Nearly Perfect Fluidity: From Atoms to Quarks

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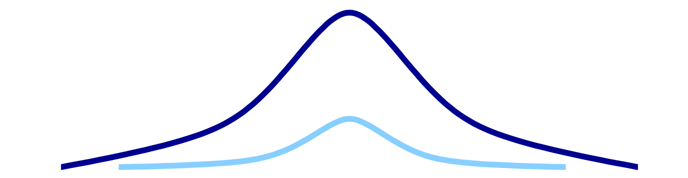
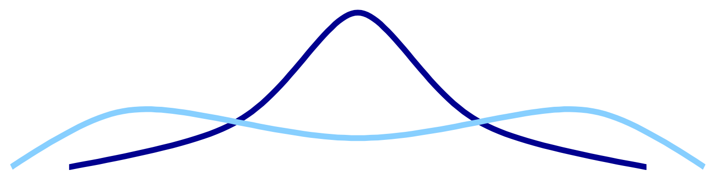
See T. Schaefer, D. Teaney, “Perfect Fluidity” [arXiv:0904.3107]

Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



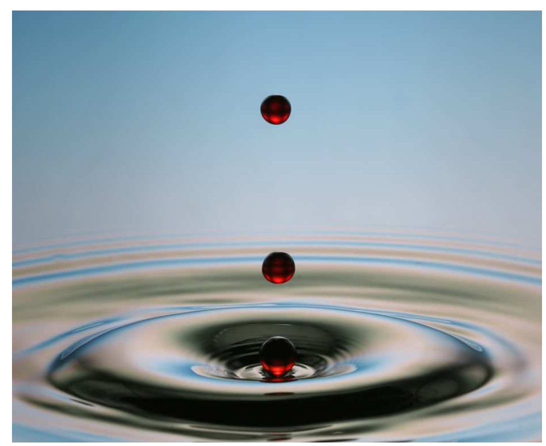
$\tau \sim \tau_{micro}$



$\tau \sim \lambda$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



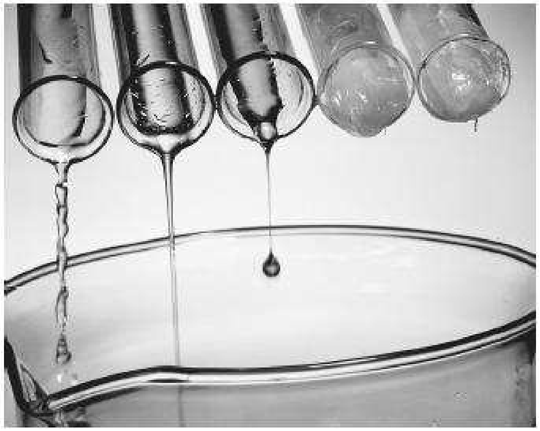
Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$



Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

reactive　　dissipative　　2nd order

Expansion $\Pi^0_{ij} \gg \delta \Pi^1_{ij} \gg \delta \Pi^2_{ij}$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$Re \quad = \quad \frac{\hbar n}{\eta} \quad \times \quad \frac{m v L}{\hbar}$$

fluid property   flow property

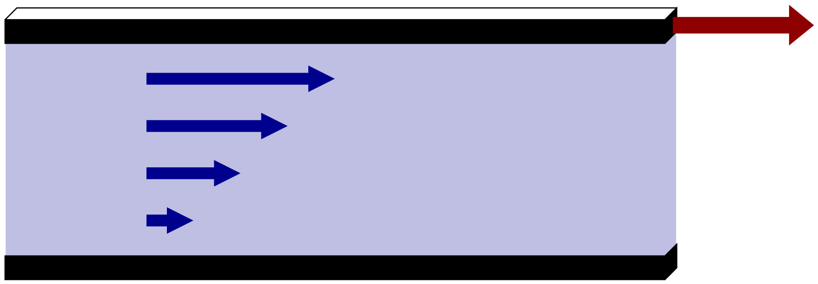
Kinetic theory estimate: $\eta \sim n p l_{mfp}$

$$Re^{-1} = \frac{v}{c_s} Kn \qquad Kn = \frac{l_{mfp}}{L}$$

expansion parameter $Kn \ll 1$

Shear viscosity

Viscosity determines shear stress ("friction") in fluid flow

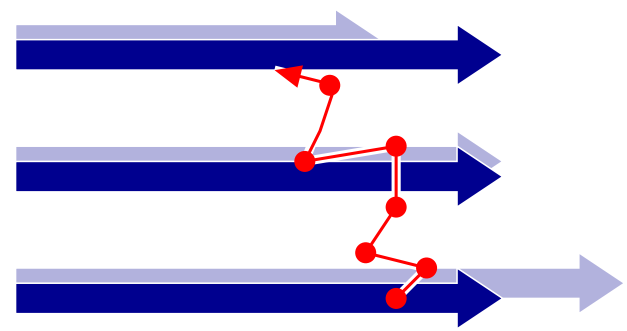


$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$



Dilute, weakly interacting gas: $l_{mfp} \sim 1/(n\sigma)$

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

independent of density!

Shear viscosity

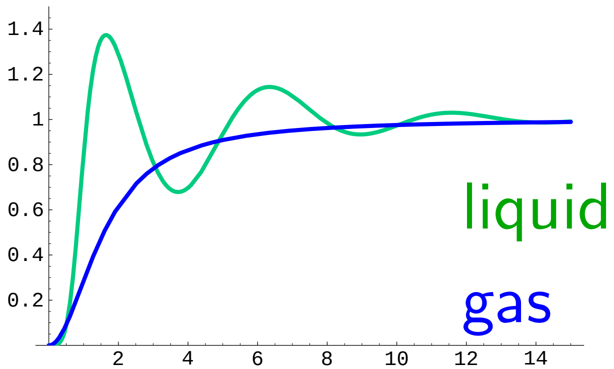
non-interacting gas ($\sigma \to 0$): $\eta \to \infty$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

strongly interacting gas: $\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$

but: kinetic theory not reliable!

what happens if the gas condenses into a liquid?



Eyring, Frenkel:

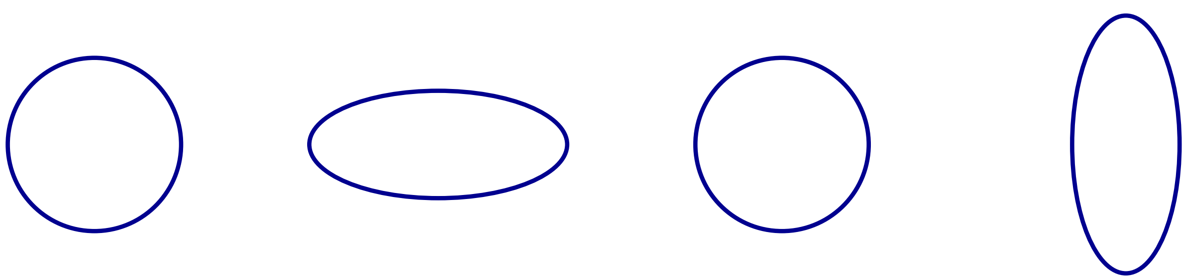
$$\eta \simeq hn \exp(E/T) \geq hn$$

Holographic Duals: Transport Properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}} \qquad\qquad g_{\mu\nu} = g^0_{\mu\nu} + \gamma_{\mu\nu}$$



Holographic Duals: Transport Properties

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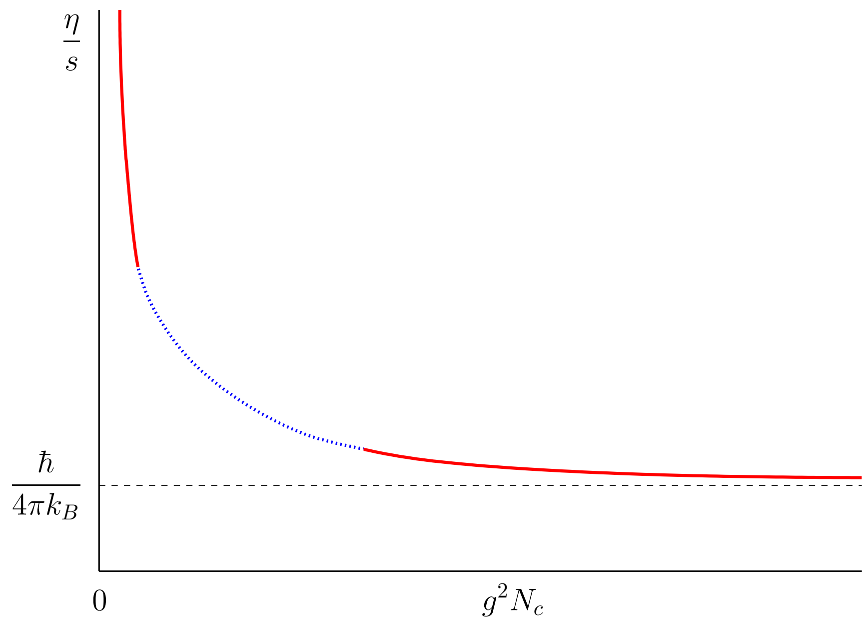
Strong coupling limit

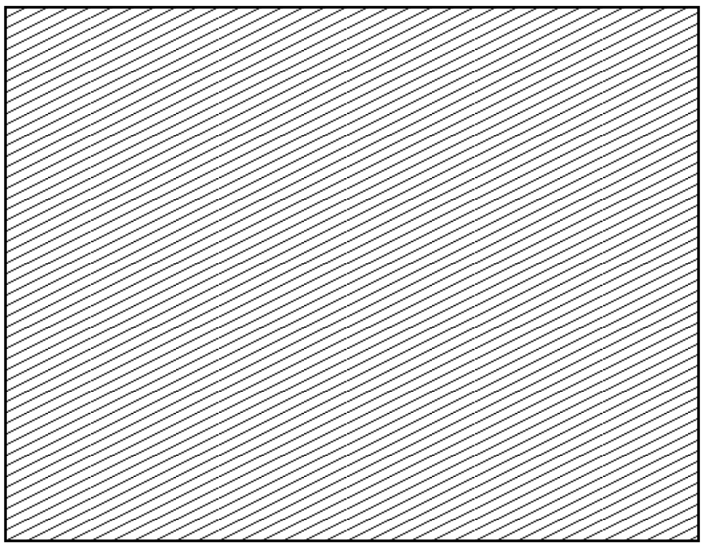
$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets (2001)

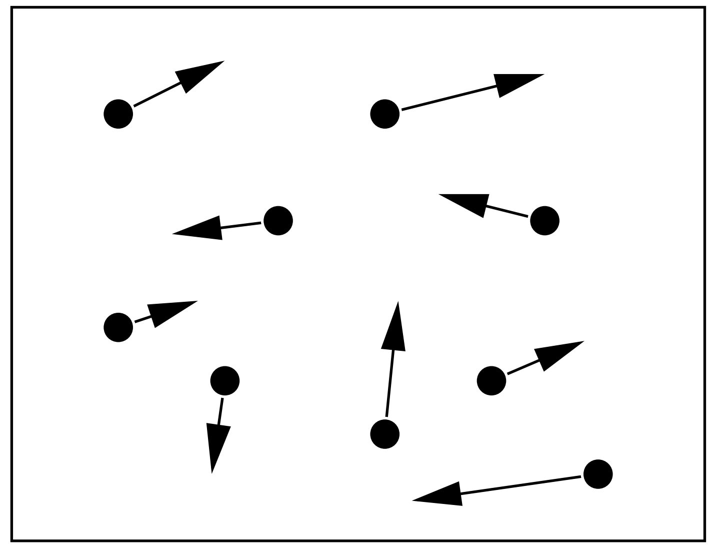


Strong coupling limit universal? Provides lower bound for all theories?

Kinetics vs No-Kinetics



AdS/CFT low viscosity goo



pQCD kinetic plasma

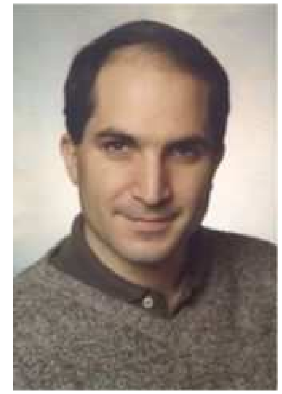
Effective theories for fluids (Here: Weak coupling QCD)

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p = C[f_p] \qquad (\omega < T)$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \qquad (\omega < g^4 T)$$

Effective theories (Strong coupling)

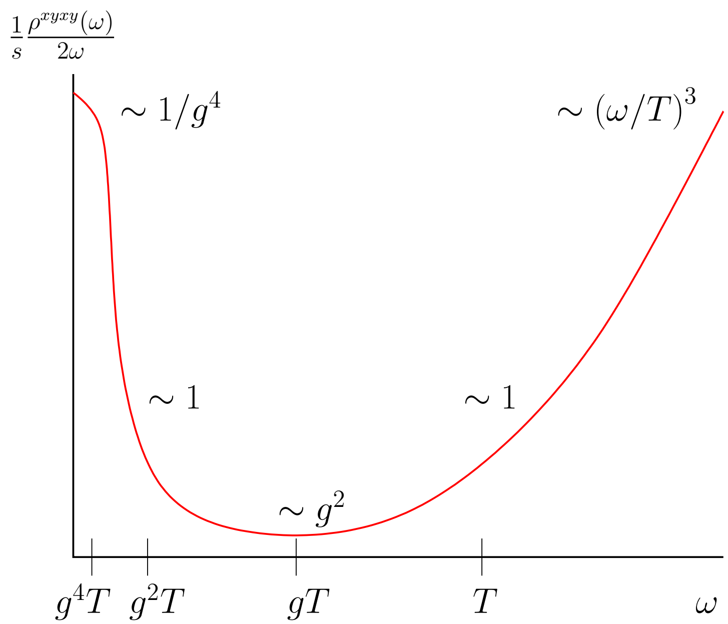


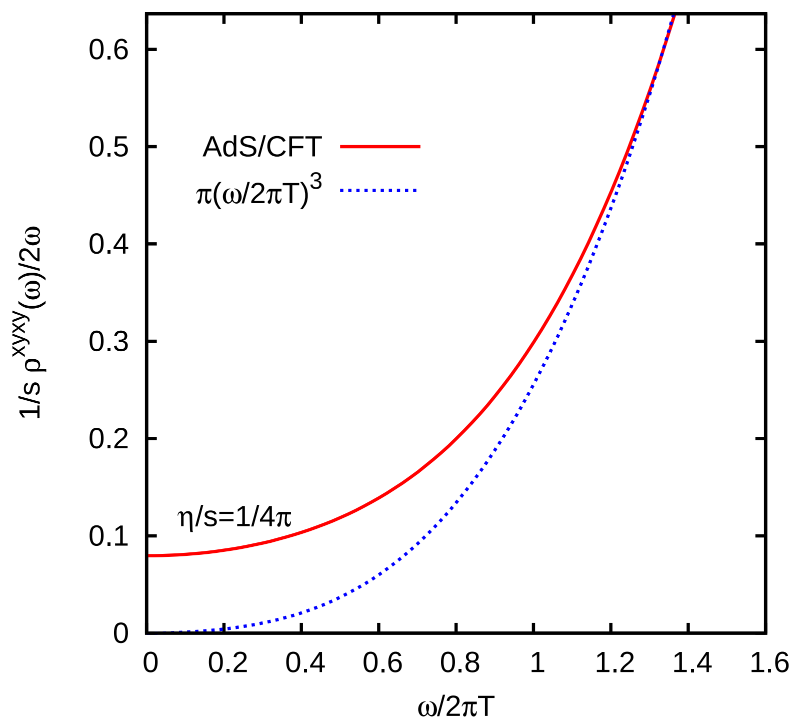
$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \Leftrightarrow S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

Kinetics vs No-Kinetics

Spectral function $\rho(\omega) = \mathrm{Im} G_R(\omega, 0)$ associated with T_{xy}

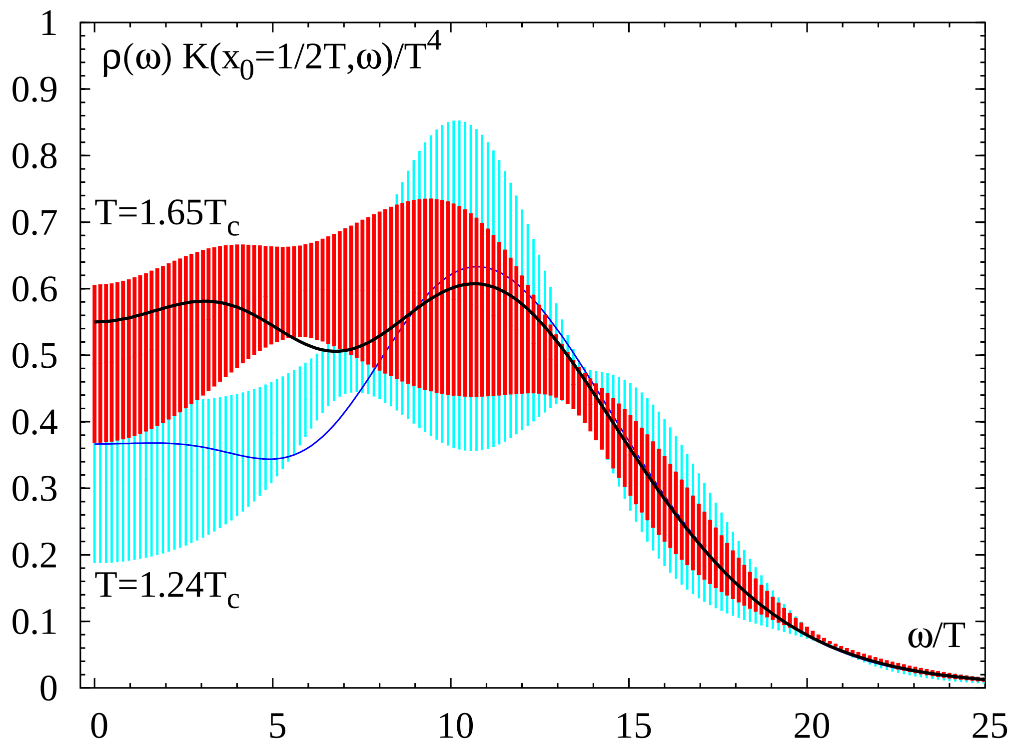




weak coupling QCD | strong coupling AdS/CFT

transport peak vs no transport peak

Spectral function (lattice QCD)



T	1.02 T_c	1.24 T_c	1.65 T_c
η/s		0.102(56)	0.134(33)
ζ/s	0.73(3)	0.065(17)	0.008(7)

H. Meyer (2007)

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

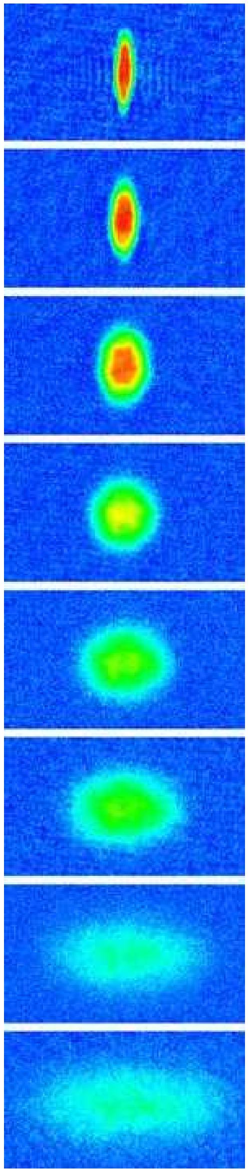
Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

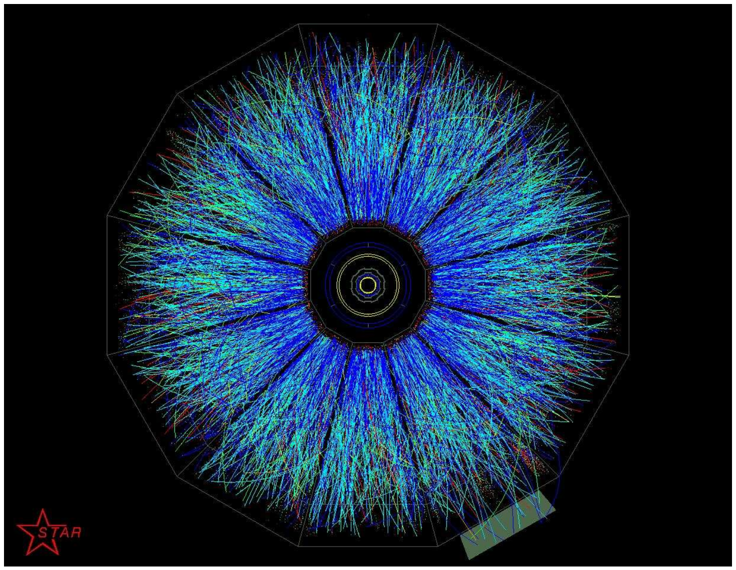
(Almost) scale invariant systems

Perfect Fluids: The contenders



Trapped Atoms

(T=0.1 neV)

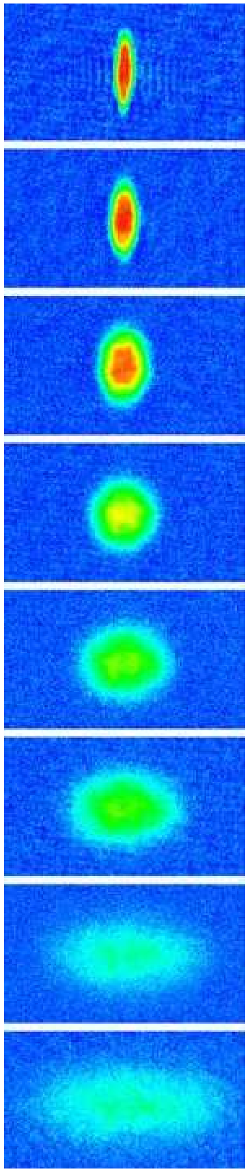


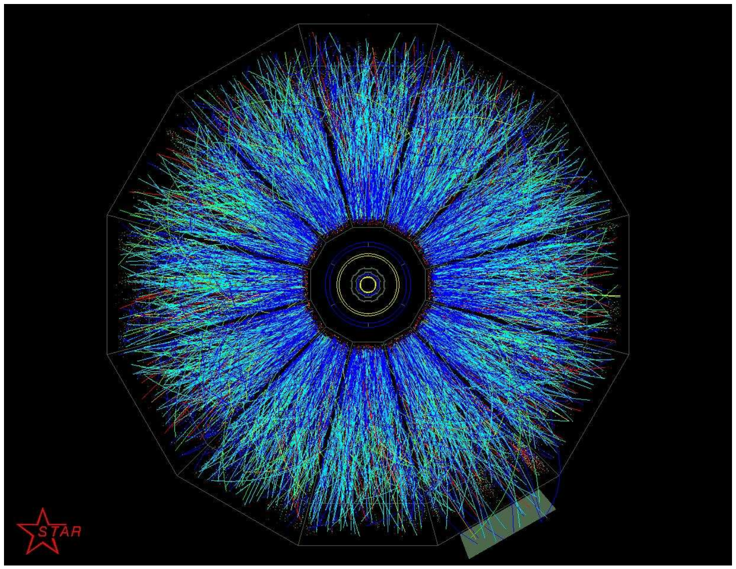
QGP (T=180 MeV)

Liquid Helium

(T=0.1 meV)

Perfect Fluids: The contenders





QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$

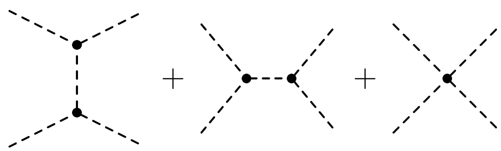
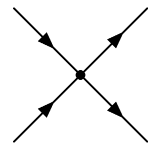
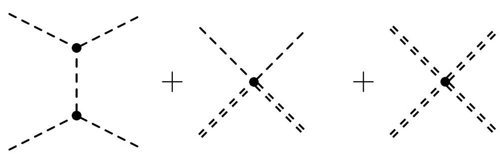
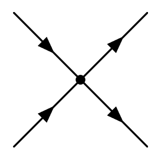
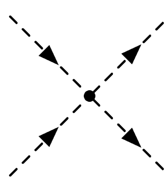
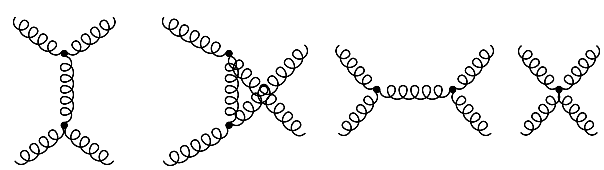
Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

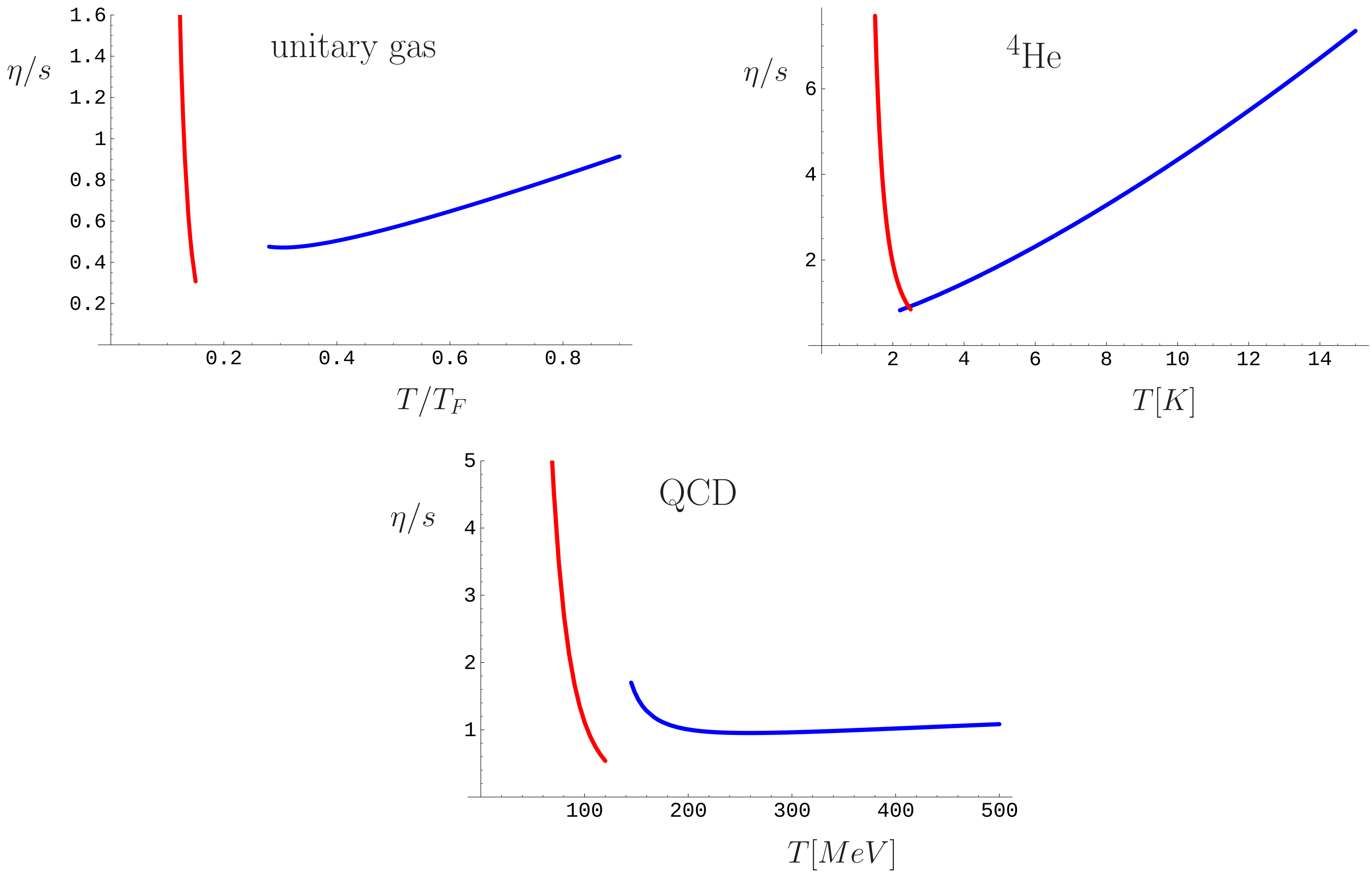
Consider ratios

η / s

Kinetic Theory: Quasiparticles

	low temperature	high temperature
unitary gas	phonons 	atoms 
helium	phonons, rotons 	atoms 
QCD	pions 	quarks, gluons 

Theory Summary



I. Experiment (Liquid Helium)

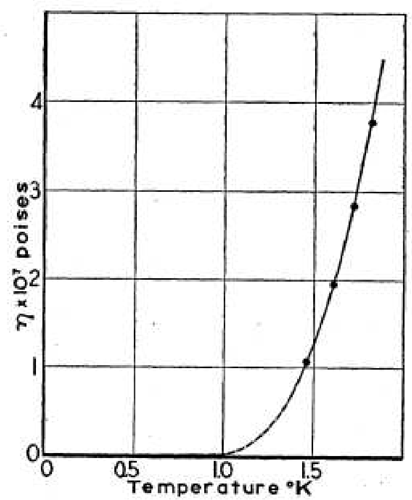
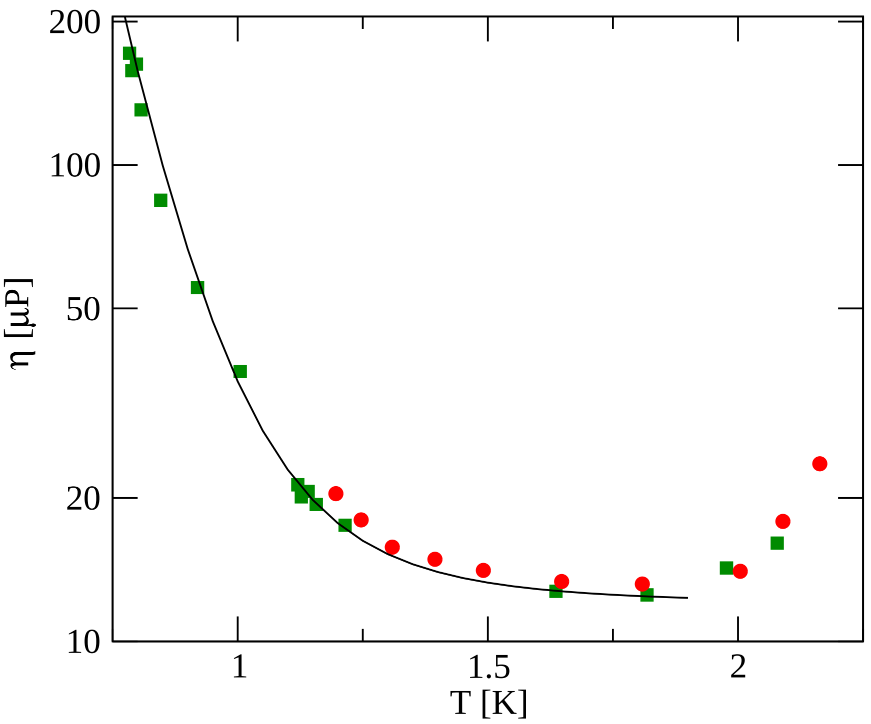


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.



Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

Hollis-Hallett (1955)

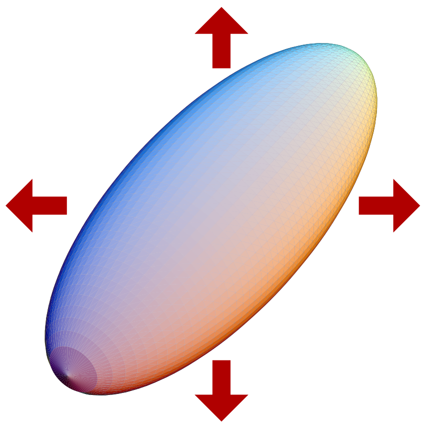
roton minimum, phonon rise

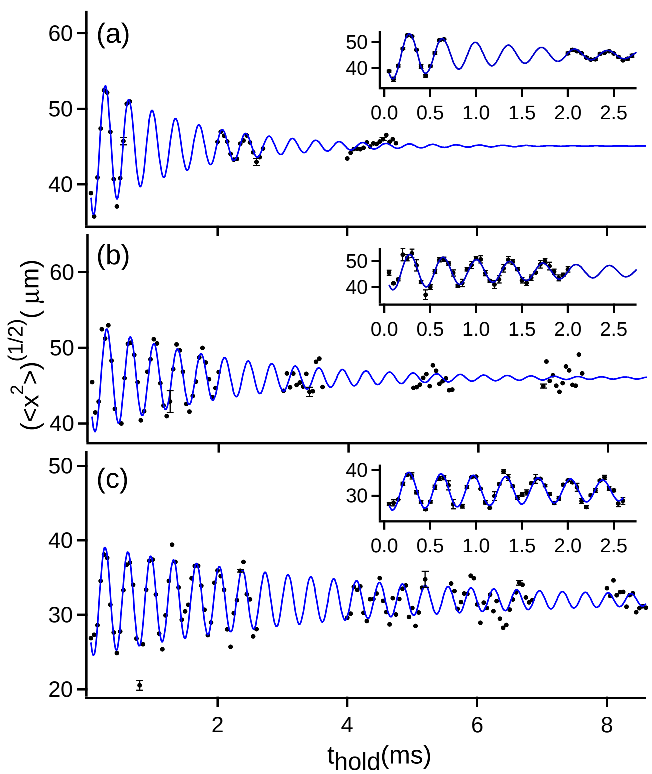
rotation viscometer

$$\eta/s \simeq 0.8\,\hbar/k_B$$

II. Hydrodynamics (Cold atoms)

Radial breathing mode





Ideal fluid hydrodynamics ($P \sim n^{5/3}$)

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} = -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m}$$

Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}}\,\omega_\perp$$

Damping small, depends on T/T_F.

experiment: Kinast et al. (2005)

Viscous Hydrodynamics

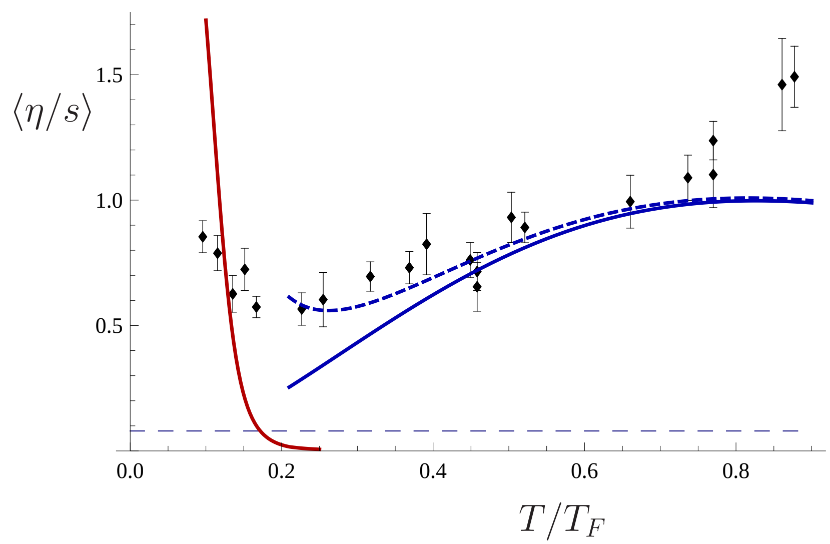
Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

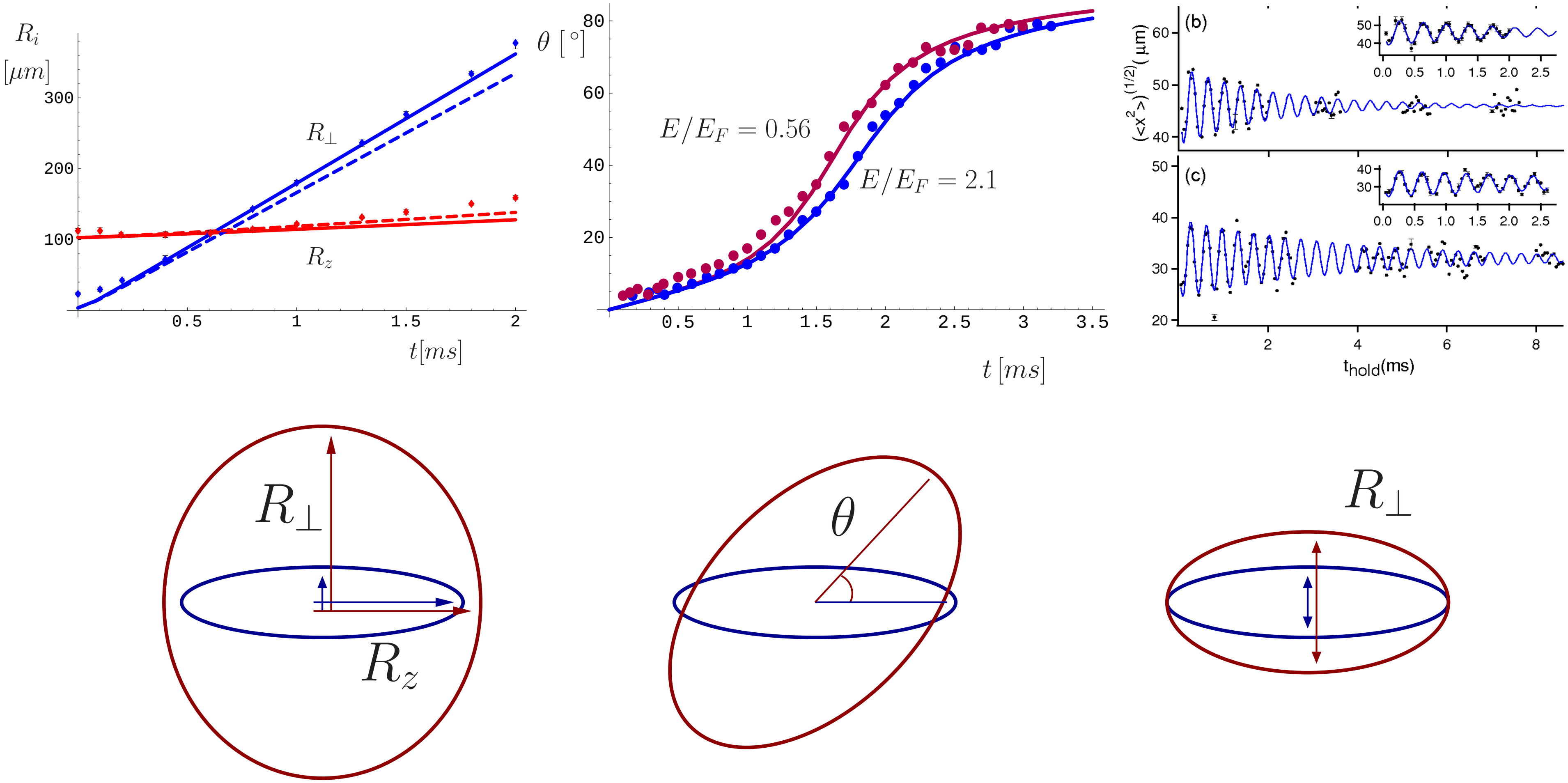
$$\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}}\,\frac{\Gamma}{\omega_\perp}\,\frac{E_0}{E_F}\,\frac{N}{S}$$



Schaefer (2007), see also Bruun, Smith

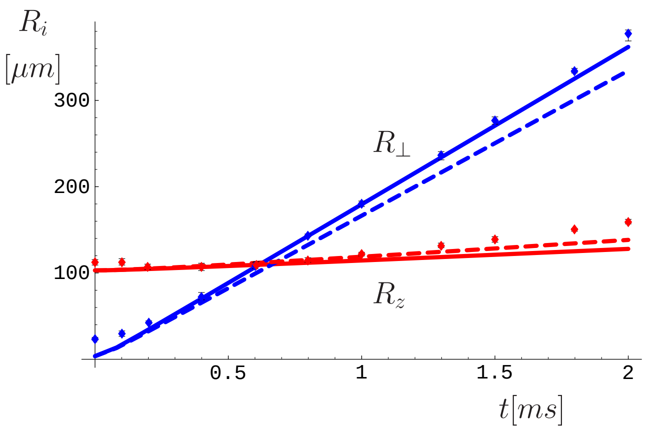
$T \ll T_F$ $\quad$ $T \gg T_F,\ \tau_R \simeq \eta/P$

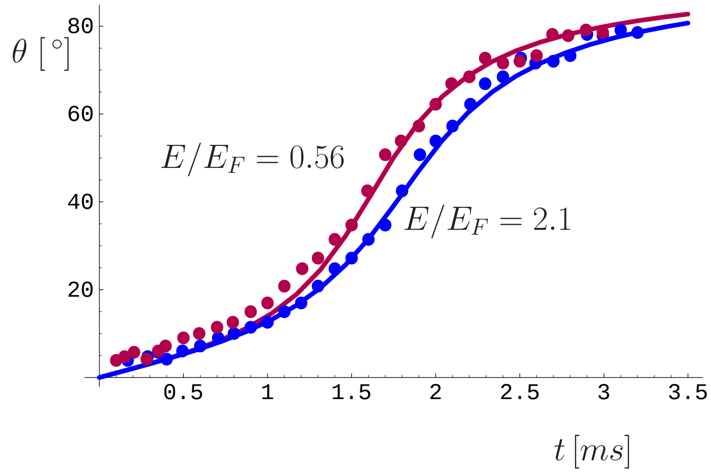
Dissipation

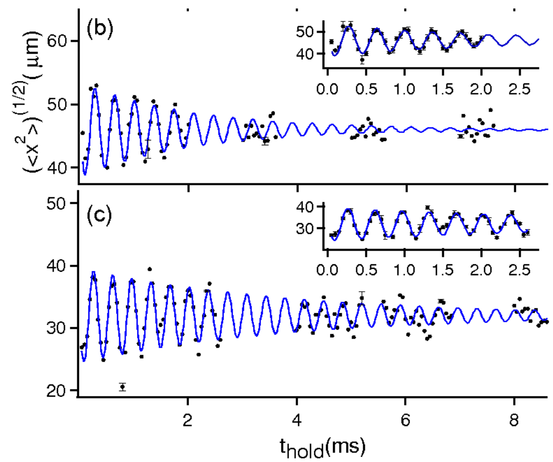


O'Hara et al (2002), Kinast et al (2005), Clancy et al (2007)

Dissipation





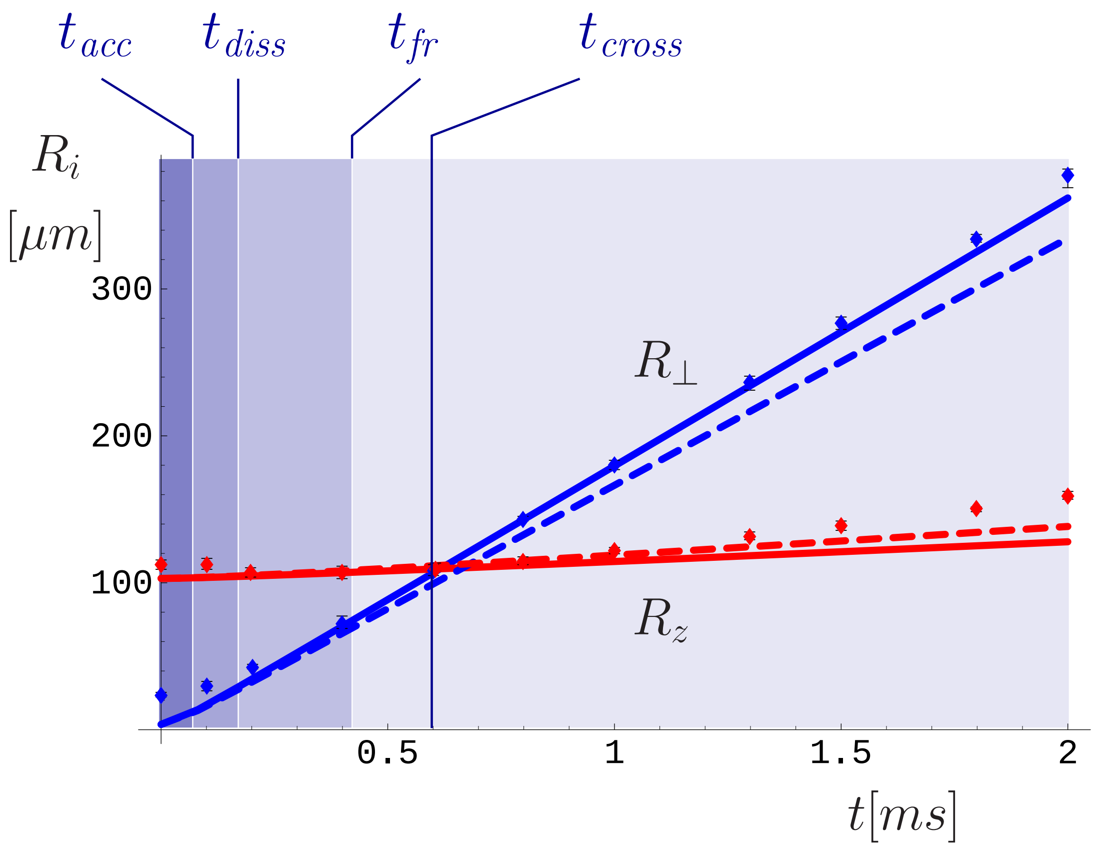


$$\left.\begin{array}{c}(\delta t_0)/t_0 \\ (\delta a)/a\end{array}\right\} = \left\{\begin{array}{c}0.008 \\ 0.024\end{array}\right\} \left(\frac{\langle\alpha_s\rangle}{1/(4\pi)}\right) \left(\frac{2\cdot 10^5}{N}\right)^{1/3} \left(\frac{S/N}{2.3}\right) \left(\frac{0.85}{E_0/E_F}\right)$$

t_0: "Crossing time" $(b_\perp = b_z,\ \theta = 45^\circ)$

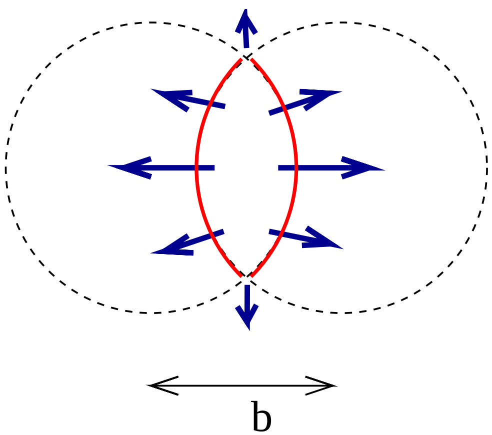
a: amplitude

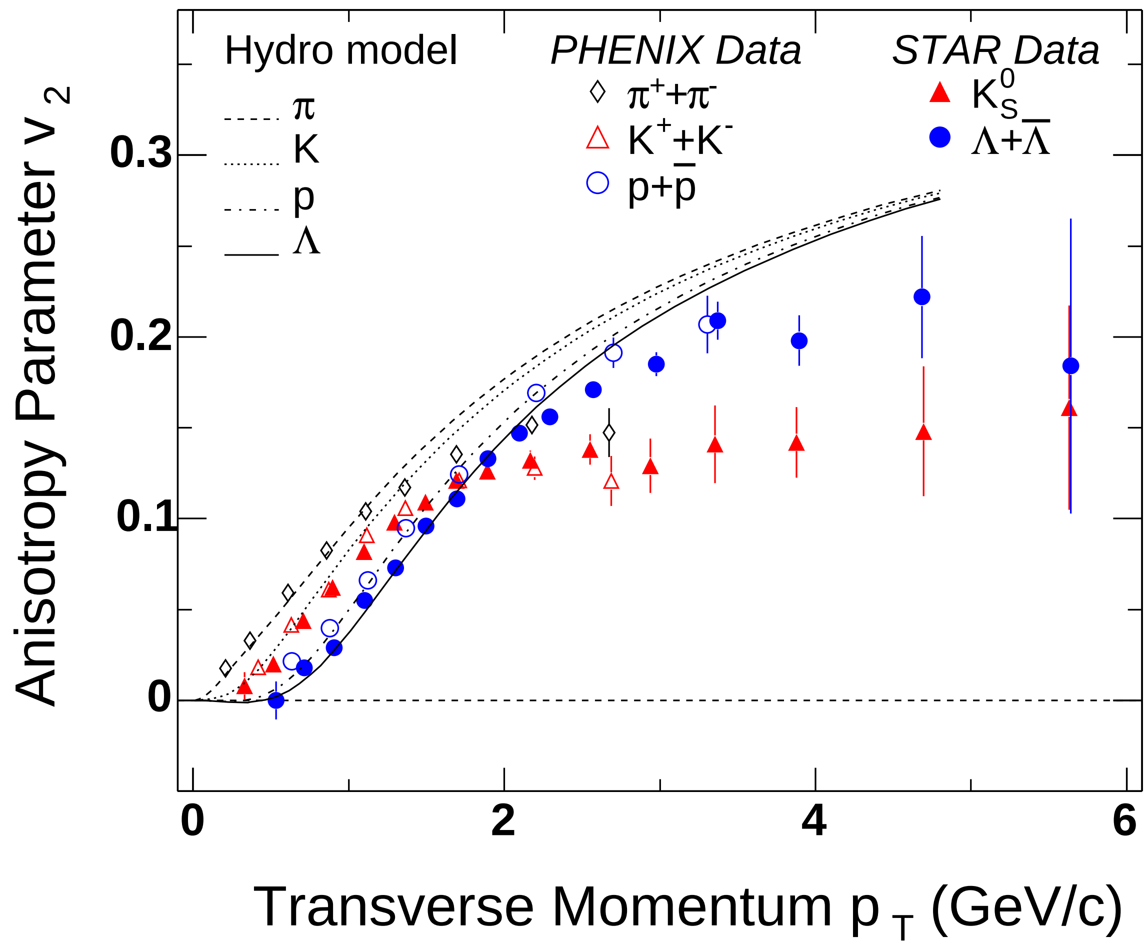
Time Scales



III. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

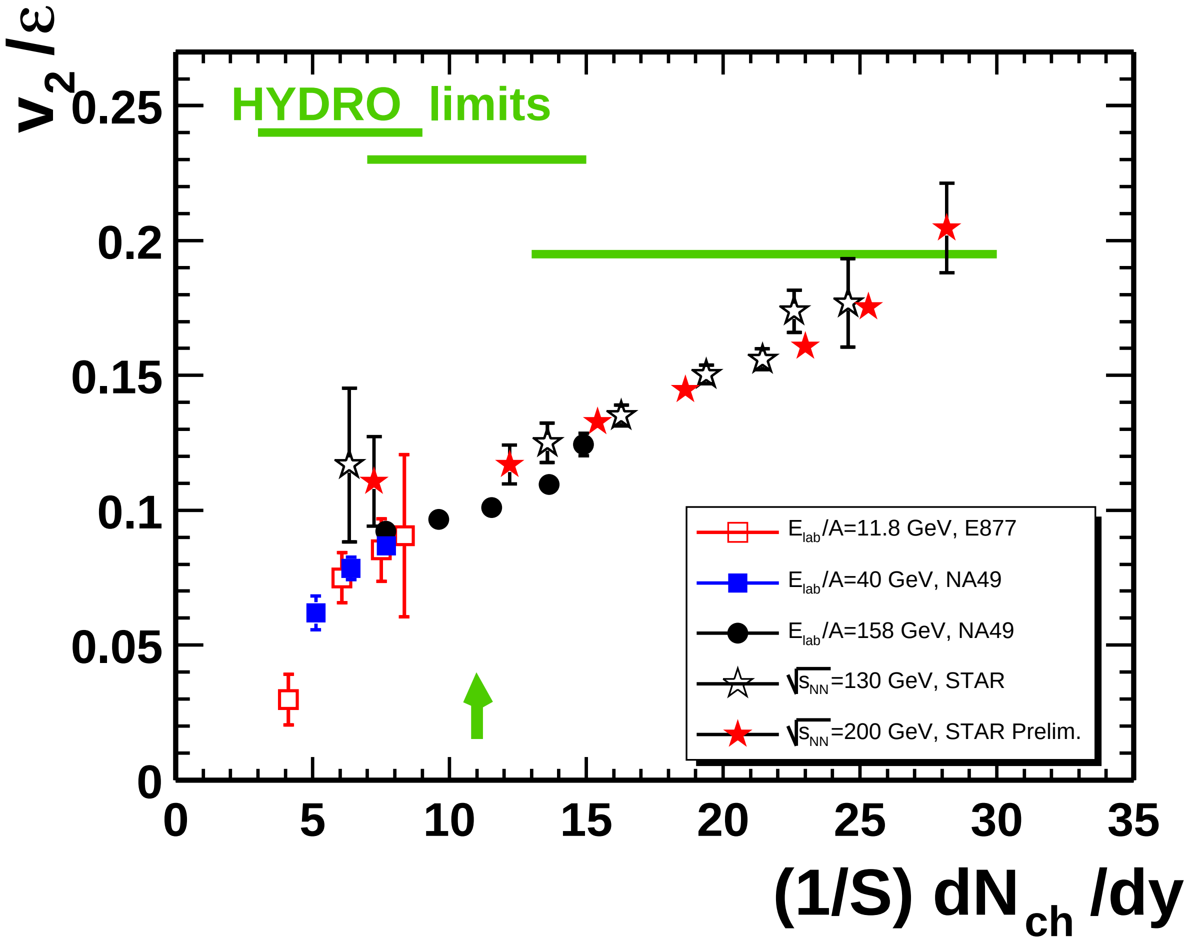




source: U. Heinz (2005)

$$p_0 \frac{dN}{d^3p}\bigg|_{p_z=0} = v_0(p_\perp)\,(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots)$$

Elliptic flow: initial entropy scaling



source: U. Heinz (2005)

Viscosity and Elliptic Flow

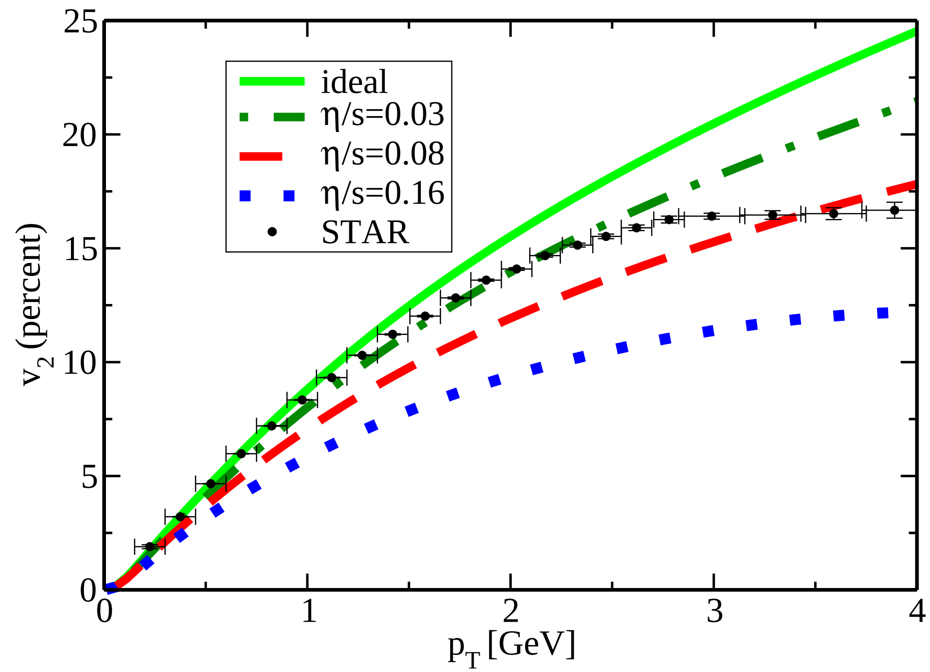
Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$

(applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm



Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

conservative bound $\frac{\eta}{s} < 0.4$

The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.