Hydro EFTs

I) Diffusion

$$\partial_0 \psi + \vec{\nabla} \vec{j} = 0$$

$$\vec{j} = - D \vec{\nabla} \psi + \dots$$

Introduce noise & non-linear interactions

$$\partial_0 \psi = \kappa \nabla^2 \frac{\delta \mathcal{F}}{\delta \psi} + \xi$$

$$\mathcal{F} = \int d^3x \left\{ \frac{\gamma}{2} (\nabla \psi)^2 + \alpha \psi^2 + \lambda \psi^3 + \dots \right\}$$

$$\langle \xi \xi \rangle = \overset{\propto \kappa}{D}\, T\, \nabla^2 \delta(x - x') \delta(t - t')$$

$$\rightsquigarrow P[\psi] \sim \exp\left(- \mathcal{F}/k_B T\right)$$

STOCHASTIC FIELD THEORY (MSR)

$$\langle \phi \ldots \phi \rangle = \frac{1}{Z} \int \mathcal{D}\xi \, e^{-\int \xi L_0 \xi}$$

NOISE AVERAGE

$$\cdot \int \mathcal{D}\phi \; \delta(\partial_0 \phi - \ldots) \, \phi \ldots \phi$$

EQU. OF MOTION

$$L_0 \equiv D T \nabla^2$$

NOISE KERNEL

$$= \frac{1}{Z} \int \mathcal{D}\tilde{\phi} \, \mathcal{D}\phi \, \mathcal{D}\xi \, e^{-\int \xi L_0 \xi} \cdot e^{-\int \tilde{\phi}(\partial_0 \phi - \ldots)} \, \phi \phi \ldots$$

AUXILIARY FIELD

$$= \frac{1}{Z} \int \mathcal{D}\tilde{\phi} \, \mathcal{D}\phi \, e^{-S}$$

$$S = \int d^4x \int \tilde{\phi} (\partial_0 - D \nabla^2) \phi$$

DIFF.

$$+ \tilde{\phi} D T \nabla^2 \tilde{\phi}$$

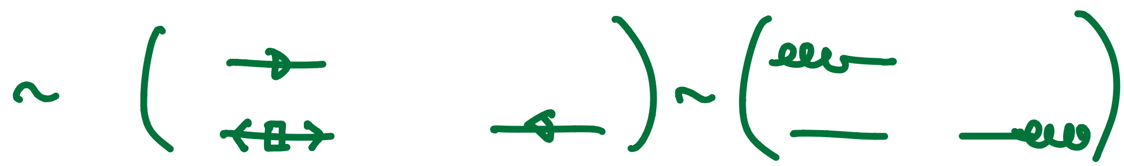
NOISE

$$+ \tilde{\psi} D \lambda \nabla^2 \psi^2 + \ldots \}$$ INT.

MATRIX PROP.

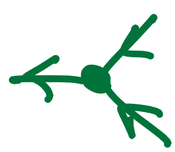
$$\begin{pmatrix} \langle \tilde{\psi} \psi \rangle & \langle \tilde{\psi} \tilde{\psi} \rangle \\ \langle \psi \psi \rangle & \langle \psi \tilde{\psi} \rangle \end{pmatrix}$$

$$\sim \begin{pmatrix} D_R & \sigma \\ D_S & D_A \end{pmatrix}$$



INTERACTION VERTEX

$\lambda D k^2$



NOTES & QUESTIONS

- FIND KELDYSH STRUCTURE!
- WHAT IS THE EXPANSION? WHAT ARE THE RULES FOR EXTRA INTERACTIONS?

(RELATED: WHAT ARE THE SYMM.?)

a) EXPANSION:

NON-CRIT FLUID: $(k\xi) < 1$

$\rightsquigarrow$ GRAD. EXP.

CRITICAL FLUID: $\xi \to \infty$

$\rightsquigarrow$ FIXED POINT, PARAM. BY FINITE NUMBER OF COUP.

b) SYMMETRIES

$$\frac{P(\psi_1 \to \psi_2)}{P(\psi_2 \to \psi_1)} = e^{-\Delta F / k_B T}$$

$\rightsquigarrow$ T-REVERSAL

$$\psi(t) \to \psi(-t)$$

$$\tilde{\psi}(t) \to -\left[\tilde{\psi}(-t) + \frac{\delta F}{\delta \psi}\right]$$

$$\mathcal{L} \to \mathcal{L} + \frac{d}{dt} F$$

WARD IDENTITIES $\rightsquigarrow$ FD RELATIONS
(REQUIRES EINSTEIN: NOISE FIXED)

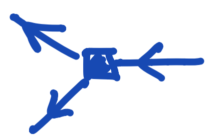
AT THIS ORDER, ALLOWS ONE NEW INTERACTION

$$D \to D(\psi) = D_0 \left[1 + \tilde{\lambda}\psi + \dots\right]$$

DENS. DEP. DIFF.

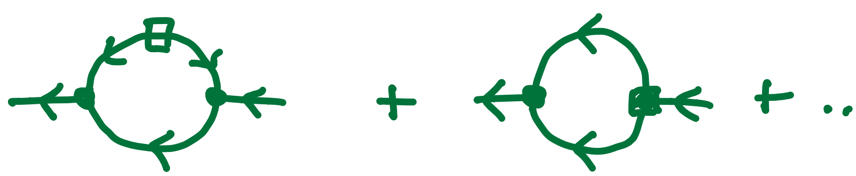
T- INVARIANCE FIXES NOISE

$$D_0 \tilde{\lambda}\, \psi\, (\vec{\nabla}\tilde{\psi})^2$$



$$D_0 \tilde{\lambda}\, (k_1 \cdot k_2)$$

NON-CRIT FLUID

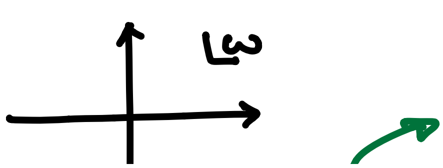


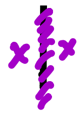
$$\Sigma(\omega,k) = i D k^2$$

NEW

$$+ \frac{\lambda}{32\pi^2}\left[i\lambda\omega k^2 + \tilde{\lambda}(i\omega - Dk^2)k^2\right]$$

$$\times \left[k^2 - \frac{2i\omega}{D}\right]^{1/2}$$





' DIFFUSIVE CUT

= HYDRO TAIL

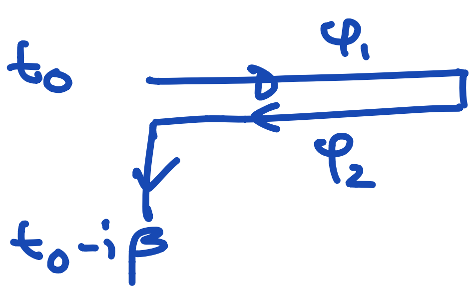
II) MODERN TECHNOLOGY

NON-DISSIPATIVE: GWW, SON-WING.

$$\mu \to X = \mu + \partial_0 \varphi - \frac{(\nabla \varphi)^2}{2m}$$

$$\mathcal{L} = P(X) \sim f^2\left((\partial_0\varphi)^2 - c_s^2 (\nabla\varphi)^2\right) + \dots$$

NOW PUT THIS ON KELDYSH CONTOUR (GLORIOSO & LIU, HARTNOLL, ..)



$$\varphi_R = \frac{1}{2}(\varphi_1 + \varphi_2)$$

$$\varphi_a = \varphi_2 - \varphi_1$$

IMPOSE KMS & T-REVERSAL

$$\varphi_1 \to \varphi_2(-t + i\theta)$$

$$\varphi_2 \to \varphi_2(-t - i(\beta - \theta))$$

TAKE SEMI CL. LIMIT

$$\tilde{\varphi}_R = \varphi_R \qquad \tilde{\varphi}_A = \hbar\, \varphi_A \qquad \hbar \ll 1$$

$$\begin{cases} \varphi_R \to \varphi_R(-t) \\ \varphi_A \to -\varphi_A(-t) + i\partial_0 \varphi_R \end{cases}$$

"KMS" SYMMETRY

EFT

$$\mathcal{L} = P'(\mu) B_a^t + i T D B_a^i \cdot (B_a^i + i\partial_0 B_R^i)$$

$$B_r^t = \mu = \partial_t \varphi_R + \dots$$

$$B_a^t = \partial_t \varphi_a + \dots$$

USE $P'(\mu) = n$, $\mu = \frac{\delta F}{\delta n}$

$$\rightsquigarrow \mathcal{L} = \varphi_a (\partial_t n + D\nabla^2 n) + \dots$$

AGREES WITH USR. BUT

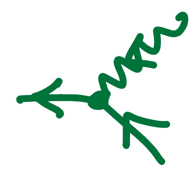
MORE POWERFUL FOR CONSTR.
HIGHER ORDERS
(KOVTUN: FIND "BEYOND CLAS. HYDRO" INTERACTIONS)

III) MODEL H

INCLUDE ADVECTION BY $\vec{\pi}$

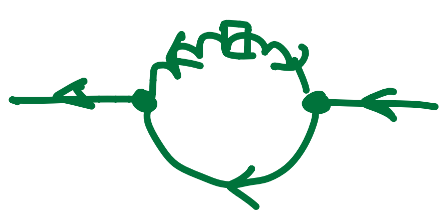
$$\mathcal{L} = \vec{\pi}\,(\partial_0 - \gamma_0 \nabla^2)\,\vec{\pi} \qquad \gamma_0 = \eta/sT$$

$$\mathcal{L} = \frac{1}{\omega}\,\tilde{\psi}\,\vec{\pi}\cdot\vec{\nabla}\psi$$



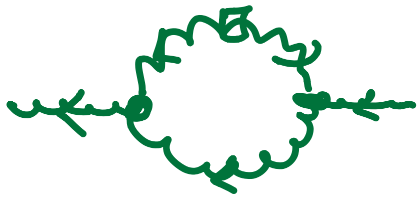
ADVECTION

IMPORTANT: ORDER PAR. RELAXATION



CRIT. FLUID "KAWASAKI FCT"

SHEAR TAIL (RENORMALIZED η)



NEW INGREDIENT: POISSON BRACKETS

$$\partial_t \psi \sim \{\mathcal{H}, \psi\} \sim \int dx \, \frac{\delta F}{\delta \pi} \{\pi, \psi\}$$

$$\sim \frac{1}{w} \pi \cdot \nabla \psi$$

SOME CONCLUSIONS

- BOTH IN MODEL B & H FIND ONE NEW COUPLING, WITH SIMPLE PHYS. INTERPRETATION

$$\kappa \to \kappa(\psi) = \kappa_0 (1 + \tilde{\lambda} \psi + \dots)$$

$$\eta \to \eta(\psi) = \eta_0 (1 + \lambda_\eta \psi + \dots)$$

AT HIGHER ORDER ADDITIONAL (" NON-CL HYDRO) COUPLINGS

APPEAR

- MODIFY HYDRO TAILS QUANT., BUT NOT QUALITATIVELY

- TO DO: N-POINT FCTS, ETC.