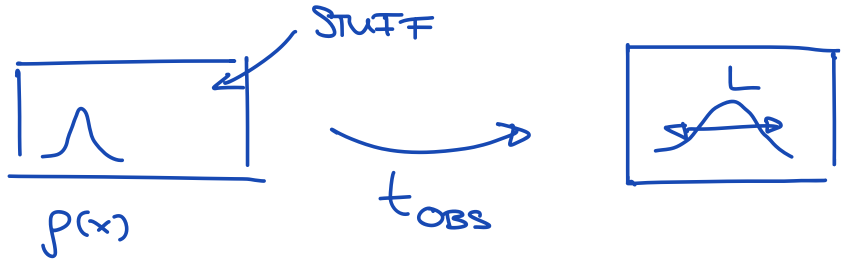
Fluctuating Fluid Dynamics

Fluid Dynamics



$$t_{OBS} \gg t_{MICRO} \qquad L \gg \ell_{mic}$$

Observation:

- Non-conserved density $\tau_{DEC} \sim t_{MICRO}$
- Conserved density $\tau_{DEC} \sim c_s L \gg t_{mic}$

$$\left.\begin{aligned} &\partial_0 \rho + \nabla \cdot j = 0 \\ &\partial_0 \pi_i + \nabla_j \pi_{ij} = 0 \\ &\partial_0 \varepsilon + \nabla \cdot j^{\epsilon} = 0 \end{aligned}\right\} \quad \partial_\mu T^{\mu\nu} = 0$$

Const. Relations: Grad. Exp.

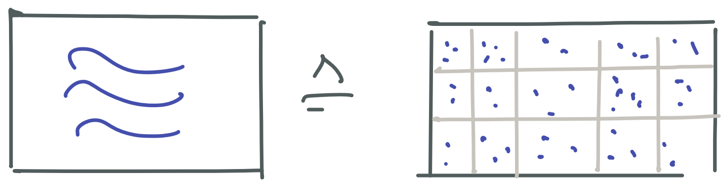
$$\pi_{ij} = \pi^0_{ij} + \pi^1_{ij} + \dots$$

$$\Pi^0_{ij} = \delta_{ij} P + \rho v_i v_j$$

$$\Pi^1_{ij} = \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\, \partial\cdot v\right) + \dots$$

Why Fluctuations?

Coarse Grainin



$$\langle (\delta\rho)^2 \rangle \sim \frac{k_B T \chi}{(\Delta V)}$$

Important if (ΔV) small or χ large $\rightsquigarrow$ Phase Transitions

Formal Reason: Respect FD-Relations ($\rightsquigarrow$ Required for correct $t \to \infty$ Behavior)

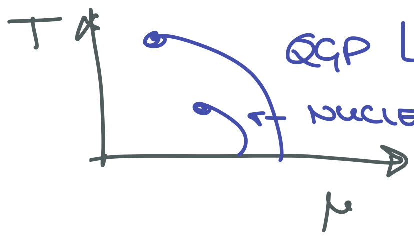
How? Landau – Lifshitz

$$\partial_0 \pi_i + \nabla_j \pi_{ij} = 0$$

$$\pi_{ij} = \pi^0_{ij} + \pi^1_{ij} + \Xi^1_{ij} + \dots$$

$$\langle \Xi_{ij} \rangle = 0 \qquad \langle \Xi_{ij} \Xi_{kl} \rangle = \eta T \Delta_{ijkl} \, \delta^3(x-x') \, \delta(t-t')$$

Example: (Non) Critical Diffusion



$$\psi \sim s/n$$

$$\partial_0 \psi + \nabla \cdot j = 0 \qquad j = -D \nabla \psi$$

Fick's Law

Non-lin. Stoch. Diffusion

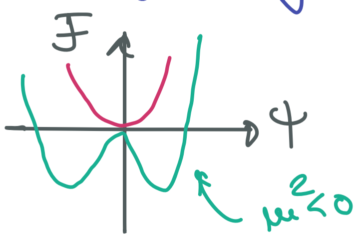
Free energy

$$\partial_0 \psi = \kappa \nabla^2 \frac{\delta F}{\delta \psi} + \zeta$$

Noise

Conductivity

$$\mathcal{F} = \int d^3x \left\{ \frac{1}{2}(\nabla\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \lambda\varphi^4 \right\}$$



$\rightsquigarrow D = \alpha m^2$

$\chi = 1/m^2$

$\xi \simeq 1/m$

$$\langle \theta \rangle = 0$$

$$\langle \theta\theta \rangle = \alpha T \nabla^2 \delta^3(x-x')\,\delta(t-t')$$

I) ANALYT. APPROACH

COR. FCT (OR RESP. FCT)

NOISE: $L_0 = \alpha T \nabla^2$

$$\langle \varphi\varphi \rangle = \int \mathcal{D}\theta \; e^{-\theta L_0^{-1} \theta}$$

$$\cdot \int \mathcal{D}\varphi \; \varphi\varphi \; \delta\left(\partial_t\varphi - \alpha\nabla^2 \frac{\delta\mathcal{F}}{\delta\varphi} - \theta\right)$$

EQU. OF MOTION

AUXILIARY FIELD

$$\delta(\mathrm{EOM}) = \int \mathcal{D}(i\tilde{\varphi}) \; e^{\tilde{\varphi}(\mathrm{EOM})}$$

$$= \int \mathcal{D}\sigma \int \mathcal{D}\phi \mathcal{D}\tilde{\phi} \; e^{-\sigma L_0^{-1} \sigma} \, e^{-\tilde{\phi}(\partial_0 \phi + \dots)}$$

$$= \int \mathcal{D}\phi \mathcal{D}\tilde{\phi} \; e^{-\mathcal{S}}$$

$$\mathcal{S} = \tilde{\phi}(\partial_0 - D\nabla^2)\phi - wT \tilde{\phi} \nabla^2 \tilde{\phi} - w\lambda \tilde{\phi} \nabla^2 \phi^3$$

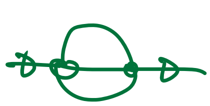
GREEN FCT

$$\begin{pmatrix} \langle \tilde{\phi}\tilde{\phi} \rangle & \langle \tilde{\phi}\phi \rangle \\ \langle \phi\tilde{\phi} \rangle & \langle \phi\phi \rangle \end{pmatrix} = \begin{pmatrix} 0 & G_R \\ G_A & G_S \end{pmatrix}$$

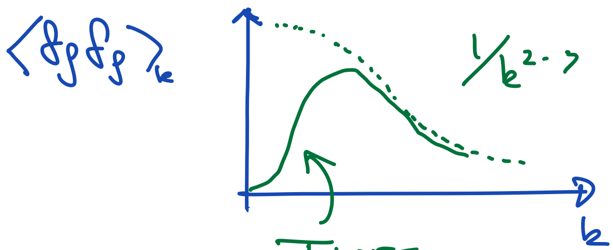
$\rightsquigarrow$ ANALYT. STR. OF WS GREEN FCT

RESULTS: (NON-CRIT)

 e^{-tDk^2}

 $\sim 1/t^{3/2}$ "LONG TIME TAIL"

CRIT. FLUID $\qquad \tau \sim L^z \qquad z \simeq 3$



FINITE
QUENCH RATE $\Rightarrow$ KZ- SCALING

II) NUMERICAL SIMULATION

CONSIDER MODEL A : $\partial_0 \psi = -\kappa \dfrac{\delta F}{\delta \psi} + \theta$

$$\psi(t+\Delta t) = \psi(t) + (\Delta t)\left\{ \left[\kappa \nabla^2 \frac{\delta F}{\delta \psi}\right]_a + \left(\frac{\kappa T}{a^3 \Delta t}\right)^{1/2} \xi \right\}$$

$\langle \xi^2 \rangle = 1$

$\Rightarrow$ NOISE DIVERGES ; BUT: IN EQUILIBRIUM

$$P[\psi] \sim \exp\left(- F/k_B T\right)$$

IDEA: METROPOLIS ALGORITHM

$$\varphi(t+\Delta t) = \varphi(t) + \sqrt{2 \kappa T \Delta t} \cdot \xi$$

$$P_{ACC} = \mathrm{MIN}\left(1, e^{-\Delta F / k_B T}\right)$$

$$P_{REJ} = 1 - P_{ACC}$$

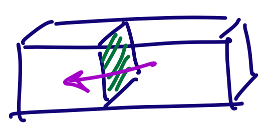
DIFFUSIVE

$$\rightarrow \quad \langle [\varphi(t+\Delta t) - \varphi(t)] \rangle = -(\Delta t)\, \kappa \frac{\delta F}{\delta \varphi}$$

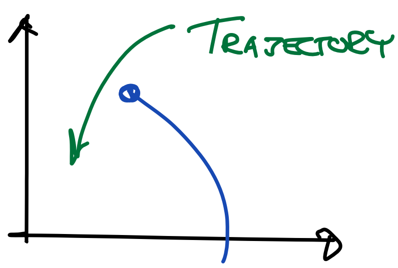
$$\langle [\varphi(t+\Delta t) - \varphi(t)]^2 \rangle = (\Delta t)\, 2 \kappa T$$

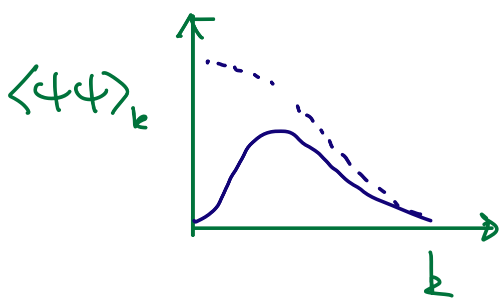
NOISE

FOR MODEL B: APPLY METROPOLIS TO FLUXES



APPLICATION:





OUTLOOK: FULL (REL?) FLUID DYNAMICS