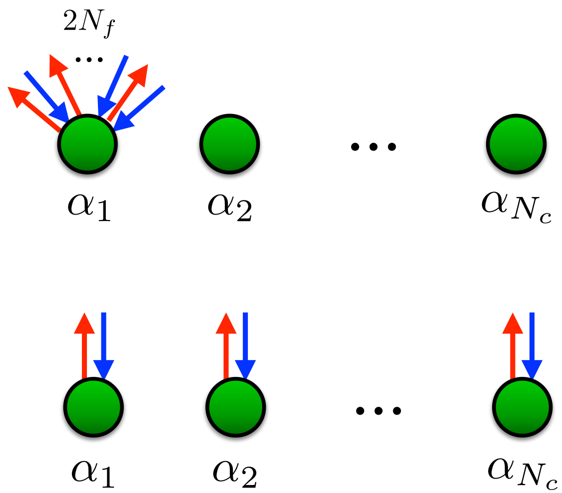
Confinement and Chiral Symmetry Breaking from Monopoles and Duality

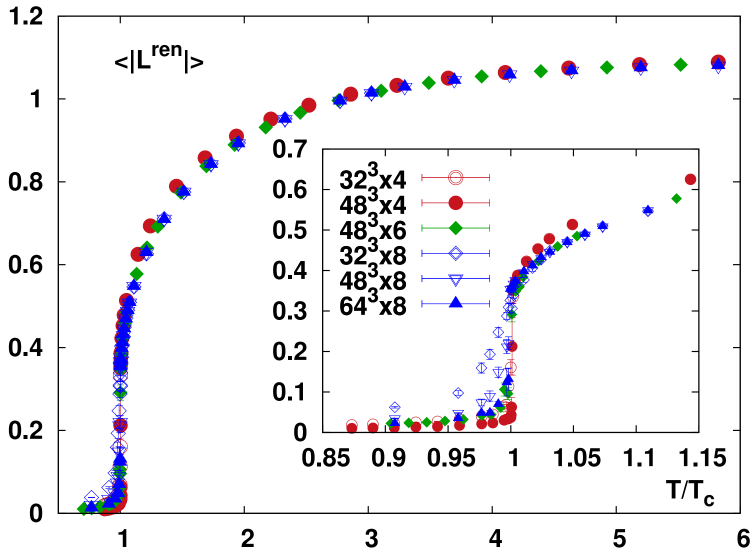
Thomas Schaefer, North Carolina State University

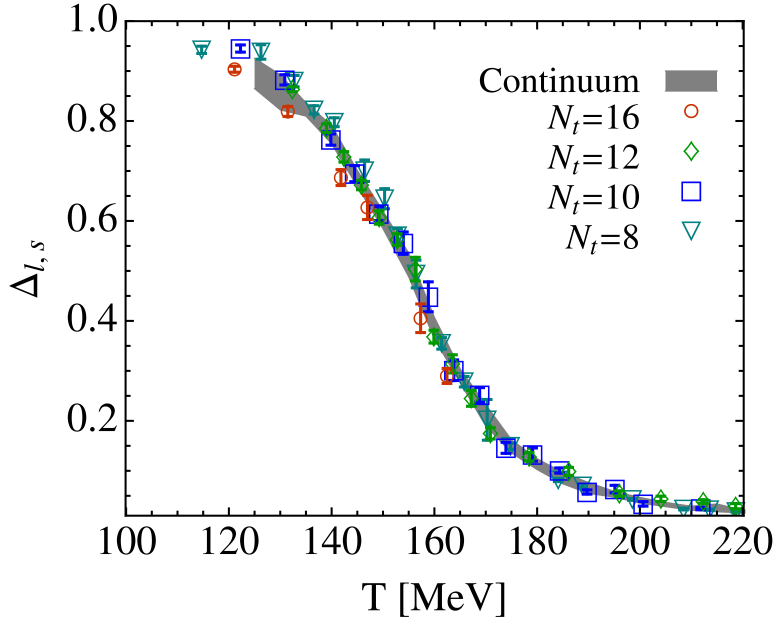


with A. Cherman and M. Unsal, PRL 117 (2016) 081601

Motivation

Confinement and chiral symmetry breaking are well established





Goal: Find deformations of QCD, continuously connected to the full theory, that exhibit χSB and confinement in weak coupling.

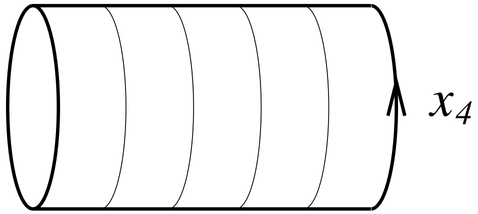
Bali et al. (2000), Borsanyi et al (2011)

Background: Confinement in Weak Coupling

Consider $SU(2)$ gauge theory with $N_f^{ad} = 1$ on $R^3 \times S_1$

$$\mathcal{L} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{i}{g^2} \lambda^a \sigma \cdot D^{ab} \lambda^b + \frac{m}{g^2} \lambda^a \lambda^a$$

$$A^a_\mu(0) = A^a_\mu(L)$$
$$\lambda^a_\alpha(0) = \lambda^a_\alpha(L)$$



Large mass limit: Pure YM. Small mass limit: SUSY YM.

Small S_1 and m: Confinement can be studied using semi-classical methods, based on monopole-instantons, instantons, and bions.

Theory abelianizes. Low energy fields: Holonomy b and dual photon σ. Perturbative potential vanishes.

Non-perturbative effects

Topological classification on $R^3 \times S_1$ (GPY)

1. Topological charge

$$Q_{top} = \frac{1}{16\pi^2} \int d^4x \, F\tilde{F}$$

2. Holonomy (eigenvalues q^α of Polyakov line at spatial infinity)

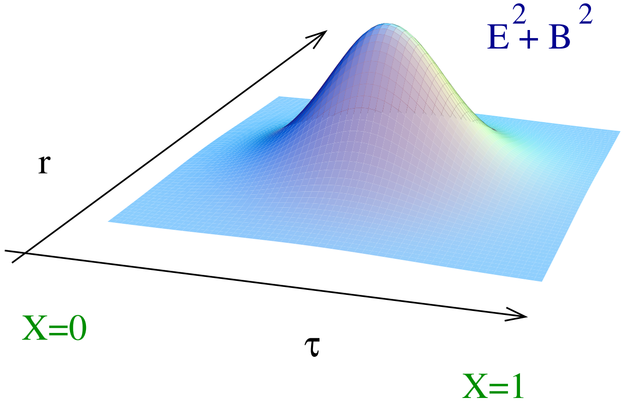
$$\langle \Omega(\vec{x}) \rangle = \left\langle \mathrm{Tr} \exp \left[i \int_0^\beta A_4 dx_4 \right] \right\rangle$$

3. Magnetic charges

$$Q_M^\alpha = \frac{1}{4\pi} \int d^2S \, \mathrm{Tr} \left[P^\alpha B \right]$$

Periodic instantons (calorons)

Instanton solution in R^4 can be extended to solution on $R^3 \times S^1$



$$Q_{top} = \pm 1$$

$$\Omega_\infty = 1 \quad Q_M^\alpha = 0$$

$SU(2)$ solution has $1+3+1+3=8$ bosonic zero modes

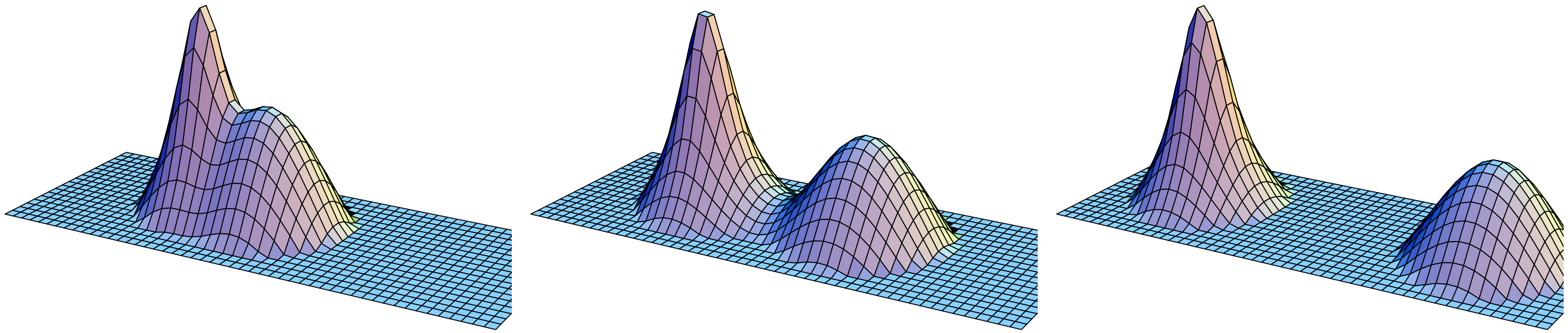
$$\int \frac{d\rho}{\rho^5} \int d^3x\, dx_4 \int dU\, e^{-2S_0} \qquad 2S_0 = \frac{8\pi^2}{g^2}$$

$4n_{adj}$ fermionic zero modes

$$\int d^2\zeta d^2\xi$$

Calorons at finite holonomy: monopole constituents

KvBLL (1998) construct calorons with non-trivial holonomy



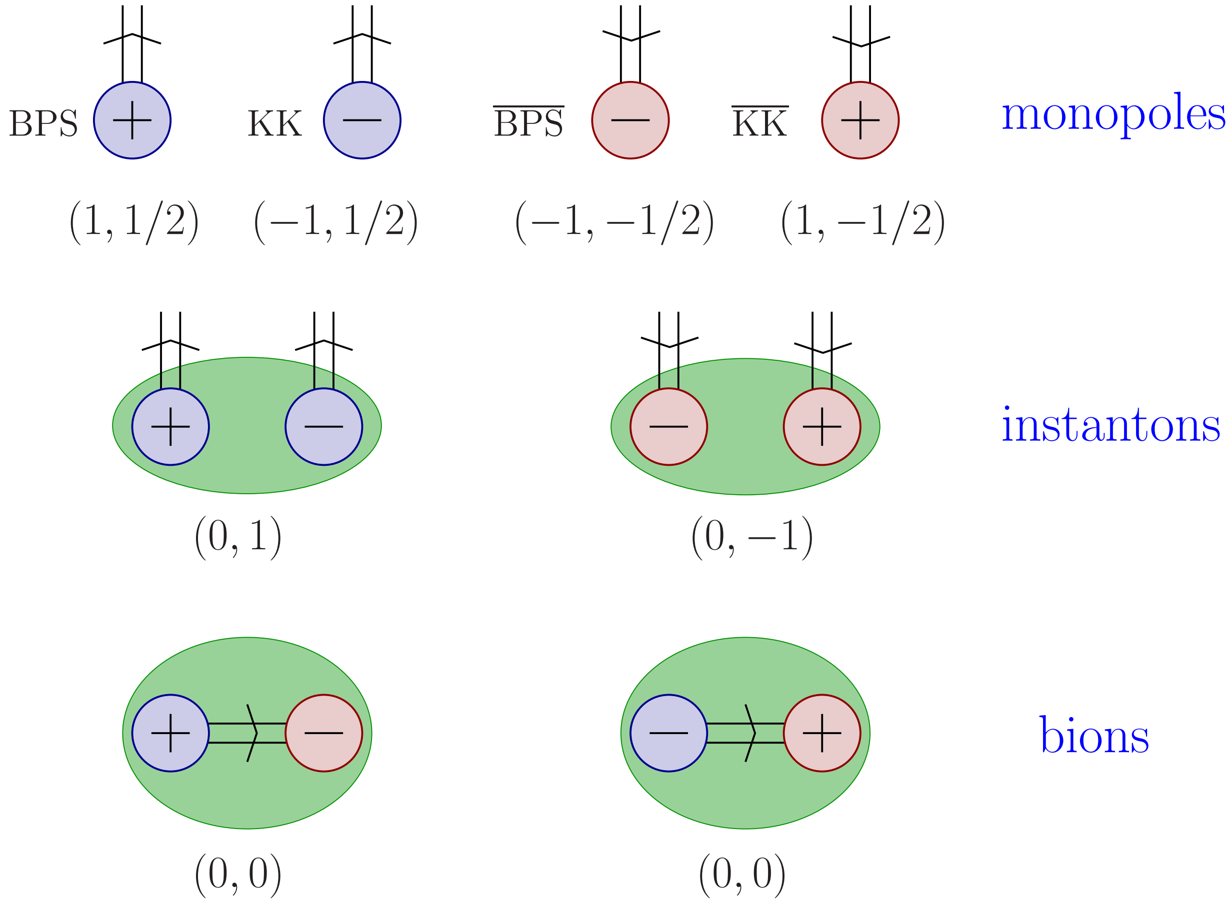
BPS and KK monopole constituents. Fractional topological charge, $1/2$ at center symmetric point.

$2 \times (3+1) = 8$ bosonic zero modes, 2×2 fermionic ZM.

$$\int d\phi_1 \int d^3x_1 \int d^2\zeta \, e^{-S_1} \int d\phi_2 \int d^3x_2 \int d^2\xi \, e^{-S_2}$$

Topological objects

$$(Q_M, Q_{top}) = (\int_{S_2} B \cdot d\Sigma, \int_{R^3 \times S_1} F\tilde{F})$$



Note: BPS/KK topological charges in Z_2 symmetric vacuum. Also have $(2, 0)$ (magnetic) bions.

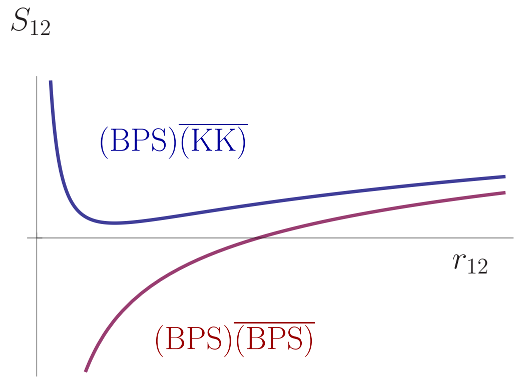
Effective potential

Instantons and monopoles: Exact solutions, but $V(b,\sigma) = 0$.

Bions: Approximate solutions

$$V_{BPS,\overline{BPS}} \sim e^{-2b} e^{-2S_0} \int d^3r \, e^{-S_{12}(r)}$$

$$S_{12}(r) = \frac{4\pi L}{g^2 r}(q_m^1 q_m^2 - q_b^1 q_b^2) + 4\log(r)$$



Saddle point integral after analytic continuation $g^2 \to -g^2$ (BZJ)

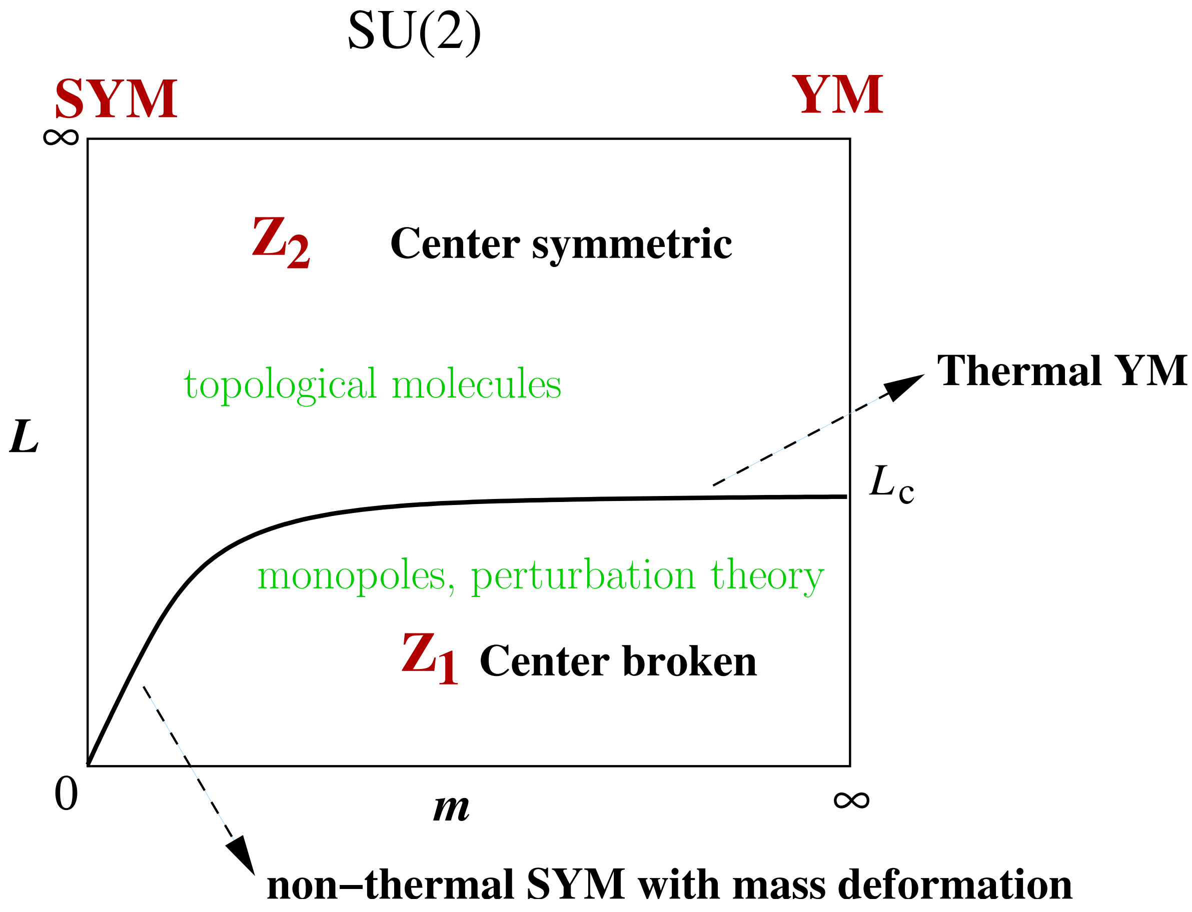
$$V(b,\sigma) \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6} \left[\cosh\left(2\left(b-b_0\right)\right) - \cos(2\sigma)\right]$$

Center symmetric vacuum $\mathrm{tr}(\Omega) = 0$ preferred

Mass gap for dual photon $m_\sigma^2 > 0$ ($\to$ confinement)

$SU(2)$ YM with $n_f^{adj} = 1$ Weyl fermions on $R^3 \times S_1$

Phase diagram in L-m plane



Direct calculation at $m = \infty$: See Shuryak, Liao, et al.

What about chiral symmetry breaking?

Original setup: One adjoint fermion, chiral symmetry is discrete.

$$\langle \bar{\lambda}\lambda \rangle \neq 0 \qquad Z_{2N_c} \to Z_2$$

Light fundamental fermions: Need strong coupling.

$$\mathcal{L} \sim G \det_{N_f}(\bar{\psi}_L \psi_R) + \text{h.c.}$$

Heavy fundamental fermions: Study explicit breaking of Z_N center symmetry.

Role of Boundary Conditions

Consider flavor twisted boundary conditions

$$\psi(\tau+\beta) = \Omega_F \psi(\tau) \qquad \Omega_F = \mathrm{diag}(1, e^{2\pi i/N_f}, \ldots, e^{2\pi i(N_f-1)/N_f})$$

Flavor holonomy Ω_F has several interesting properties:

1. $N_f = N_c$: Respects Z_{N_c} center symmetry.

2. Large L: Breaks flavor symmetry, but in a controlled fashion.

3. Small L: New semi-classical picture of chiral symmetry breaking: Distributed zero modes and color-flavor transmutation.

Large L expectations

Flavor holonomy corresponds imaginary flavor (isospin) chemical potential $\tilde{\mu}_F \sim i/L$.

Can be studied using chiral Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{Tr}\left[\nabla_\mu U \nabla^\mu U^\dagger\right] - B\mathrm{Tr}\left[MU + h.c\right]$$

with $\nabla_0 U = \partial_0 U + i[\tilde{\mu}_F T_F, U]$.

Consider $N_f = 2$ (isospin chemical potential)

$$m_{\pi^0}^2 = m_\pi^2 \qquad m_{\pi^\pm}^2 = m_\pi^2 + \tilde{\mu}_I^2$$

$N_f - 1$ exact Goldstone modes (m=0), others acquire gaps.

Small L theory: Perturbation theory

Consider center symmetric gauge holonomy (add double trace deformation). For $LN_c \lesssim \Lambda^{-1}$ theory abelianizes

$$SU(N_c) \to [U(1)]^{N_c-1}$$

Gapless (Cartan) gluons described by dual photon $\vec{\sigma}$

$$S = \frac{g^2}{8\pi^2 L} \int d^3x \, (\partial_\mu \vec{\sigma})^2$$

with $F^i_{\mu\nu} = \frac{g^2}{2\pi L} \epsilon_{\mu\nu\alpha} \partial^\alpha \sigma^i$.

Remain gapless to all orders in perturbation theory due to emergent shift symmetry $\vec{\sigma} \to \vec{\sigma} + \vec{\epsilon}$.

Small L theory: Semiclassical objects

Center symmetric background, no fermions: Instanton fractionalize into N_c constituents

$$\mathcal{M}_i \sim e^{-S_0} e^{i\vec{\alpha}_i\cdot\vec{\sigma}} \qquad S_0 = \frac{8\pi^2}{g^2 N_c} \qquad \vec{\alpha}_i \;\; SU(N_c) \text{ root vectors}$$

In the ground state these objects proliferate: The monopole-anti-monopole gas.

$$V(\vec{\sigma}) \sim m_W^3 e^{-S_0} \sum_i \cos(\vec{\alpha}_i \cdot \vec{\sigma})$$

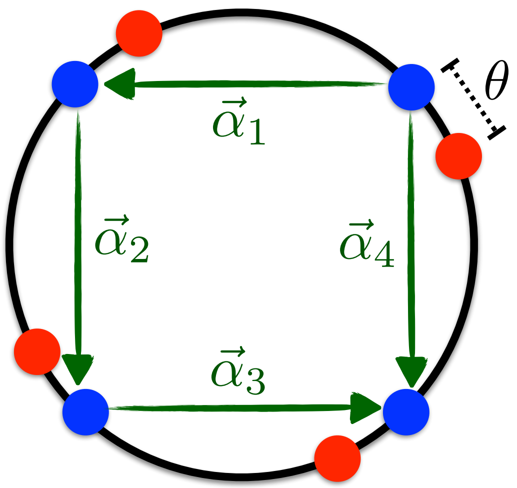
Mass gap for the dual photon, continuous shift symmetry broken.

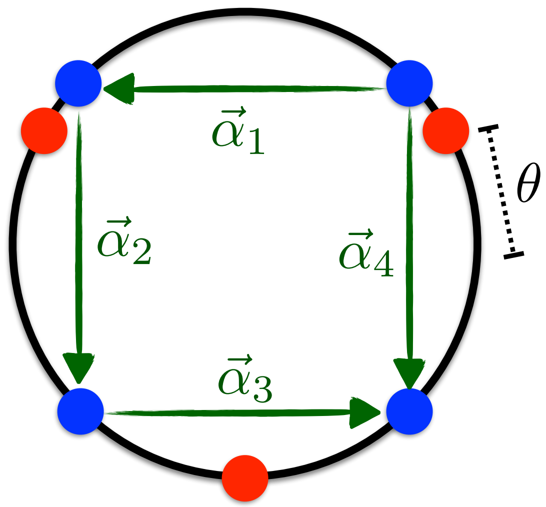
Massless fermions: Take into account fermion zero modes.

Small L theory: Fermion zero modes

Many eigenvalue circles: Polyakov line   Flavor holonomy

Instanton-monopoles θ flavor singlet twist





$N_c = N_f = 4$ $N_c = 4\ N_f = 3$

Zero modes localize on monopoles jumping over flavor eigenvalues

Bruckmann, Nogradi, van Baal (2003); Moore et al. (2014)

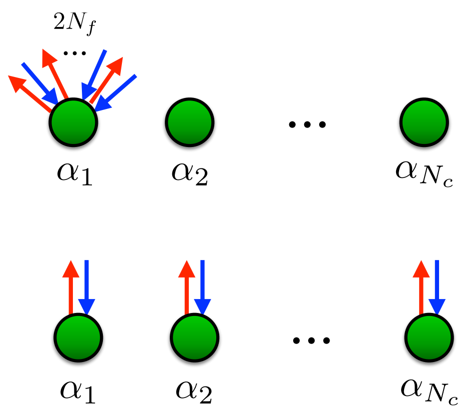
Two basic scenarios $(N_c = N_f)$

No flavor twist: Standard 't Hooft vertex carried by one monopole

$$\mathcal{M}_1 \sim e^{-S_0}\, e^{i\vec{\alpha}_1\cdot\vec{\sigma}}\, \det{}_F(\bar{\psi}_L^f \psi_R^g) \qquad \mathcal{M}_{i>1} \sim e^{-S_0}\, e^{i\vec{\alpha}_i\cdot\vec{\sigma}}$$

Center symmetric flavor holonomy: Single flavor 't Hooft vertex carried by each monopole

$$\mathcal{M}_i \sim e^{-S_0}\, e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\, (\bar{\psi}_L^i \psi_R^i)$$



trivial flavor holonomy

center symmetric holonomy

Spontaneous symmetry breaking

Unbroken symmetries of flavor twisted theory

$$[U(1)_J]^{N_c-1} \times [U(1)_V]^{N_f-1} \times [U(1)_A]^{N_f-1} \times U(1)_Q$$

Shift symmetry — Exact flavor symmetry

Symmetries of monopole vertex

$$\mathcal{M}_i \sim e^{-S_0}\, e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\, (\bar{\psi}_L^i \psi_R^i)$$

Preserves vectorial symmetry $[U(1)_V]^{N_f-1} \times U(1)_Q$. Breaks axial symmetry

$$[U(1)_A]^{N_f-1}: \qquad (\bar{\psi}_L^f \psi_R^f) \to e^{i\epsilon_i} (\bar{\psi}_L^i \psi_R^i)$$

Spontaneous symmetry breaking, continued

Monopole vertex is invariant provided $[U(1)_A]^{N_f-1}$ is combined with $[U(1)_J]^{N_c-1}$ shift symmetry

$$[\tilde{U}(1)_A]^{N_f-1}: \quad \begin{cases} (\bar{\psi}_L^f \psi_R^f) & \to \quad e^{i\epsilon_i}(\bar{\psi}_L^i \psi_R^i) \\ e^{i\vec{\alpha}_i\cdot\vec{\sigma}} & \to \quad e^{-i\epsilon_i} e^{i\vec{\alpha}_i\cdot\vec{\sigma}} \end{cases}$$

Ground state $\langle e^{i\vec{\alpha}_i\cdot\vec{\sigma}}\rangle \to 1$. Breaks

$$[U(1)_V]^{N_f-1} \times [\tilde{U}(1)_A]^{N_f-1} \to [U(1)_V]^{N_f-1}$$

For $m = 0$ the ground state is degenerate. Massless Goldstone boson

$$S_\sigma = L \int d^3x \left\{ \frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] - B\,\mathrm{Tr}\left[M\Sigma + h.c.\right] \right\}$$

Microscopically $\Sigma = e^{i\Pi/f_\pi}$ with $\Pi = \pi^a T^a$ and $\pi^a = \frac{g}{2\pi L}\sigma^a$

Color-flavor transmutation

Discrete symmetries and anomaly matching

Discrete symmetries

$Z_{2N_f} \in U(1)_A$ $\qquad$ $Z_D \in Z_{N_c} \times Z_{N_f}^{perm}$

't Hooft vertex $\qquad$ color-flavor center

Mixed $[Z_d]^2 \times Z_{2N_f}$ anomaly can be studied along the lines of Gaiotto et al. Introdude 1 and 2-form gauge fields, obtain anomaly

$$\mathcal{A} = -\frac{N}{2\pi} \int B_c^{(1)} \wedge B_f^{(2)} \in \frac{2\pi}{N} Z$$

Matching requires Z_d or Z_{2N_f} to be broken (or more exotic phases)

Here: Z_d preserved, Z_{2N_f} broken, and $U(1)_L^{N-1} \times U(1)_R^{N-1}$ breaking comes along for the ride.

Tanizaki et al. 1711.10487, Cherman et al. 1711.10567.

Chiral Lagrangian

Chiral lagrangian has calculable coefficients

$$S_\sigma = L \int d^3x \left\{ \frac{f_\pi^2}{4} \mathrm{Tr}\left[\partial_\mu \Sigma \partial^\mu \Sigma^\dagger\right] - B \mathrm{Tr}\left[M\Sigma + h.c.\right] \right\}$$

$$f_\pi^2 = \left(\frac{g}{\sqrt{6}\pi L} \right)^2 = \frac{N_c \lambda m_W^2}{24\pi^2}$$

$$B = -\frac{1}{2}\langle \bar{\psi}\psi \rangle \sim m_W^{-3} e^{-\frac{8\pi^2}{\lambda}}$$

Also note: VEV of monopole operator can be viewed as effective constituent quark mass

$$m_Q \sim m_W e^{-\frac{8\pi^2}{\lambda}}$$

Conclusions and Outlook

Calculable mechanism for chiral symmetry breaking and confinement in compactified versions of QCD.

Results consistent with continuity between large L, m (full QCD) and small L, m theory.

Mechanism based on monopole instantons and color flavor transmutation.

Study: Relation between χSB and confinement? Chiral phase transition?