# The Beam Energy Scan Theory Collaboration


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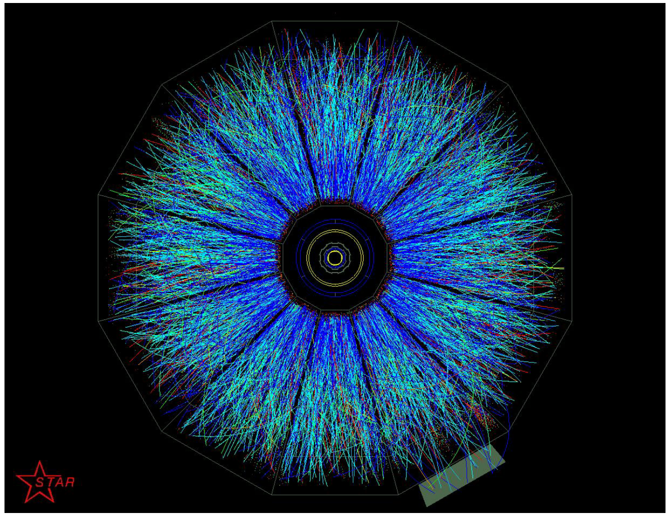
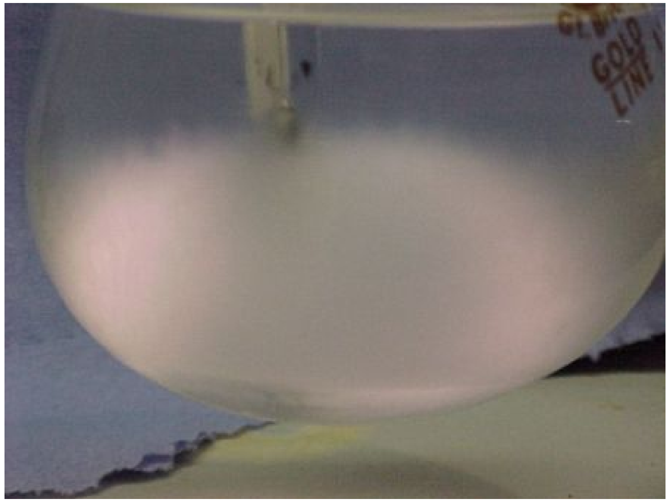
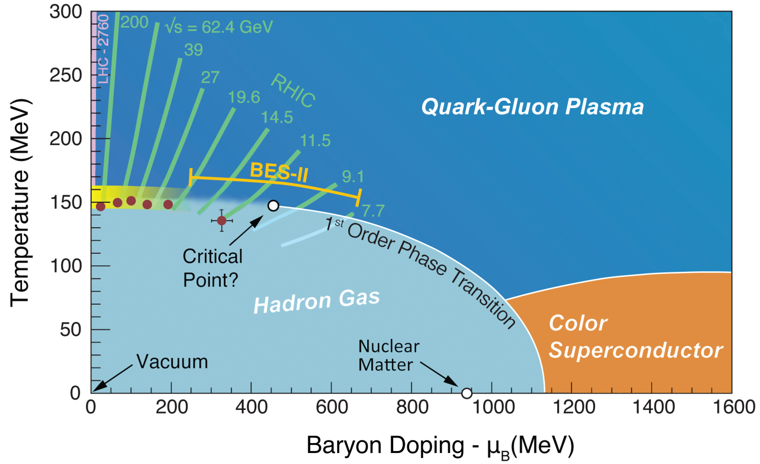


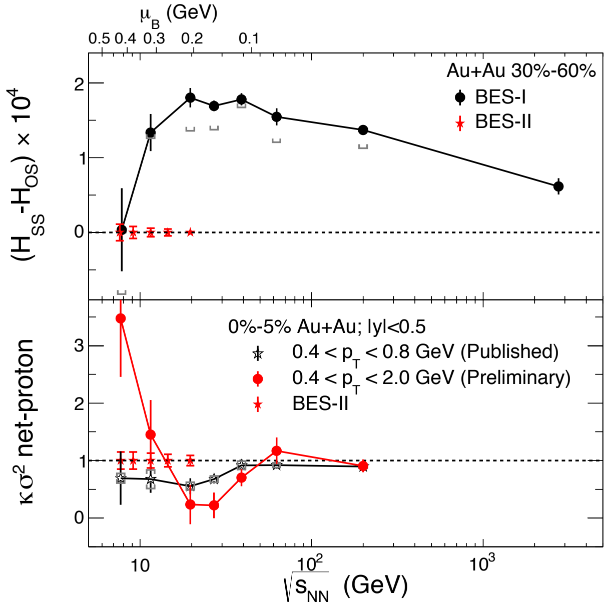



Xin An et al., Nucl.Phys.A 1017 (2022) 122343 [2108.13867 [nucl-th]]

# RHIC beam energy scan

Can we locate the QCD phase transition, either by locating a critical point, or identifying a first order transition? Can we identify anomalous transport phenomena in the QGP?




Provide theoretical tools for analyzing BES II data.

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# The BEST framework for the search for the QCD critical point and the chiral magnetic effect

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Also see: https://bitbucket.org/bestcollaboration/

# Outline:

1. Lattice QCD results
2. Critical fluctuations and anomalous transport
3. EoS with 3D-Ising model critical point
4. Initial conditions
5. Hydrodynamics
6. Non-critical contributions to proton number fluctuations
7. Particlization and kinetic transport
8. Global modeling and analysis framework

# Outline:

1. Lattice QCD results
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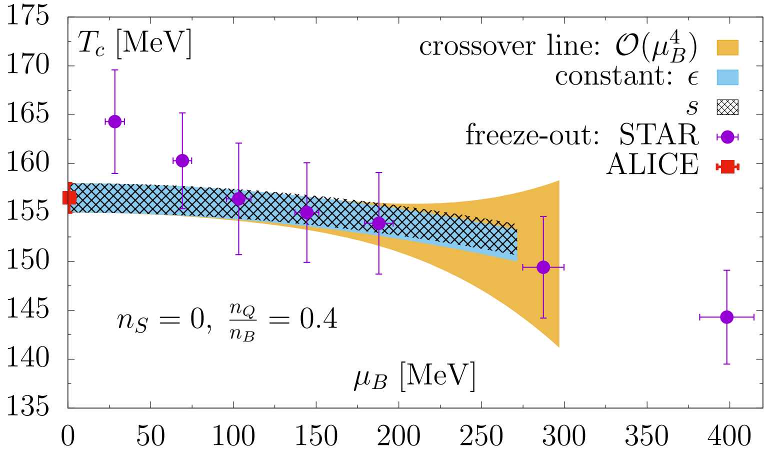
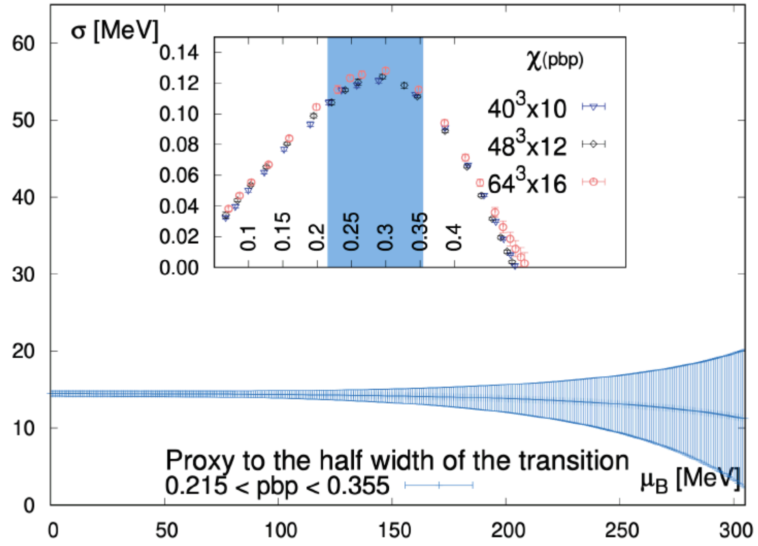
6. Particlization and kinetic transport

# 1. Lattice results: Crossover transition



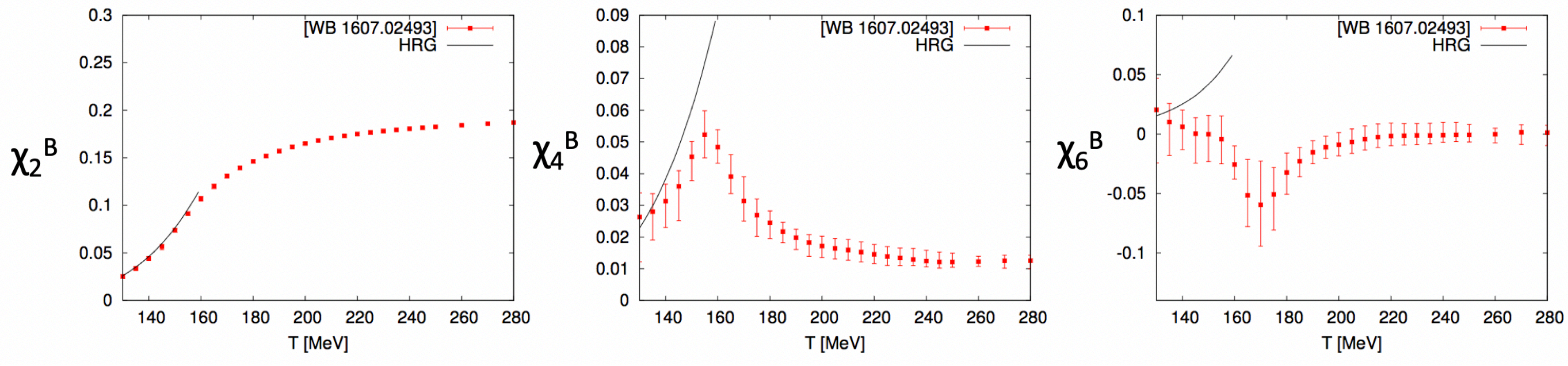

Curvature of crossover transition.

Width of transition $\Delta \sim 15$ MeV.

Roughly constant, no hint of sharpening.

Bazavov et al. [1812.08235], Borsanyi et al. [2002.02821].

# Lattice results: Baryon number cumulants ($\mu_B = 0$)



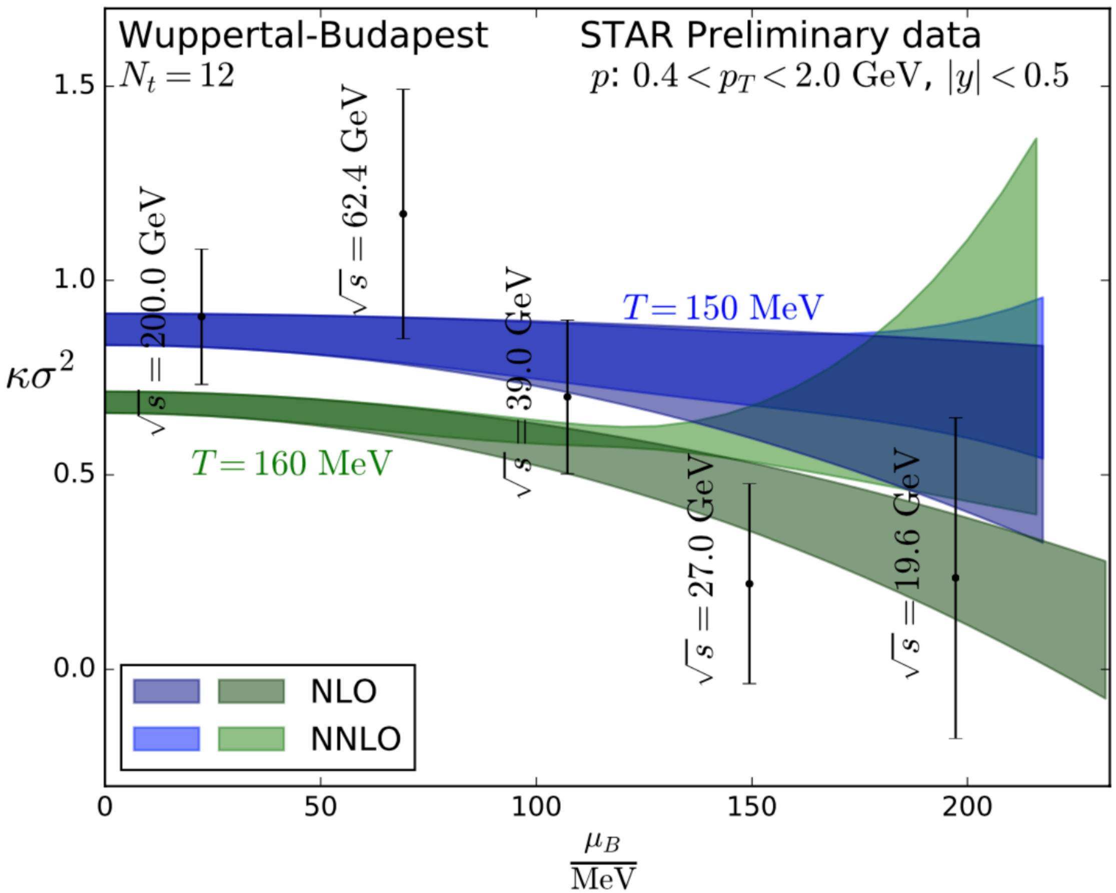

Baryon number (as well as charge and strangeness) cumulants

$$\chi_n^i = \frac{\partial^n(\beta^4 P)}{\partial(\beta\mu_i)^n} \qquad (i = B, Q, S)$$

link the EOS and fluctuation observables, and can be computed at zero (or small) chemical potential

J. Guenther et al [1607.02493]. Here: $\langle n_s \rangle = 0$, $\langle n_Q \rangle = 0.4 \langle n_B \rangle$.

# Ratio of cumulants: Lattice vs. BES data




Borsanyi [1805.04445]

STAR [1503.02558]

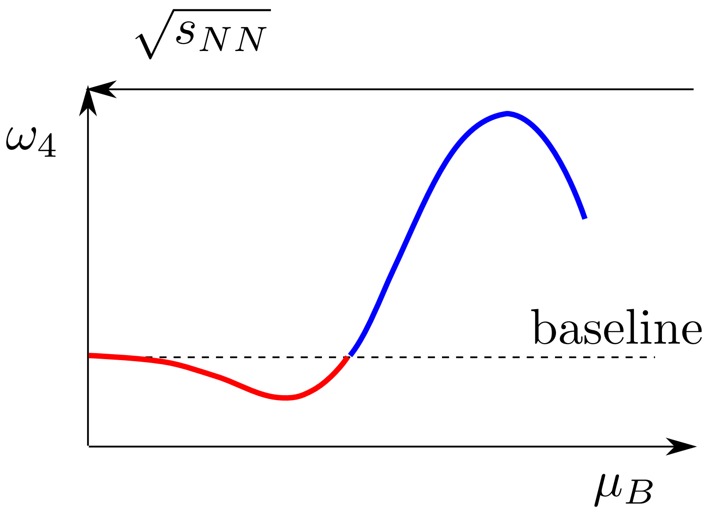
Also: $R_{42}$ vs $R_{31}$.

Bazavov [2001.08530]

Ratios of cumulants can be compared to data at freezeout

$$\kappa\sigma^2 = \frac{\langle(\Delta N_B)^4\rangle_c}{\langle(\Delta N_B)^2\rangle}$$

Simple discovery mode: Non-monotonic variation




Real world may well be more complicated:

- Critical point beyond the regime probed by lattice.
- Non-equilibrium effects important.
- Freezeout, resonances, global charge conservation, etc.

Motivates BEST (non-lattice) theory effort.

Figure from Bzdak et al. [1906.00936]

## 2. Equilibrium fluctuations

Consider an Ising-like system with order parameter $\psi$. Fluctuations governed by an entropy functional

$$Prob[\psi, \epsilon] \sim \exp(S[\psi, \epsilon]) \qquad\qquad S = \int d^3x \, s(\psi, \epsilon)$$

energy density $\epsilon$, order parameter $\psi$

Conjugate variables

$$x^A = (\epsilon, \psi) \qquad\qquad X_A = -\frac{\partial s}{\partial x^A} = (r, h)$$

reduced temperature $r$, magnetic field $h$

QCD: Canonical pair

$$x^a = (e, n) \qquad\qquad X_a = (-\beta, \beta\mu)$$

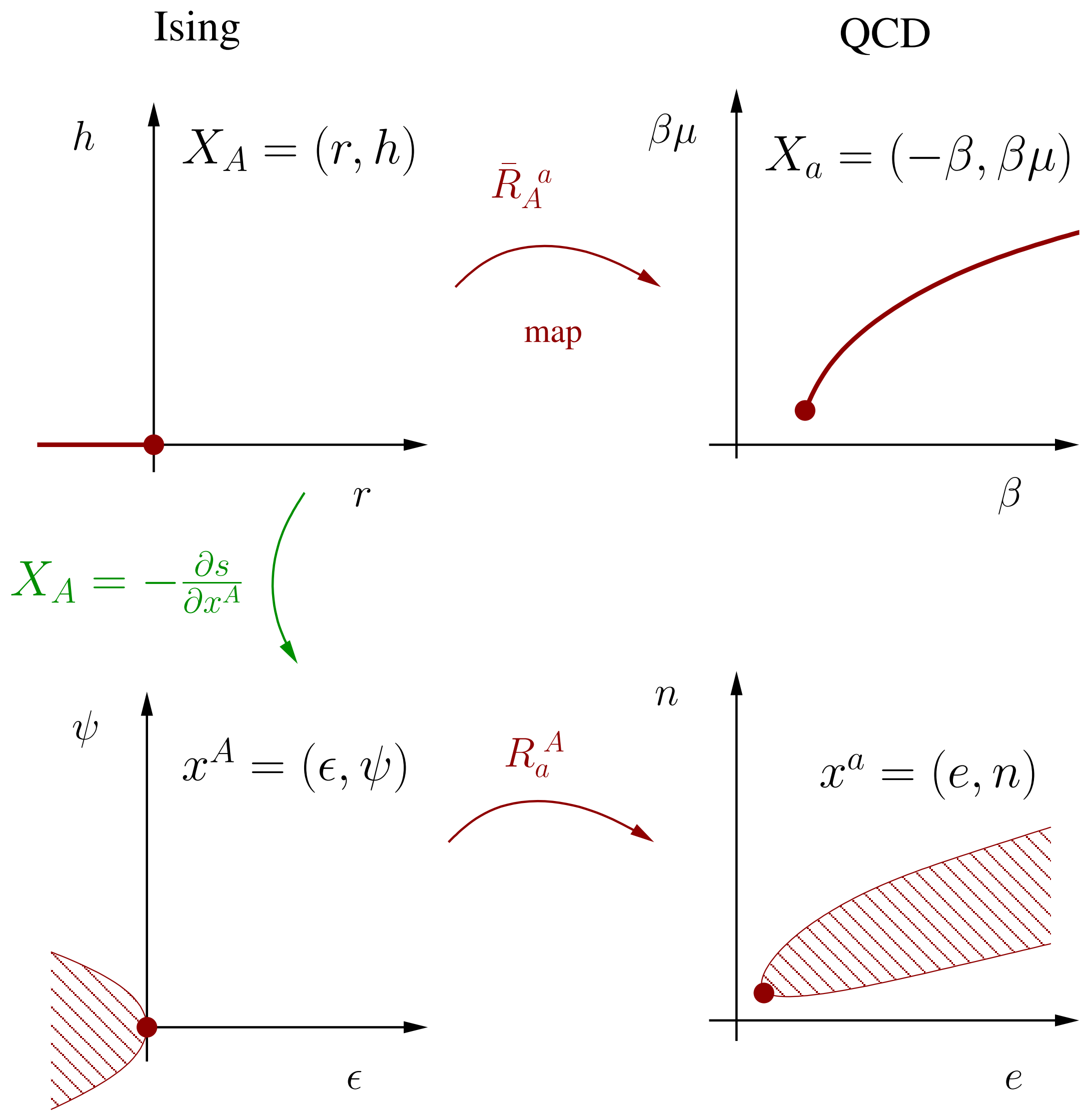
energy density $e$, baryon density $n$

inverse temperature $\beta$, chemical potential $\mu$

# Mapping the Ising EOS to QCD

intensive variables

densities




$$R_b^A = (\partial x^A)/(\partial x^b) \qquad \bar{R}_A^b = (\partial X_A)/(\partial X_b) \qquad R_b^A \bar{R}_C^b = \delta_C^A$$

# 3. BEST equation of state

Parotto et al. write

$$P(T,\mu_B) = T^4 \sum_n c_{2n}^{reg}(T)(\beta\mu_B)^{2n} + P^{crit}(T,\mu_B)$$

where $c_{2n}^{reg}$ is adjusted to reproduce lattice $\chi_n^B(T)$ and the critical part is determined by a linear map to the Ising EOS (parameterized by Zinn-Justin)

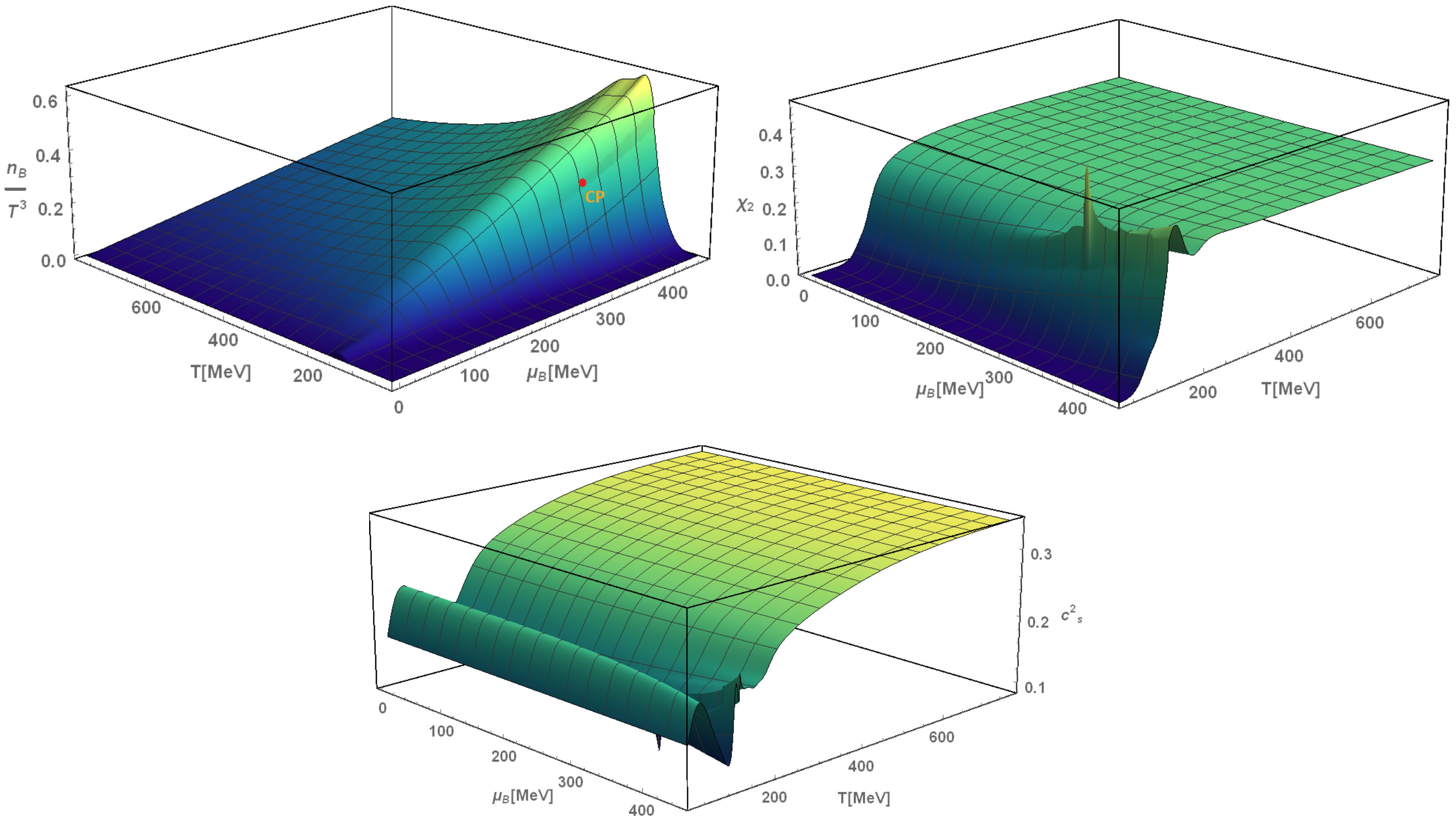
$$\begin{aligned} \frac{T-T_c}{T_c} &= \bar{w}\,(r\bar{\rho}\sin\alpha_1 + h\sin\alpha_2) \\ \frac{\mu-\mu_c}{T_c} &= \bar{w}\,(-r\bar{\rho}\cos\alpha_1 - h\cos\alpha_2) \end{aligned}$$

parameters

$(\mu_c, T_c, \bar{w}, \bar{\rho}, \alpha_1, \alpha_2)$

Connect to hadron gas at low $T$, and impose thermodynamic constraints.

# A critical equation of state for QCD




Baryon density, compressibility, speed of sound.

Parotto et al. [1805.05249]

# Application: Critical bulk viscosity

Bulk viscosity from order parameter relaxation

$$\zeta \sim (\gamma n T R_n^\epsilon)^2 \int d^3k \; \frac{2T\chi_k^2}{-i\omega + 2\Gamma_k}\bigg|_{\omega\to 0} \sim (\gamma n T R_n^\epsilon)^2 \xi^4$$

Critical bulk viscosity

$$\frac{\zeta}{s} = \sin^2(\alpha_1)\left(\frac{4\pi}{s/\eta}\right)\left(\frac{\xi}{\xi_0}\right)^{2.8}\begin{cases} 3.4\cdot 10^{-2} & r>0 \\ 2.2\cdot 10^{-1} & r<0 \end{cases}$$

$z \simeq 3$ dynamical critical exponent.

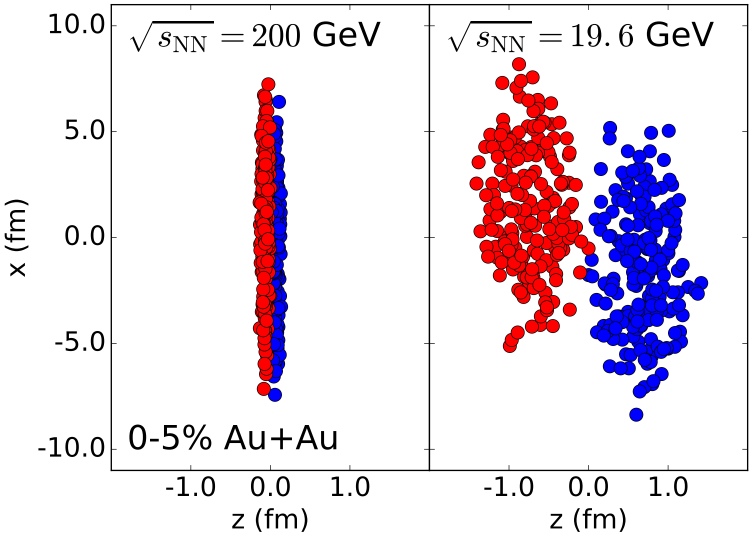
$\sin(\alpha_1)$: angle between Ising $r$ and QCD temperature.

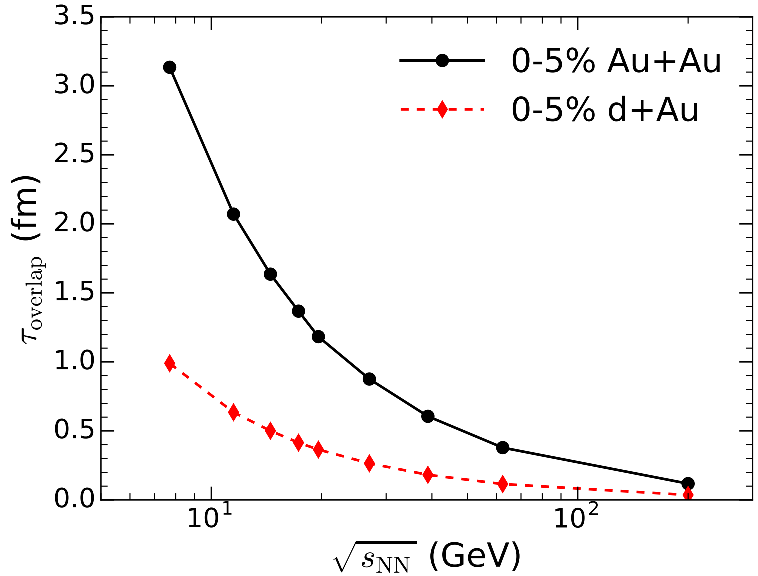
[Note: For $\sin(\alpha_1) \sim 0$ get $\zeta/s \sim (n/s)^2$.]

Amplitude ratio $(\gamma_-/\gamma_+)^2 \simeq 6$.

Martinez, T.S., Skokov [1906.11306]; see also Stephanov & Yin [1712.10305], An et al. [1912.13456], and Onuki, PRE 55 403 (1997).

# 4. Initial conditions




Issue: Extended overlap phase, cannot cleanly separate initial state from hydrodynamic evolution.

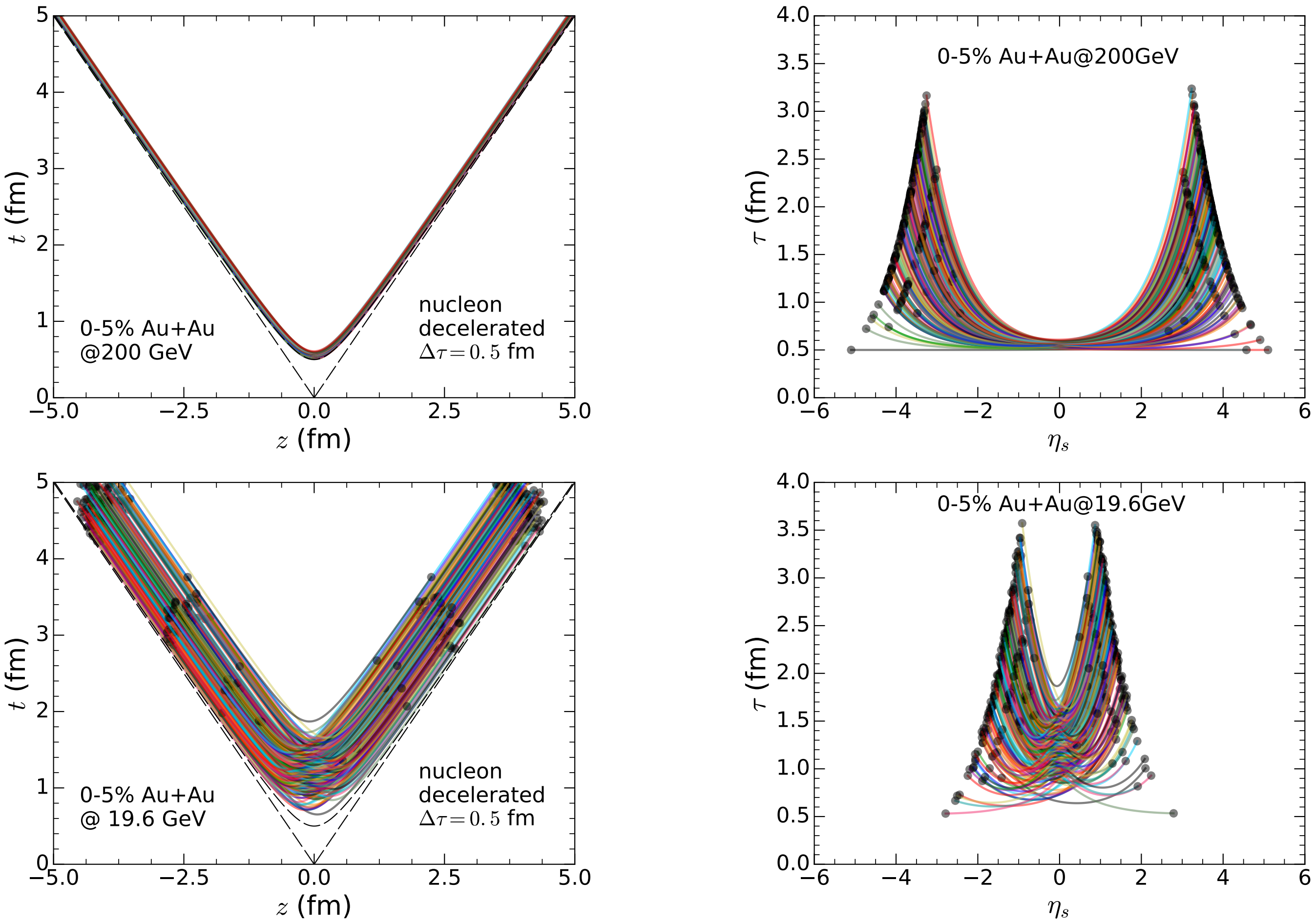
C. Shen, B. Schenke [1710.00881].

# String model

200 GeV

19.6 GeV

SS [1710.00881].

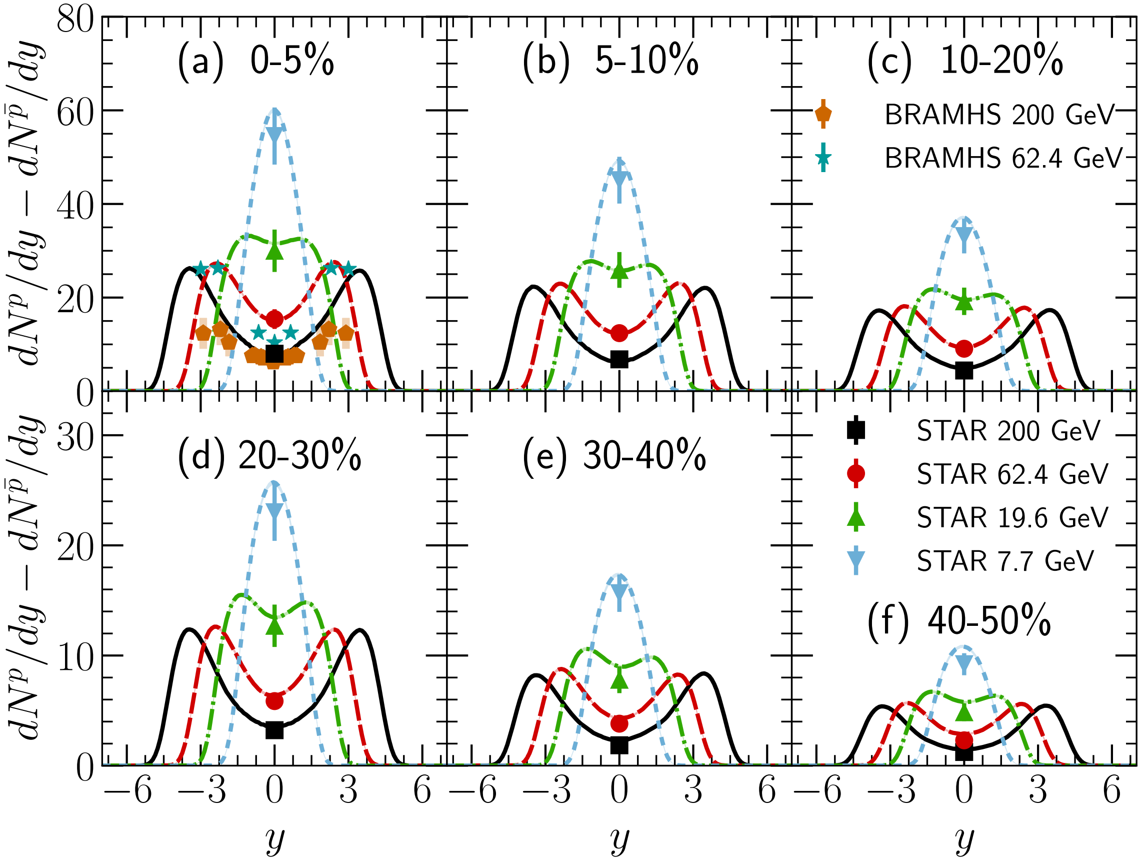



Space-time ($t - z$ and $\tau - \eta_s$) distribution of sources.

High energy/central rapidity: Single $\tau_0$ hyper-surface.

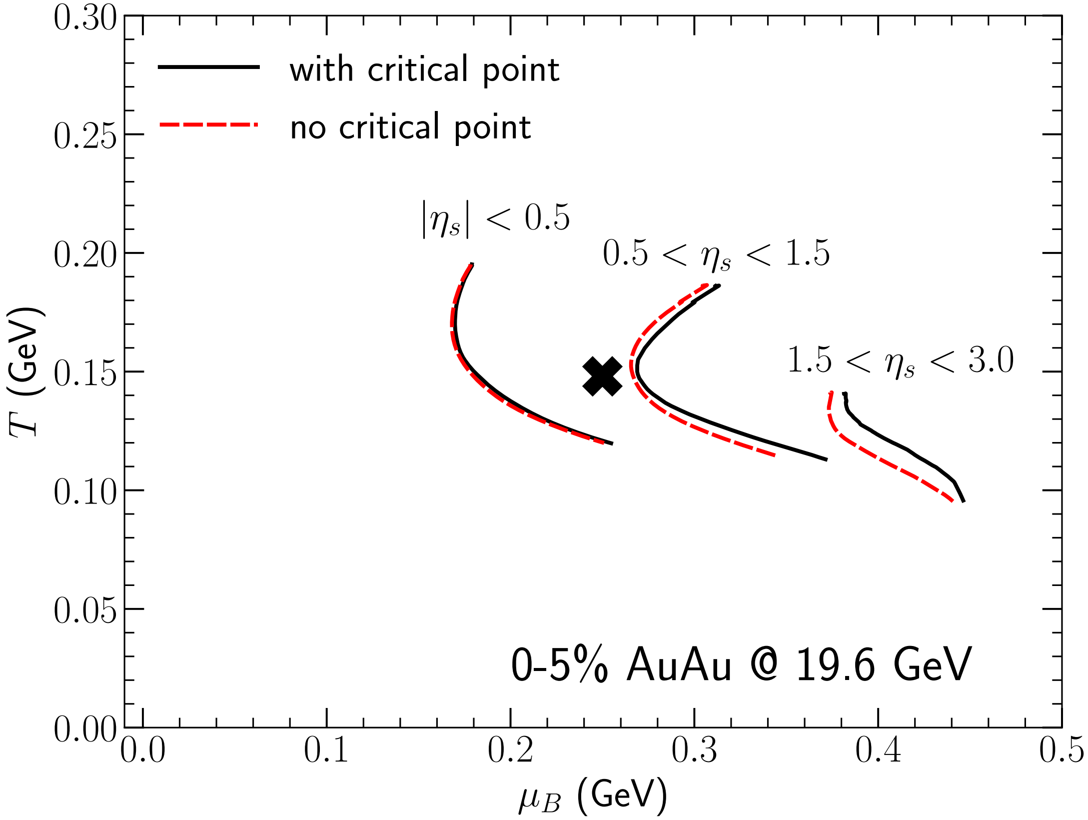
Low energy/forward: Distribution of $\tau$.

# Compare to STAR/BRAHMS net proton $dN/dy$




Significant evolution of $dN_{p-\bar{p}}/dy$ with energy. Shen, Schenke [2108.04987] and to appear.

# 5. Hydrodynamics




Average trajectories from 3+1 viscous hydro Shen [2003.05852]

BEShydro: 3+1d hydrodynamics with baryon diffusion Du, Heinz [1906.11181].

# Hydrodynamic equation for critical mode

Equation of motion for critical mode $\psi$ coupled to momentum density $\vec{\pi}$ ("model H")

$$\frac{\partial\psi}{\partial t} = \lambda\nabla^2\frac{\delta\mathcal{F}}{\delta\psi} - g\vec{\nabla}\psi\cdot\frac{\delta\mathcal{F}}{\delta\vec{\pi}_T} + \zeta_\psi$$

Diffusion Advection Noise

Free energy functional: Order parameter $\psi$, momentum density $\vec{\pi} = w\vec{v}$

$$\mathcal{F} = \int d^dx\,\left[\frac{1}{2w}\vec{\pi}^2 + \frac{\kappa}{2}(\vec{\nabla}\psi)^2 + \frac{m^2}{2}\psi^2 + \frac{v}{4}\psi^4\right] \qquad D = m^2\lambda$$

Noise average (noise kernel $L = DT\nabla^2$)

$$\langle O\rangle = \frac{1}{Z}\int D\zeta_\psi\; O(\psi(x,t))\;\exp\left(-\frac{1}{4}\int d^3x\,\zeta_\psi L^{-1}\zeta_\psi\right)$$

# Approaches to stochastic fluid dynamics

| Stochastic fluid dynamics | Stochastic Effective Actions |
|---|---|
| $\partial_t \psi = \lambda \nabla^2 \frac{\delta \mathcal{F}}{\delta \psi} + \zeta$ <br> $\langle \zeta(x,t)\zeta(x',t')\rangle = 2\lambda T \delta_{xx'}\delta_{tt'}$ | $\mathcal{L} = \tilde{\psi}\left(\partial_t - D\nabla^2\right)\psi - \tilde{\psi} D T \nabla^2 \tilde{\psi}$ <br> $+\, v\, \tilde{\psi} D \nabla^2 \psi^3 + \ldots$ |
| **Hydro Kinetics** | **Hydro+** |
| $W_k(x,t) = \int d^3y\, e^{-iy\cdot k}$ <br> $\langle \psi_t(x+y)\psi_t(x-y)\rangle$ <br> $\partial_t W_k(x,t) = -2\Gamma_k\big[W_k(x,t)$ <br> $- W_k^0(x,t)\big]$ | $\Big(\partial_\tau + u\cdot\nabla\Big) W_k(x,t) =$ <br> $-2\Gamma_k\left[W_k(x,t) - W_k^0(x,t)\right]$ <br> $T^{\mu\nu} = \epsilon u^\mu u^\nu$ <br> $+\, p_{(+)}(\epsilon, W_k)\left[g^{\mu\nu} + u^\mu u^\nu\right]$ |

# Hydro+: Hydro plus slow mode

Relaxation equation for two-point function

$$\left(\partial_\tau + u\cdot\nabla\right) W_k(x,t) = -2\Gamma_k \left[W_k(x,t) - W_k^0(x,t)\right]$$

Non-equilibrium energy momentum tensor

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + p_{(+)}(\epsilon, W_k)\left[g^{\mu\nu} + u^\mu u^\nu\right] + \textit{dissipative terms}$$

Non-equilibrium entropy

$$S = \frac{1}{2}\int d^3k \; \left[\log(W_k/W_k^0) - (W_k/W_k^0) + 1\right]$$

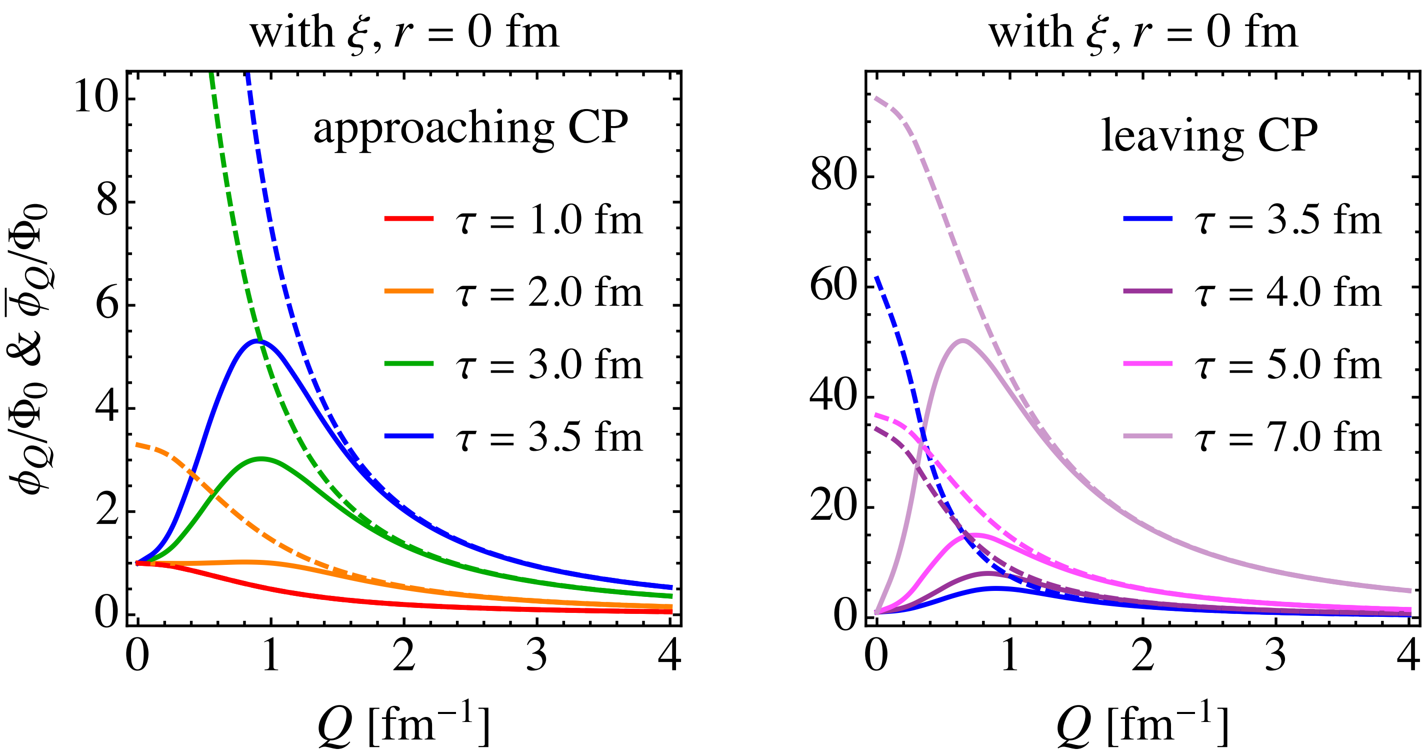
Equilibrium $W_k^0$ and $\Gamma_k$ from universality

$$W_k^0 = \frac{c_P}{n^2}\left(\frac{\xi}{\xi_0}\right)^2 \frac{1}{1+(k\xi)^2} \qquad \Gamma_k = 2\frac{\lambda\xi_0^2}{c_P\xi^4}(k\xi)^2\left[1+(k\xi)^2\right]$$

Stephanov, Yin [1712.10305]; Du et al. [2004.02719].

# Hydro+ simulations

dashed: $W_Q^0$ solid: $W_Q$



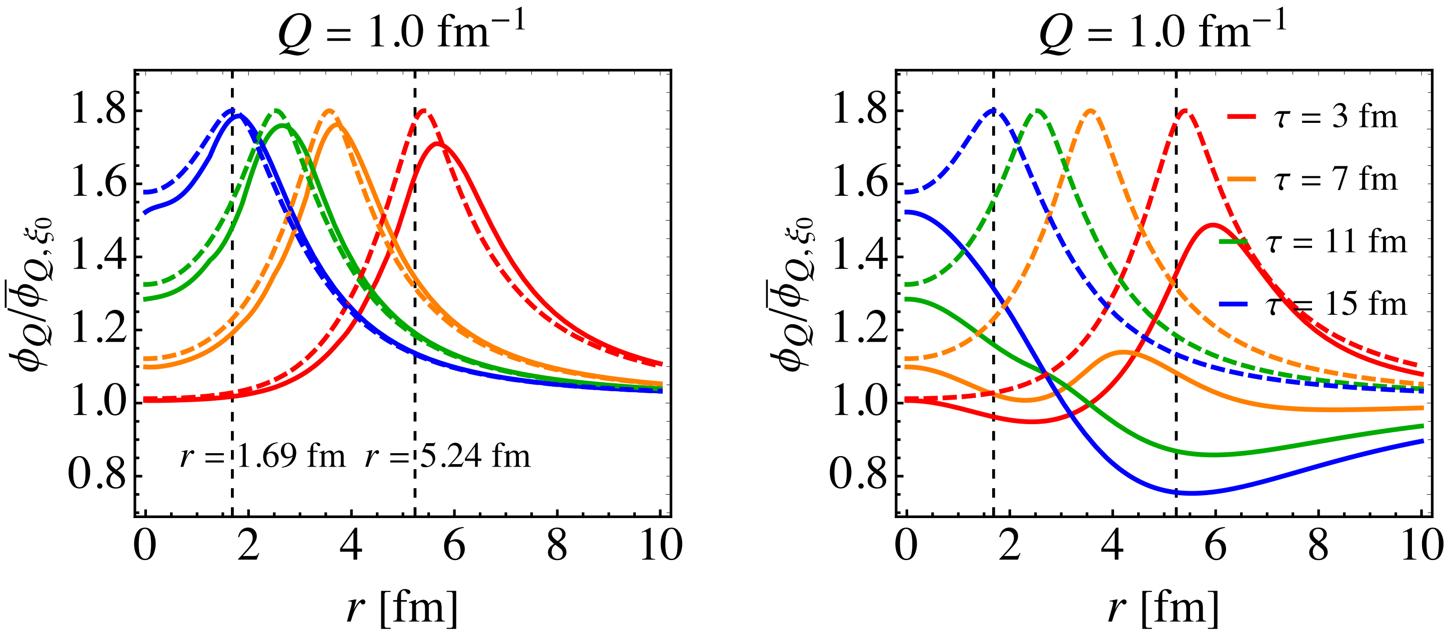

2-pt function $\phi_Q = W_Q(r=0, \tau_i)$ at the center of the fireball.

$\xi(T(\tau))$ reaches a maximum at $\tau \sim 3.5$ fm.

Background Gubser flow. Du et al. [2004.02719].

# Hydro+ simulations

dashed: $W_Q^0$ solid: $W_Q$




No advection. Advection included.

2-pt function $\phi_Q = W_Q(r, \tau_i)$ as a function of $r$ for different $\tau_i$.

Other findings: Backreaction $(\Delta s/s) \sim 10^{-4}$ small.

Background Gubser flow. Du et al. [2004.02719].

# 6. Particlization

Hydro+ particlization: Critical mode $\hat{s} = s/n$ couples to $f_A(x,p)$

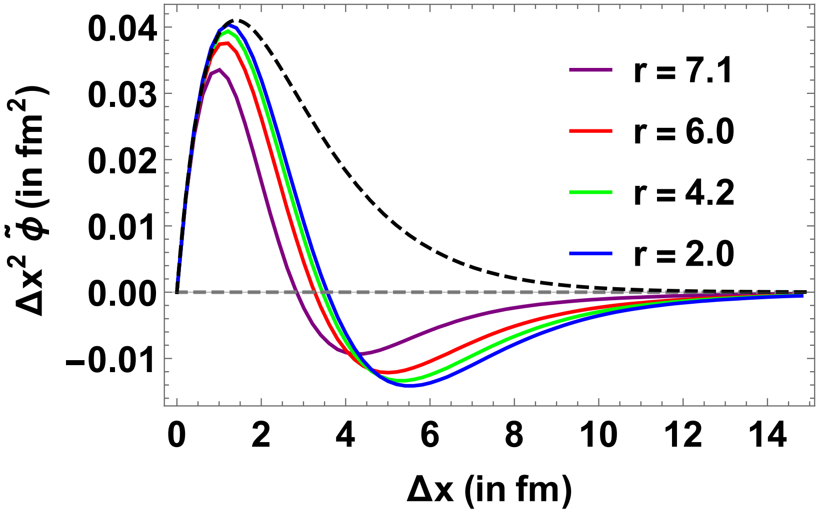
$$f_A(x,p) = \langle f_A(x,p)\rangle + g_A \frac{\partial\langle f_A(x,p)\rangle}{\partial m_A}\,\hat{s}(x)$$

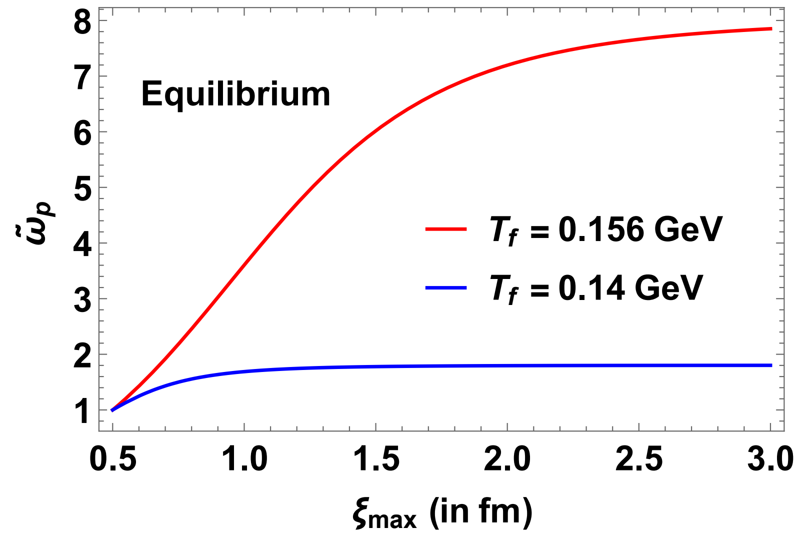
Variance of particle multiplicity

$$\langle \delta N_A^2\rangle = g_A^2 \int d\Sigma_\mu\, d\Sigma_\nu\; J_A^\mu(x_+) J_A^\nu(x_-)\; \langle \hat{s}(x_+)\hat{s}(x_-)\rangle$$

$$J_A^\mu = 2\, d_A \int \frac{d^4p}{(2\pi)^3}\delta(p^2 - m_A^2)\, p^\mu\, \frac{\partial\,\langle f_A\rangle}{\partial m_A}$$

# Hydro+ Particlization




$\langle \hat{s}(x_+)\hat{s}(x_-)\rangle$ as a function of $\Delta x = [\Delta x_\perp^2 + (\tau\Delta\eta)^2]^{1/2}$.

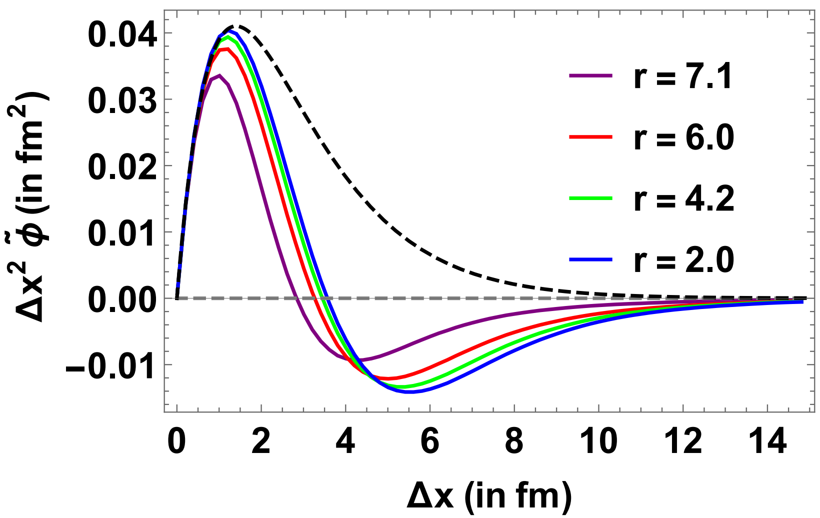
Labeled by $r$ on freezeout surface.

$\tilde{\omega}_p = \langle \delta N_p^2\rangle / \langle N_p\rangle$ vs maximum correlation length.

Equilibrium $\tilde{\omega}_p$ for different $T_{freeze}$.

Pradeep et al. [2109.13188]. Dashed curve: equilibrium expectation.

# Hydro+ Particlization

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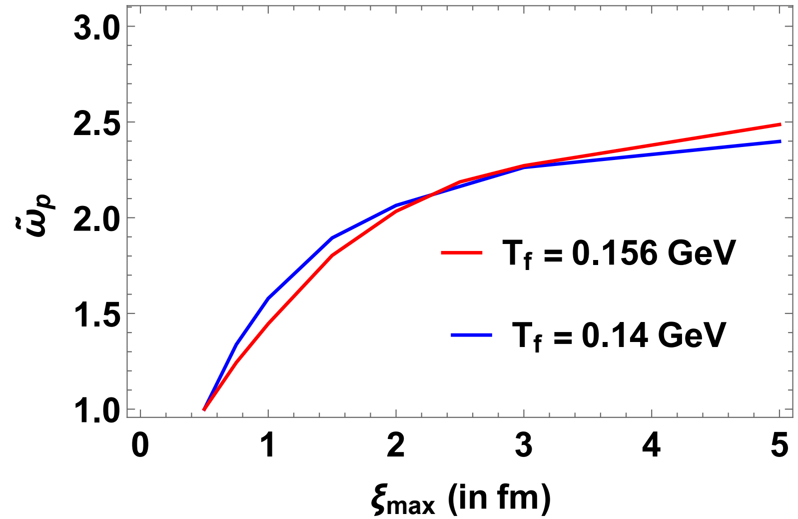


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Hydro+ $\tilde{\omega}_p$ for different $T_{freeze}$.

Pradeep et al. [2109.13188]. Dashed curve: equlibrium expectation.

# Other work

## Stochastic simulations.

Bluhm et al. [1804.05728], Singh et al., in preparation.

## Local micro-canonical particlization of fluctuation hydrodynamics.

D. Oliinychenko, V. Koch, [1902.09775].

## Hadronic afterburners with mean-field effects.

A. Sorensen, V. Koch, [2011.06635].

## Non-critical contributions.

V. Vovchenko, V. Koch, C. Shen, [2107.00163].

## BEST Bayesian analysis frame work.

In progress, see J.E. Bernhard, et al., Nat.Phys.15 (2019) 1113.

## Summary

We are looking forward to BESII data.

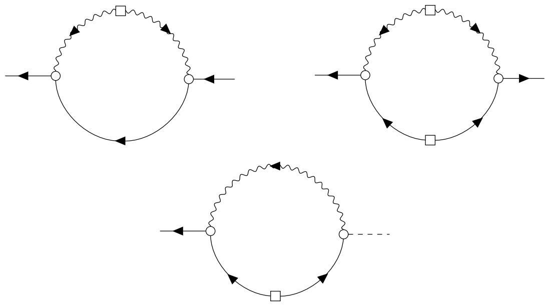
We developed a set of tools to study fluid dynamics at BES energies: Initial state, 3d hydro (incl baryon diffusion), after-burners with mean fields, micro-canonical particlization.

Also investigated tools for non-equilibrium evolution of fluctuations. Some simplifications: Back-reaction not large, growth in correlation length modest. Still working on higher order cumulants.

Integrated cumulants not the end of the story. May have to look at correlations for suitable cuts $k\xi \sim 1$.

# Further developments, other approaches

## Stochastic effective actions and multilicative noise



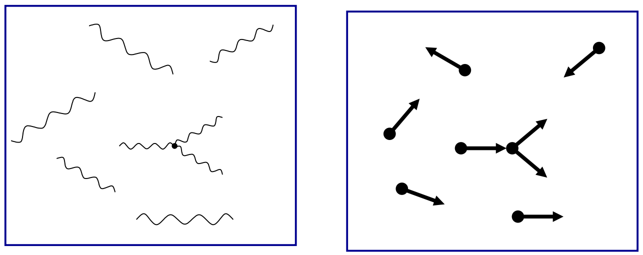
Chao, Schaefer, [2008.01269]

## Covariant Wigner-function for order parameter 2pt function

$$W(x,q) = \int d^4y\, \delta(u(x)\cdot y) e^{-iy\cdot q} \left\langle \psi\left(x+\frac{y}{2}\right)\psi\left(x-\frac{y}{2}\right)\right\rangle$$

An et al. [1912.13456]

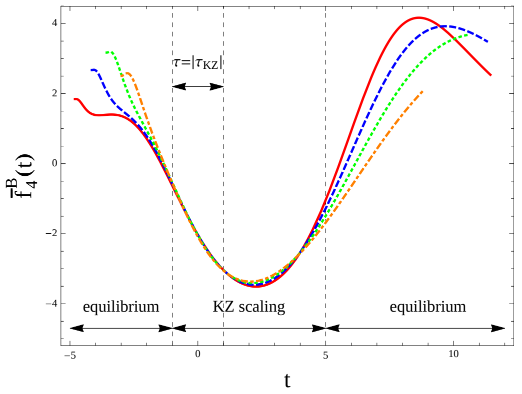
## Kinetics of hydro fluctuations



$$W(x,q) = \sum_n w_n \delta(x-x_n)\delta(q-q_n)$$

An et al. [1912.13456]

## Evolution of 4th order cumulant




S. Mukherjee et al. [1605.09341], An et al. [2009.10742]