Bulk viscosity, spectra, and flow in heavy ion collisions

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RHIC serves the perfect fluid

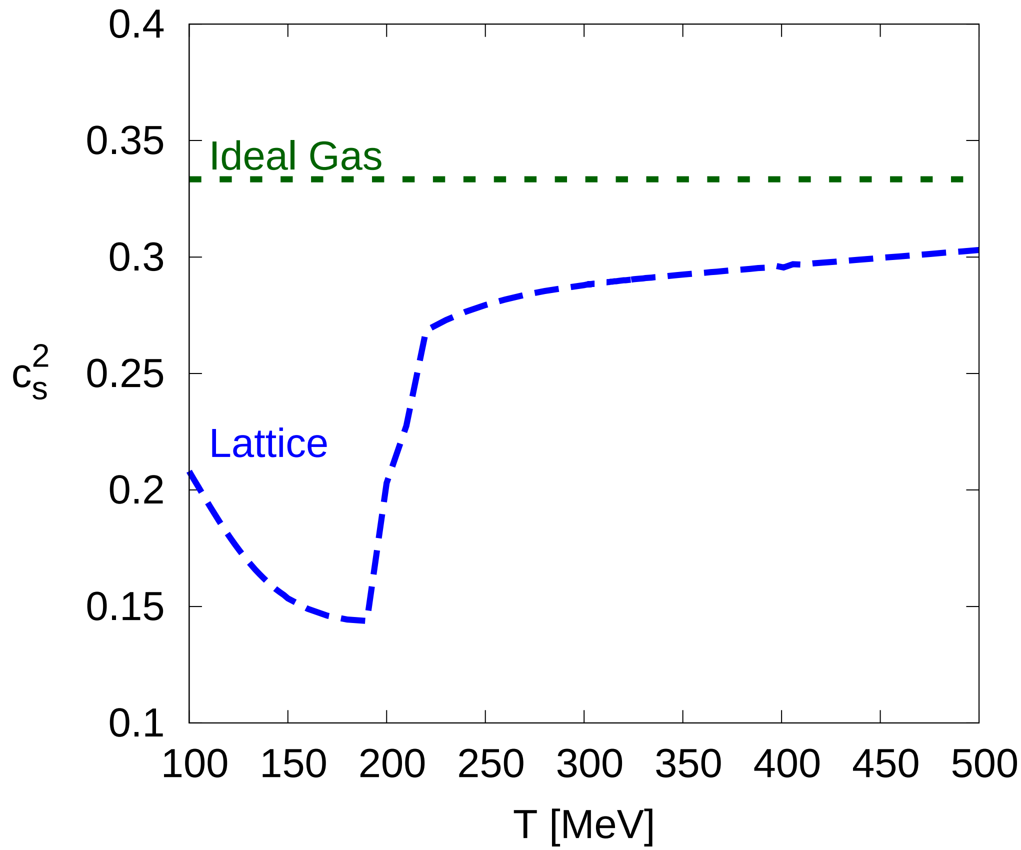
Experiments at RHIC/LHC are consistent with the idea that a thermalized plasma is produced, and that the equation of state is that of a weakly coupled gas of quarks and gluons.

But: Transport properties of the system (primarily viscosity and energy loss) are in dramatic disagreement with expectations for a weakly coupled QGP.

The plasma must be very strongly coupled.

$$\frac{\eta}{s} \sim \frac{\hbar}{4\pi} \qquad\qquad \frac{\hat{q}}{T^3} \gg \frac{1}{\hbar}$$

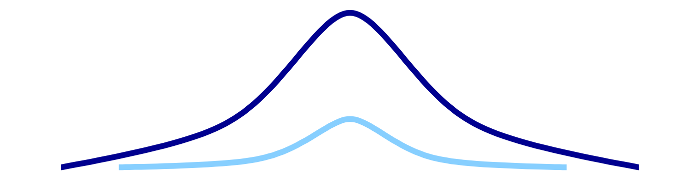
Why bulk viscosity?



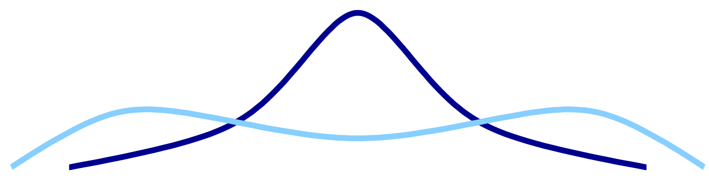
Real QCD is not scale invariant, and $\zeta \neq 0$. Usually, this is treated as a nuisancance – it leads to uncertainties in the extraction of η. Here, I want to estimate ζ from data and see what (if anything) we can learn.

Fluids: Gases, liquids, plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved charges (or spontaneously broken symmetry fields).



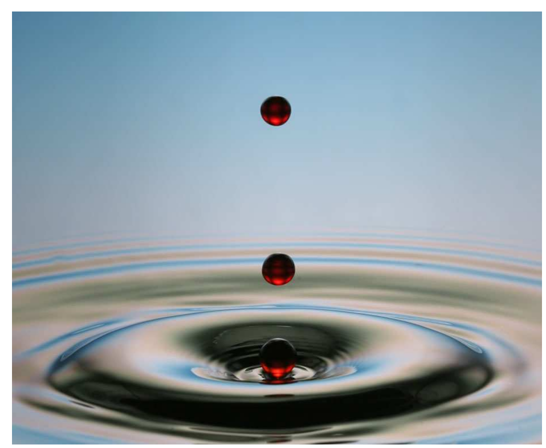
$\tau \sim \tau_{micro}$



$\tau \sim \lambda$

Historically: Water

$$(\rho, \epsilon, \vec{\pi})$$



Relativistic fluid

Conservation laws for baryon number and energy-momentum

$$\partial^\mu j^B_\mu = 0$$

$$\partial^\mu \Pi_{\mu\nu} = 0$$

Constitutive relations: Stress tensor

$$\Pi_{\mu\nu} = (\epsilon + P) u_\mu u_\nu + P\eta_{\mu\nu} - \eta\sigma_{\mu\nu} - \zeta\eta_{\mu\nu}(\partial\cdot u) + O(\partial^2)$$

reactive dissipative 2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

$$\sigma^{\mu\nu} = \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3}\eta_{\alpha\beta}\partial\cdot u\right), \qquad \Delta_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial u)}{(\epsilon + P)u^2} = \frac{\eta}{sT\tau} \ll 1$

$$\frac{1}{Re} = \frac{\eta}{\hbar s} \times \frac{\hbar}{\tau T}$$

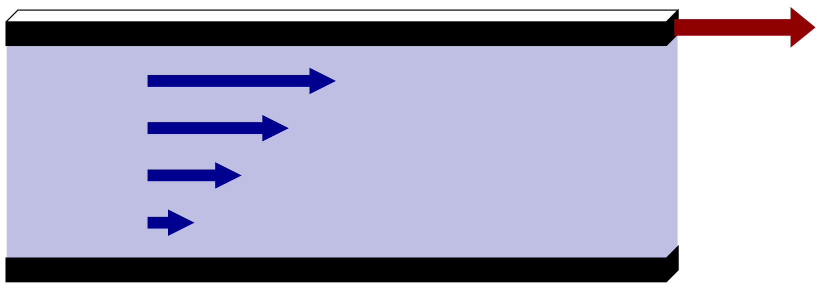
	fluid property	flow property

Bath tub : $mvL \gg \hbar$ hydro reliable

Heavy ions : $\tau T \sim \hbar$ need $\eta < \hbar s$

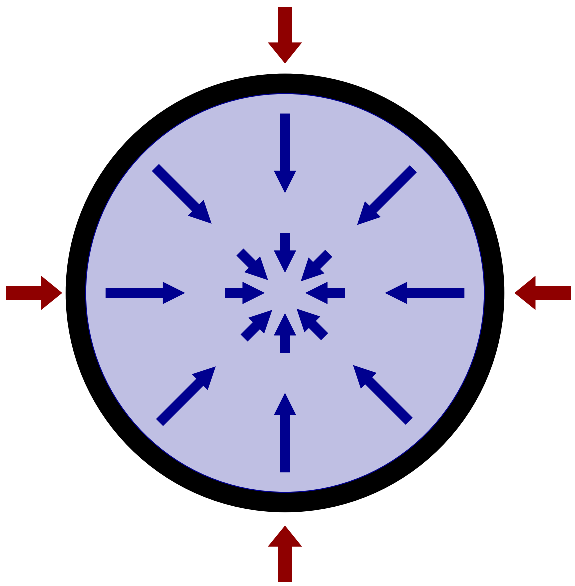
Viscosity and dissipative forces

Shear viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Bulk viscosity controls non-equlibrium pressure



$$P = P_0 - \zeta(\partial \cdot v)$$

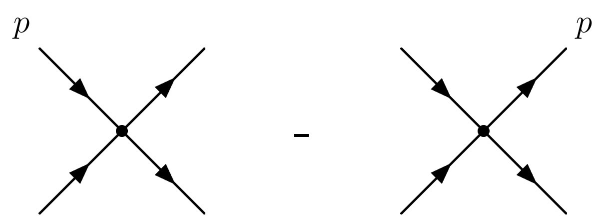
Kinetic theory

Kinetic theory: conserved quantities carried by quasi-particles.

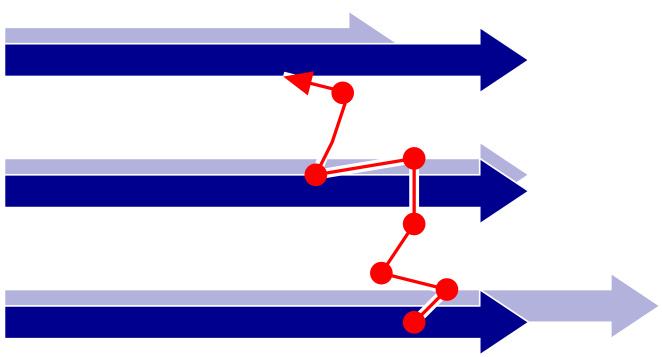
Quasi-particles described by distribution functions $f(x, p, t)$.

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = -C[f_p]$$

$$C[f_p] \; =$$



Shear viscosity corresponds to momentum diffusion



$$\eta \sim \frac{1}{3} n \, \bar{p} \, l_{mfp}$$

Bulk viscosity and scale invariance

Consider scale invariant theory $j^s_\mu = x^\nu \Pi_{\mu\nu}$

$$\partial^\mu j^s_\mu = 0 \quad \Rightarrow \quad \Pi^\mu_\mu = 0 \quad \Rightarrow \quad \zeta = 0$$

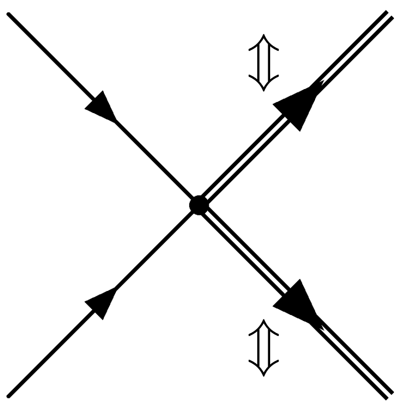
(Indirect) consequence: No simple kinetic theory estimate for bulk viscosity due to elastic $2 \leftrightarrow 2$ scattering in relativistic $(E_p \sim p)$ or non-relativistic limit $(E_p \sim p^2)$.

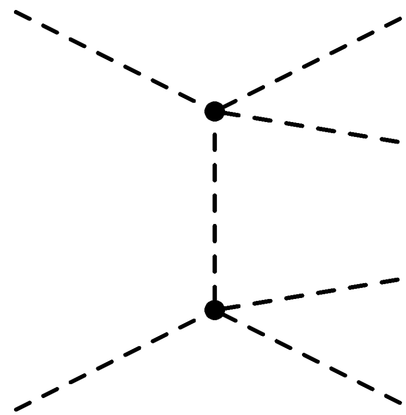
$$\frac{f^0}{E_p T}\left(\frac{p^2}{3} - c_s^2 E_p \frac{\partial(\beta E_p)}{\partial\beta}\right)(\partial\cdot u) = -C[\delta f]$$

Use $c_s^2 = \frac{1}{3}$, $E_p = p$ and $C[\delta f = f^0 E_p] = 0$: Get $0 = 0$.

Bulk viscosity in kinetic theory

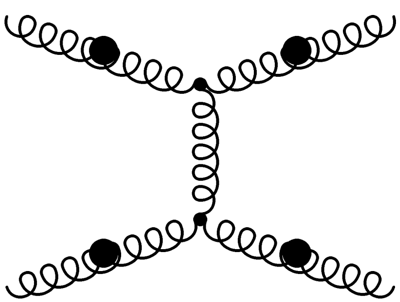
From air to the dilute pion gas: inelastic scattering

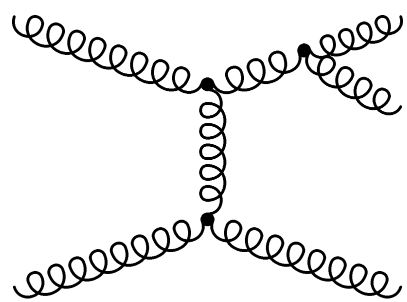




$O_2 + O_2 \to O_2^* + O_2^* \;\; (\Delta E = \hbar\omega n)$ $\qquad$ $\pi + \pi \to 4\pi$

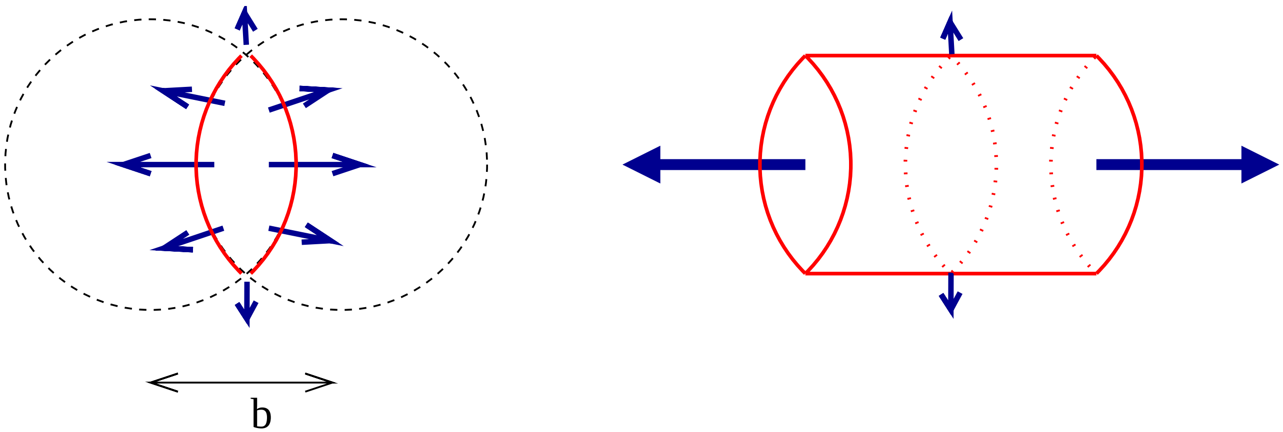
QCD: Elastic vs inelastic reactions





$g + g \to g + g \;\; (m_g^2 \sim g^2 T^2)$ $\qquad$ $g + g \to g + g + g$

Shear and bulk viscosity in heavy ion collisions (first guess)



$$E_p \left. \frac{dN}{d^3p} \right|_{p_z=0} = v_0(p_\perp)\left(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots\right)$$

η suppresses v_2, enhances v_0

ζ suppresses v_0, (typically) enhances v_2

Note: v_0 also sensitive to eos, freezeout, hadronic phase.

Differential elliptic flow from dissipative hydrodynamics

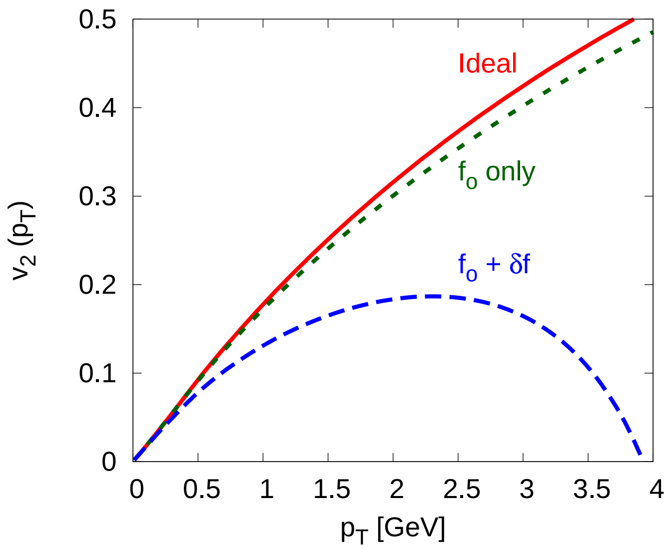
Spectra computed on freeze-out surface ("Cooper-Frye")

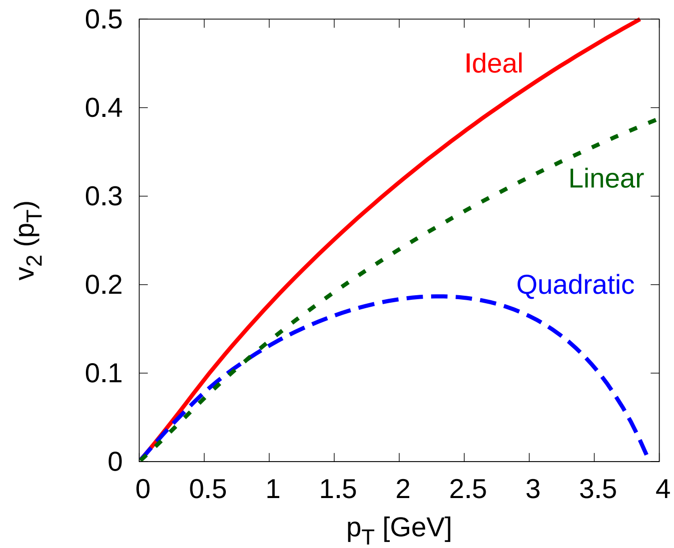
$$E_p \frac{dN}{d^3p} = \frac{1}{(2\pi)^3} \int_\sigma f(E_p) p^\mu d\sigma_\mu$$

Write $f = f^0 + \delta f$ and match to hydrodynamics

$$\delta\Pi^{\mu\nu} = \int d\Omega_p \, p^\mu p^\nu \delta f(E_p)$$

Only moments of δf fixed by η, ζ. Need kinetic models.





δf from Chapman-Enskog & relaxation time approximation

Linear response to slowly varying thermodynamic variables

$$f_p^0 = \left[\exp\left(-\beta(x)\left(P^\mu(\beta)u_\mu(x)\right)\right) \mp 1\right]^{-1} \qquad P^\mu = (E_p, \vec{p})$$

Drift term proportional to "driving term" $\partial \cdot u$

$$LHS = \frac{n_p(1 \pm n_p)}{E_p T}\left(\frac{p^2}{3} - c_s^2 E_p \frac{\partial(\beta E_p)}{\partial \beta}\right)(\partial \cdot u)$$

Linearized collision operator $f_p = n_p - n_p(1 \pm n_p)\chi_p \quad (n_p = f_p^0)$

$$RHS = -C[f_p] \simeq -n_p(1 \pm n_p) C_L[\chi_p]$$

Relaxation time approximation

$$C_L[\chi_p] \simeq \frac{\chi_p}{\tau(E_p)}$$

Relaxation time approximation

Bulk viscosity second order in conformal breaking parameter δc_s^2

$$\zeta = 15\eta \left(c_s^2 - \frac{1}{3} \right)^2$$

Weinberg (1972)

Distribution function is first order in conformal breaking

$$\delta f \sim f_p^0 \frac{\eta}{sT} \frac{p^2}{T^2} \left(c_s^2 - \frac{1}{3} \right) (\partial \cdot u)$$

Near conformal fluids: Bulk viscous correction dominated by δf

Also note: RTA consistent with energy conservation only for very specific choices of $\tau(E_p)$

$$\delta\epsilon = 0 = \int d\Omega_p \, \tilde{E}_p^2 \delta f_p$$

Distribution function in QGP

elastic $2 \leftrightarrow 2$ can be written as Fokker-Planck equation (diffusion equation in momentum space)

$$(\partial \cdot u)\left(\frac{p^2}{3} - c_s^2 E_p \frac{\partial\left(\beta E_p\right)}{\partial \beta}\right) = \frac{T\mu_A}{n_p}\frac{\partial}{\partial p^i}\left(n_p \frac{\partial}{\partial p^i}\left[\frac{\delta f_p}{n_p}\right]\right) + \dots$$

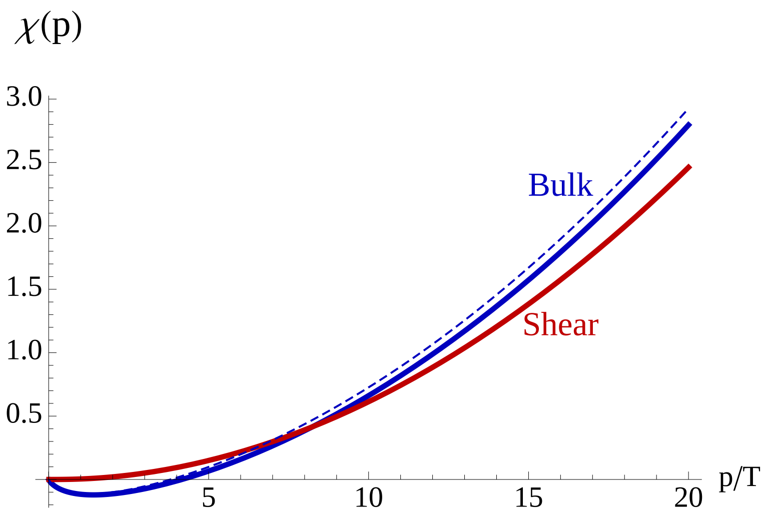
drag coefficient $\mu_A = \frac{g^2 C_A m_D^2}{8\pi} \log\left(\frac{T}{m_D}\right)$

Find $\chi_B \sim \left(\frac{1}{3} - c_s^2\right)\chi_S$ and (pure glue)

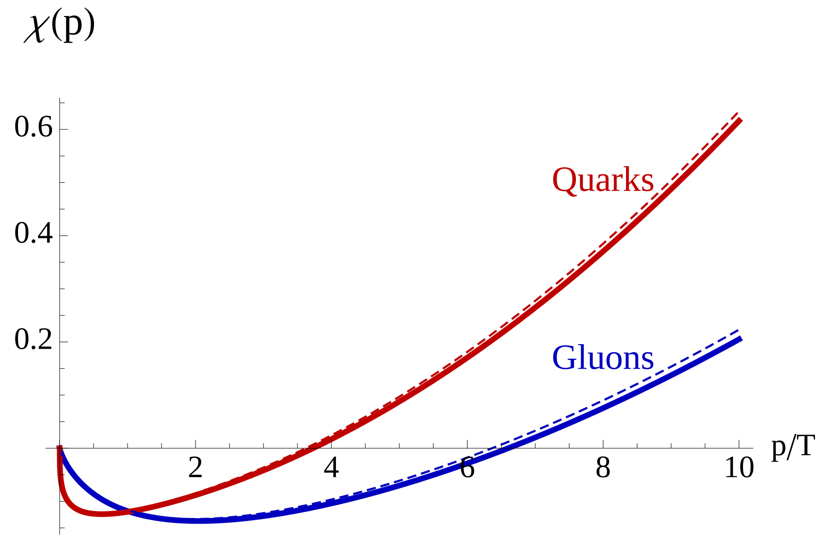
$$\zeta = \frac{0.44\alpha_s^2 T^3}{\log(\alpha_s^{-1})} \qquad \zeta \sim 47.9\left(\frac{1}{3} - c_s^2\right)^2 \eta$$

Arnold, Dogan, Moore (2006)

Distribution function in QGP



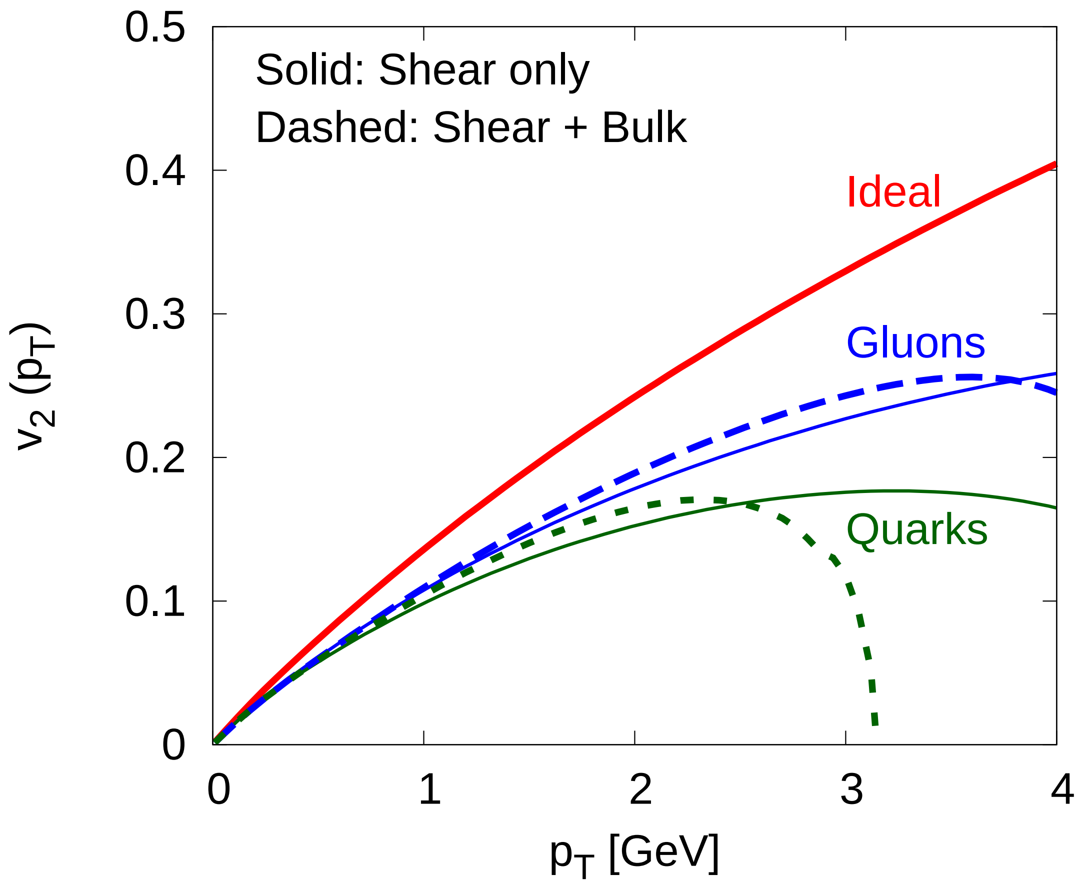
Pure glue: shear vs bulk



QGP: quarks vs gluons

$$\delta f_p = -n_p(1 \pm n_p)\left[\chi_S(p)\hat{p}_i\hat{p}_j\sigma_{ij} + \chi_B(p)(\partial \cdot u)\right]$$

Spectra and flow (pure QGP, no hadronic phase)



$$\eta/s = 0.16 \quad \zeta/s = 0.04$$

large coupling: $m_D = 2.9T$, $c_s^2 = 0.2$

Pion gas

Pion gas: Bulk viscosity governed by chemical non-equilibration

$$\delta f_p = n_p(1+n_p)\left(\frac{\delta\mu}{T} + \frac{E_p \delta T}{T^2}\right) = -n_p(1+n_p)(\chi_0 + \chi_1 E_p)(\partial \cdot u)$$

More formal: χ_0 is a "quasi zero mode" which dominates C^{-1}

Inelastic rate determines χ_0, energy conservation fixes χ_1

$$\chi_0 = \frac{\zeta}{\mathcal{F}} \qquad \zeta = \frac{\beta \mathcal{F}^2}{4\Gamma_{2\pi\to 4\pi}}$$

where we have defined $\mathcal{F} = \int d\Omega_p \left(\frac{p^2}{3} - c_s^2 E_p \frac{\partial(\beta E_p)}{\partial \beta}\right) n_p(1+n_p)$

$$\zeta \simeq 12285 \frac{f_\pi^8}{m_\pi^5} \exp\left(-\frac{2m_\pi}{T}\right)$$

Lu, Moore (2011)

Hadron resonance gas (model)

Hadron gas: Assume bulk viscosity dominated by chemical relaxation

$$\delta f_p^a = -n_p(1 \pm n_p)\left(\chi_0^a - \chi_1 E_p\right)(\partial \cdot u)$$

χ_0^a determined by rates, χ_1 fixed by energy conservation

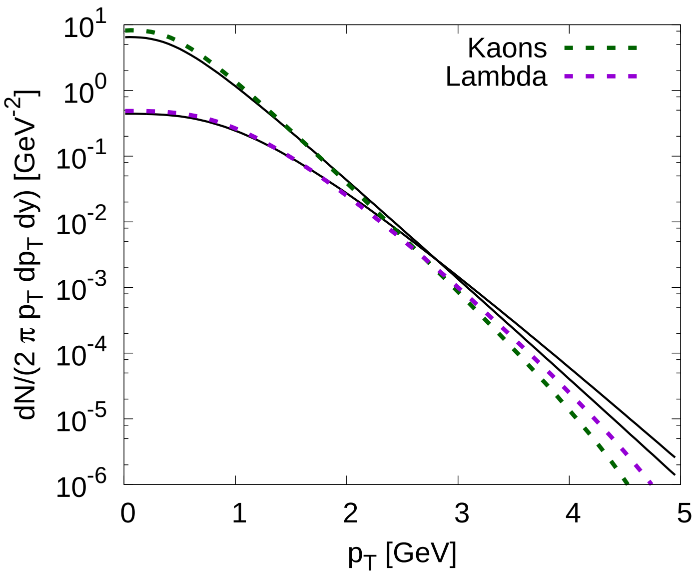
Slowest rate determines ζ, other rates fix $\delta\mu^a/\delta\mu_\pi$. Simple model

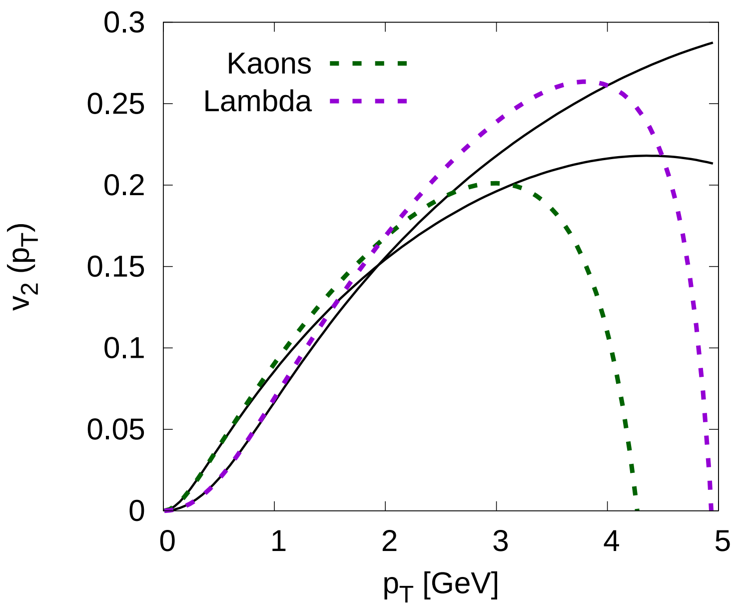
$$\chi_0^a \simeq \chi_0^\pi \begin{cases} 2 & mesons \\ 2.5 & baryons \end{cases}$$

inspired by $\mu_\rho = 2\mu_\pi$ and $2\mu_N = 5\mu_\pi$. Find

$$\zeta/s = 0.05 \quad \Rightarrow \quad \delta\mu_\pi = 20\,\mathrm{MeV}$$

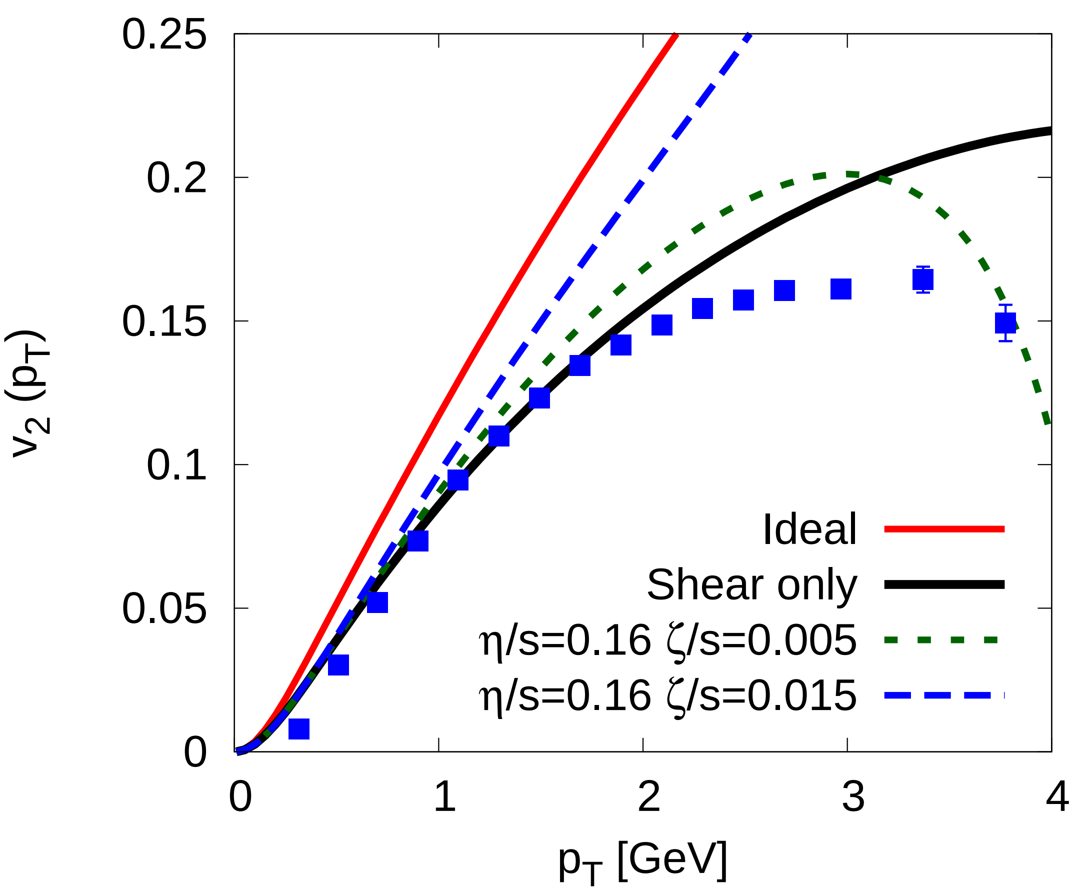
Spectra and flow: Pions and Protons



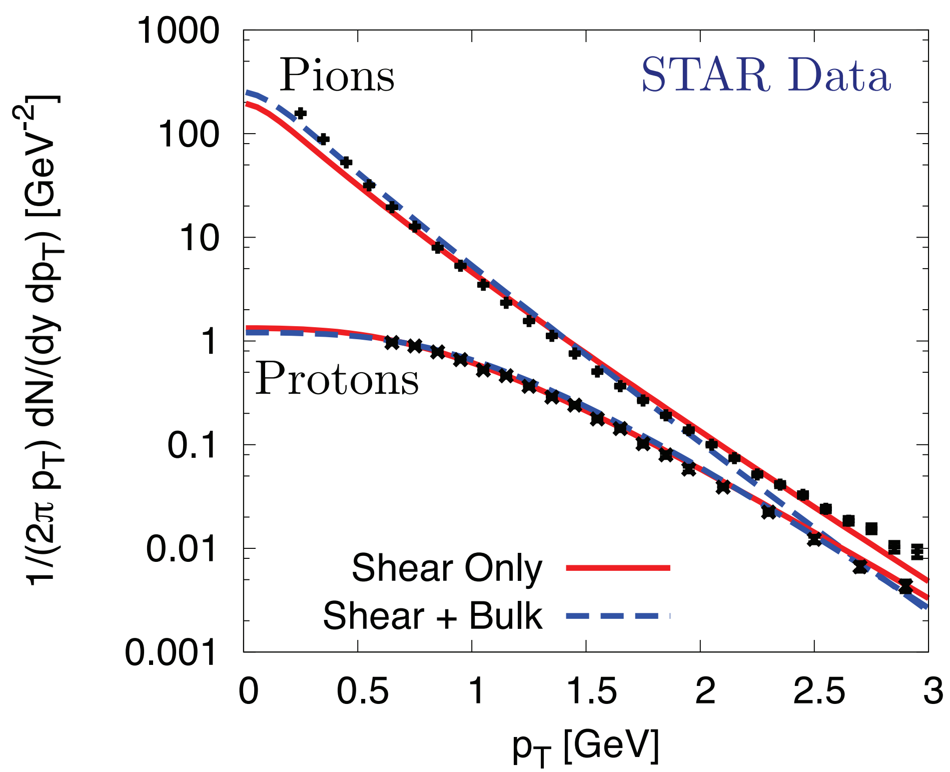


$\eta/s = 0.16$ $\zeta/s = 0.04$

Bounds on ζ/s from differential v_2 (here: K_s)

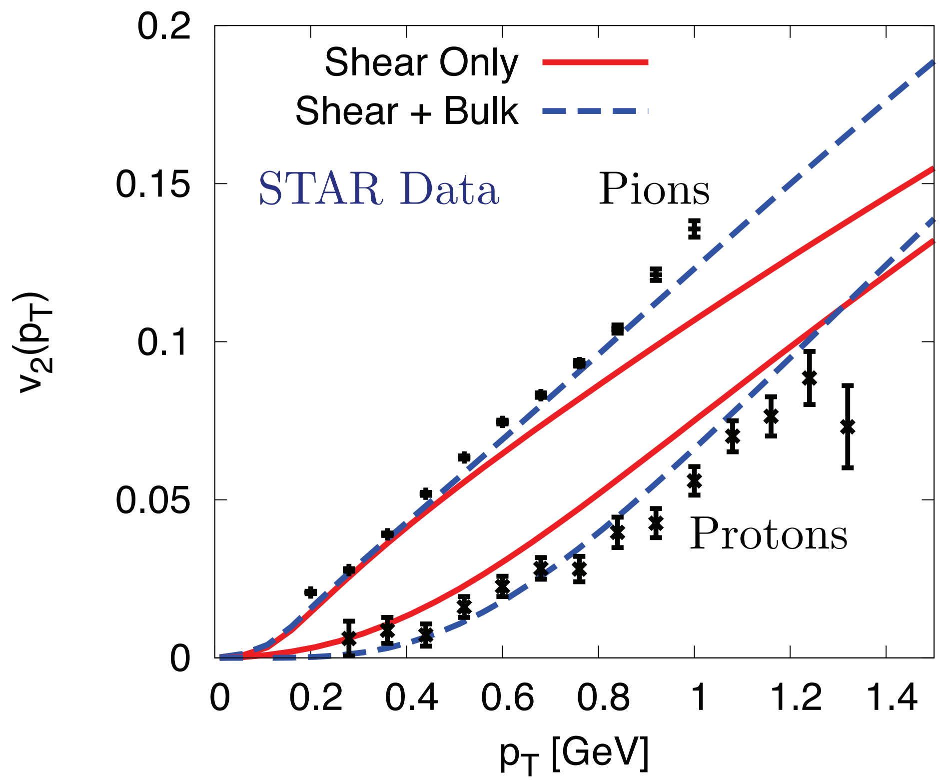


Pion/Proton p_T spectra (low P_T)



Kevin Dusling (2012)

Pion/Proton differential $v_2(p_T)$ spectra (low p_T)



Kevin Dusling (2012)

Conclusions

Bulk viscous corrections dominated by freezeout distributions

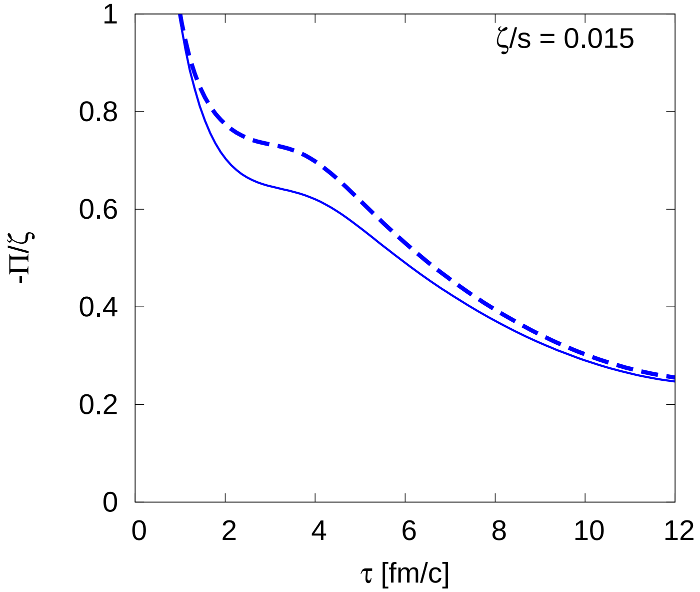
QGP: ζ controlled by momentum rearrangement
Hadron gas: ζ determined by chemical non-equilibration

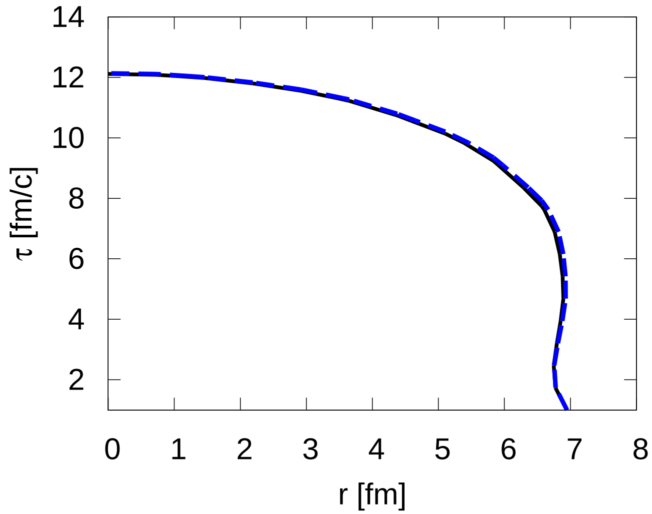
A new way to look at fugacity factors in thermal fits?

RHIC spectra seem to require $\zeta/s \lesssim 0.05$

Bulk viscosity not zero: Spectra prefer $\delta\mu$, fine structure of v_2 improves

Extras: Second order hydrodynamics





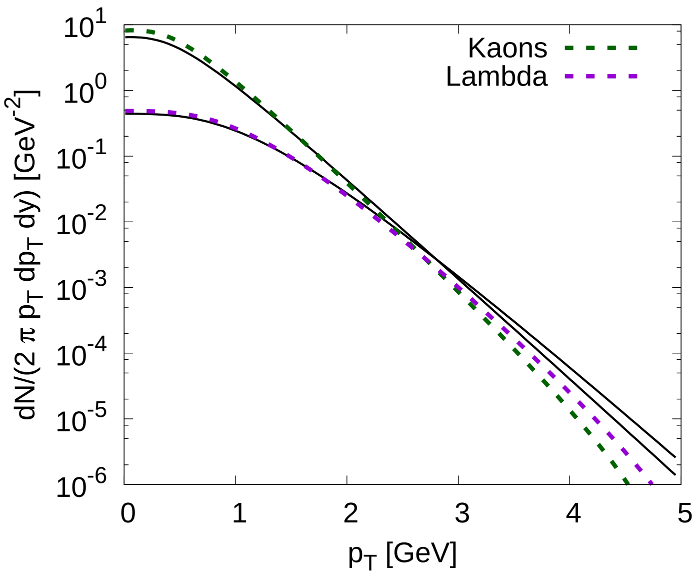
gradient expansion

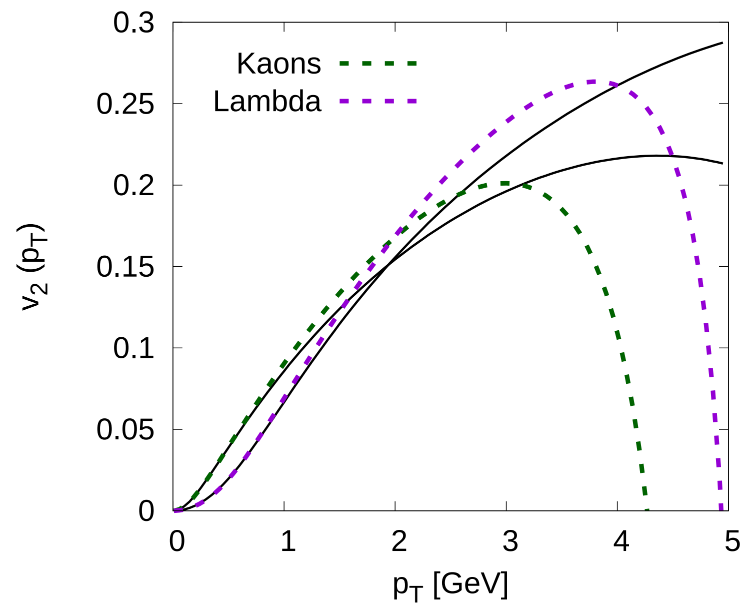
(bulk stress)

freeze out surface

(w/o bulk viscosity)

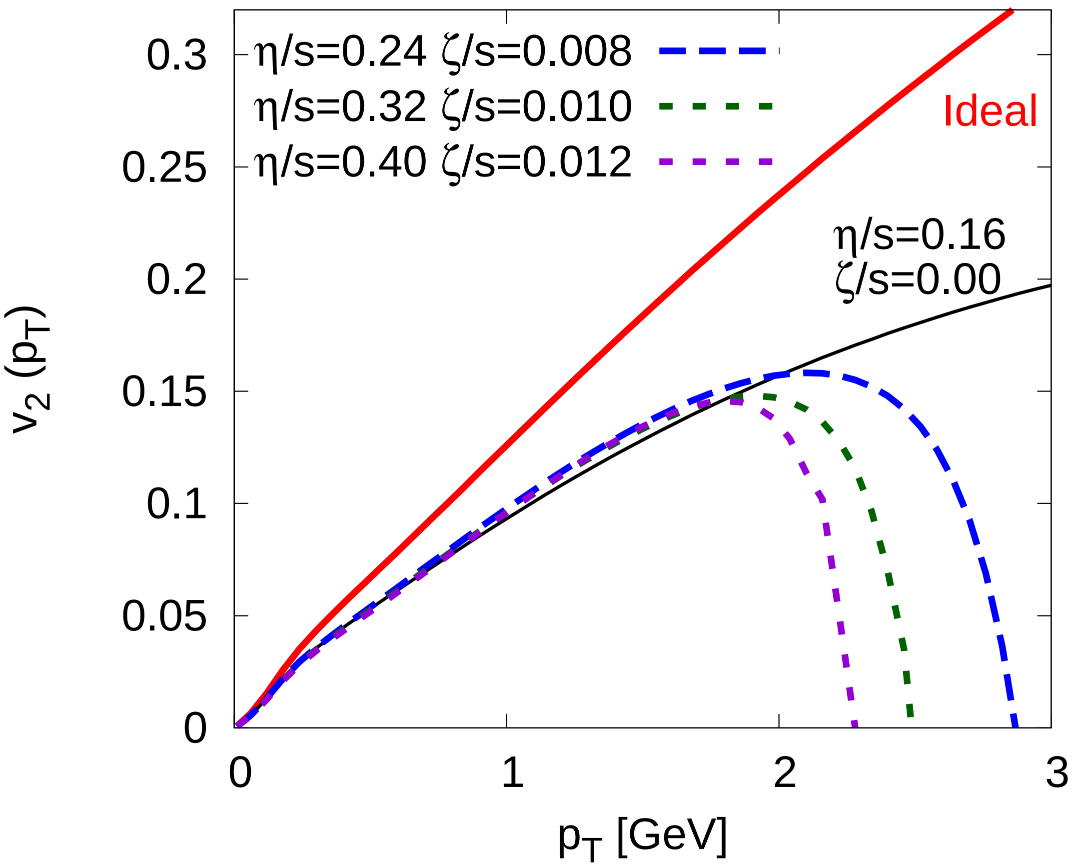
Spectra and flow: Kaons and Lambdas



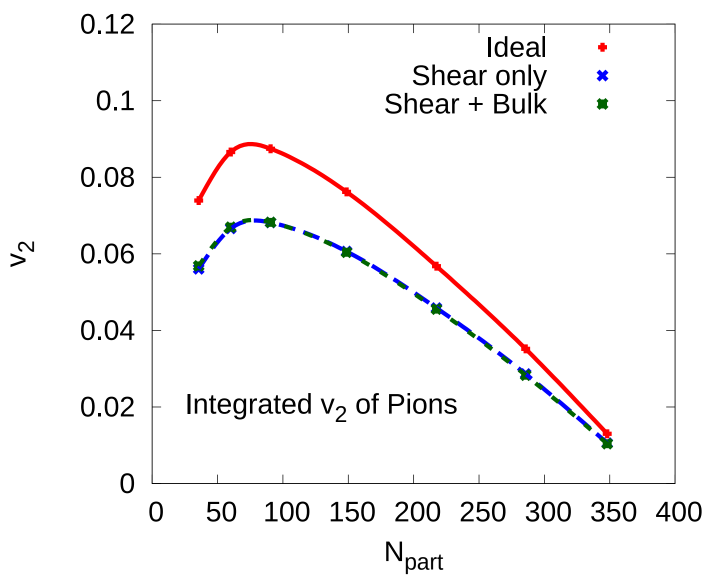


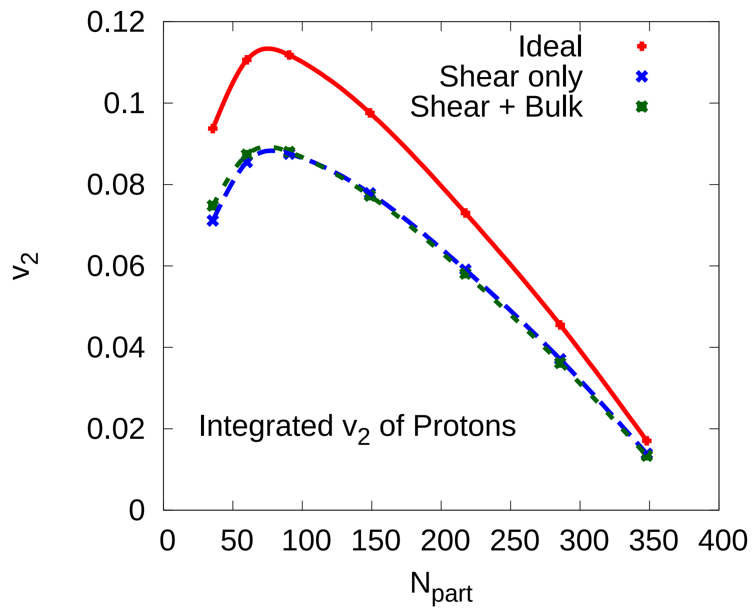
$\eta/s = 0.16$ $\zeta/s = 0.04$

Flow: Interplay between shear and bulk viscosity



Integrated v_2 versus centrality





Distribution functions: Signs

Consider four-velocity u_α with $u^2 = -1$ ($g_{\alpha\beta} = (-1, 1, 1, 1)$)

$$\delta f_p = -n_p \chi_S p^\alpha p^\beta \langle \partial_\alpha u_\beta \rangle - n_p \chi_B (\partial \cdot u)$$

Asymptotic behavior $\chi_{S,B} \sim p^2$.

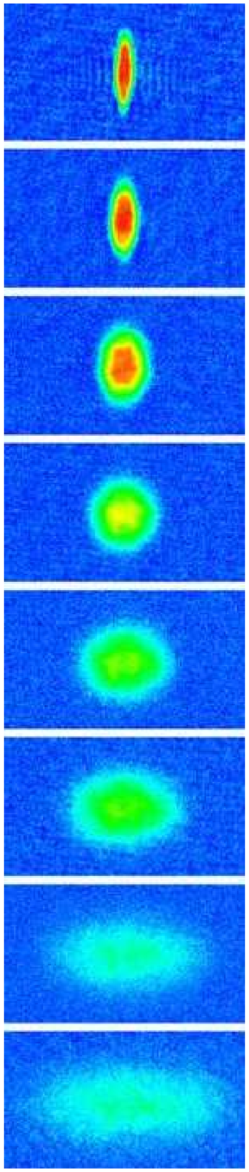
Consider BJ flow: $p^\alpha p^\beta \langle \partial_\alpha u_\beta \rangle \sim -\frac{p_T^2}{\tau}$ and $\partial \cdot u \sim \frac{1}{\tau}$.

$$\delta f_p \sim \frac{\eta}{s} \left(\frac{p_T}{T}\right)^2 \frac{1}{\tau T} - \frac{\zeta}{s} \left(\frac{p_T}{T}\right)^2 \frac{1}{\tau T}$$

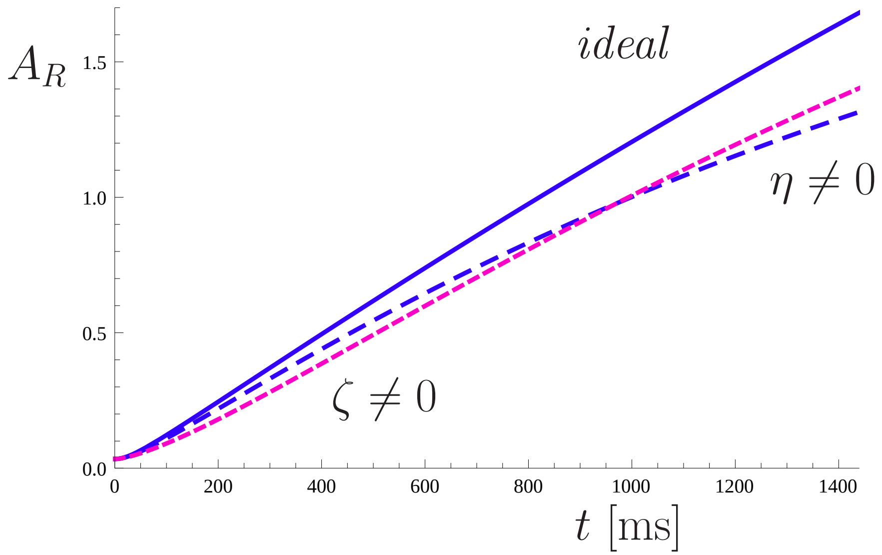
Elliptic flow

$$\langle v_2 \rangle = \frac{\int d\phi \, [f(\phi) + \delta f(\phi)] \cos(2\phi)}{\int d\phi \, [f(\phi) + \delta f(\phi)]} \simeq \langle v_2^0 \rangle + \langle \delta v_2 \rangle - \langle v_2^0 \rangle \langle \delta v_0 \rangle$$

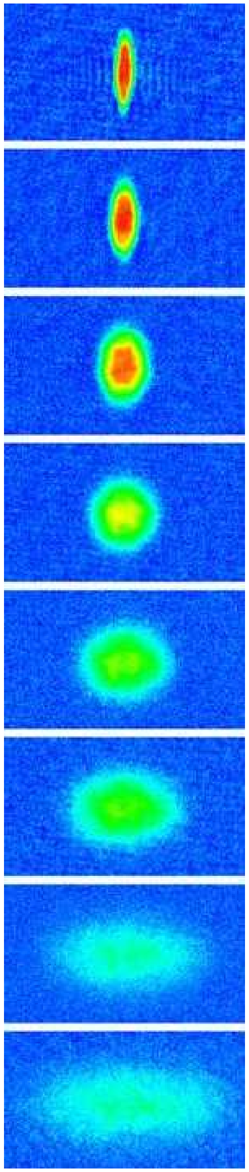
Elliptic flow: Shear vs bulk viscosity



Dissipative hydro with both η, ζ

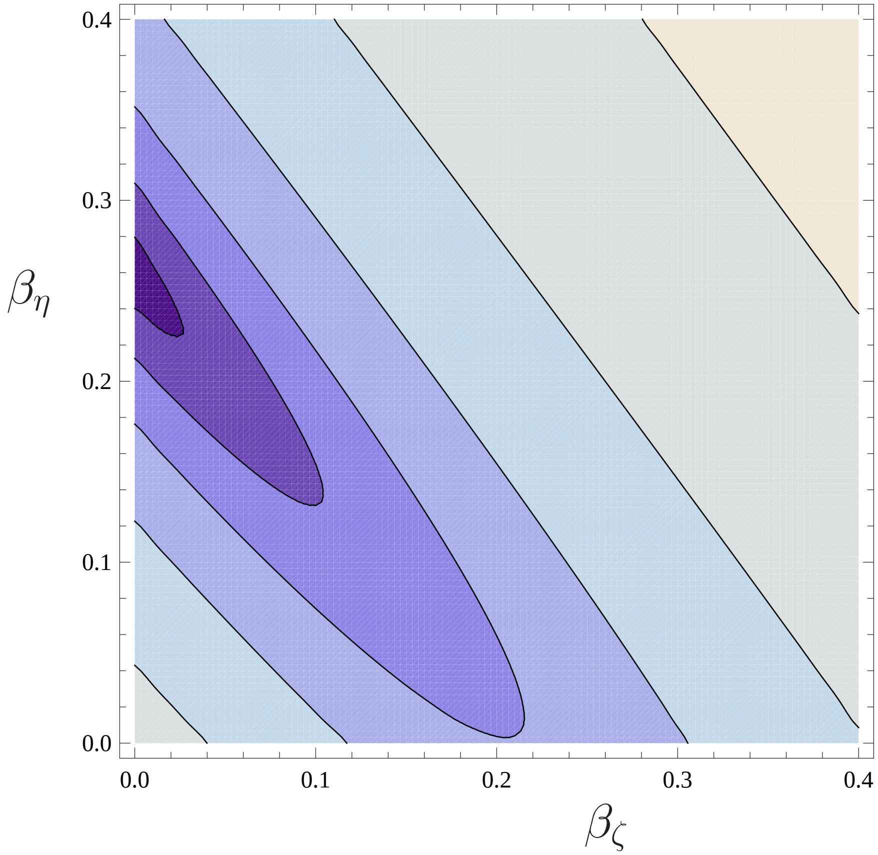


Elliptic flow: Shear vs bulk viscosity



Dissipative hydro with both η, ζ

$$\beta_{\eta,\zeta} = (\eta, \zeta)\frac{E_F}{E}\frac{1}{(3\lambda N)^{1/3}}$$



$\eta \gg \zeta$

Dusling, Schaefer (2010)