Xtreme QCD

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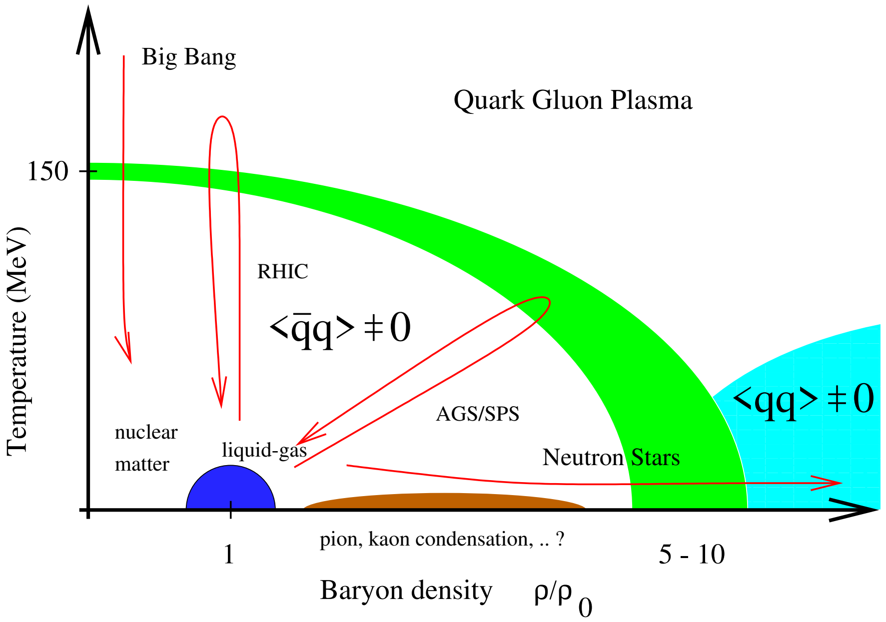
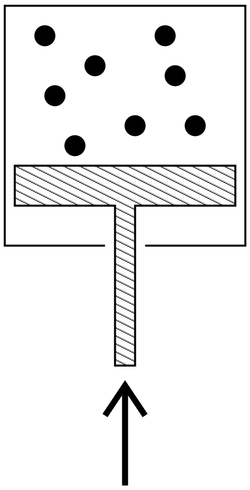
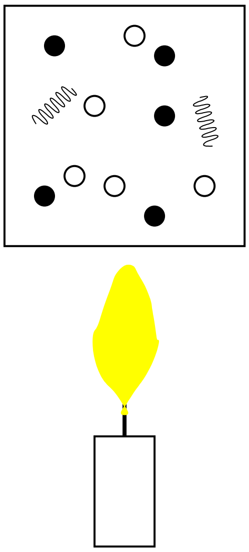


QCD in Xtreme Conditions

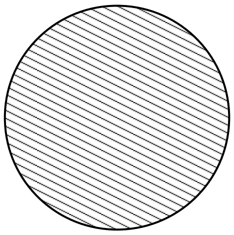
High Density



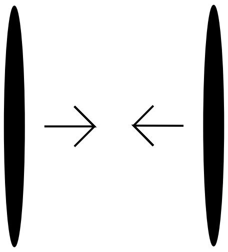
High Temperature



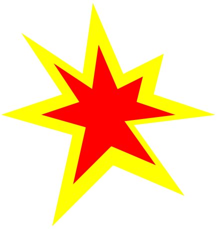
more practical:



neutron
stars



relativistic
heavy–ion
collisions



big bang

WHY?

- These conditions exist in the universe:

Neutron Stars, Big Bang

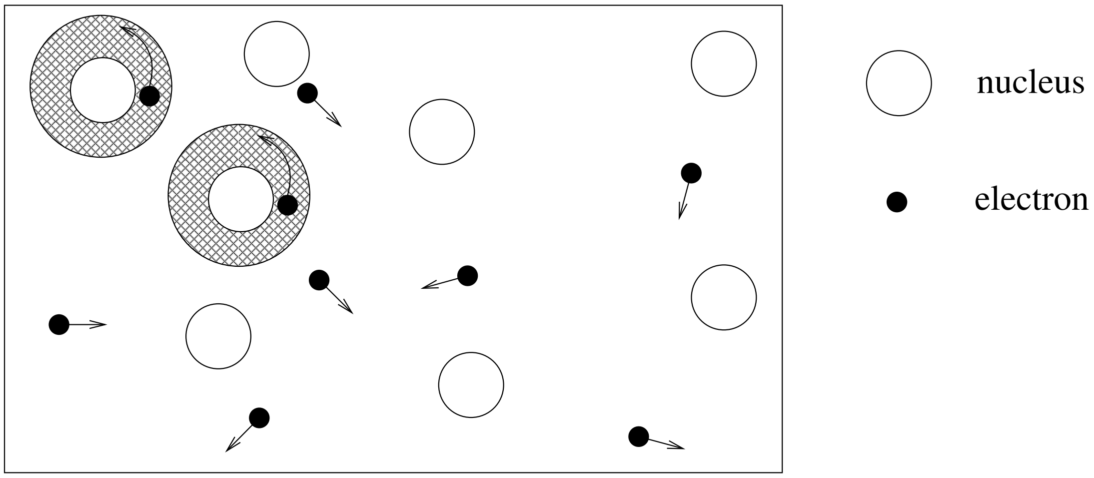
- Exploring the entire phase diagram is important to understanding the phase that we happen to live in

We cannot understand the structure of hadrons without understanding the non-trivial QCD vacuum.
We cannot understand the vacuum state without understanding how it can be modified.

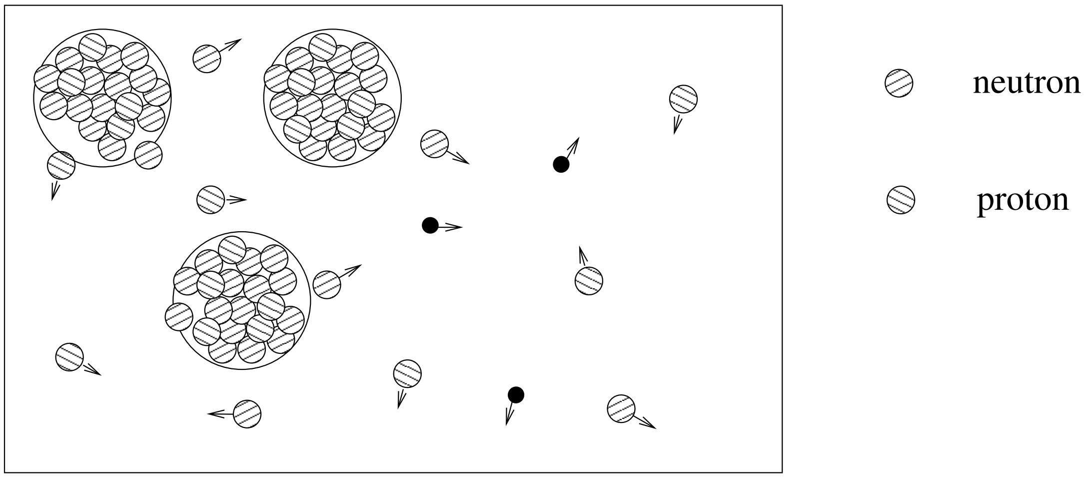
- QCD simplifies in extreme environments:

Study QCD matter in a regime where quarks and gluons are indeed the correct degrees of freedom.

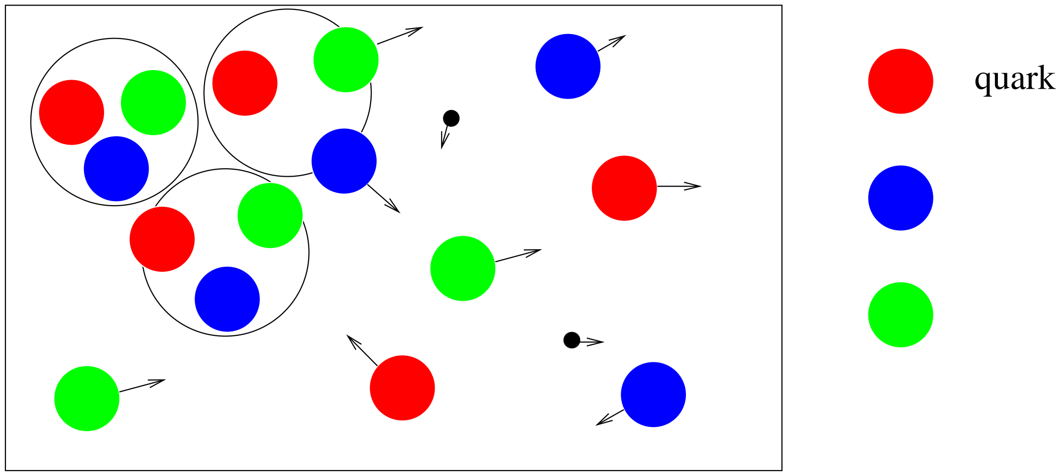
Dense Matter



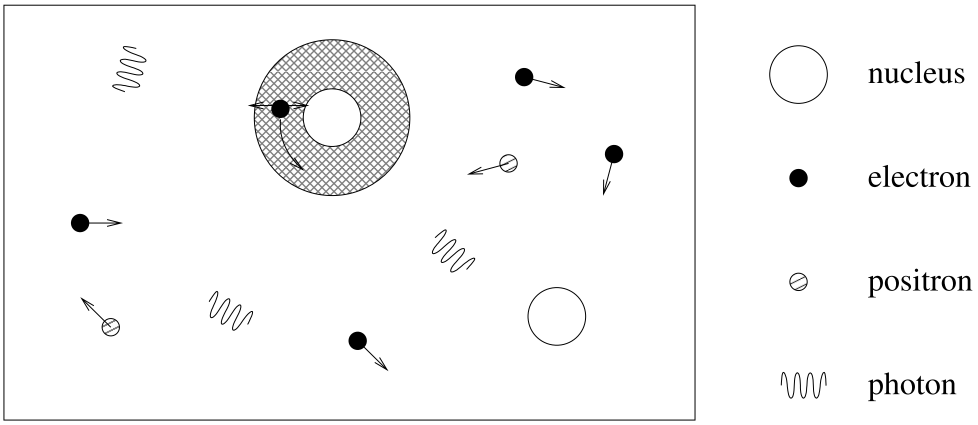
white dwarf matter



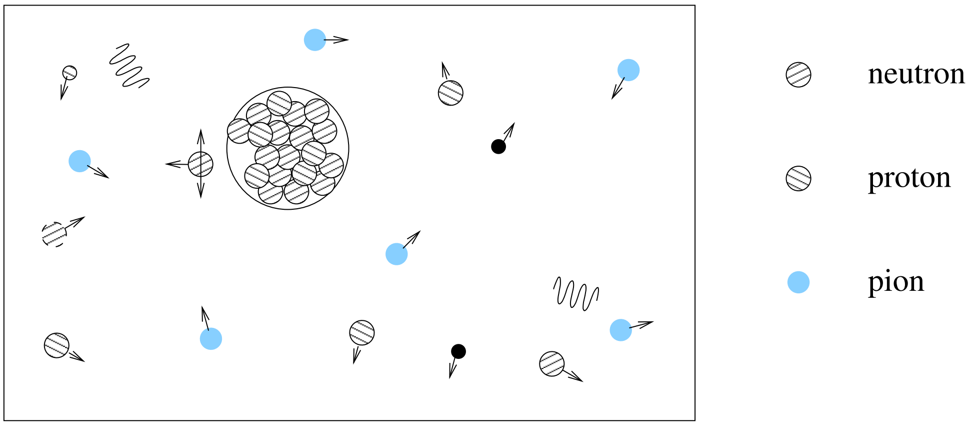
neutron matter



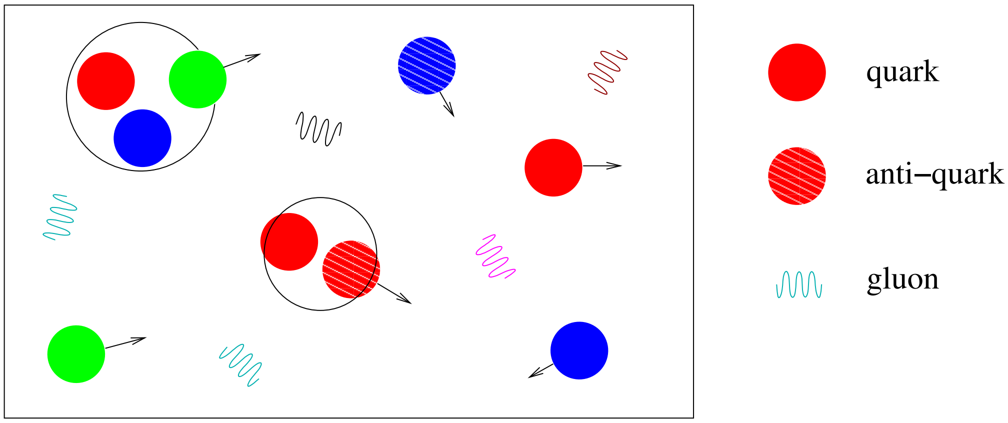
Hot Matter



Plasma



Hadron Gas



Quark Gluons Plasma

Quantum Chromo Dynamics

- elementary fields

$$\text{quarks} \qquad (q_\alpha)^a_f \qquad \begin{cases} \text{color} & a = 1,\ldots,3 \\ \text{spin} & \alpha = 1,2 \\ \text{flavor} & f = u,d,s,c,b,t \end{cases}$$

$$\text{gluons} \qquad A^a_\mu \qquad \begin{cases} \text{color} & a = 1,\ldots,8 \\ \text{polarization} & \epsilon^\pm_\mu \end{cases}$$

- dynamics: gauge theory

$$\mathcal{L} = \sum_f^{N_f} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

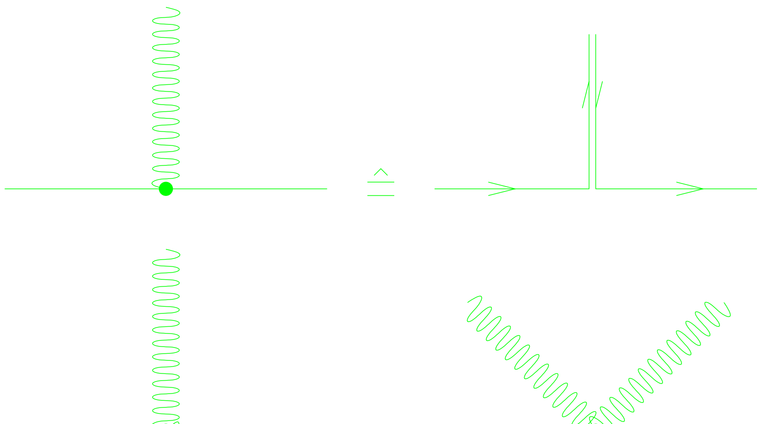
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

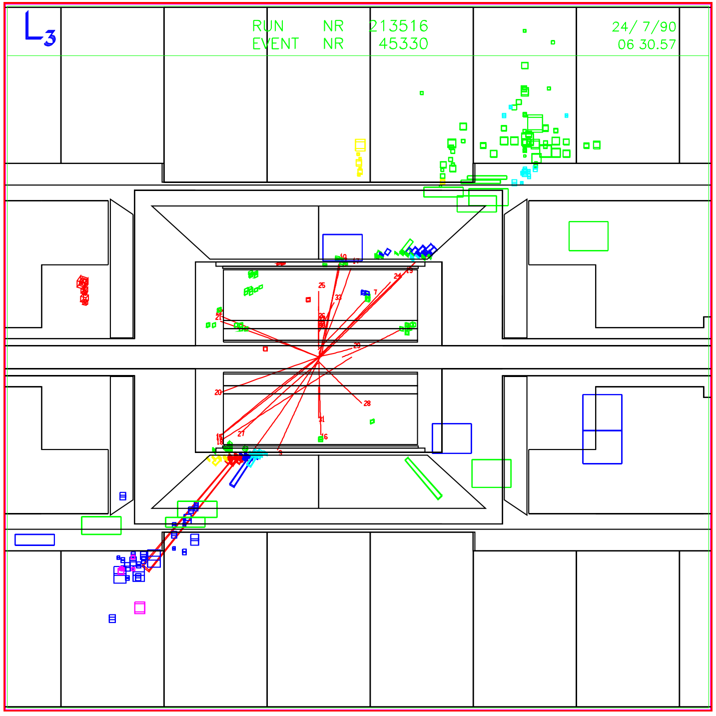
$$i\not{D} q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu \frac{\lambda^a}{2} \right) q$$

- quantum mechanics: path integral (partition function)

$$Z = \int DA_\mu \;\; \mathrm{Det}(i\not{D} - m) \;\; \exp(i \int d^4x \mathcal{L}_g)$$

- dynamics in diagrams (Feynman)

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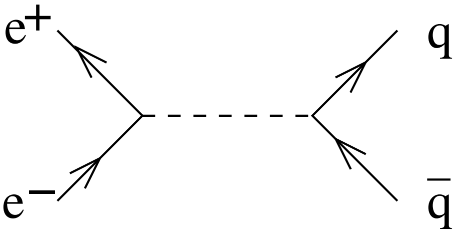
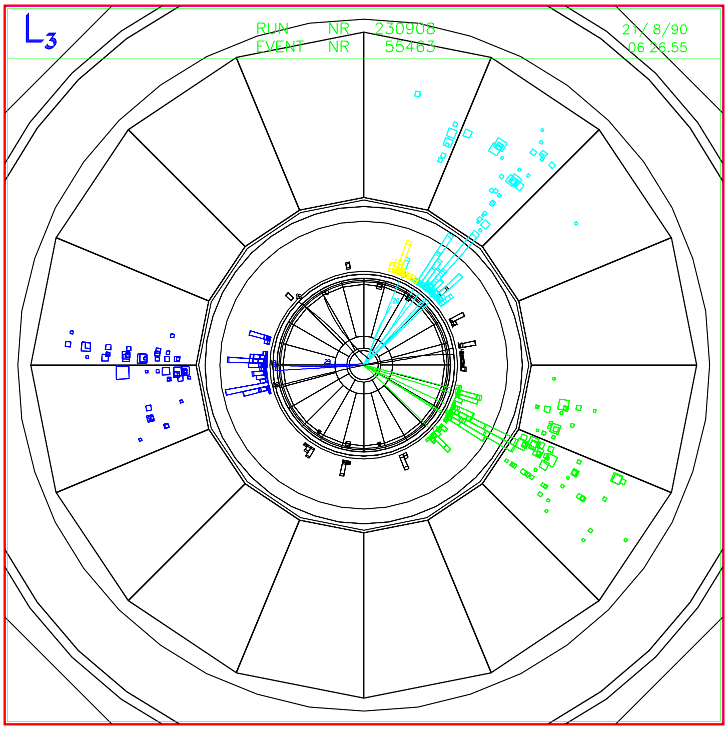
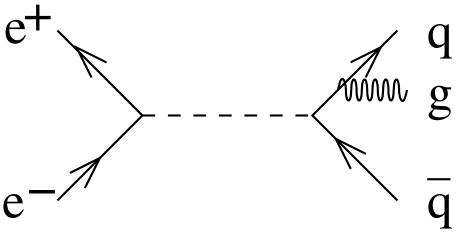
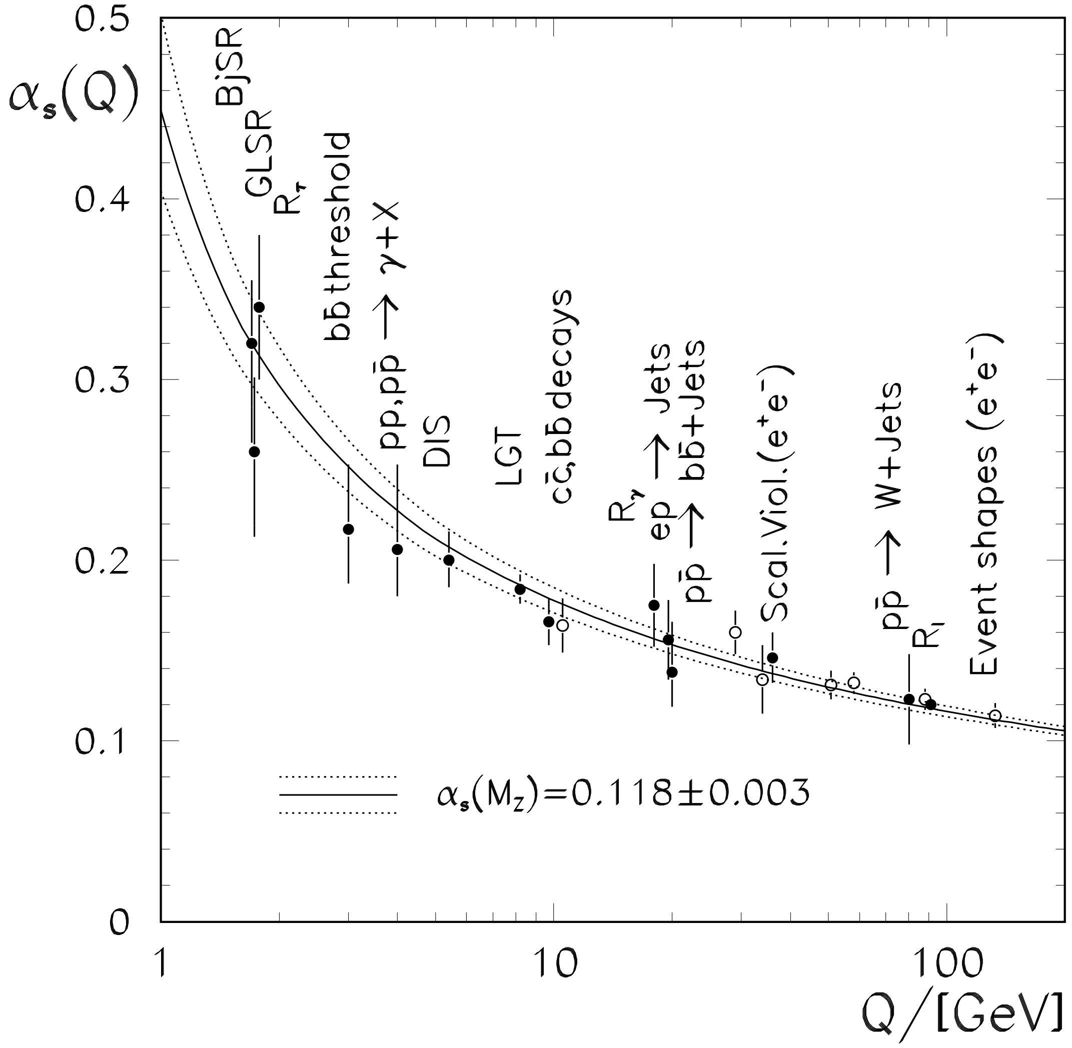
RUN NR 213516
EVENT NR 45330
24/ 7/90
06 30.57


e+
e−
q
$\bar{q}$


L3
RUN NR 230908
EVENT NR 55463
21/ 8/90
06 26.55


e+
e−
q
g
$\bar{q}$


$\alpha_s(Q)$
0.5
0.4
0.3
0.2
0.1
0
1
10
100
Q/[GeV]
BjSR
GLSR
R_τ
$b\bar{b}$ threshold
$pp, p\bar{p} \rightarrow \gamma + X$
DIS
LGT
$c\bar{c}, b\bar{b}$ decays
R_γ
$ep \rightarrow$ Jets
$p\bar{p} \rightarrow b\bar{b}$+Jets
Scal.Viol.(e^+e^-)
$p\bar{p} \rightarrow$ W+Jets
R_l
Event shapes (e^+e^-)
$\alpha_s(M_Z)=0.118\pm0.003$

About units

- QCD is essentially parameter free: The lagrangian has a coupling constant, g, but no scale (if $m_q \simeq 0$).
- After renormalization g becomes scale dependent (g is traded for a scale parameter Λ)
- g is not a parameter, but QCD has an "urkilogramm"

  $$\Lambda_{QCD} \simeq 200\,\mathrm{MeV} \simeq 1\,\mathrm{fm}^{-1}$$

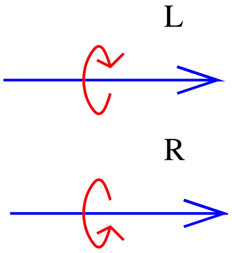
  $\Lambda^{-1} \simeq 1\,\mathrm{fm}$ is the distance at which the coupling becomes strong
- In practice, perturbation theory breaks down at much shorter distances

  $$x \sim \Lambda_\chi^{-1} \sim 1\,\mathrm{GeV}^{-1} \sim 0.2\,\mathrm{fm}$$
- Note: $m_\rho \sim m_p \sim 1\,\mathrm{GeV}$, $r_p \sim 1\,\mathrm{fm}$.

Chiral Symmetry

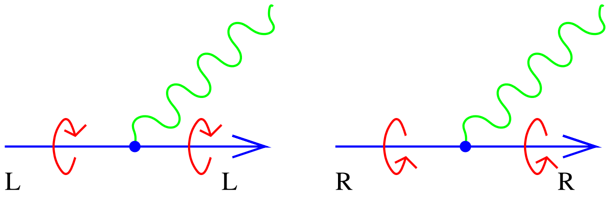
- define L/R fields

$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

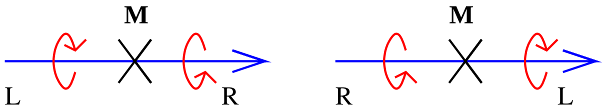


- fermionic lagrangian

$$\mathcal{L} \;=\; \bar{\psi}_L(i\not{D})\psi_L + \bar{\psi}_R(i\not{D})\psi_R$$



$$+\,\bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$



$$M \;=\; \mathrm{diag}(m_u, m_d, m_s)$$

- $m_u = m_d = m_s$: Flavor symmetry $V \in SU(3)$

$$\psi_L \to V\psi_L, \qquad \psi_R \to V\psi_R$$

- $m_u = m_d = m_s = 0$: Chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

$$\psi_L \to L\psi_L, \qquad \psi_R \to R\psi_R$$

The Fate of Chiral Symmetry

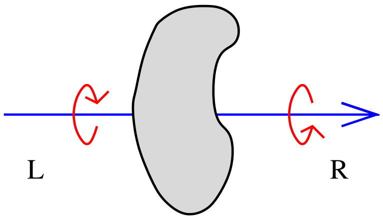
- Chiral symmetry implies parity doubling, e.g.

$$\begin{array}{rcl} \rho(LL+RR) & \leftrightarrow & a_1(LL-RR) \\ \text{expt}\ \ m_\rho = 770\,\text{MeV} & & m_{a_1} = 1230\,\text{MeV} \end{array}$$

too large in order to be explained by quark masses $m_u, m_d \sim 10\,\text{MeV}$

- chiral symmetry is spontaneously broken

$$\langle \bar\psi_L \psi_R + \bar\psi_L \psi_R \rangle \simeq -(230\,\text{MeV})^3$$

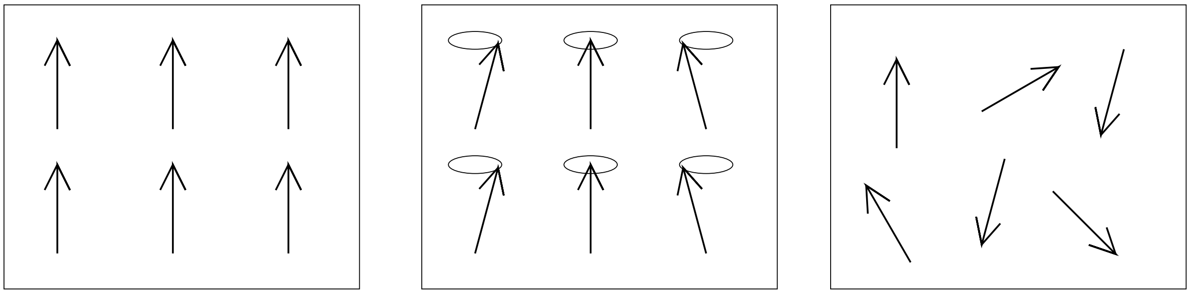


- consequences: dynamical mass generation $m_Q = 300\,\text{MeV} \gg m_q$

$$\begin{array}{lccc} m_p = & 890\,\text{MeV} & + & 45\,\text{MeV} \\ & (\text{QCD},\ 95\%) & & (\text{Higgs},\ 5\%) \end{array}$$

- existence of Goldstone modes

$$\langle \bar\psi^f_L \psi^g_R \rangle = B\,(\sigma + i\vec\tau\vec\pi)^{fg}$$

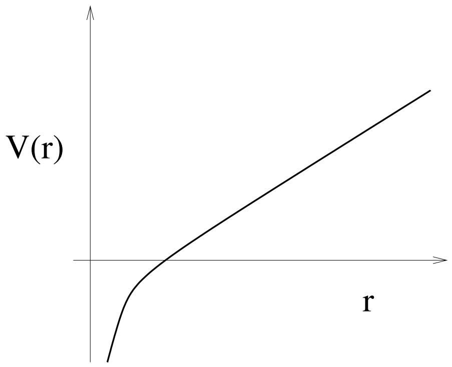


ordered | spin wave | disordered

chiral symmetry broken | pion | chiral symmetry restored

Confinement

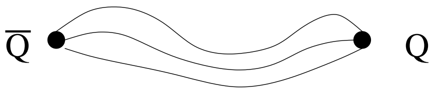
- heavy quark potential



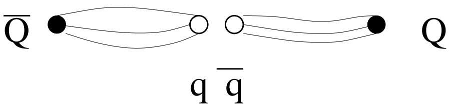
$$V(r) \sim \frac{\alpha}{r} + kr$$

$$k \sim 1\mathrm{GeV/fm}$$

- picture: gluon field channeled into flux tubes

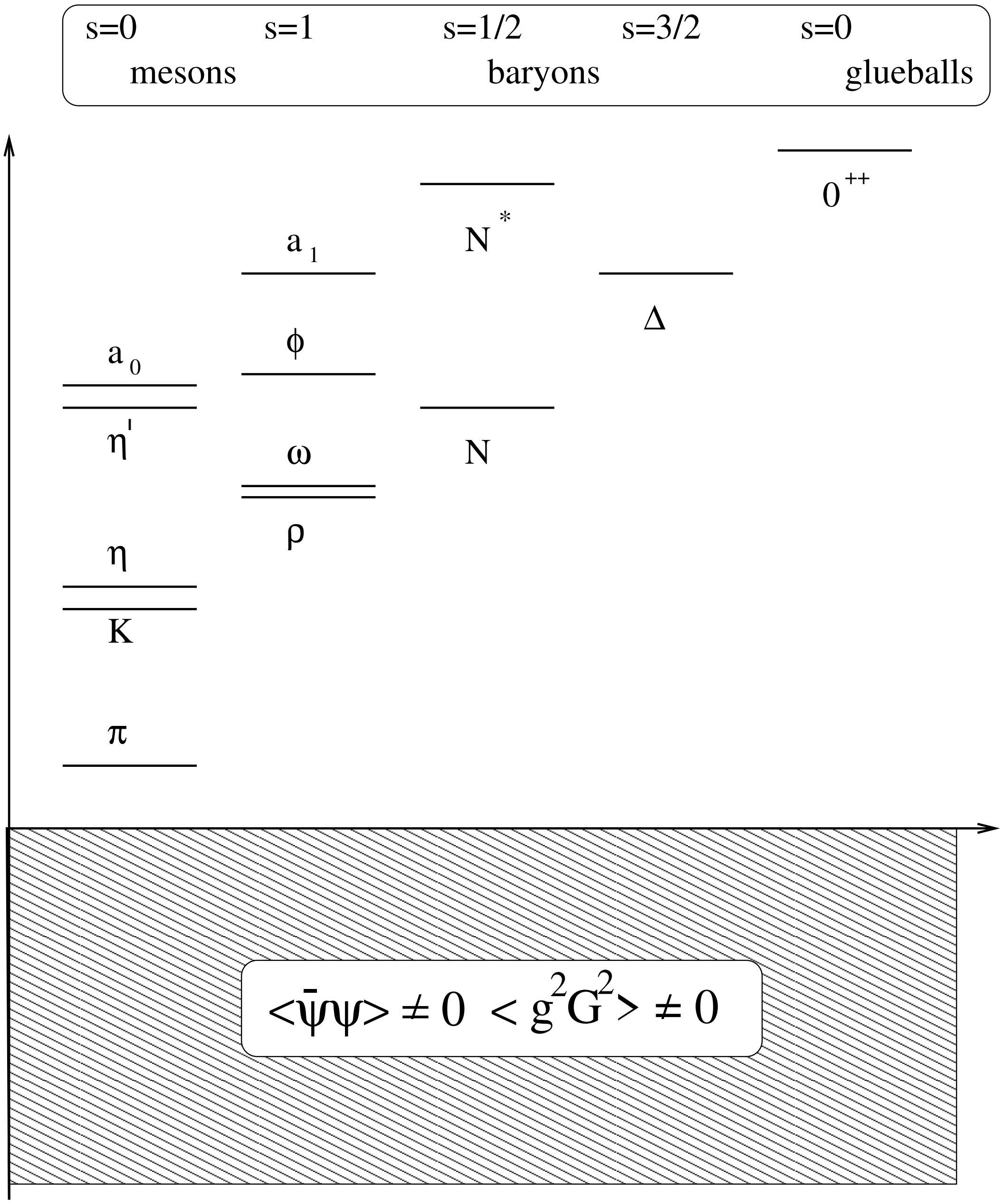


- light quarks: string can break



$$J/\psi(\bar{c}c) \rightarrow D(\bar{c}u) + \bar{D}(\bar{u}c)$$

QCD Ground State



Chiral Symmetry restoration

- vacuum expectation value

$$\langle\sigma\rangle = f_\pi \qquad f_\pi = 93\,\mathrm{MeV}$$

- thermal pion fluctuations

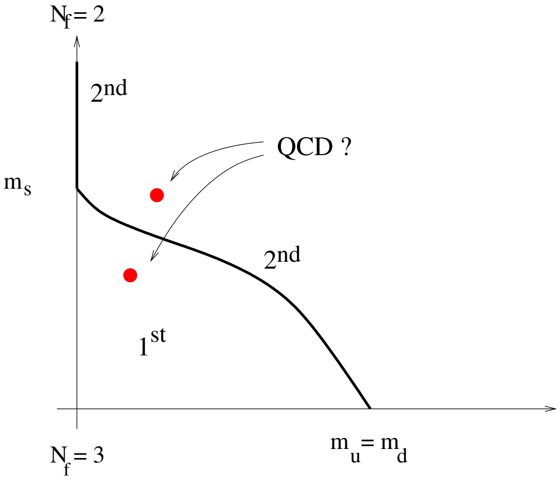
$$\langle\sigma\rangle = \langle\sigma\rangle|_{T=0}\left\{1 - \frac{N_f^2-1}{3N_f}\left(\frac{T^2}{4f_\pi^2}\right) + \ldots\right\}$$

$$\rightarrow T_c \sim \sqrt{8} f_\pi \sim 200\,\mathrm{MeV}$$

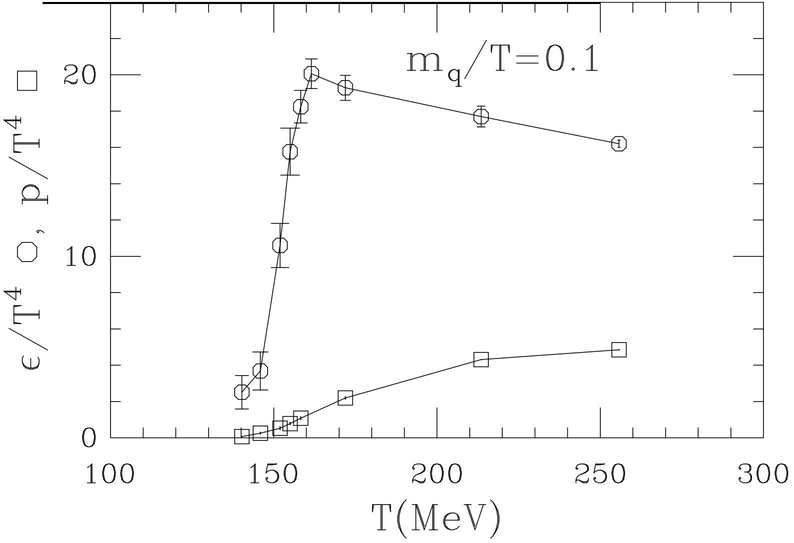
- universality: critical behavior near 2nd order phase transition is independent of microscopic details

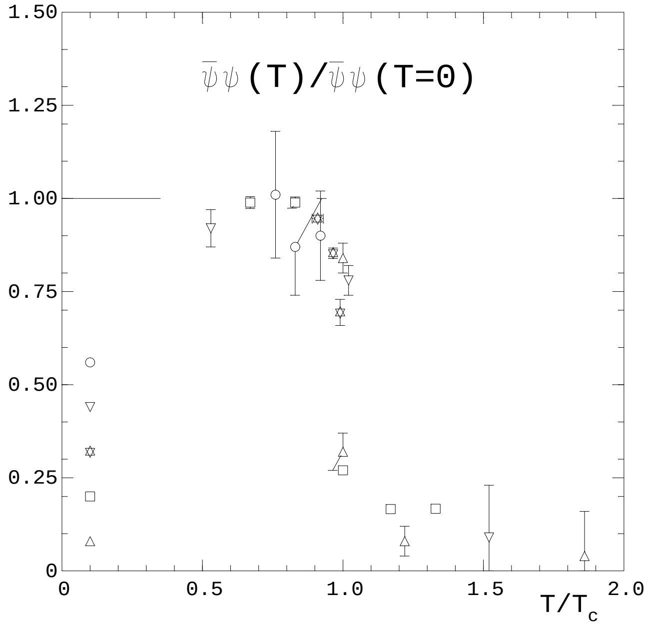
$N_f = 2$ 2nd order, $O(4)$ critical exponents

$N_f = 3$ 1st order



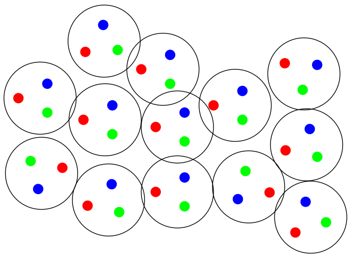
The QCD phase transition





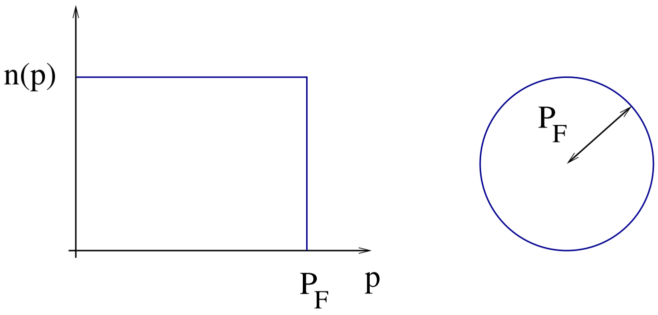
Dense Hadronic Matter

- consider baryon density $n_B \gg 1\,\mathrm{fm}^{-3}$



quarks can move freely

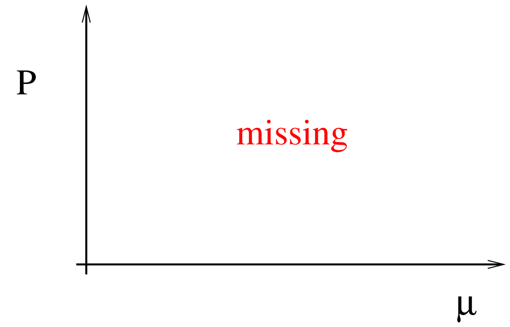
- groundstate: cold quark matter (quark fermi liquid)

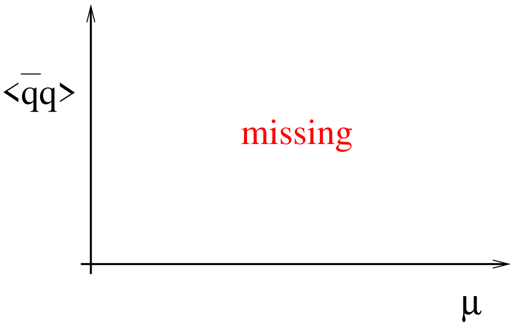


- only quarks with $p \sim p_F \gg \Lambda_{QCD}$ scatter
- $p_F \gg \Lambda_{QCD}$ weak → coupling weak

→ no chiral symmetry breaking
→ no confinement
→ no dynamically generated masses

Lattice Results



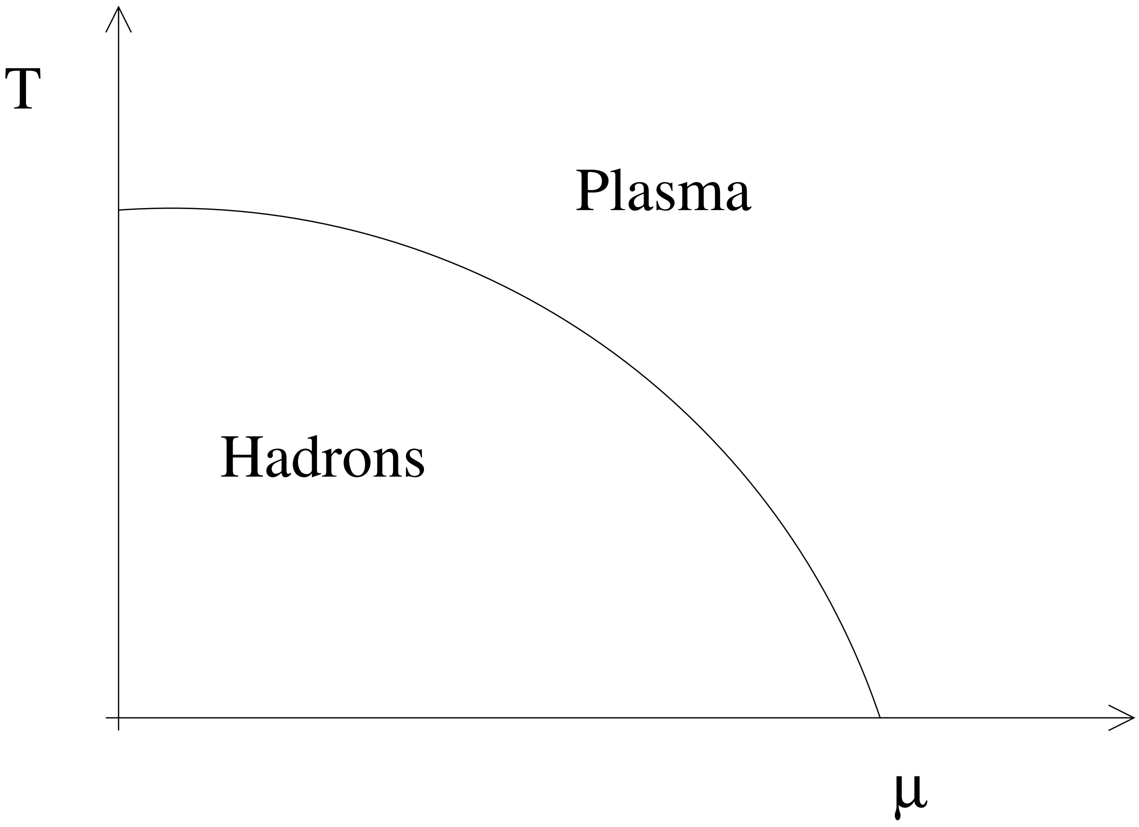


- Problem: No lattice data

$$Z = \int dA \, \underbrace{\det(\not{D} + i\mu\gamma_4)}_{\text{complex}} \, e^{-S}$$

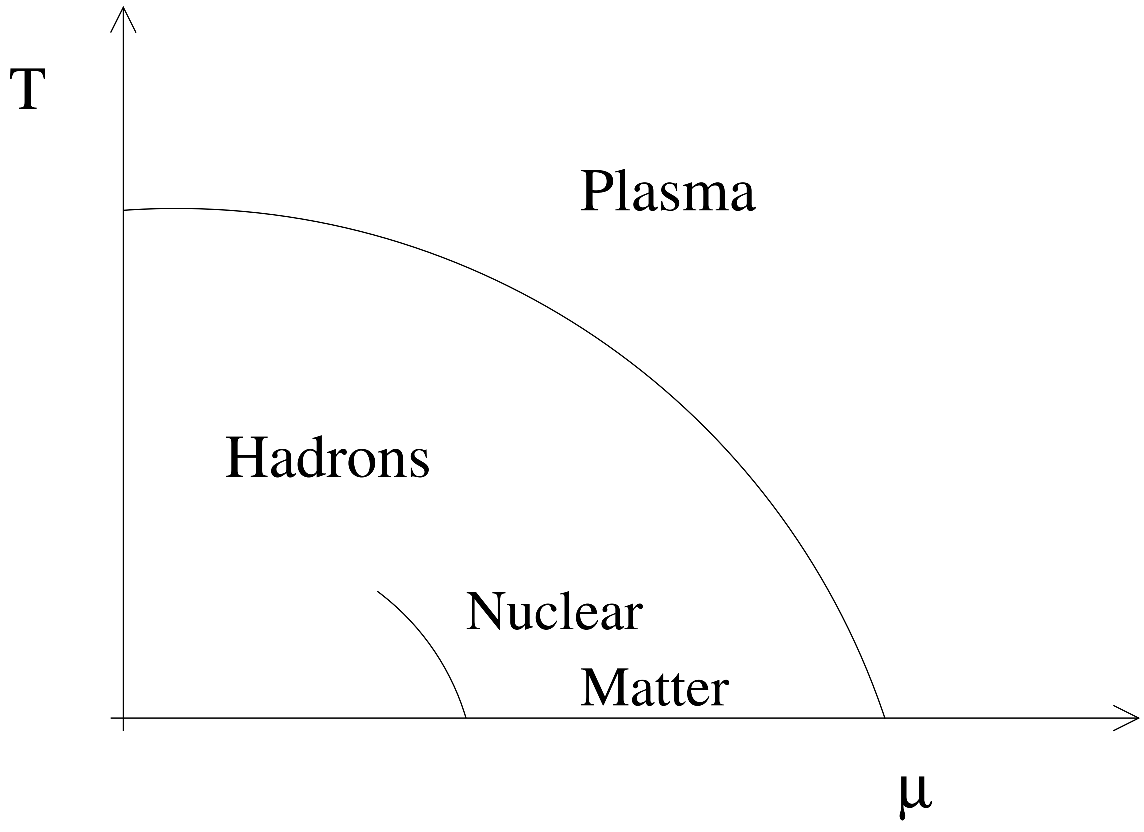
- "sign" problem: no importance sampling

Phase Diagram: First Try



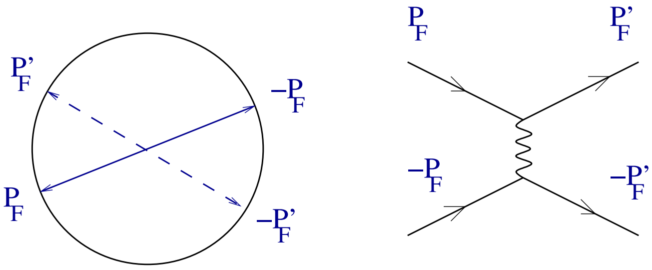
Where are we?

Phase Diagram: Second Try



Superfluidity

- dominant interaction

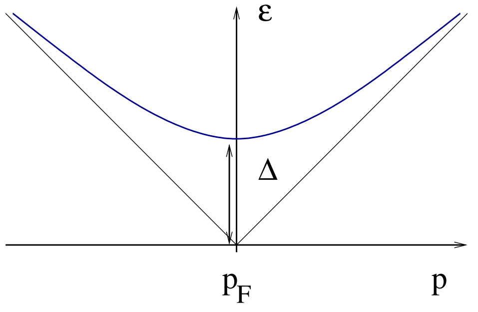


Uses the whole
Fermi surface
coherently

- very degenerate perturbation theory

$\Rightarrow$ instability (BCS) in weak coupling
$\Rightarrow$ $q(p_F)q(-p_F)$ condensate

- arbitrarily weak (attractive!) coupling leads to superfluidity/superconductivity (order parameter neutral/charged)



$$\varepsilon(p) = \sqrt{(p - p_F)^2 + \Delta^2}$$

gap in excitation spectrum, transport without dissipation, . . .

- QCD: gluon exchange attractive in $\bar{3}$ channel

$3 \times 3 = 6_S + 3_A$ Flux reduced $\Rightarrow$ attractive

- spin-flavor-color wave function

$$(\uparrow\downarrow - \downarrow\uparrow) \times (ud - du) \times (rb - br) \qquad s = 0,\ I = 0,\ c = \bar{3}$$

order parameter: $\Phi^a = \epsilon^{abc}\langle q^b C\gamma_5\tau_2 q^c\rangle$

- chiral symmetry is not broken

$$\Phi \sim q_L q_L - q_R q_R$$

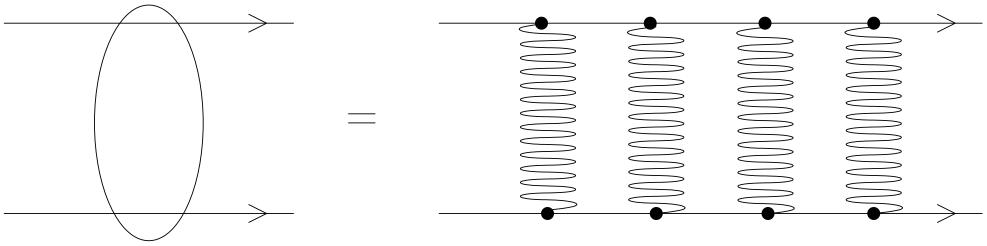
- color symmetry broken by Higgs mechanism (Meissner effect)

$$\Phi^a \in [\bar{3}];\ \Rightarrow\ SU(3) \to SU(2)$$

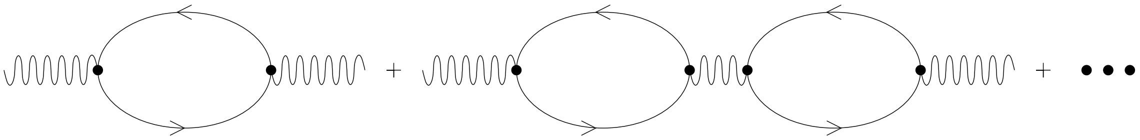
  - 5/8 gluons acquire mass via Higgs mechanism
  - $SU(2)$ is confined

Gap Equation

- $\mu \gg \Lambda_{QCD}$: perturbative forces dominate



- gluon exchange: small angle scattering dominates → medium effects important



$D_E = \frac{1}{\vec{q}^2 + 2m^2}$ $\quad m^2 = \frac{N_f}{4\pi^2} g^2 \mu^2$ Debye screening

$D_M = \frac{1}{\vec{q}^2 + i\frac{\pi}{2} m \frac{\omega}{q}}$ $\quad \omega < q$ Landau damping

- consider $\omega \simeq \Delta$. Typical momenta

$$q_E \simeq g\mu, \qquad q_M \simeq (g^2\mu^2\Delta)^{1/3}$$

Superconductivity driven by magnetic forces!

- Note: Perturbation theory is self-consistent

$$\mu \to \infty \quad \Rightarrow \quad q_M \gg \Lambda_{QCD}$$

Eliashberg Equation

- retardation important: Eliashberg theory

$$\Delta(p_0) = \frac{g^2}{18\pi^2}\int dq_0 \left\{ \log\left(\frac{b_M}{|p_0-q_0|}\right) + \ldots \right\} \frac{\Delta(q_0)}{\sqrt{q_0^2+\Delta(q_0)^2}}$$

colinear log BCS log

- double logarithmic behavior

$$\Delta_0 \sim \frac{g^2}{18\pi^2}\Delta_0\left[\log\left(\frac{\mu}{\Delta_0}\right)\right]^2$$

$$\to \Delta_0 \sim \exp\left(-\frac{c}{\sqrt{\alpha_s}}\right)$$

compared to BCS result $\Delta_0 \sim \exp\left(-\frac{c}{\alpha_s}\right)$

- more careful analysis gives $c = 3\pi^2/\sqrt{2}$
- condensation energy

$$\epsilon \simeq N\Delta_0^2\left(\frac{\mu^2}{4\pi^2}\right)$$

- typical numbers

$$\begin{aligned} \Delta_0 &\simeq 100 \ \ \text{MeV} \ \ (\rho \sim 5\rho_0) \\ T_c &\simeq 50 \ \ \ \text{MeV} \end{aligned}$$

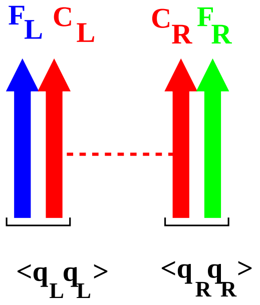
Beautiful Case: $N_f = 3$

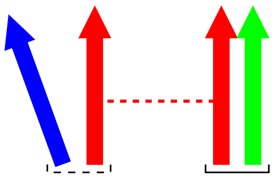
- quarks q_i^a with $i = (u, d, s)$ and $a = (r, b, g)$. Order parameter?

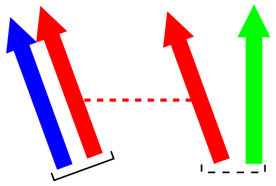
$$\langle q_i^a q_j^b \rangle = \phi \left(\delta_i^a \delta_j^b - \delta_j^a \delta_i^b \right) \quad \text{Color} - \text{Flavor} - \text{Locking}$$

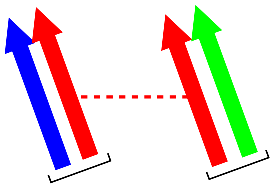
$$\begin{array}{ll} \langle ud \rangle = \langle us \rangle = \langle ds \rangle & \text{Color and Flavor} \\ \langle rb \rangle = \langle rg \rangle = \langle bg \rangle & \text{direction aligned} \end{array}$$

- Symmetries: $SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \to SU(3)_{C+F}$




Rotate left flavor


Compensate by rotating color


... have to rotate right flavor also !

- Novel mechanism for Chiral Symmetry Breaking:

$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

Breaks chiral $SU(3)_L \times SU(3)_R$ symmetry because

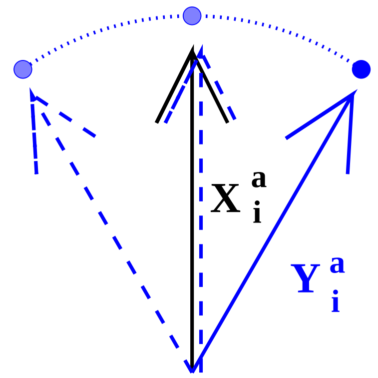
$$SU(3)_L \overset{Lock}{\longleftrightarrow} SU(3)_C \overset{Lock}{\longleftrightarrow} SU(3)_R$$

- All quarks and gluons acquire a mass gap

Goldstone Bosons

- How to set up a color-flavor wave

a)

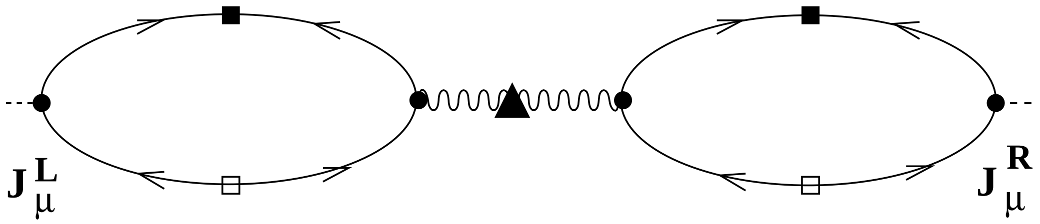


$$X_i^a = \epsilon_{ijk}\epsilon^{abc}\langle(\psi_L)_j^b C(\psi_L)_k^c\rangle$$

$$Y_i^a = \epsilon_{ijk}\epsilon^{abc}\langle(\psi_R)_j^b C(\psi_R)_k^c\rangle$$

$$K^0 \sim \epsilon^{abc}\epsilon_{ade}(\bar{u}_R^b C\bar{s}_R^c)(d_L^d C u_L^e)$$

b)



- low energy degrees of freedom

$$\Sigma = \exp\left(\frac{i\phi^a\lambda^a}{f_\pi}\right) \qquad \phi^a = (\pi, K, \eta, \eta')$$

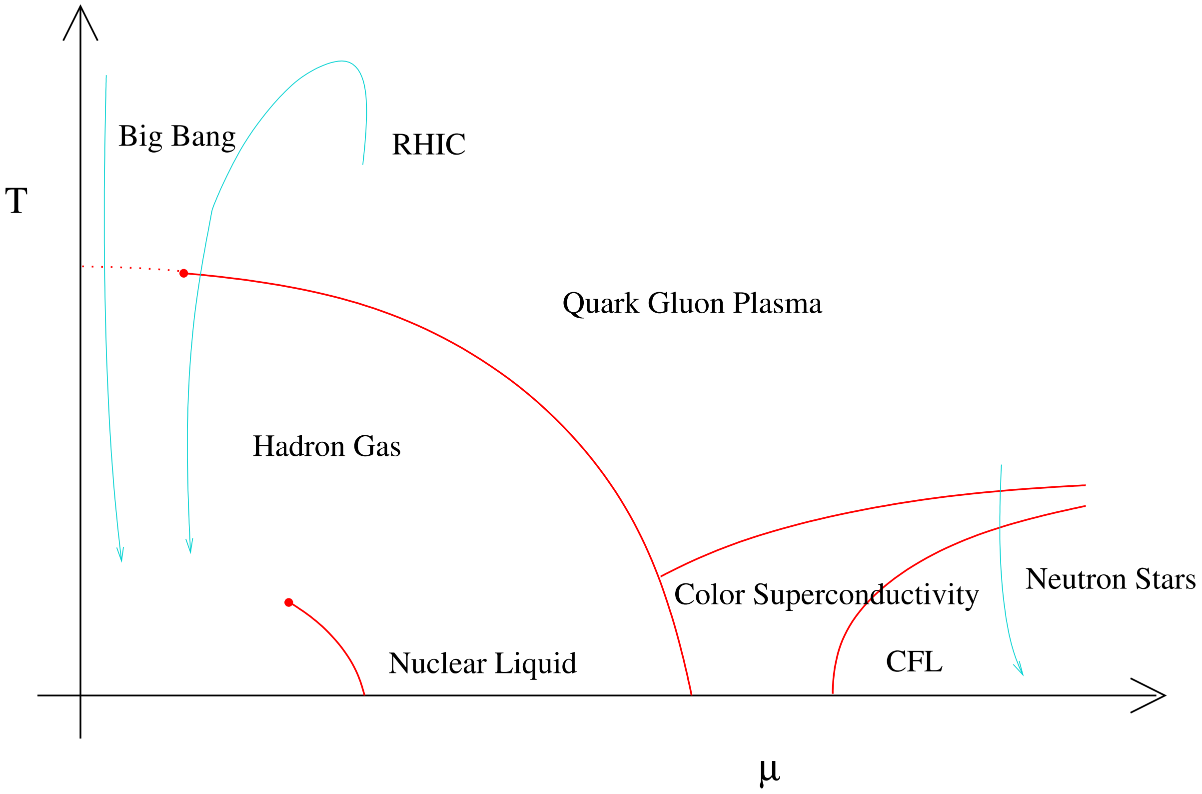
- effective theory

$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\nabla_0\Sigma\nabla_0\Sigma^\dagger - v^2\vec{\nabla}\Sigma\vec{\nabla}\Sigma^\dagger\right] + \ldots$$

- parameters calculable in perturbative QCD!

$$f_\pi^2 = \frac{21 - 8\log(2)}{18}\left(\frac{\mu^2}{2\pi^2}\right) \qquad m_\pi^2 = \frac{3\Delta^2}{4f_\pi^2}(m_u + m_d)m_s$$

Summary: The Many Phases (Faces?) of QCD



- Interesting new phases of QCD at non-zero baryon density: Color Superconductivity, Color-Flavor-Locking
- Theory: Weak coupling realization of chiral symmetry breaking and mass gaps
- Phenomenology: Neutron Stars (cooling, neutrino burst, glitches?), Heavy Ions (Tri-critical point, precursor phenomena)