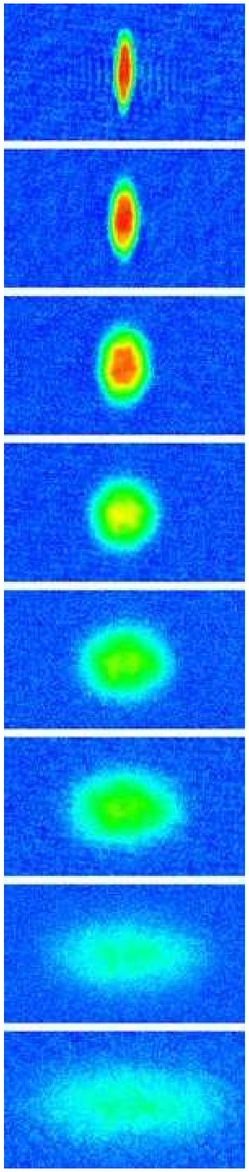
Perfect Fluids:
From Nano to Tera

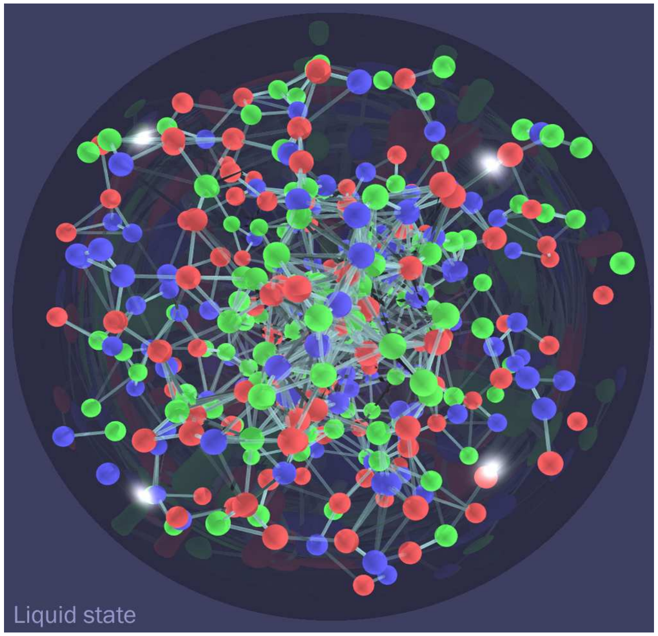
Thomas Schaefer

North Carolina State University

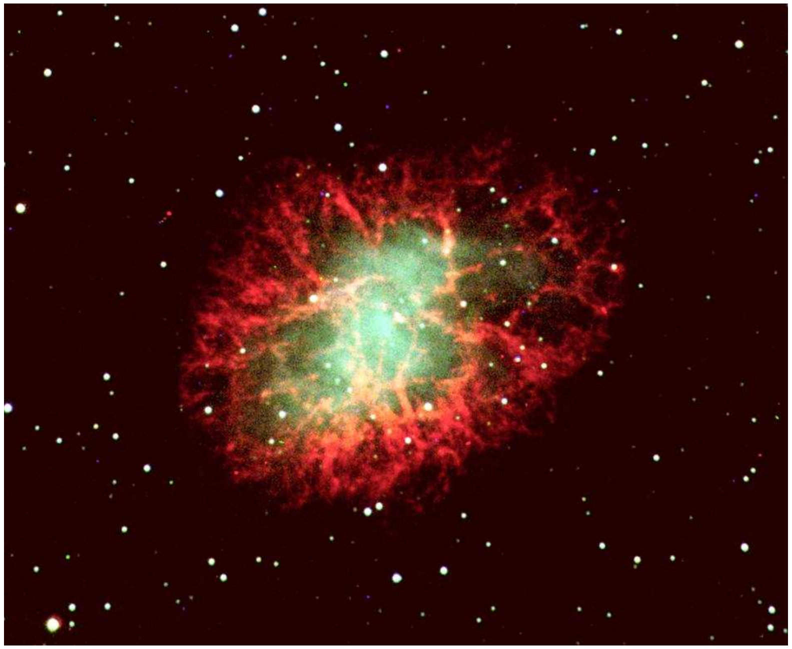

"RHIC serves the perfect liquid"

Perfect Fluids





sQGP (T=180 MeV)

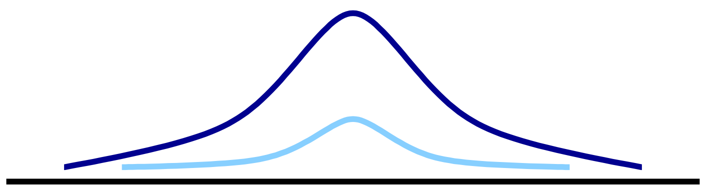


Neutron Matter (T=1 MeV)

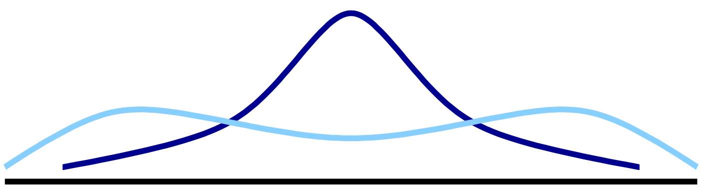
Trapped Atoms
(T=0.1 neV)

Hydrodynamics

Long-wavelength, low-frequency dynamics of conserved or spontaneoulsy broken symmetry variables.



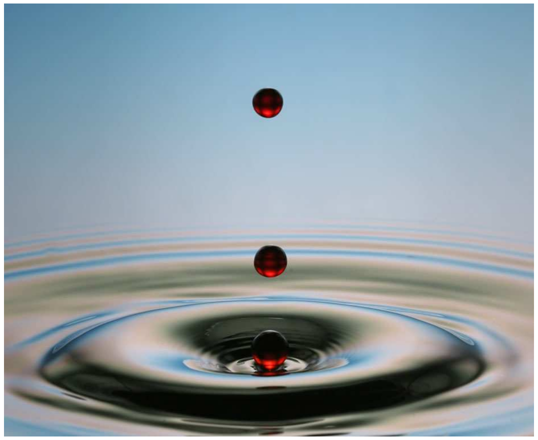
$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Example: Simple Fluid

Continuity equation

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

Euler (Navier-Stokes) equation

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0$$

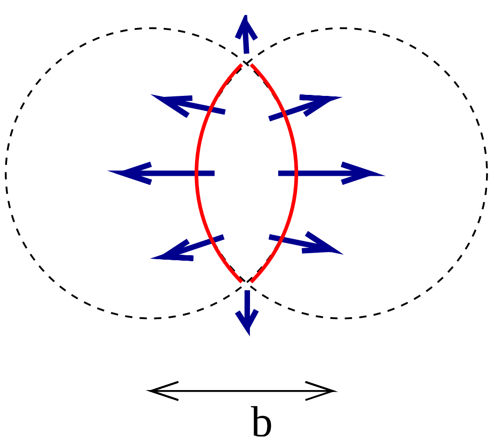
Energy momentum tensor

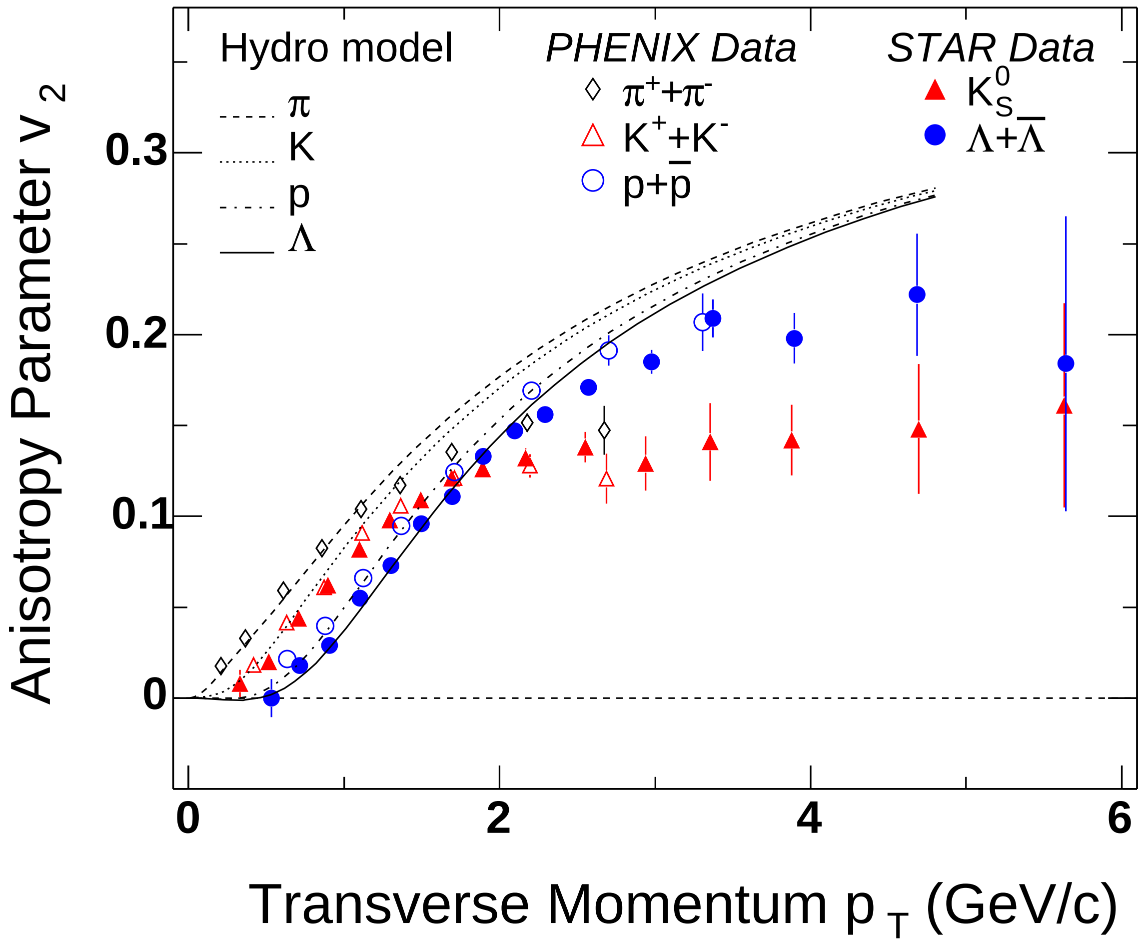
$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right) + \ldots$$

reactive dissipative

Elliptic Flow

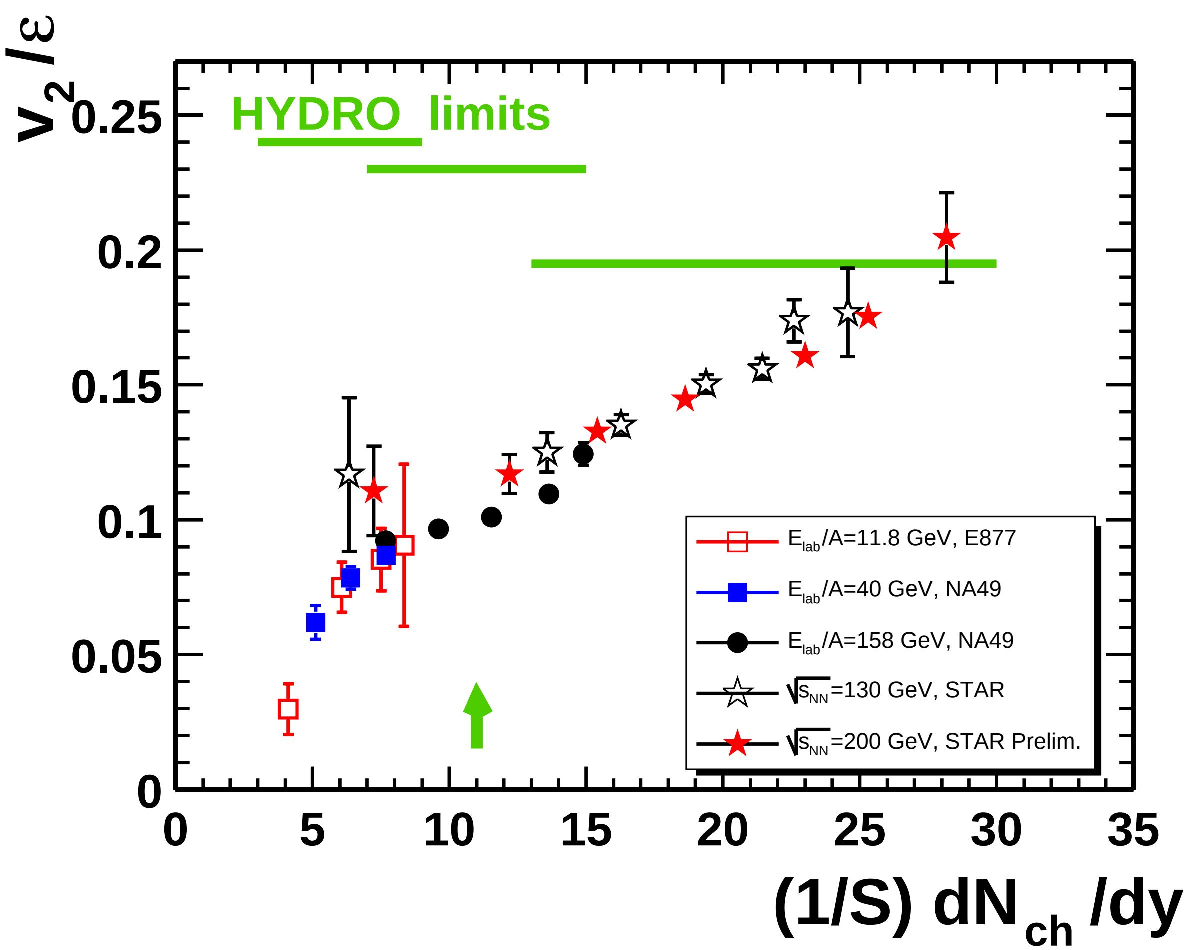
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





source: U. Heinz (2005)

Elliptic Flow II



Requires "perfect" fluidity ($\eta/s < 0.1$?)

(s)QGP saturates (conjectured) universal bound $\eta/s = 1/(4\pi)$?

Viscosity Bound: Rough Argument

Kinetic theory estimate of shear viscosity

$$\eta \sim \frac{1}{3} n \, \bar{p} \, l_{mfp} \qquad\qquad (\mathrm{Note}: l_{mfp} \sim 1/(n\sigma))$$

Entropy density: $s \sim k_B n$. Uncertainty relation implies

$$\frac{\eta}{s} \sim \frac{n \, \bar{p} \, l_{mft}}{k_B n} \geq \frac{\hbar}{k_B}$$

Validity of kinetic theory as $\bar{p} \, l_{mfp} \sim \hbar$?

Why η/s? Why not η/n?

Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

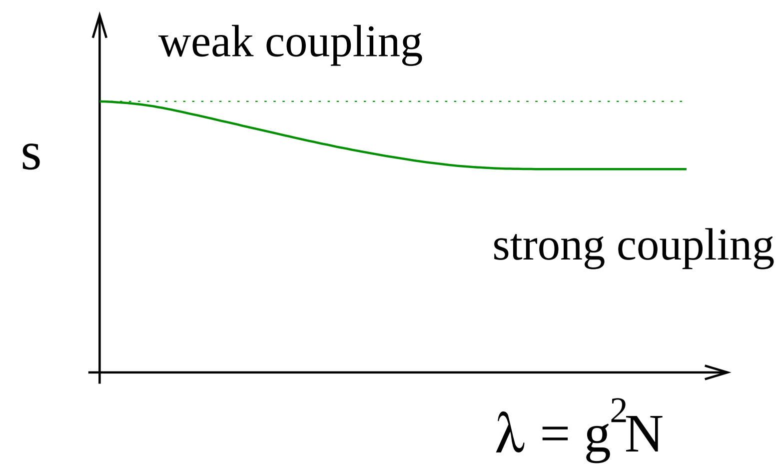
CFT temperature $\Leftrightarrow$ Hawking temperature of black hole

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy $\sim$ area of event horizon

Strong coupling limit

$$s = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0$$

Gubser and Klebanov



Holographic Duals: Transport Properties

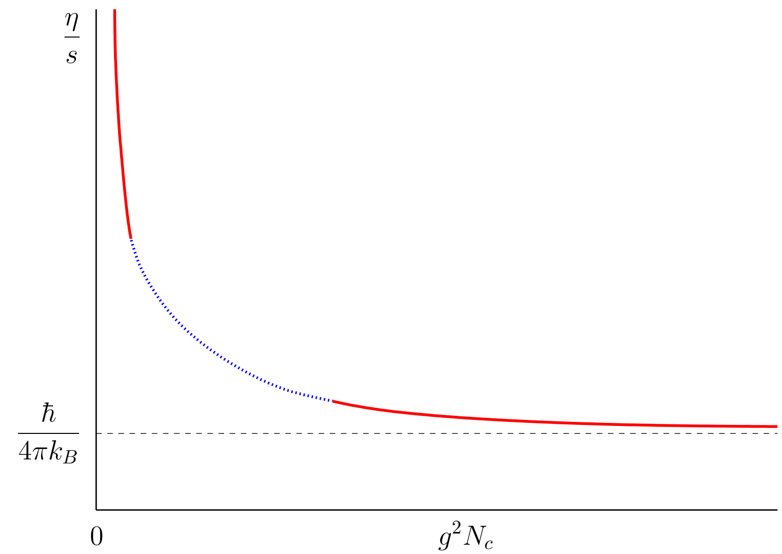
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

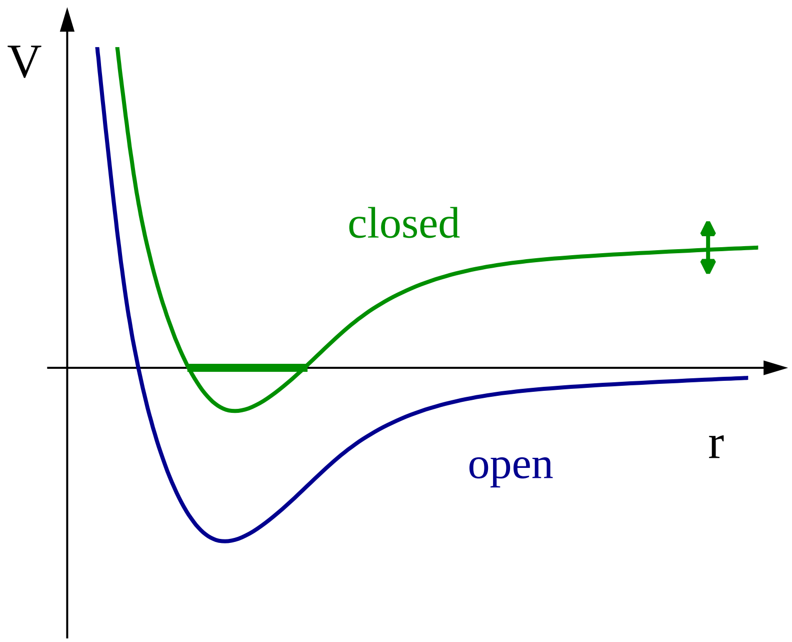
Son and Starinets

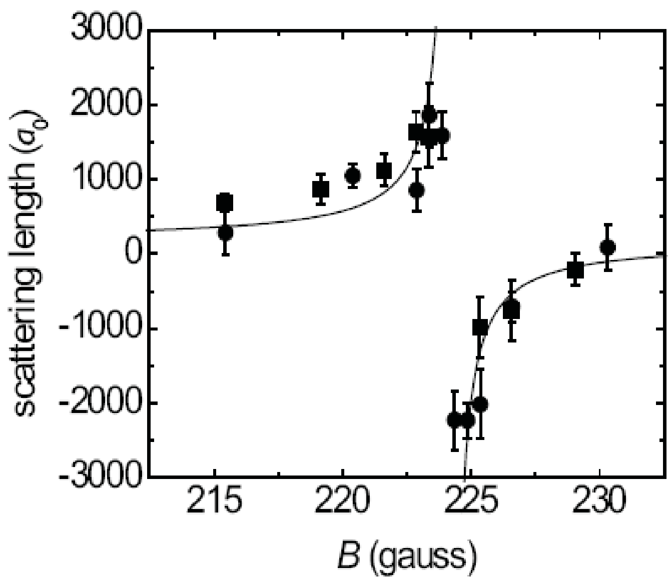


Strong coupling limit universal? Provides lower bound for all theories?

Designer Fluids

Atomic gas with two spin states: "↑" and "↓"





Feshbach resonance

$$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

"Unitarity" limit $a \to \infty$

$$\sigma = \frac{4\pi}{k^2}$$

Why are these systems interesting?

System is intrinsically quantum mechanical

cross section saturates unitarity bound

Scale (and conformally) invariant at unitarity

$$\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0, \qquad \text{OPE, power laws, } \ldots$$

System is strongly coupled but dilute

$$(k_F a) \to \infty \qquad (k_F r) \to 0$$

Strong hydrodynamic elliptic flow observed experimentally

Questions

✓ Equation of State, Quasi-Particles, . . .

Critical Temperature

✓ Transport: Shear Viscosity, . . .

Stressed Pairing

I. EOS, Quasi-Particles

Microscopic Effective Field Theory

Effective field theory for pointlike, non-relativistic fermions

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \Big[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \Big] + \ldots$$

Match to effective range expansion

$$C_0 = \frac{4\pi a}{M}, \qquad C_2 = \frac{4\pi a^2}{M} \frac{r}{2}$$

Unitarity limit

$$C_0 \to \infty, \qquad C_2 \to 0$$

Effective Lagrangian

Fermions are paired $\langle\psi\psi\rangle \neq 0$. Energy gap

$$\omega \sim \Delta \sim E_F$$

Low energy degrees of freedom: phase of condensate

$$\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$$

Effective lagrangian

$$\mathcal{L} = f^2\left(\dot{\varphi}^2 - v^2(\vec{\nabla}\varphi)^2\right) + \ldots$$

Effective Lagrangian

Low energy ($\omega < \Delta \sim E_F$) effective lagrangian

$$\mathcal{L} = c_0 m^{3/2} X^{5/2} + c_1 m^{1/2} \frac{(\vec{\nabla} X)^2}{\sqrt{X}} + \frac{c_2}{\sqrt{m}} \left[\left(\nabla^2 \varphi\right)^2 - 9m\nabla^2 A_0 \right] \sqrt{X}$$

$$X = \mu - A_0 - \dot{\varphi} - \frac{(\vec{\nabla}\varphi)^2}{2m}$$

variables

φ: phase $\psi\psi = e^{2i\varphi}\langle\psi\psi\rangle$

μ: chemical potential

A_0: gauge potential

constrained by

$U(1)$ invariance

Galilean invariance

Scale invariance

Conformal invariance

Greiter et al. (1989); Son, Wingate (2005)

Effective Lagrangian

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Scale invariance

Conformal invariance

Effective lagrangian determines

Coupling to external fields

Energy density functional

Phonon interactions

Superfluid hydrodynamics

Non-perturbative physics in $c_0, c_1, c_2, \ldots$

Use epsilon ($\epsilon = d - 4$) expansion

Upper and lower critical dimension

Zero energy bound state for arbitrary d

$$\psi''(r) + \frac{d-1}{r}\psi'(r) = 0 \qquad (r > r_0)$$

d=2: Arbitrarily weak attractive potential has a bound state

$$\xi(d{=}2) = 1$$

d=4: Bound state wave function $\psi \sim 1/r^{d-2}$. Pairs do not overlap

$$\xi(d{=}4) = 0$$

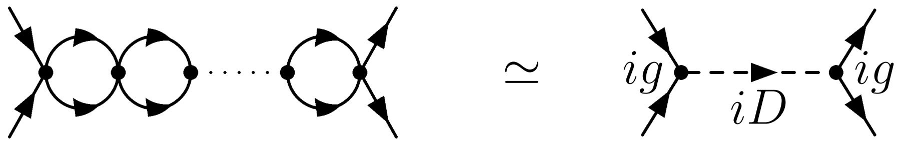
Conclude $\xi(d{=}3) \sim 1/2$?

Try expansion around $d = 4$ or $d = 2$?

Nussinov & Nussinov (2004)

Epsilon Expansion

EFT version: Compute scattering amplitude ($d = 4 - \epsilon$)



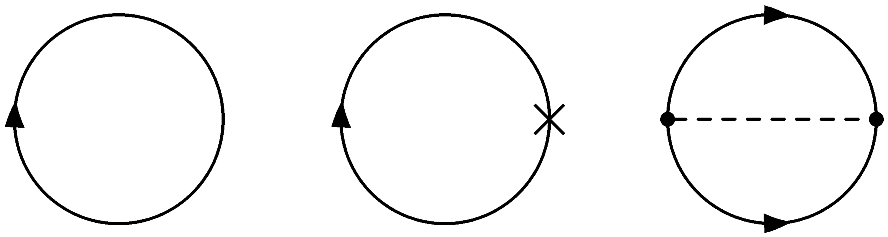
$$T = \frac{1}{\Gamma\left(1-\frac{d}{2}\right)} \left(\frac{m}{4\pi}\right)^{-d/2} \left(-p_0 + \frac{\epsilon_p}{2}\right)^{1-d/2} \simeq \frac{8\pi^2\epsilon}{m^2} \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

$$g^2 \equiv \frac{8\pi^2\epsilon}{m^2} \qquad D(p_0, p) = \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

Weakly interacting bosons and fermions

Matching Calculations

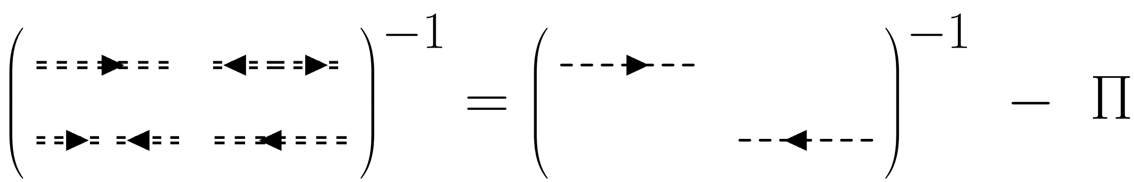
Effective potential



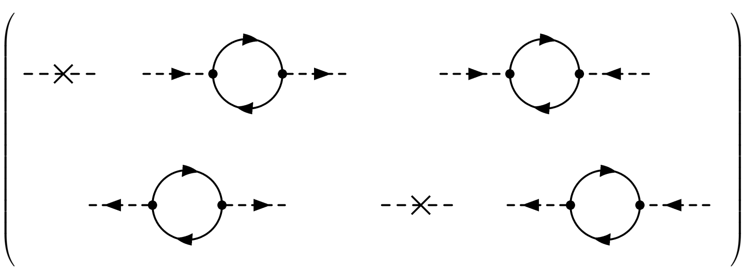
$O(1)$ $O(1)$ $O(\epsilon)$

$$P = \#(2m)^{d/2}\mu^{d/2+1}$$

Phonon Propagator



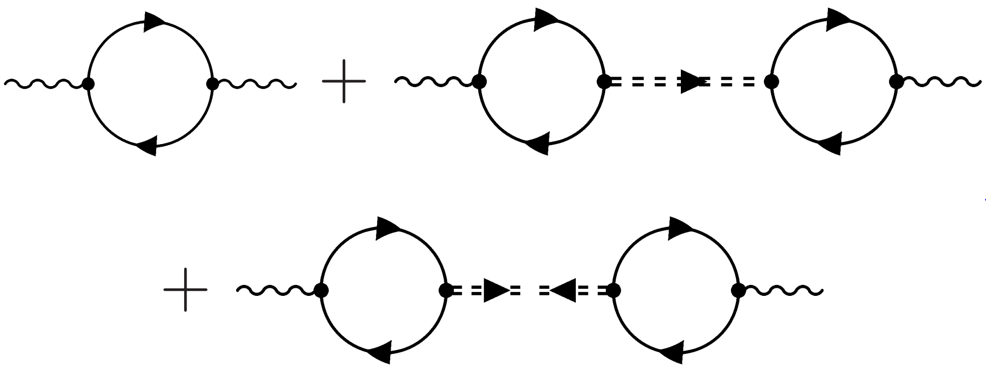
$-\Pi =$



$$\omega = c_s p \left\{ 1 + \# \left(\frac{p^2}{m\mu} \right) + \ldots \right\}$$

Matching (continued)

Static susceptibility $\chi(q) = \int d^3x\, e^{iqx} \langle \psi^\dagger\psi(x)\psi^\dagger\psi(0)\rangle$



$$\chi(q) = \chi(0)\left\{1 - \#\left(\frac{q^2}{m\mu}\right) + \ldots\right\}$$

Nishida, Son (2007)

Rupak, Schaefer (2008)

Low Energy Constants: Result

Low energy constants

$$c_0 \simeq 0.116, \qquad c_1 \simeq -0.021, \qquad c_2 \simeq 0$$

Density functional in unitarity limit

$$\mathcal{E}(x) = 1.364\,\frac{n(x)^{5/3}}{m} + 0.032\,\frac{\left(\nabla n(x)\right)^2}{mn(x)} + O(\nabla^4 n)$$

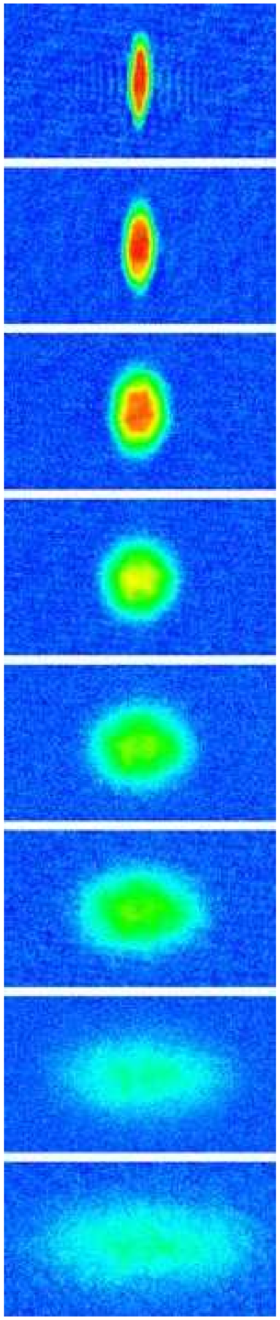
Compare: Free Fermions

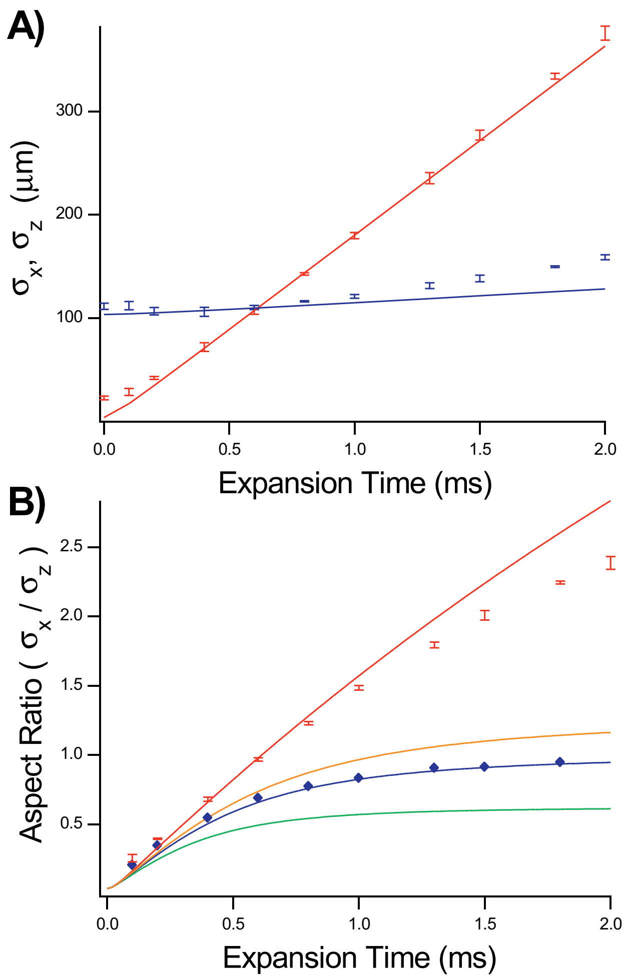
$$\mathcal{E}(x) = 2.871\,\frac{n(x)^{5/3}}{m} + 0.014\,\frac{\left(\nabla n(x)\right)^2}{mn(x)} + O(\nabla^4 n)$$

Volume energy reduced, surface energy increased

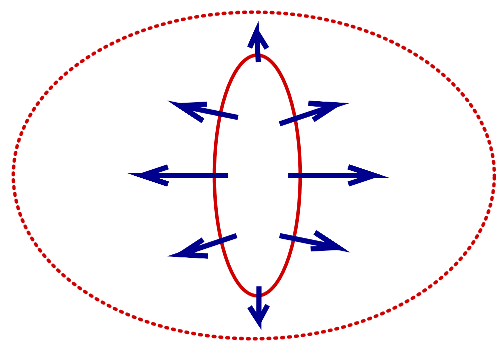
II. Transport Properties

Elliptic Flow



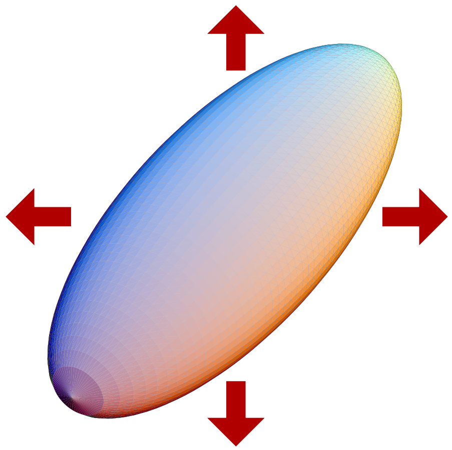


Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

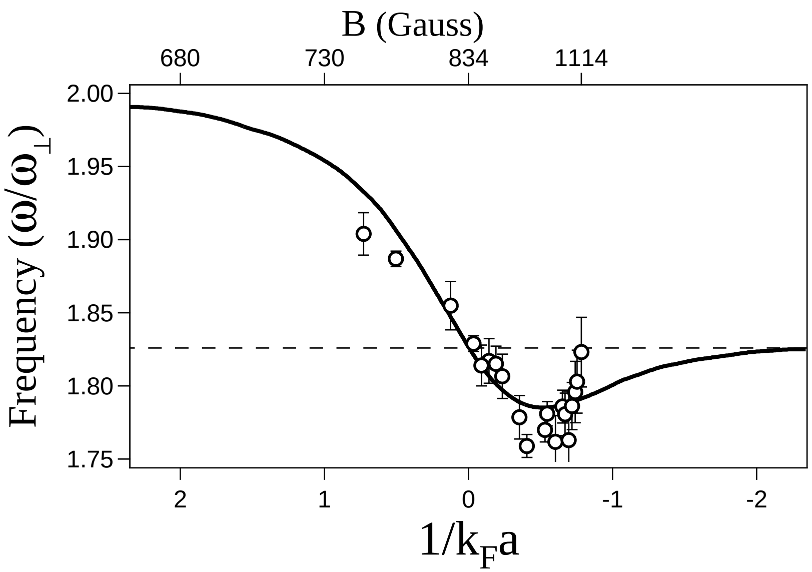


Collective Modes

Radial breathing mode



Kinast et al. (2005)

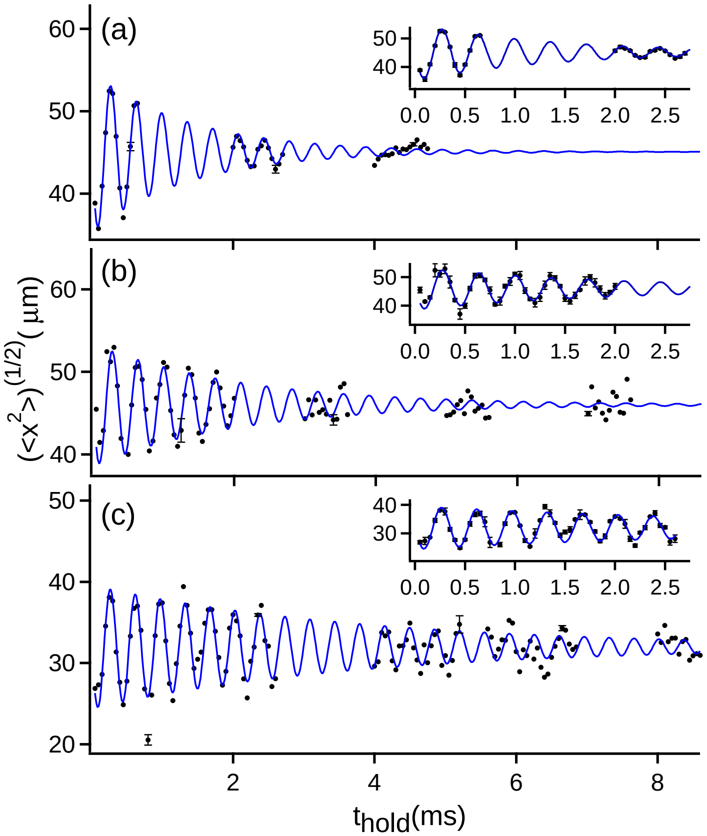


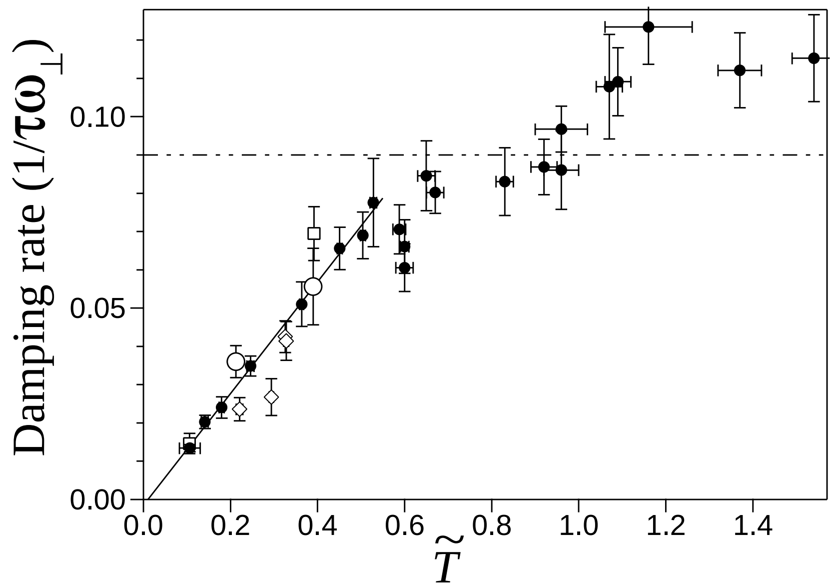
Ideal fluid hydrodynamics, equation of state $P \sim n^{5/3}$

$$\begin{aligned} \frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) &= 0 \\ \frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} &= -\frac{1}{mn}\vec{\nabla}P - \frac{1}{m}\vec{\nabla}V \end{aligned}$$

$$\omega = \sqrt{\frac{10}{3}}\omega_\perp$$

Damping of Collective Excitations





$T/T_F = (0.5, 0.33, 0.17)$

$\tau\omega$: decay time $\times$ trap frequency

Kinast et al. (2005)

Viscous Hydrodynamics

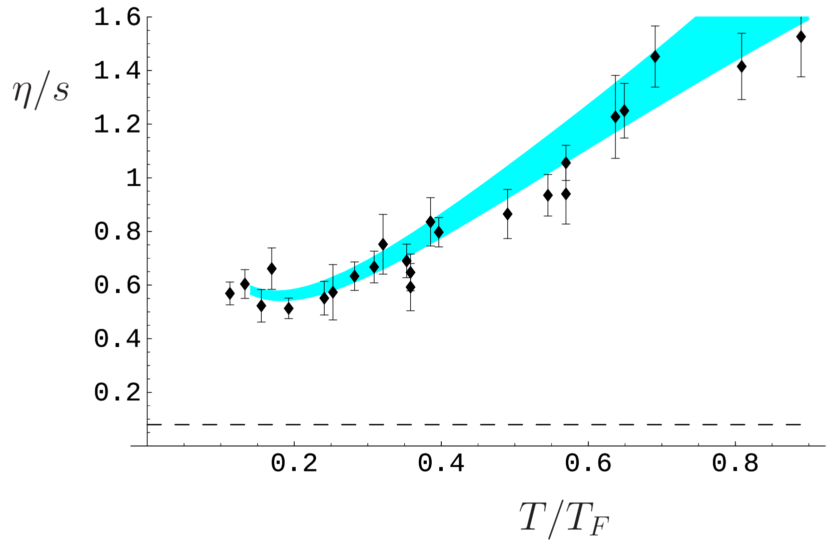
Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} \; = \; -\frac{\eta}{2}\int d^3x \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2$$
$$-\,\zeta \int d^3x \, (\partial_i v_i)^2 - \frac{\kappa}{T}\int d^3x \, (\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

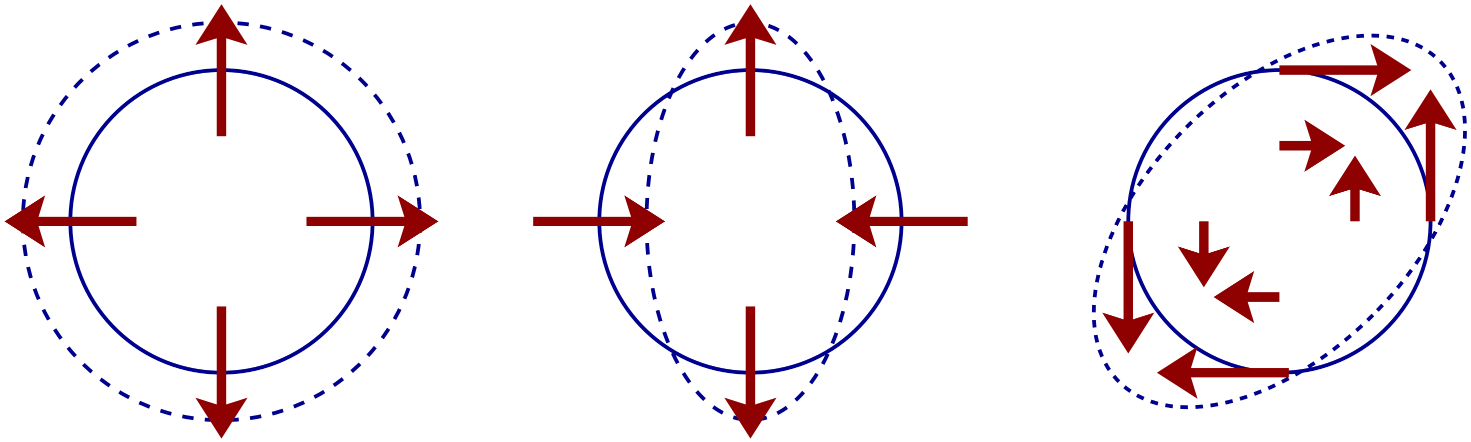
$$\frac{\eta}{s} = \frac{3}{4}\xi^{\frac{1}{2}}(3N)^{\frac{1}{3}}\,\frac{\Gamma}{\omega_\perp}\,\frac{\bar{\omega}}{\omega_\perp}\,\frac{N}{S}$$



see also Bruun, Smith, Gelman et al.

Damping dominated by shear viscosity?

Study dependence on flow pattern



Study particle number scaling

viscous hydro: $\Gamma \sim N^{-1/3}$ Boltzmann: $\Gamma \sim N^{1/3}$

Role of thermal conductivity?

suppressed for scaling flows: $\delta T \sim T(\delta n/n) \sim const \Rightarrow \nabla(\delta T) = 0$

Kinetic Theory

Quasi-Particles: Kinetic Theory

$$T_{ij} = \int d^3p \, \frac{p_i p_j}{E_p} f_p ,$$

Boltzmann equation

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

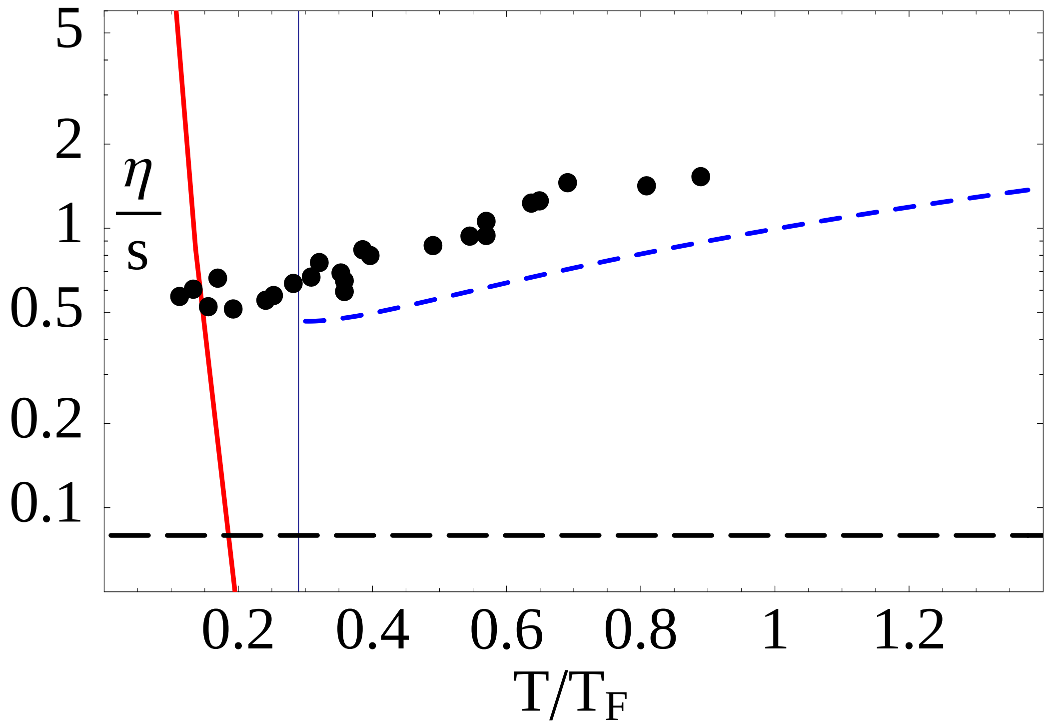
Linearized theory (Chapman-Enskog): $f_p = f_p^0 (1 + \chi_p / T)$

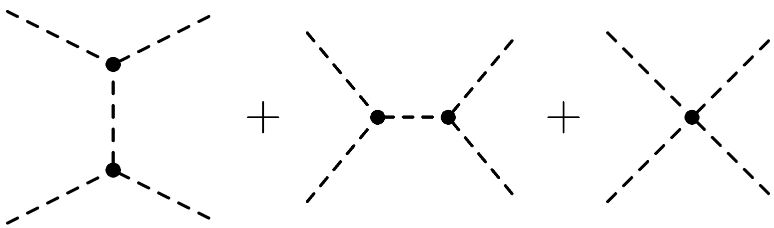
$$\eta \geq \frac{\langle \chi | X \rangle^2}{\langle \chi | C | \chi \rangle} \qquad \langle \chi | X \rangle = \int d^3p \, f_p^0 \, \chi_p \, p_{ij} v_{ij}$$

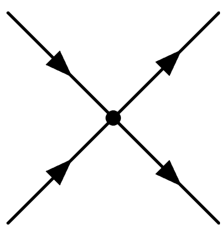
$$v_{ij} = v^2 \delta_{ij} - 3 v_i v_j$$

Low T: Phonons

High T: Atoms







$$\frac{\eta}{s} \sim \left(\frac{T_F}{T}\right)^8$$

$$\frac{\eta}{s} \sim \left(\frac{T}{T_F}\right)^{3/2} \log\left(\frac{T}{T_F}\right)^{-1}$$

Outlook

Transport Theory near T_c: Relation to Viscosity Bound Conjecture?

Other experimental constraints: Observation of "irrotational flow"?

Other uses of conformal symmetry? OPE? Braaten, Platter (2008)

AdS/Cold Atom correspondence? Son (2008), Balasubramanian & McGreevy (2008)