0.011\,\frac{M_s^4}{T}$$	phonons
$$\zeta = \frac{C\gamma_K}{\omega^2+\gamma_K^2} \quad (\gamma_K \sim G_F^2 f_K^2)$$	weak kaon decay
$$\epsilon_\pi \sim A G_F^2 f_\pi^2 m_\pi^2 n_\pi$$	pion (kaon) decay
$$\kappa \simeq \,?$$	phonons, kaons

Summary

Systematic weak coupling expansion for $\Delta/m_D, T/m_D \ll 1$.
(Non-Fermi Liquid Regime)

$\Delta \gg \delta\mu$: Color-flavor-locked (CFL) phase

Regime $\delta\mu < \Delta$ controlled by chiral symmetry, but the regime $\delta\mu \sim \Delta$ is complicated

s and p-wave meson condensation

Issues not covered in this talk: Transition to nuclear matter, nuclear exotics, etc.

Constraints from compact star phenomenology