Euclidean Correlation Functions in a Holographic Model of QCD

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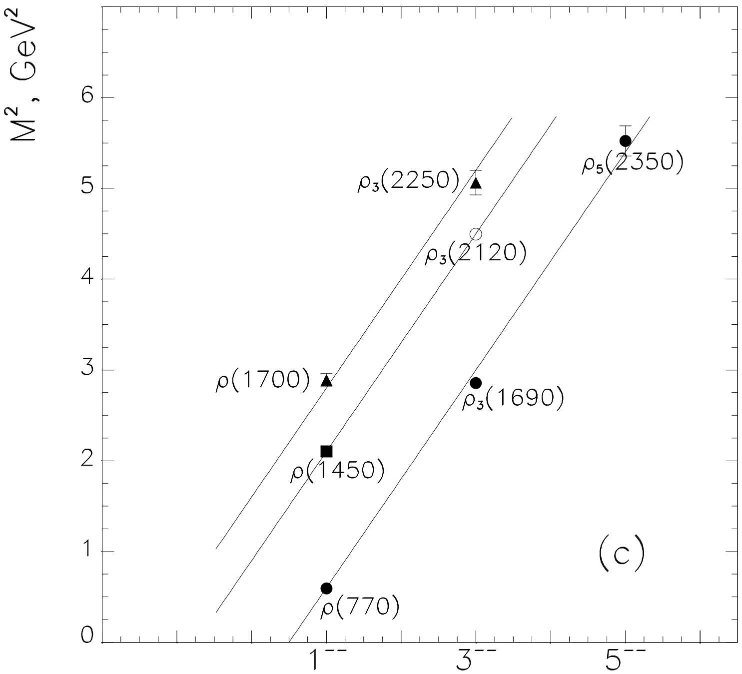
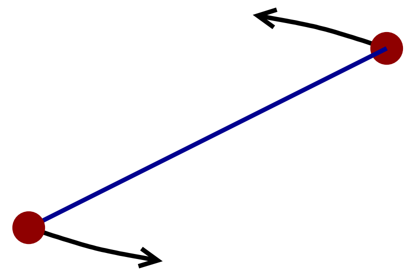
QCD and Strings: Pre-History



Anisovich et al. (2000)



$$J \sim \alpha' M^2$$

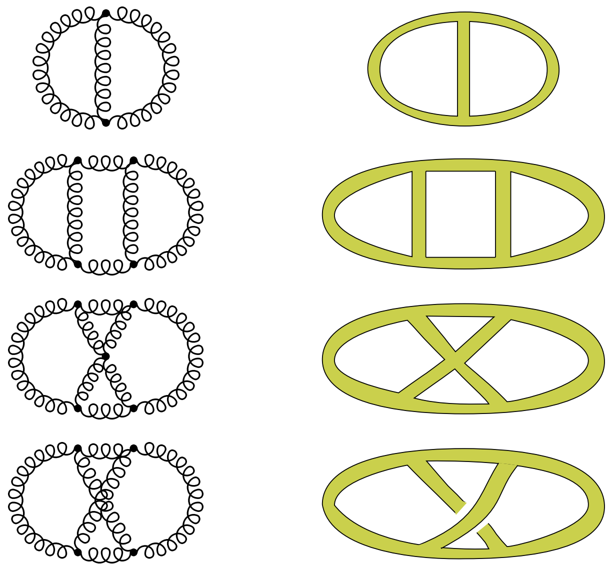
$$T_{str} = \frac{1}{2\pi\alpha'} = 1 \text{ GeV/fm}$$

Regge trajectories, pomerons, dual models, Veneziano amplitude, . . .

faded away with the advent of QCD

QCD and Strings: Pre-History

't Hooft: large N_c expansion ($\lambda = g^2 N_c = const$)

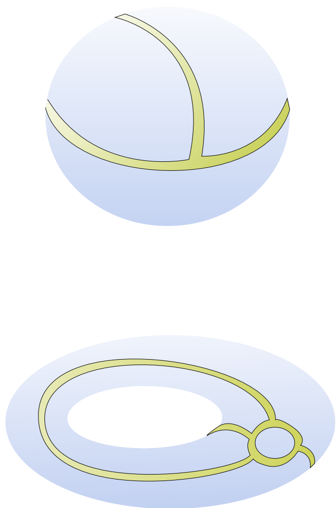


$\sim (g^2_{\rm YM})^{3-2}\, N_c^3 = \lambda\, N_c^2$

$\sim (g^2_{\rm YM})^{6-4}\, N_c^4 = \lambda^2\, N_c^2$

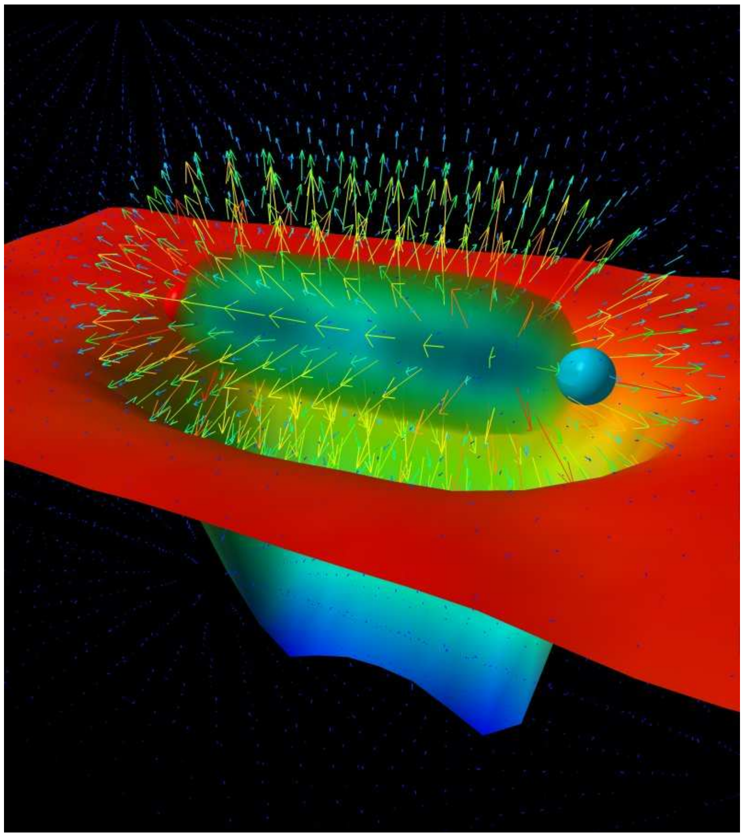
$\sim (g^2_{\rm YM})^{8-5}\, N_c^5 = \lambda^3\, N_c^2$

$\sim (g^2_{\rm YM})^{6-4}\, N_c^2 = \lambda^2$

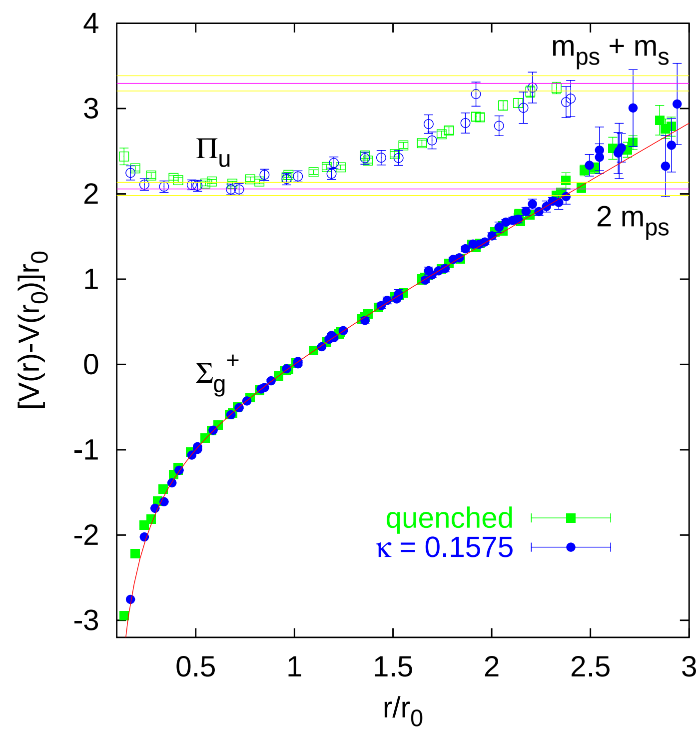


Large N_c limit: topological expansion (string theory?)

QCD and Strings: Pre-History



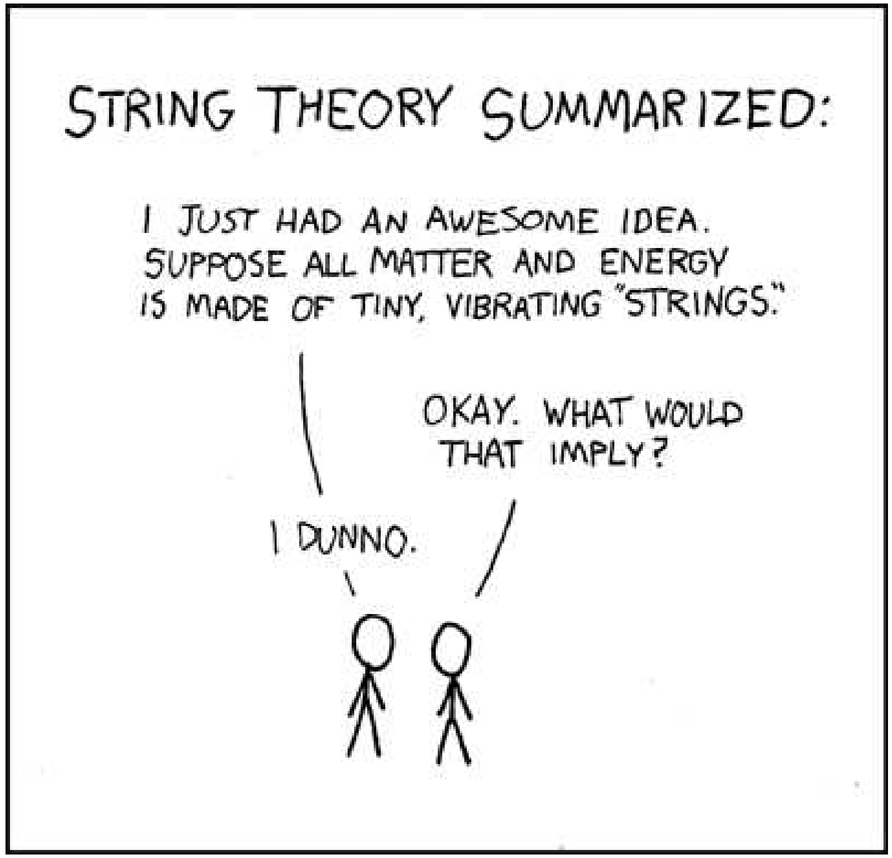
Leinweber (2001)



Bali (2001)

QCD: flux tubes and string potentials

QCD and Strings: History



Polyakov (1980), . . ., Polchinski (1995),Maldacena (1997),

Gubser, Klebanov, Polyakov (1998)

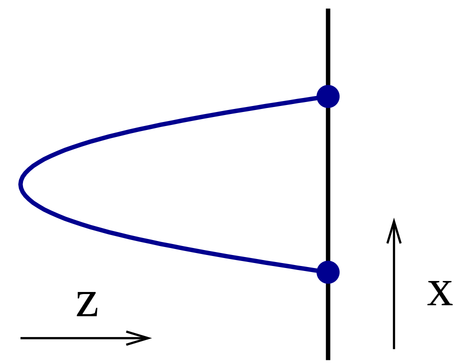
QCD and Strings: Holography

The AdS/CFT duality relates

$\mathcal{N}=4$ large N_c gauge theory in 4 dimensions	$\Leftrightarrow$	type IIb string theory on $AdS_5 \times S_5$
correlation fcts of gauge invariant operators	$\Leftrightarrow$	boundary correlation fcts of AdS fields

$$\langle \exp \int dx\ \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



The correspondence is simplest at strong coupling $g^2 N_c$

strongly coupled gauge theory $\Leftrightarrow$ classical string theory

Maldacena (1997)

$\mathcal{N} = 4$ Supersymmetric Yang-Mills Theory

Fields: Gluons, Gluinos, Higgses; all in the adjoint of $SU(N_c)$

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\lambda}^a_A \sigma^\mu (D_\mu \lambda^A)^a + (D_\mu \Phi_{AB})^a (D_\mu \Phi^{AB})^a + \ldots$$

$$A^a_\mu \qquad \lambda^a_A \ (\bar{4}_R) \qquad \Phi^a_{AB} \ (6_R)$$

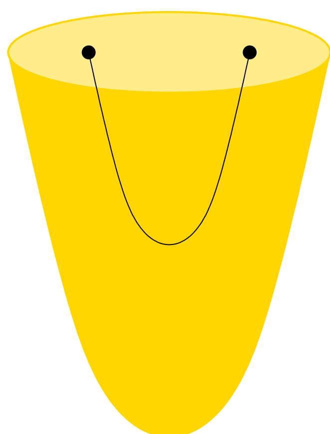
Global symmetries: Conformal and $SU(4)_R$

$$SO(4,2) \times SU(4)_R$$

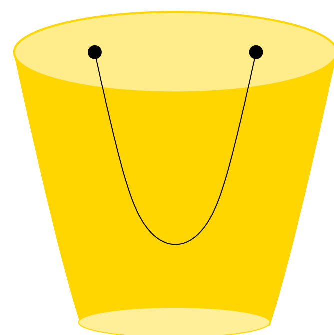
Properties: Conformal $\beta(g) = 0$, extra scalars, no fundamental fermions, no chiral symmetry breaking, no confinement

QCD and Strings: Towards QCD

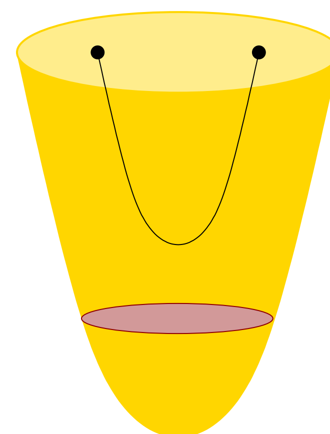
non-AdS/non-CFT correspondence, a.k.a "AdS/QCD"



AdS: conformal



cutoff AdS



AdS black hole

Example: 5d Gauge Field

Consider five dimensional action ($F_{\mu\nu} = \partial_\mu V_\nu - \partial_\mu V_\nu - i[V_\mu, V_\nu]$)

$$S_5 = -\frac{1}{4g_5^2} \int d^5x \, \sqrt{g} F^a_{\mu\nu} F^{a\,\mu\nu}$$

$$ds^2 = \frac{1}{z^2}(-dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu) \qquad\qquad 0 \le z \le z_m$$

Equ. of motion: linearize, F-trafo in x^μ, $V_5 = 0$ gauge

$$z\,\partial_z \left(\frac{1}{z} \partial_z V^a_\mu \right) + q^2 V^a_\mu = 0$$

Using equ. of motion

$$S_5 = -\frac{1}{2g_5^2} \int d^4x \, \frac{1}{z} \, V^a_\mu \partial_z V^{a\,\mu} \Big|_{z=0}$$

1. Find solution with $V(z \to 0, x) = V_0(x)$
2. Compute action $S_5[V]$.
3. Take functional derivative $\Pi_{\mu\nu} = (\delta^2 S_5)/(\delta V_0^\mu \delta V_0^\nu)$

Write $V^\mu(q,z) = V_0^\mu(q) V(q,z)$ with $V(q,0) = 1$. Then

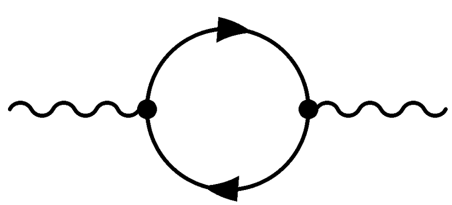
$$\Pi(Q^2) = -\frac{1}{g_5^2 Q^2} \left. \frac{\partial_z V(q,z)}{z} \right|_{z=0} \qquad Q^2 = -q^2$$

The required solution is

$$V(q,z) \simeq 1 + \frac{1}{2} Q^2 z^2 \log(Qz) + \ldots$$

$$\Pi(Q^2) = -\frac{1}{2g_5^2} \log(Q^2)$$

match to QCD:



$$g_5^2 = \frac{12\pi^2}{N_c}$$

AdS/CFT Dictionary

4d field theory	$\leftrightarrow$	5d gravitational theory
generating functional $W[O]$	$\leftrightarrow$	boundary action $S[\phi_0]$
operator $O(x)$ coupled to $\phi_0(x)$	$\leftrightarrow$	field $\phi(z,x)$ (boundary val $\phi_0(x)$)
dimension, spin of O	$\leftrightarrow$	5-d mass of ϕ
symmetry breaking:	$\leftrightarrow$	non-normalizable mode:
$\langle O \rangle \neq 0$ as $\phi_0 \to 0$		$\phi \sim \phi_0 z^{d_\phi} + A z^{d_O}$
large N_c	$\leftrightarrow$	weak coupling g_5
large Q	$\leftrightarrow$	small z

The model: Chiral Symmetry Breaking

5-d action with vector and scalar fields

$$S = \int d^5x \sqrt{g} \left\{ -\frac{1}{4g_5^2} \mathrm{Tr} \left(F_L^2 + F_R^2 \right) + \mathrm{Tr} \left(|DX|^2 + 3|X|^2 \right) \right\}$$

Erlich et al. (2005), DaRold et al (2005)

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ (for L/R)

$X \to LXR$ and $D_\mu X = \partial_\mu X - iA_{L\mu} X + iXA_{R\mu}$

Chiral symmetry breaking

$$\langle X_{ij} \rangle = \sigma_{ij} z^3 + M_{ij} z,$$

Pseudoscalar fields

$$X_{ij} = \langle X_{ij} \rangle \exp(i\pi^a t^a),$$

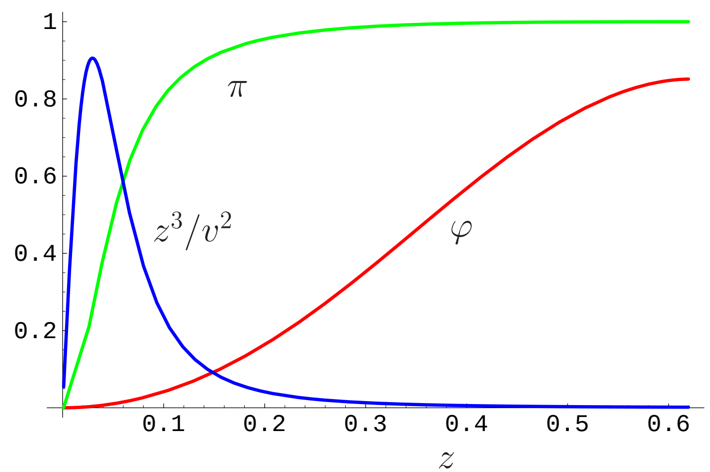
Chiral Symmetry Breaking and the Pion

Mixing between axial and pseudoscalars: $A_\mu = A_{\mu\perp} + \partial_\mu \varphi$

$$\begin{aligned}
\partial_z\left(\frac{1}{z}\partial_z A^a_\perp\right) + \frac{q^2}{z}A^a_\perp - \frac{g_5^2 v^2}{z^3}A^a_\perp &= 0 \\
\partial_z\left(\frac{1}{z}\partial_z \varphi^a\right) + \frac{g_5^2 v^2}{z^3}(\pi^a - \varphi^a) &= 0\,. \quad [v(z) = mz + \sigma z^3] \\
-q^2\partial_z\varphi^a + \frac{g_5^2 v^2}{z^2}\partial_z \pi^a &= 0\,.
\end{aligned}$$

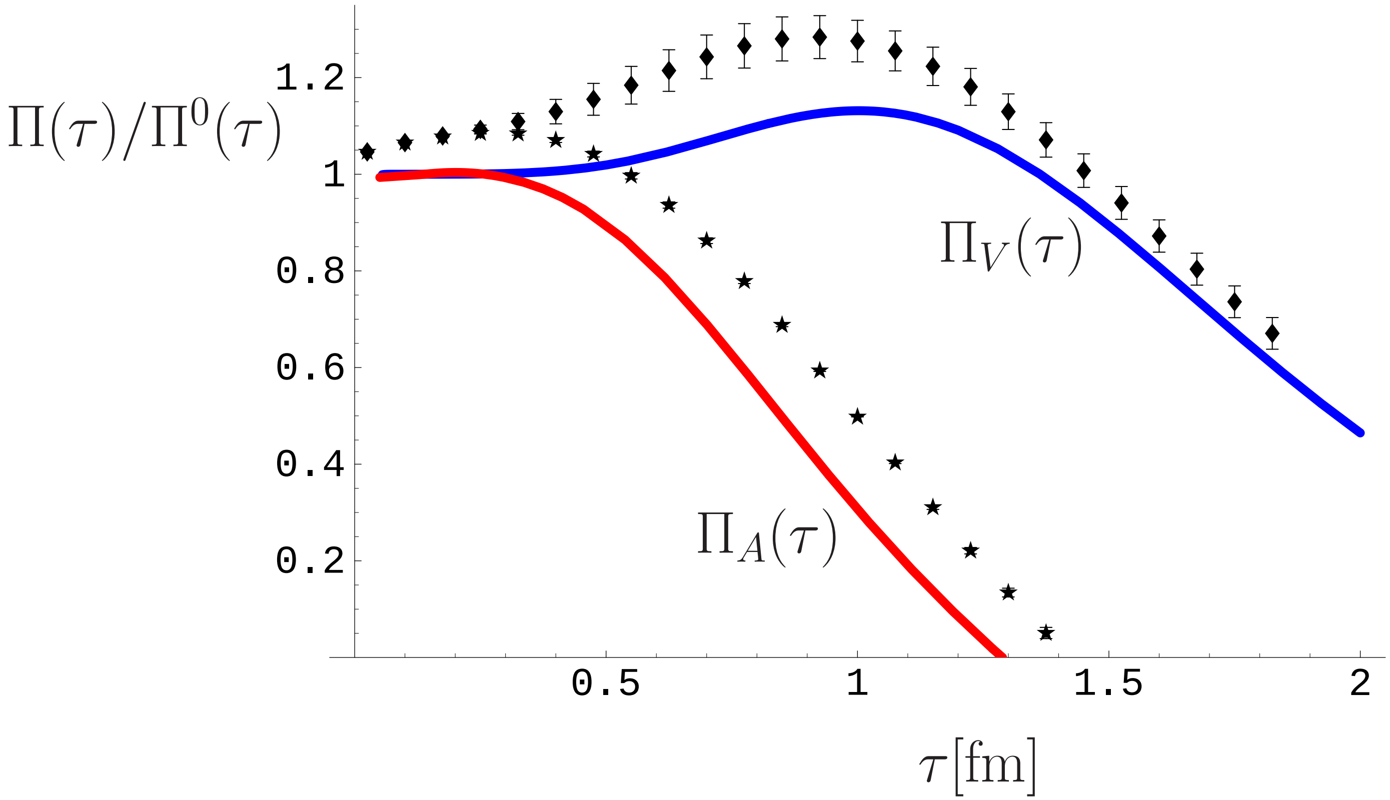
Goldstone mode: Define $f_\pi^2 = -\frac{1}{g_5^2}\left.\frac{\partial_z A(z,0)}{z}\right|_{z=0}$ (b.c. $A(0,q) = 1$)

$$\begin{aligned}
\phi(z) &= A(0,z) - 1 \ \ (\pi(z) = 1) \\
\pi(z) &= q^2 \int_0^z d\bar{z}\, \frac{\bar{z}^3}{v(\bar{z})^2}\frac{\partial_{\bar{z}} A(0,\bar{z})}{g_5^2\,\bar{z}}
\end{aligned}$$



$$m_\pi^2 f_\pi^2 = 2m\sigma$$

Vector/Axialvector Correlation Funtions



Data: V/A spectral functions from $\tau \longrightarrow \nu_\tau + hadrons$ (Aleph)

Flavor Singlet Axialvector

Add singlet field $Y = \langle Y \rangle e^{ia}$ (pseudoscalar glueball, "axion")

$$S = \int d^5x \sqrt{g} \left\{ \frac{1}{2}|DY|^2 + \frac{\kappa_0}{2} \left(Y^{N_f} \det(X) + h.c. \right) \right\}$$

Katz & Schwartz (2007)

$$Y = \langle Y \rangle = c + \Xi z^4 \qquad c \sim g^2, \;\; \Xi \sim G^2$$

QCD axial anomaly: $\partial^\mu j^5_\mu = 2N_f \, \frac{g^2}{32\pi^2} G\tilde{G}$

$$\int d^4x \; e^{iqx} \, \langle \partial^\mu j^5_\mu(x) \partial^\mu j^5_\mu(0) \rangle = (2N_f)^2 \frac{\alpha_s^2}{8\pi^4} Q^4 \log(Q^2) + \ldots$$

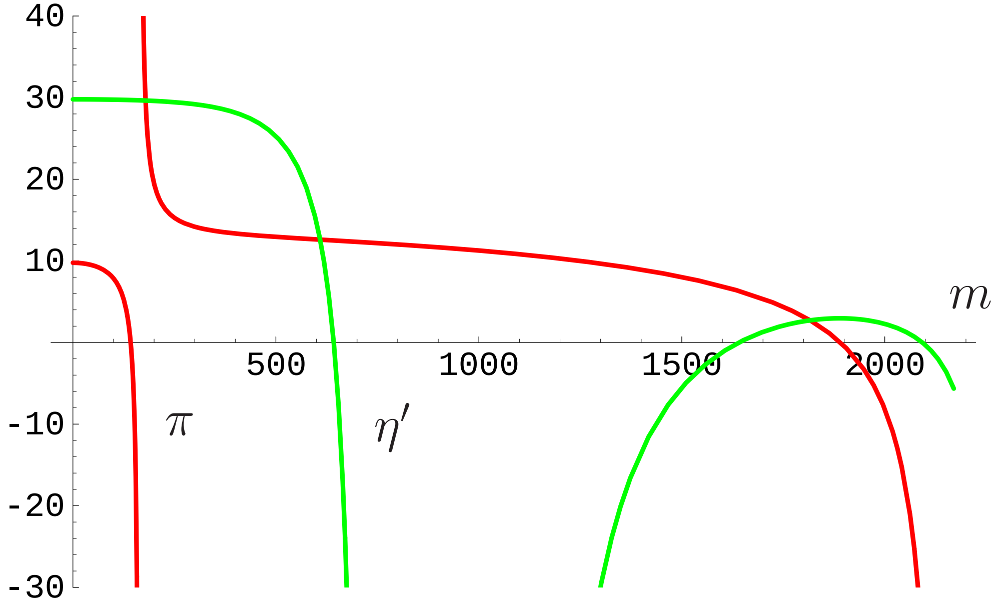
Matching: axial fields $A^0_\mu = A^0_{\mu\perp} + \partial_\mu \varphi^0$ and a

$$\int d^4x \; e^{iqx} \, \langle \partial^\mu j^5_\mu(x) \partial^\mu j^5_\mu(0) \rangle = -\frac{Q^2}{g_5^2} \; \frac{\partial_z \phi^0(z)}{z} \bigg|_{z=0}$$

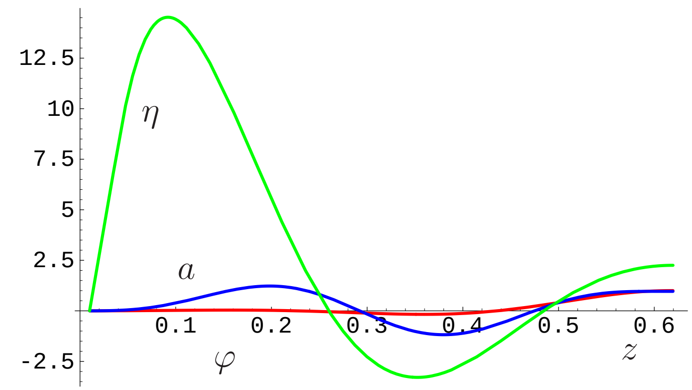
$$c = \sqrt{2N_f} \, \frac{\alpha_s}{2\pi^2} \qquad \qquad \Xi \to \mathrm{OPE} \qquad \kappa \; \mathrm{free}$$

Spectrum: Pseudoscalar Singlets

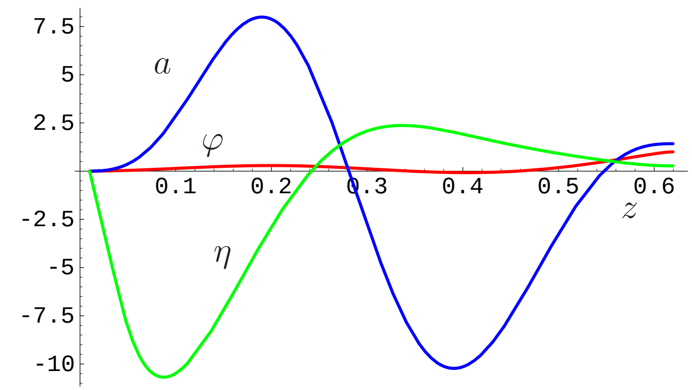
eigenvalues of (φ^0, η^0, a) and (φ, π) system



excited state (mostly $\bar{q}q$)



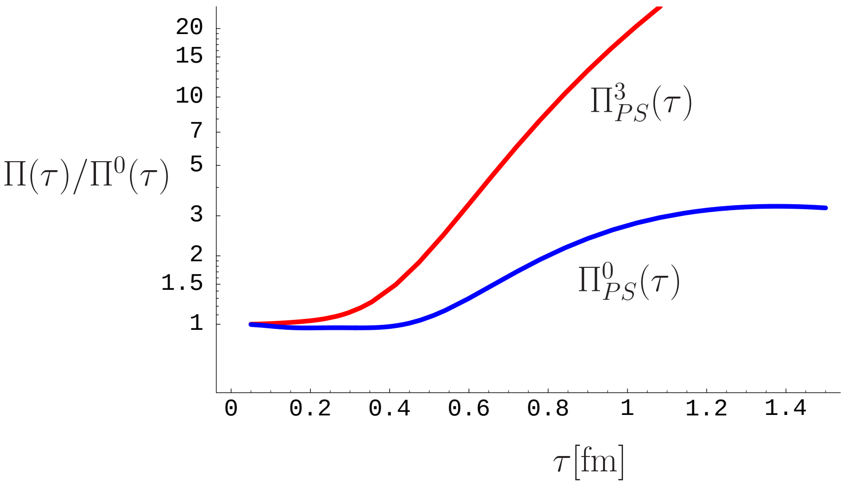
excited state (mostly $G\tilde{G}$)

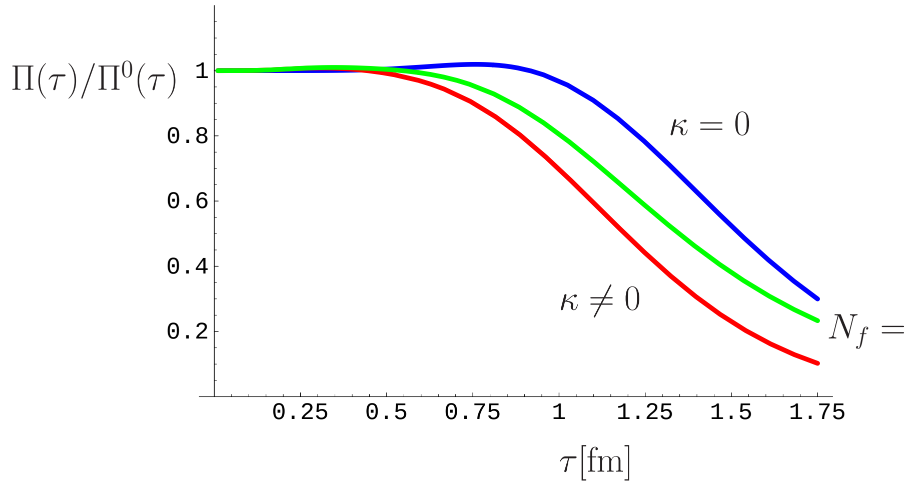


Pseudoscalar Correlation Functions

$$\Pi(x) = \left\langle \bar{q} t^a \gamma_5 q(x) \bar{q} t^b \gamma_5 q(0) \right\rangle$$

$$\Pi(x) = \left\langle g^2 G\tilde{G}(x) g^2 G\tilde{G}(0) \right\rangle$$





$$m_{\eta'} \simeq 660 \text{ MeV}$$

$$m_{0^{+-}} \simeq 1400 \text{ MeV}$$

$$\langle 0 | g^2 G\tilde{G} | \eta' \rangle \neq 0$$

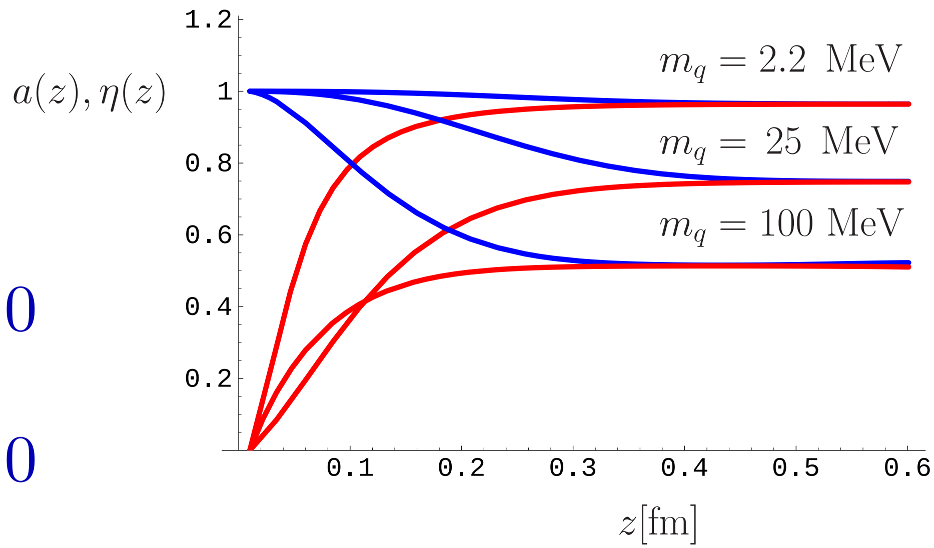
Toplogical Suscetibility

Topological susceptibility

$$\chi_{top} = \lim_{V\to\infty} \frac{\langle Q_{top}^2 \rangle}{V} = -\int d^4x \, \Pi_P(x)$$

Holography: Find solution with $q^2 = 0$ and $a(0,q) = 1$

$$\chi_{top} = -\frac{c^2}{2N_f} \frac{\partial_z a}{z^3}\Big|_\epsilon$$



$$\partial_z \left(\frac{c^2}{z^3} \partial_z a \right) + \kappa \frac{v^{N_f}}{z^5} \left(\eta^0 - a \right) = 0$$
$$v^2 \partial_z \eta^0 + c^2 \partial_z a = 0$$

Note that $\chi_{top} \sim m_q \sigma$.

Witten-Veneziano

Pure gluodynamics

$$a(z) = \frac{N_f}{2c^2}\,\chi_{top} z^4 + \ldots$$

Full QCD: (Pseudo) Goldstone modes $\eta - \varphi$

$$\eta^0(z) \simeq 1 \qquad\qquad \varphi^0(z) \simeq \frac{g_5^2}{2}\,f_\pi^2 z^2 + \ldots$$

Study coupling, use perturbation theory in c ($\sim 1/N_c$)

$$m_{\eta'}^2 z^2 \partial_z \varphi^0 - g_5^2 v^2 \partial_z \eta^0 - g_5^2 c^2 \partial_z a = 0$$

Witten-Veneziano relation

$$f_{\eta'}^2 m_{\eta'}^2 = 4 N_f \chi_{top}$$

What about instantons?

Topological charge correlator: Trear κa^2 as a perturbation

$$\Pi_P(Q) = -\frac{1}{2N_f}\int_0^{z_m}\frac{dz}{z^5}\,\bar{\kappa}\,\left[\frac{1}{2}(Qz)^2 K_2(Qz)\right]^2,$$

AdS_5 measure $\times$ (Bulk-to-boundary prop)^2

Compare to instanton result

$$\Pi_P(Q) = -2\int\frac{d\rho}{\rho^5}\,d(\rho)\,\left[\frac{1}{2}(Q\rho)^2 K_2(\rho Q)\right]^2,$$

instanton measure $\times$ (F-trafo of $G\tilde{G}_I)^2$

- AdS cutoff provides instanton size cutoff
- Correspondence extends to other correlators

Positivity and all that

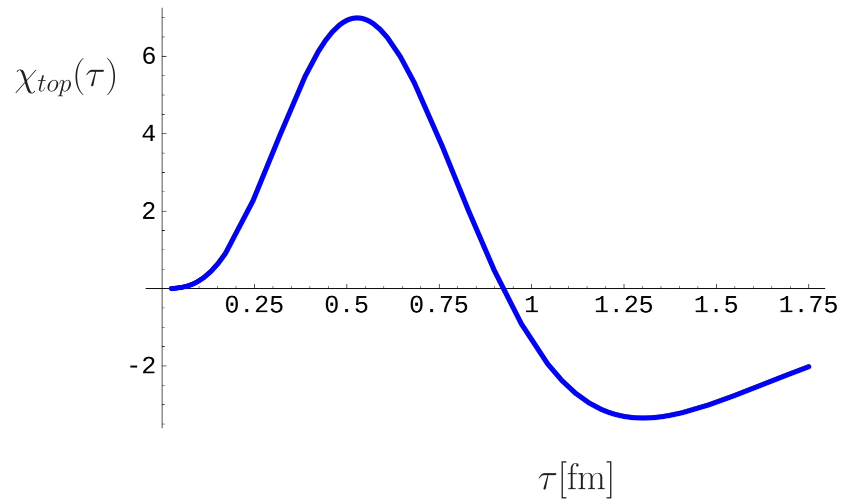
$$\chi_{top} = \lim_{V\to\infty} \frac{\langle Q^2_{top}\rangle}{V} = -\int d^4x\, \Pi_P(x)$$

Have $\chi_{top} \geq 0$ and $\Pi_P(x) \geq 0$ (spectral positivity)

How can that be? $\Pi_P(x) \sim \alpha_s^2/x^8$ singular $\Rightarrow$ need regulator

$$\Pi_P^{reg}(x) = \Pi_P^{AdS|}(x) - \Pi_P^{AdS}(x)$$

$$\int d^4x\, \Pi_P^{reg}(x) = -\frac{c^2}{2N_f}\,\frac{\partial_z a}{z^3}\bigg|_\epsilon$$



Anomaly term: $\delta\Pi_P(x) \leq 0$ $(\chi_{top} \geq 0)$

Outlook

Improved models: Asymptotic freedom? OPE?

Evans (2004), Kiritsis (2007), . . .

Top-down approach: Origin of anomaly term?

Witten (1998), Barbon, Mateos, Myers (2004), Armoni (2004)

Large N_c limit: Lattice/instanton calculations suggest that $d(\rho) \to \delta(\rho - \rho^*)$.

Teper (2003), Schäfer (2003), Shuryak (2007)

Non-zero T, μ: Better perturbative control. Holographic duals?