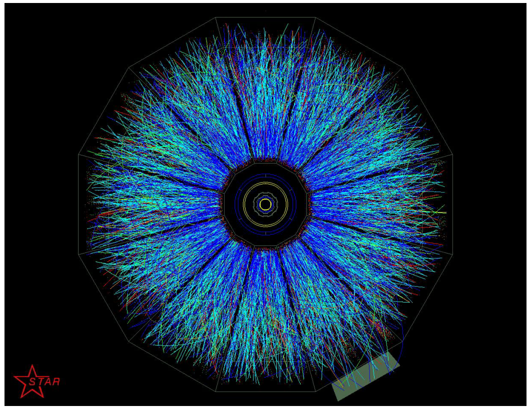
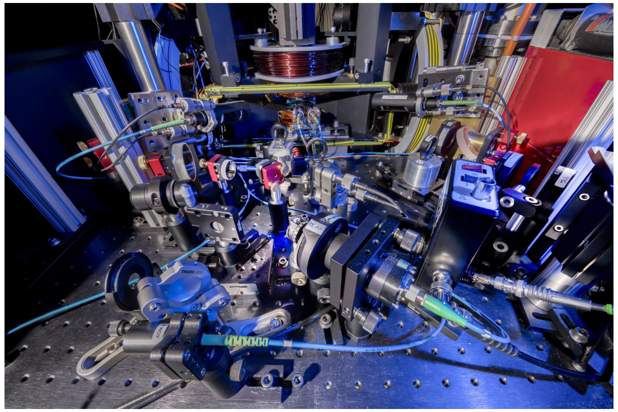
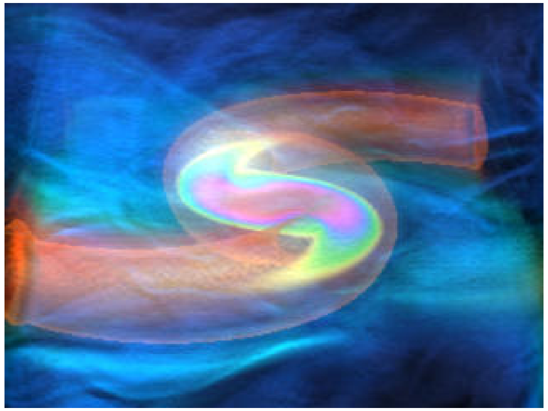
Finite Density QCD

Thomas Schaefer

North Carolina State University

Dense Matter

Dense QCD (strongly interacting) matter

$$P = P(T, \mu_B, \mu_I, \mu_Y) \quad \leftrightarrow \quad P = P(\mathcal{E}, n_B, n_I, n_Y)$$

Dense SM matter: Include leptons, charge neutrality, and weak equilibrium

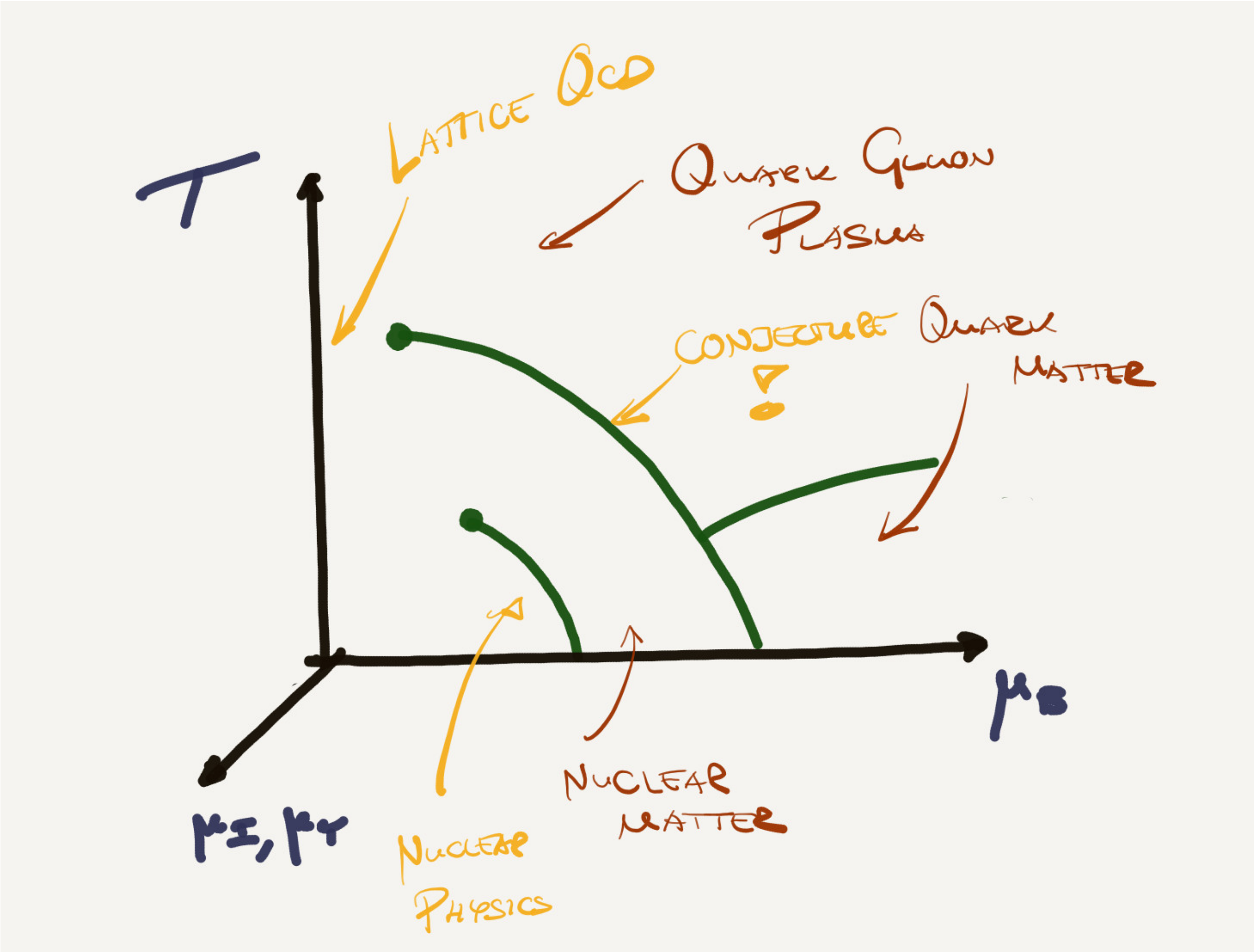
$$P = P(\mathcal{E}, n_B, n_Q = 0)$$

Dense NS matter: Only charged leptons (neutrino transparent matter)

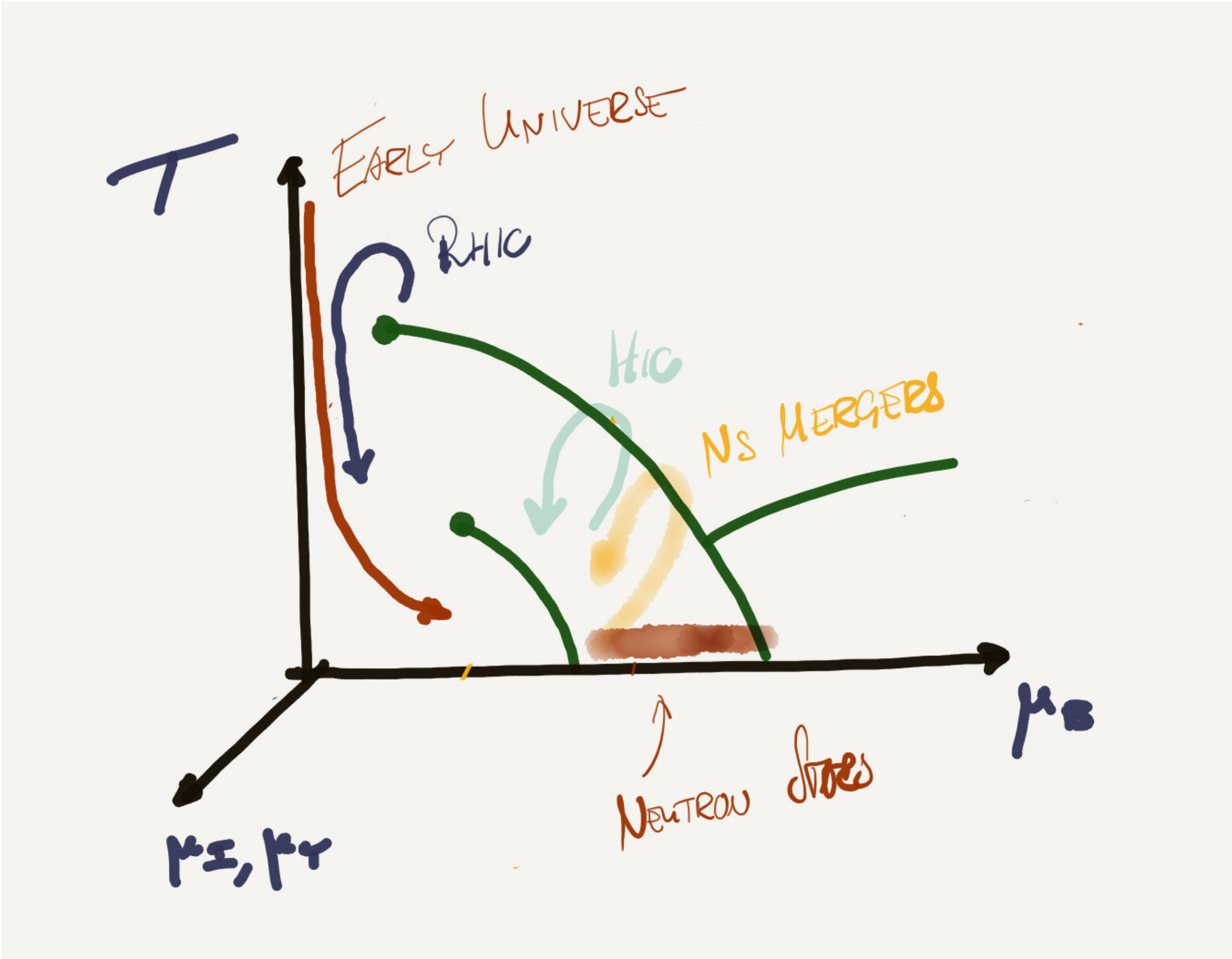
Typically: Ignore QED (other than $Q_e = 0$) and weak interactions; treat leptons as free particles: $P = P_{QCD} + P_{lept}^{free}$. Some exceptions, e.g. NS crust, NSE, ...

Partial equilibrium matter: In astrophysical environments QCD (almost) always equilibrated, but weak interaction need not be in equilibrium. Different strategies: Bulk viscosity, microscopic or macroscopic rate equations.

QCD phase diagram



QCD phase diagram



Outline

1. Cold, low (moderate) density matter.

2. Cold, (very) high density matter.

3. Astrophysical constraints and Bayesian extractions.

4. Brief remarks on heavy ion collisions.

1. Nuclear Effective Field Theory (EFT)

Non-relativistic fermions at low momentum

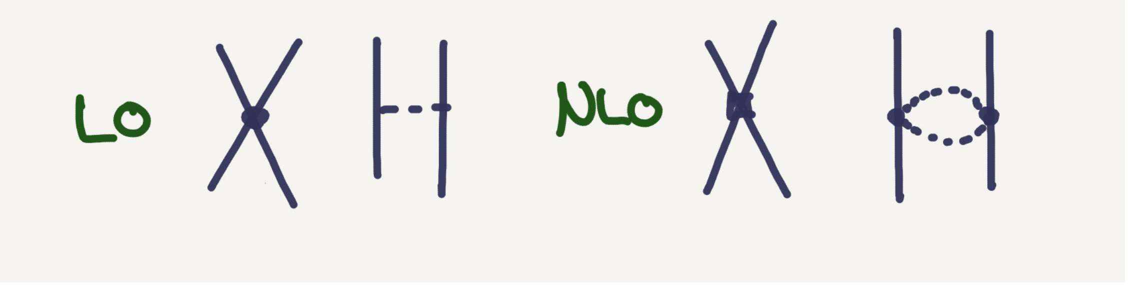
$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_S}{2} (\psi^\dagger \psi)^2 - \frac{C_T}{2} (\psi^\dagger \vec{\sigma} \psi)^2$$

$$- \frac{C_2}{4} (\psi^\dagger \nabla^2 \psi)(\psi^\dagger \psi) + \ldots$$

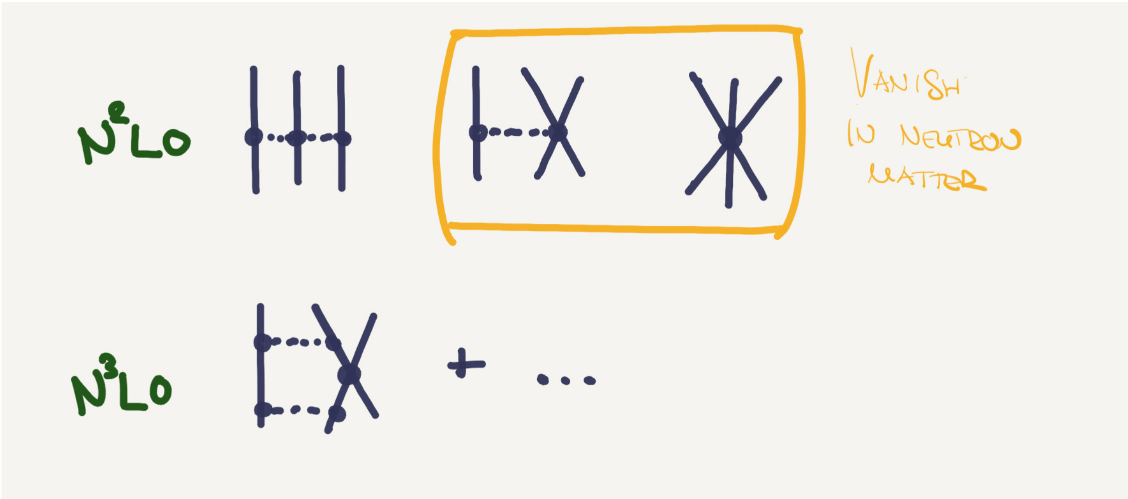
Expansion parameter $\nabla / \Lambda \sim k_F / \Lambda$.

Parameters adjusted to fit scattering observables.

At nuclear density have to include explicit pion exchange



Effective field theory automatically includes 3 and 4-body forces

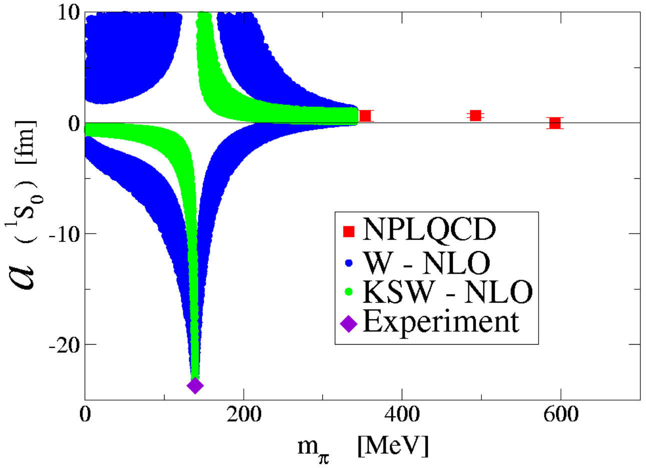


In pure neutron matter, no new parameters up to N^4LO.

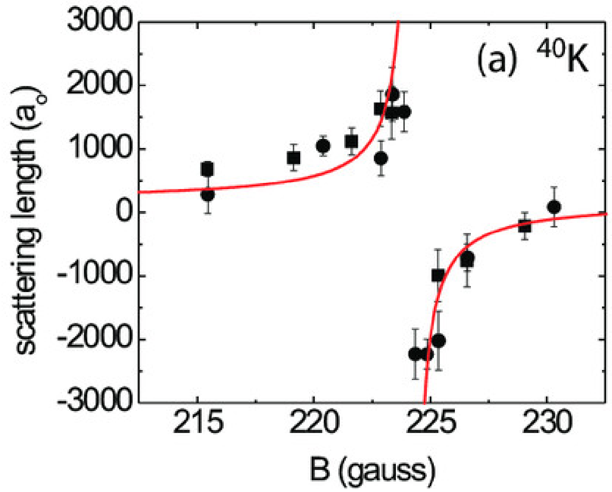
For nuclear matter, can use many-body perturbation theory, but $a_{nn} \simeq -18$ fm implies that neutron matter is non-perturbative, except at extremely small density.

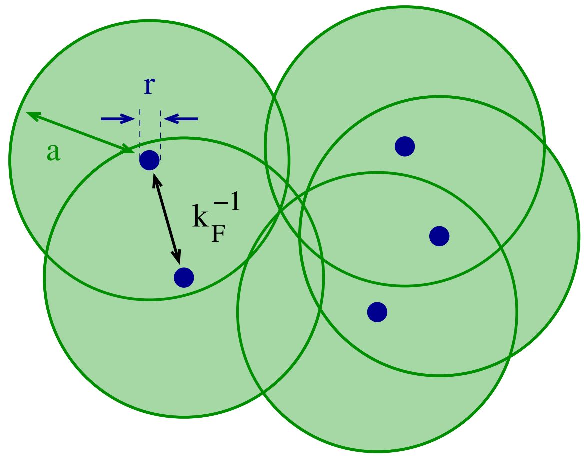
Universality: From neutrons to atoms

Neutron Matter



Feshbach resonance





What do these systems have in common?

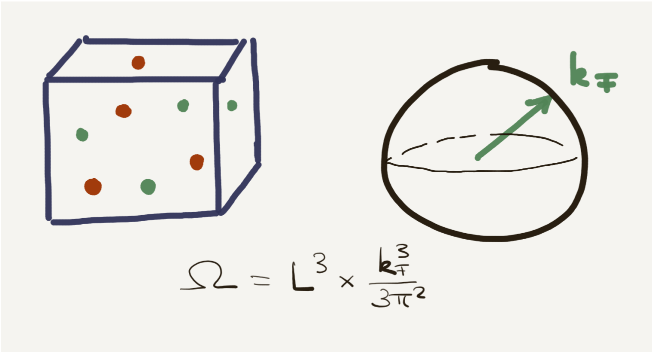
dilute: $r\rho^{1/3} \ll 1$

strongly correlated: $a\rho^{1/3} \gg 1$

Lattice calculation: Beane et al., hep-lat/0602010.

Unitary Fermi Gas: Equation of state

Free fermi gas at zero temperature



$$\frac{E}{N} = \frac{3}{5}\frac{k_F^2}{2m} \qquad \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

$$\mathcal{E} = E/V \sim (N/V)^{5/3}$$

Unitarity limit ($a \to \infty$, $r \to 0$). No dimensionful parameters.

$$\frac{E}{N} = \xi \, \frac{3}{5}\frac{k_F^2}{2m} \qquad k_F \equiv (3\pi^2 N/V)^{1/3}$$

Prize problem (George Bertsch, 1998): Determine ξ.

Is $\xi > 0$ (is the system stable)?

How to measure ξ with trapped atoms

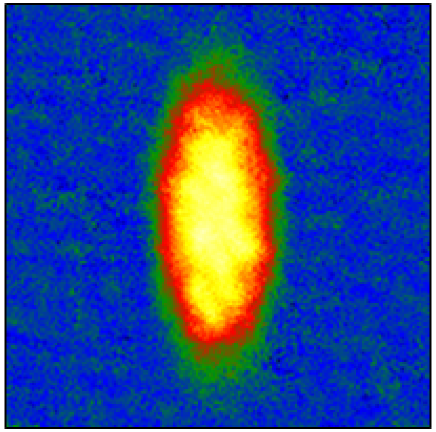
Trapped gas in hydrostatic equilibrium

$$\frac{1}{n}\vec{\nabla}P = -\vec{\nabla}V_{ext} \qquad \vec{\nabla}P = n\vec{\nabla}\mu$$

Pressure determines size of the cloud ($V_{ext} = \frac{1}{2}m\omega^2x^2$).

$$r(a=0) = \sqrt{\frac{2E_F^{trap}}{m\omega^2}} \qquad r(a=\infty) = \xi^{1/4}r(0)$$

Cloud size can be measured with a CCD camera and a ruler

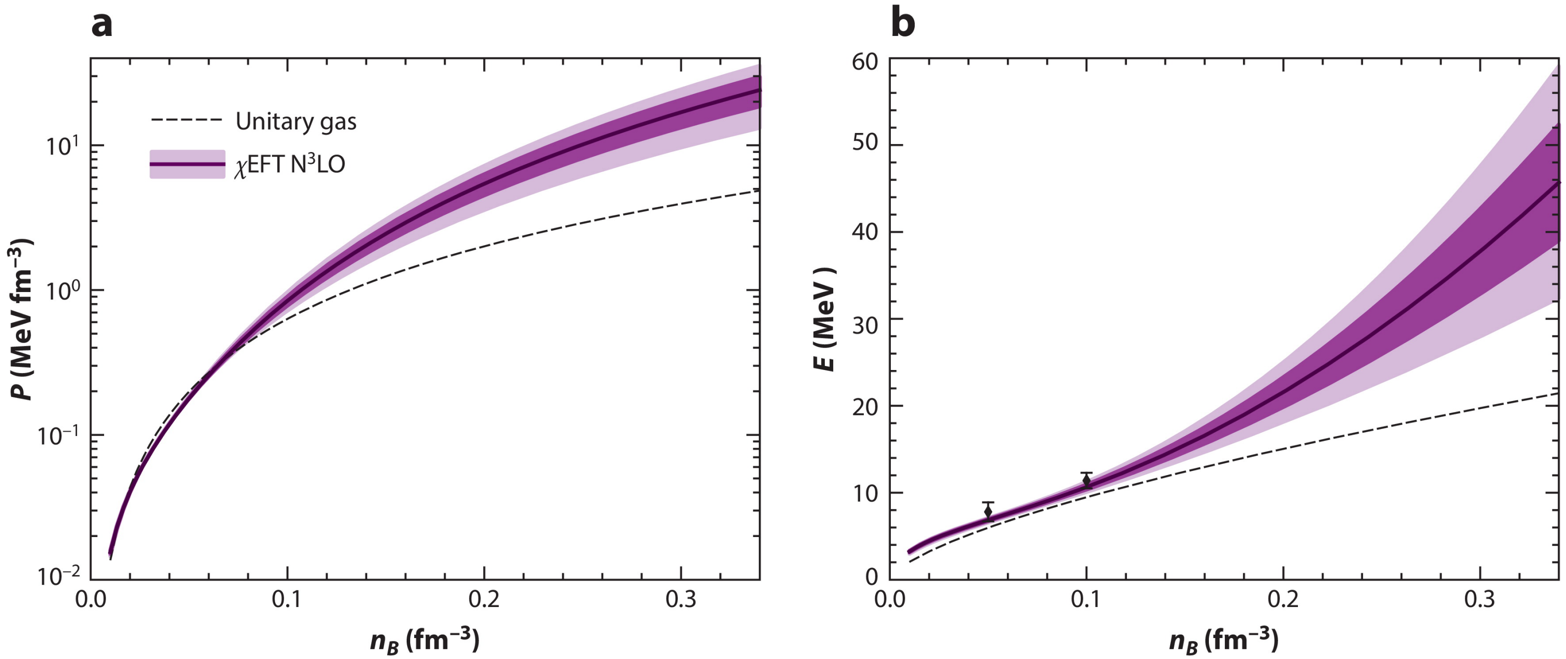


modern value

$$\xi = 0.37(5)$$

(MIT, Sommer et al.)

Neutron matter equation of state



Lattimer JM. 2021
Annu. Rev. Nucl. Part. Sci. 71:433–64

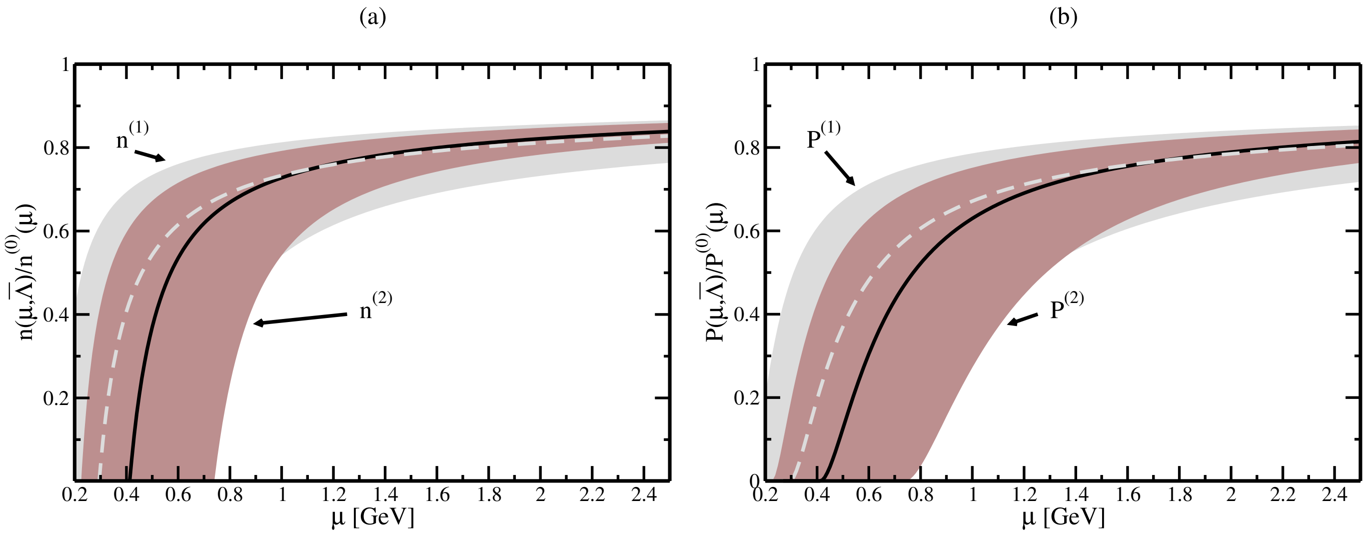
$n \lesssim 0.1\,\mathrm{fm}^{-3}$: Unitary gas with a^{-1}, r corrections.

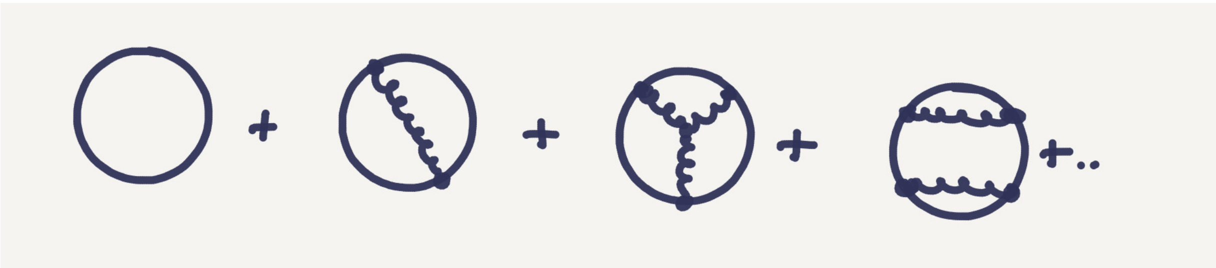
$n \gtrsim 0.1\,\mathrm{fm}^{-3}$: Repulsive 2-body, 3-body forces.

Lattimer, Ann.Rev. (2021); Based on Drischler et al. (2021); Contours show $\pm 1\sigma$, $\pm 2\sigma$.

2. Weak coupling QCD

For $\mu_B \gg m_N$ the correct degrees of freedom are quarks, and because of asymptotic freedom the coupling is weak.

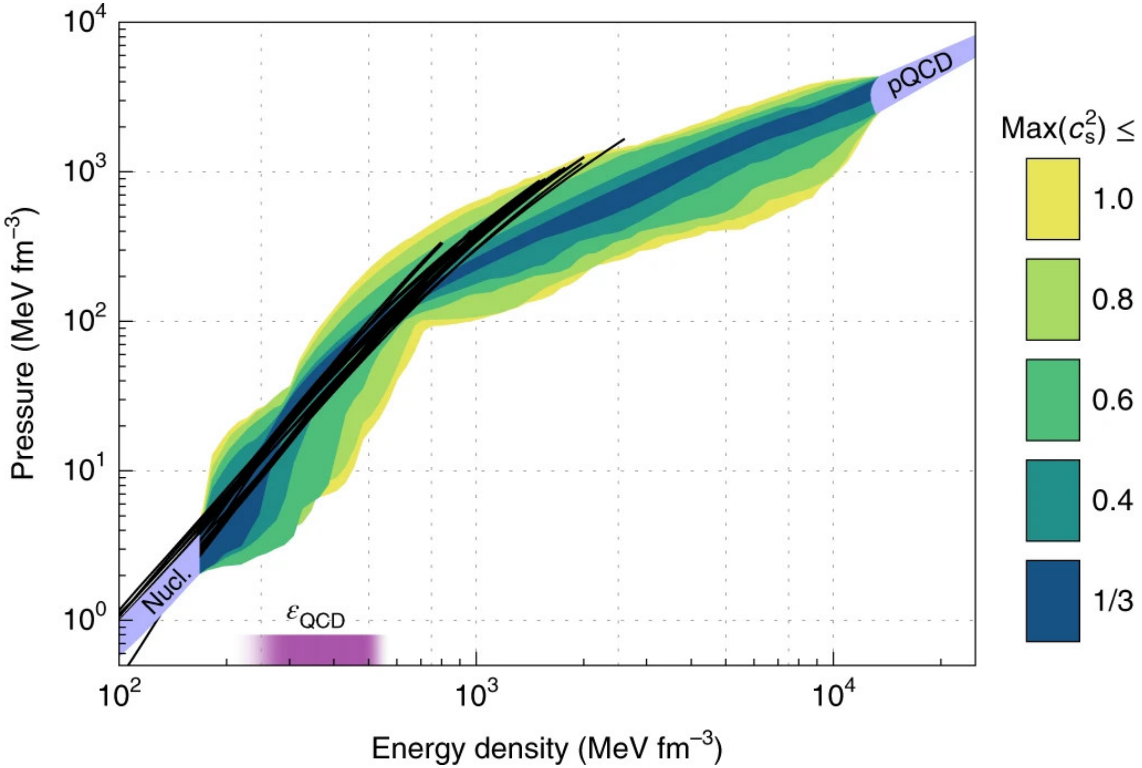




A. Kurkela et al., 0912.1856. $O(\alpha_s)$ and $O(\alpha_s^2)$ results; error from scale ambiguity.

High density constraint

Even if weak coupling is not realized in neutron stars, the EOS must eventually extrapolate to the weak coupling limit.



Annala et al., 1903.09121. Dark lines are hadronic EOS.

3. Observational constraints: Neutron Stars

Neutron star structure governed by (GR) hydrostatic equilibrium

$$\frac{dP}{dr} = -\frac{Gm(r)[\epsilon(r) + P(r)]}{r^2}\left(1 + \frac{4\pi r^3 P(r)}{m(r)}\right)\left(1 - \frac{2Gm(r)}{r}\right)^{-1}$$

Tolman-Oppenheimer-Volkov

$$m(r) = 4\pi \int_0^r d\bar{r}\, \bar{r}^2 \epsilon(\bar{r})$$

Need $P = P(\epsilon)$. Given $\epsilon(0) = \epsilon_c$ integrate to $P(R) = 0$. Obtain

$$M = m(R) \qquad \mathrm{Mass-Radius\,relation}$$

and maximum mass M_{max}.

Observational constraints

(Maximum) masses from Shapiro delays Cromartie et al. (2019), Also: PSR J0348+0432

$$\mathrm{PSR\,J0740+6620} \quad m = (2.08 \pm 0.07) m_\odot$$

Recent black widow pulsar (photometry) Romani et al. (2022).

$$\mathrm{PSR\,J0952-0607} \quad m = (2.35 \pm 0.17) m_\odot$$

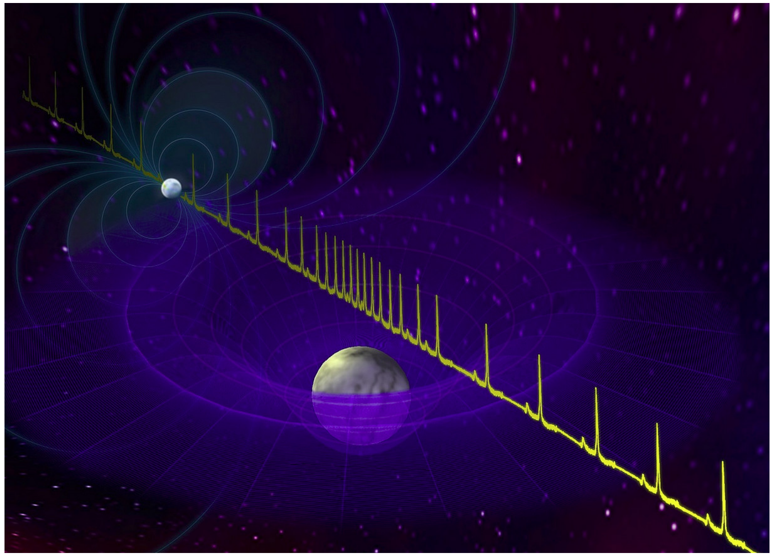
Mass-radius results from NICER (2105.06980, 1912.05705)

$$\mathrm{PSR\,J0030-0451} \quad m = (1.44 \pm 0.15) m_\odot, \quad R = 13.2^{+1.24}_{-1.06}\,\mathrm{km}$$

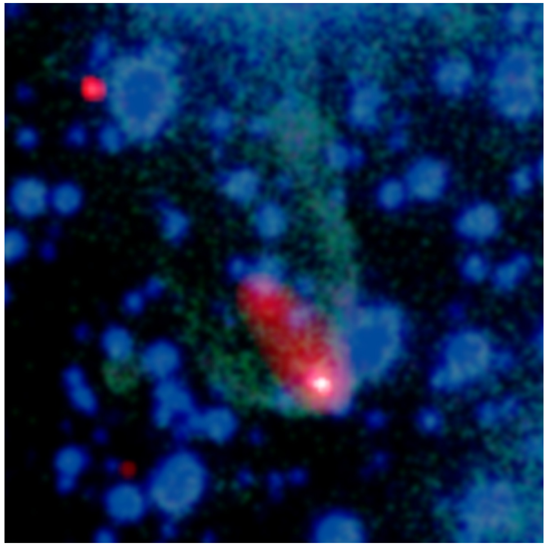
Tidal deformability from LIGO

$$\mathrm{GW170817} \quad \tilde{\Lambda} = 222^{+420}_{-138} \;\; 8.9\,\mathrm{km} \le \hat{R} \le 13.2\,\mathrm{km}$$

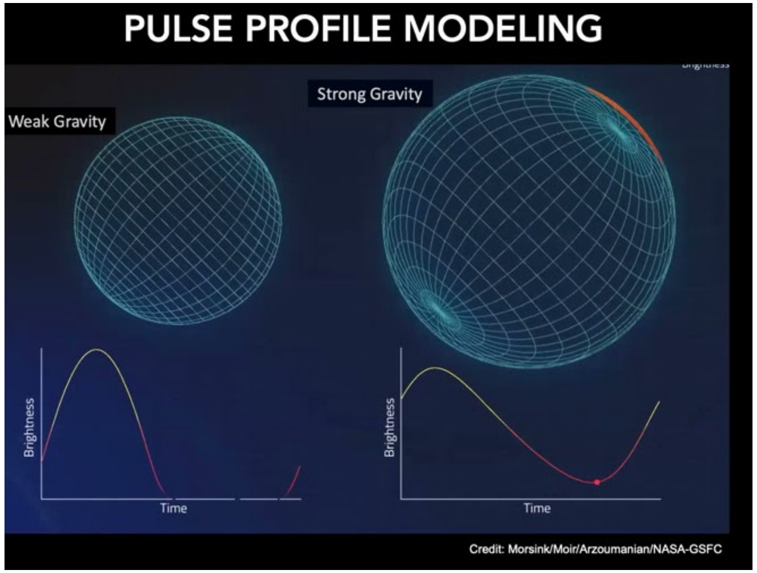
Low mass compact object in GW190814: $M = 2.6 M_\odot$.



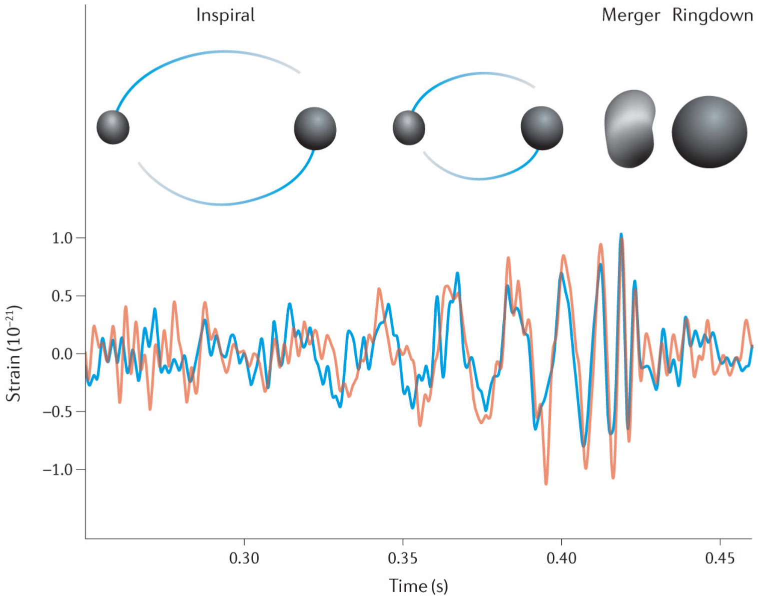
Shapiro delay



Black widow pulsar



Pulse shape (NICER)



Merger signal (LIGO)

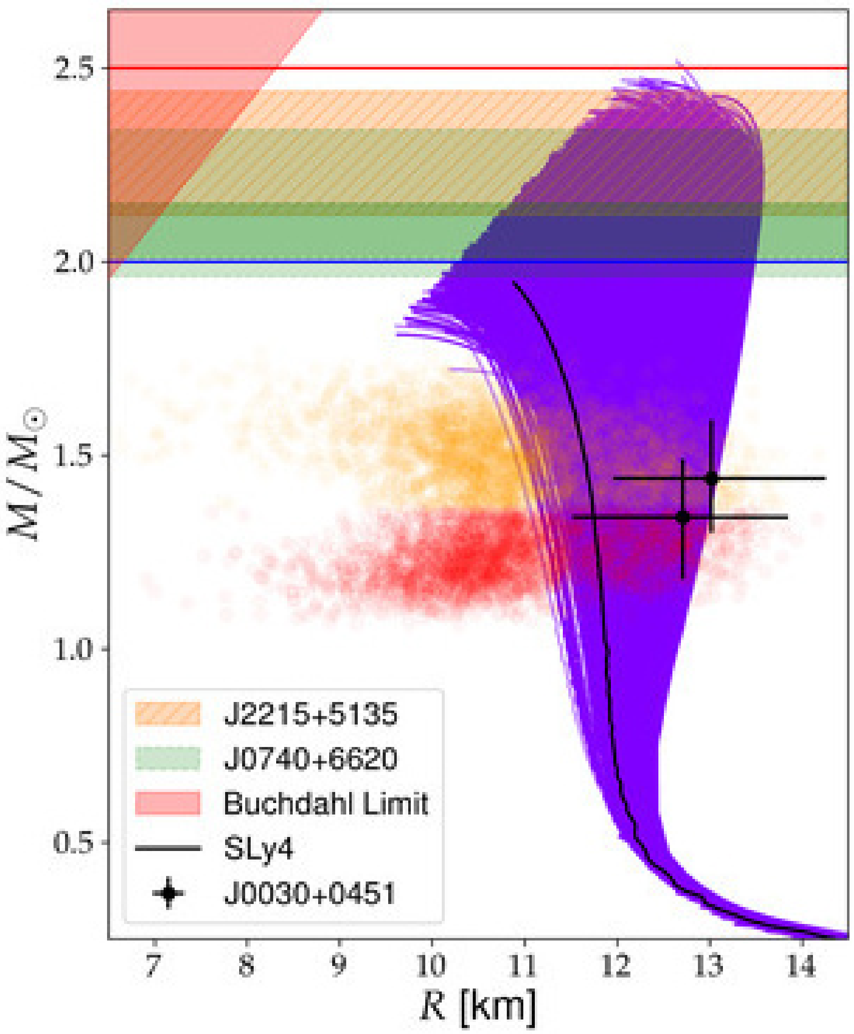
Observational constraints

Chimanski et al., 2205.01174.

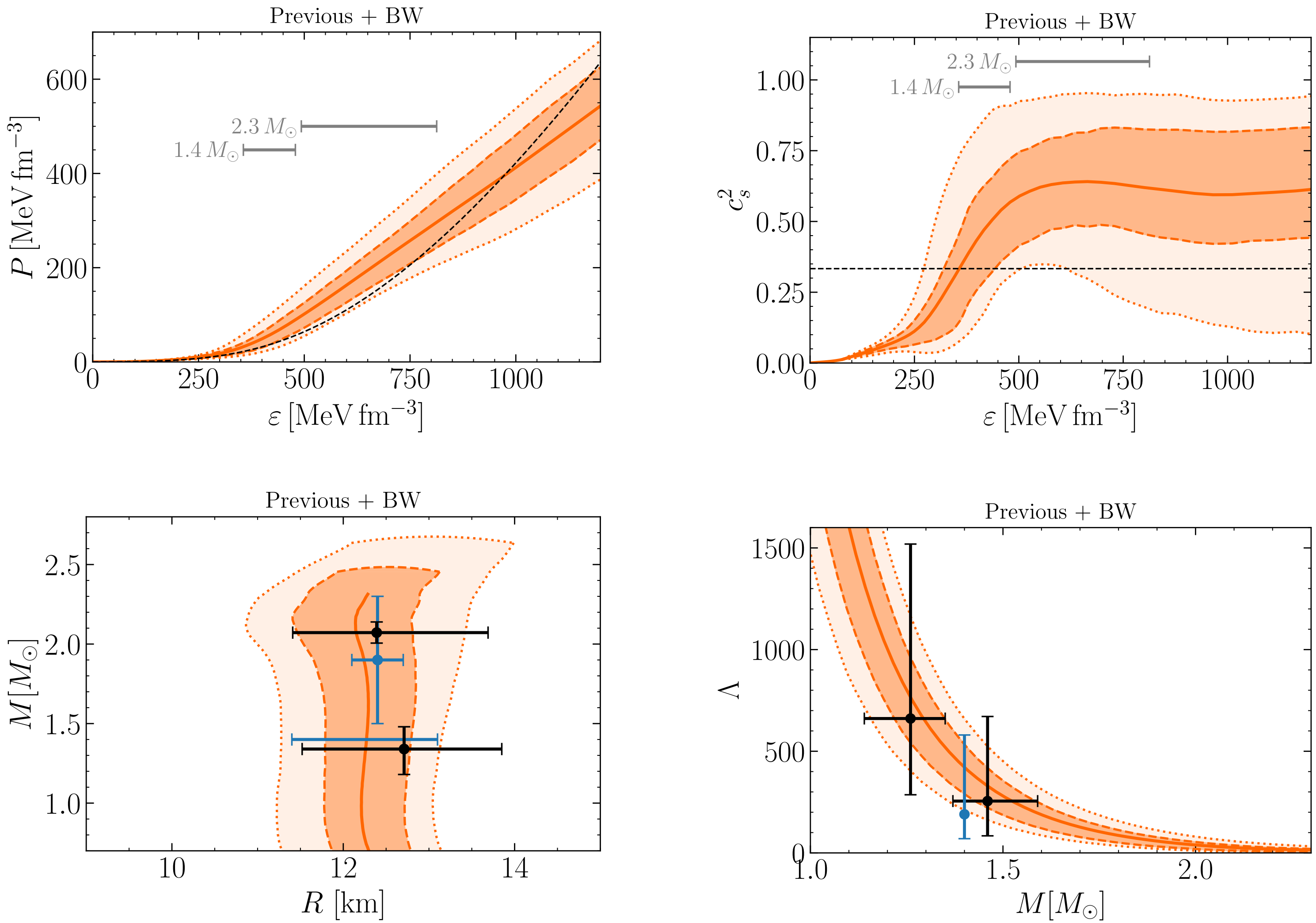
Blue: PSR J1614-2230 (Demorest et al.)

Yellow/Red: GW170817

Black Cross: PSR J0030-0451 (NICER)



Combined (Bayesian) Analysis

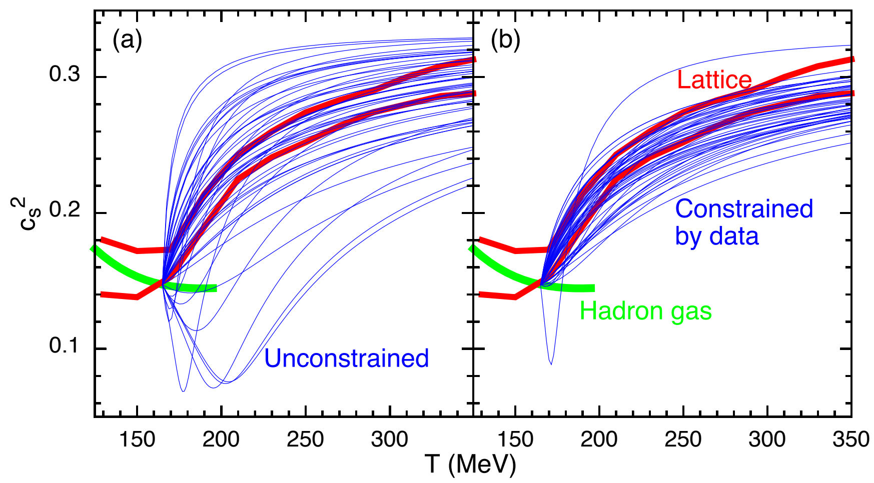


Brandes et al., 2306.06218. χEFT up to $1.3n_0$, c_s^2 based extrapolations. 68% and 95% posterior contours.

Grey: Central c_s^2 and P at 68%. $M(R)$ data from NICER, Λ from GW170817.

4. Equation of State at top RHIC energy (200 AGeV)

Hydro analysis based on $P = P(\mathcal{E})$. The speed of sound $c_s^2 = (\partial P)/(\partial \mathcal{E})$ determines the acceleration history of the fireball. Sharp phase transition: $c_s^2 = 0$. Crossover: Soft point $c_s^2(min) > 0$

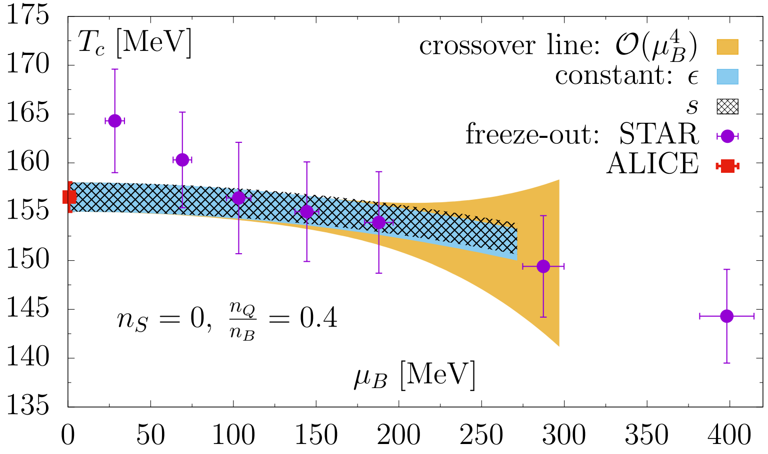


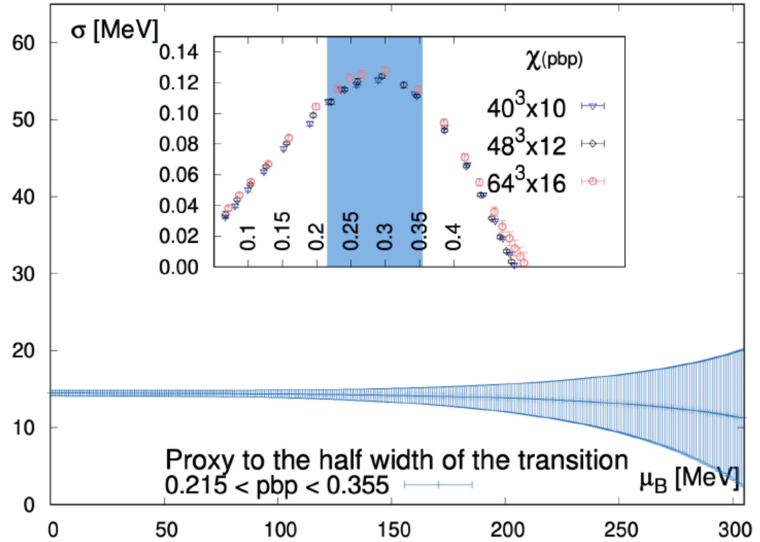
Pratt et al, PRL (2013)

Reconstruct sound speed from hydrodynamic analysis of particle spectra, flow, HBT source sizes and emission duration

Towards lower energy: The RHIC beam energy scan

Some guidance from the lattice available: Taylor expansions give susceptibilities and curvature of transition line.





Curvature of crossover transition.

Width of transition $\Delta \sim 15$ MeV.

Roughly constant, no hint of sharpening.

Bazavov et al. [1812.08235], Borsanyi et al. [2002.02821].

Equation of State at O(1) AGeV

The main tool is a (semi) phenomenological transport equation for the one-body phase space densities $f_a(x,p,t)$ $(a = N, \Delta, \pi, \ldots)$

$$\partial_t f + \nabla_p \epsilon_p \nabla_x f - \nabla_x \epsilon_p \nabla_p f = C[f]$$

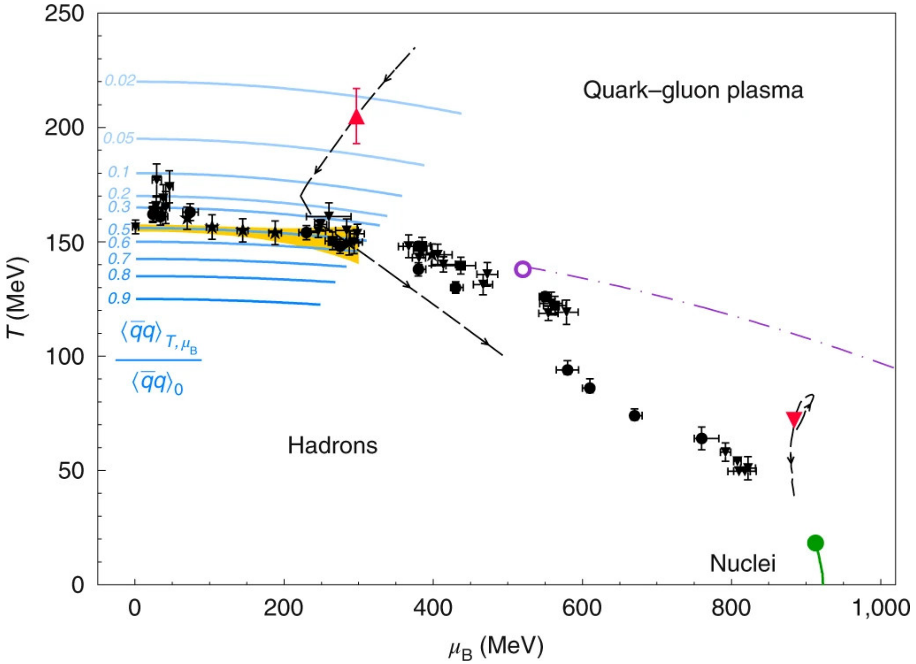
where the single particle energies are given in terms of an energy functional

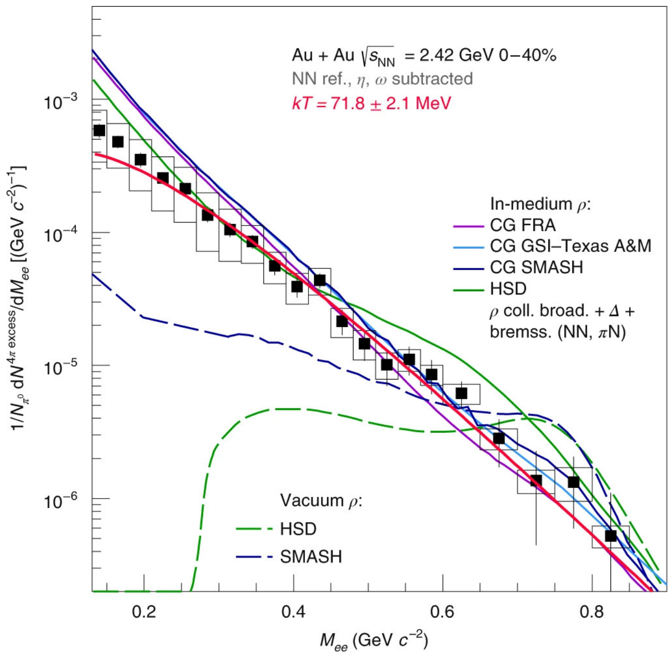
$$\epsilon_p = \frac{\delta E[f]}{\delta f(p)}$$

so that the equilibrium energy is $E[f^{eq}(p)]$.

See, for example, Sorenson et al. 2301.13253.

Phase diagram in $O(1)$ AGeV collisions





NA60: $T = 205 \pm 12$ MeV.

HADES: $T = 71.8 \pm 2.1$ MeV.

Hades, Nature Physics (2019)

Final thoughts

Cold dense matter: Many body calculations based on chiral EFT have come of age. Applicable up to 1.5, possibly 2.0 n_0.

Many observational constraints: Maximum masses, radii, tidal deformabilities.

pQCD constraints interesting, but no evidence for first order transitions or significant softening in observable NS.

Low energy HICs probe the regime of interest to NS mergers, but theory challenging.