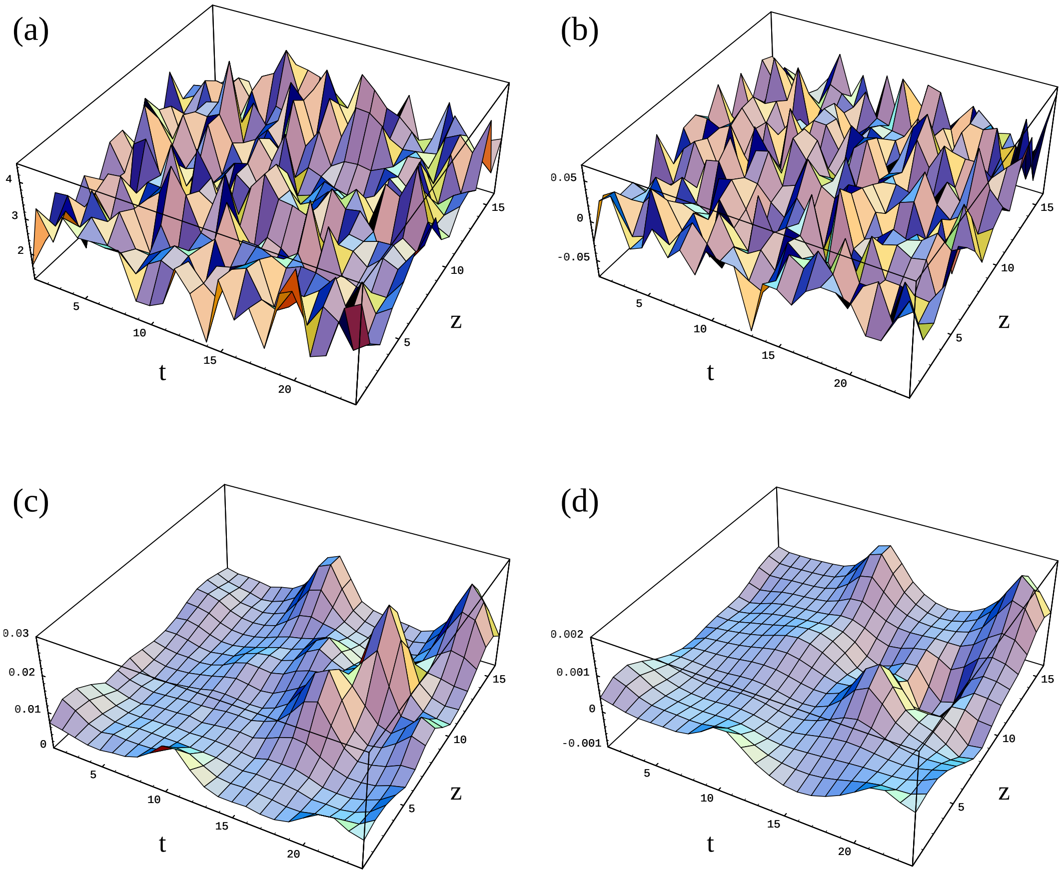
Instantons and large N_c

Thomas Schaefer

SUNY Stony Brook

and Riken BNL Research Center



<u>QCD at large N_c</u>

- QCD ($m = 0$) is a parmater free theory

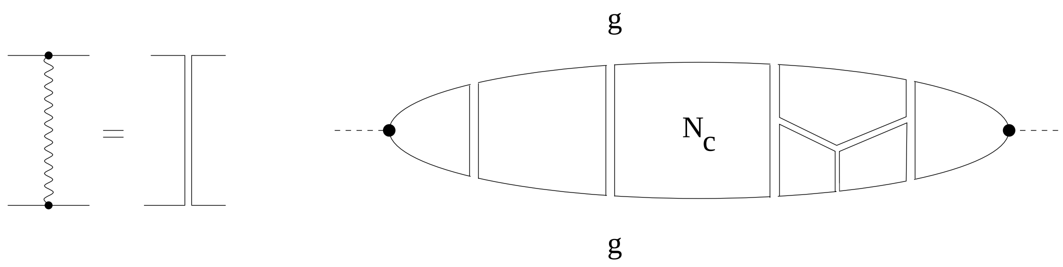
Very beautiful But: No expansion parameter

- 't Hooft: Consider $N_c \to \infty$ and use $1/N_c$ as a small parameter

$$N_c \to \infty \quad \Rightarrow \quad \text{classical master field}$$

- keep Λ_{QCD} fixed

$$\Rightarrow g^2 N_c = const$$



- Could the master field be a multi-instanton configuration?

$$\text{Witten (1979) : No} \qquad dn \sim \exp(-\frac{1}{g^2}) \sim \exp(-N_c)$$

$U(1)_A$ anomaly at large N_c

- consider θ term

$$\mathcal{L} = \frac{ig^2\theta}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu},$$

- no θ dependence in perturbation theory.

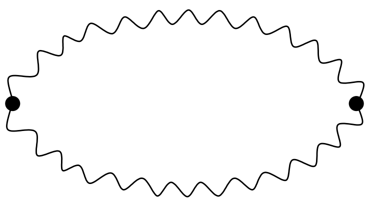
Witten: *non-perturbative θ dependence*

$$\chi_{top} = \left.\frac{d^2E}{d\theta^2}\right|_{\theta=0} \sim O(1)$$

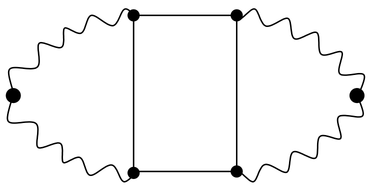
- massless quarks: toplogical charge screening

$$\lim_{m\to 0} \chi_{top} = 0$$

- How can that happen? Fermion loops are suppressed!



$g^4 N_c^2 \sim 1$



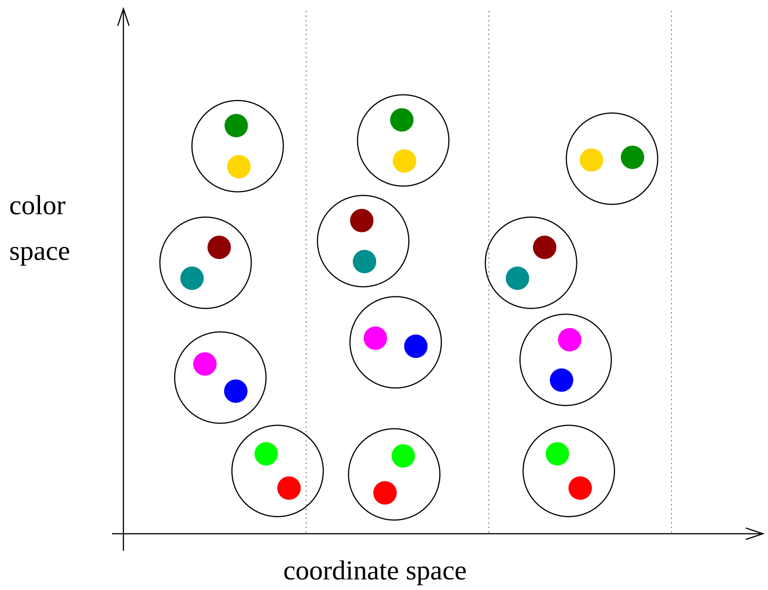
$g^8 N_c^3 \sim \frac{1}{N_c}$

Witten: *η' has to become light*

$$f_\pi^2 m_{\eta'}^2 = 2N_f\chi_{top} \quad \Rightarrow \quad m_{\eta'}^2 = O(1/N_c)$$

Instantons at large N_c

- semi-classical ensemble of instantons at large N_c



- instantons are $N_c = 2$ configurations

$$\left(\frac{N}{V}\right) = O(N_c) \quad \Rightarrow \epsilon_{vac} = O(bN_c) = O(N_c^2)$$

- instantons are semi-classical

$$\rho \simeq \rho^* = O(1) \quad S_{inst} = O(N_c)$$

- density $dn \sim \exp(-S_{inst}) = O(\exp(-N_c))$?

$$NO! \quad \textit{large entropy} \quad dn \sim \exp(+N_c)$$

- topological susceptibility $\chi_{top} \simeq (N/V) = O(N_c)$?

$$NO! \quad \textit{fluctuations suppressed} \quad \chi_{top} = O(1)$$

Instantons ensemble

- instanton ensemble

$$Z = \frac{1}{N_I! N_A!} \prod_I^{N_I+N_A} \int [d\Omega_I \, n(\rho_I)] \, \exp(-S_{int})$$

$$n(\rho) = C_{N_c} \left(\frac{8\pi^2}{g^2}\right)^{2N_c} \rho^{-5} \exp\left[-\frac{8\pi^2}{g(\rho)^2}\right]$$

$$C_{N_c} = \frac{0.466 \exp(-1.679 N_c)}{(N_c-1)!(N_c-2)!} \qquad \frac{8\pi^2}{g^2(\rho)} = -b \log(\rho\Lambda), \qquad b = \frac{11}{3} N_c$$

$$S_{int} = -\frac{32\pi^2}{g^2} |u|^2 \left\{ \frac{\rho_I^2 \rho_A^2}{R_{IA}^4} \left(1 - 4\cos^2\theta\right) + S_{core}\left(\frac{\rho_I^2 \rho_A^2}{R_{IA}^4}\right) \right\}$$

- complicated ensemble, size distribution

$$\rho^* \sim O(1) \qquad \begin{cases} \rho < \rho^* & dn \sim \exp(-N_c) \\ \rho \sim \rho_+^* & dn \sim \exp(+N_c) \end{cases}$$

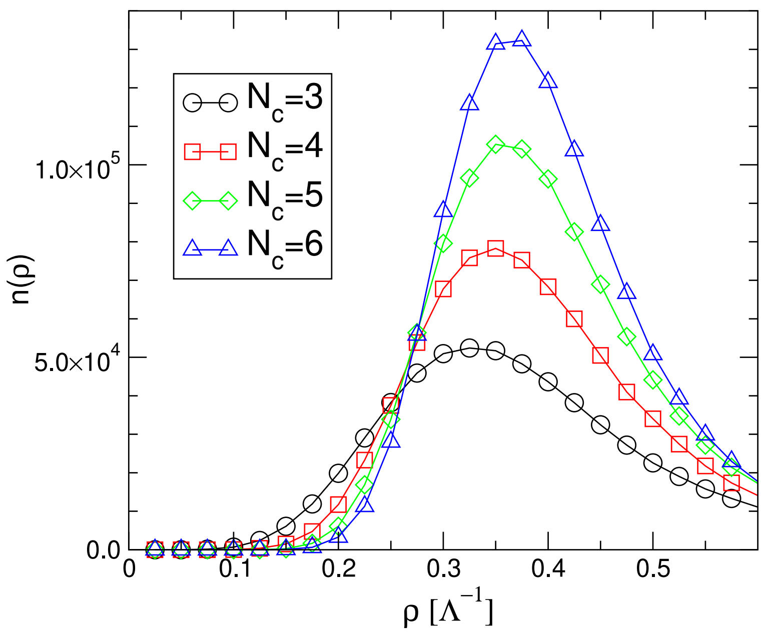
- total density determined by interactions

$$\begin{array}{ccc} S(1-body) & \sim & S(2-body) \\ N_c & \sim & N_c \;\; \times \;\; \frac{1}{N_c} \;\; \times \;\; \left(\frac{N}{V}\right) \\ classical & \sim & classical \times color\ overlap \times density \end{array}$$

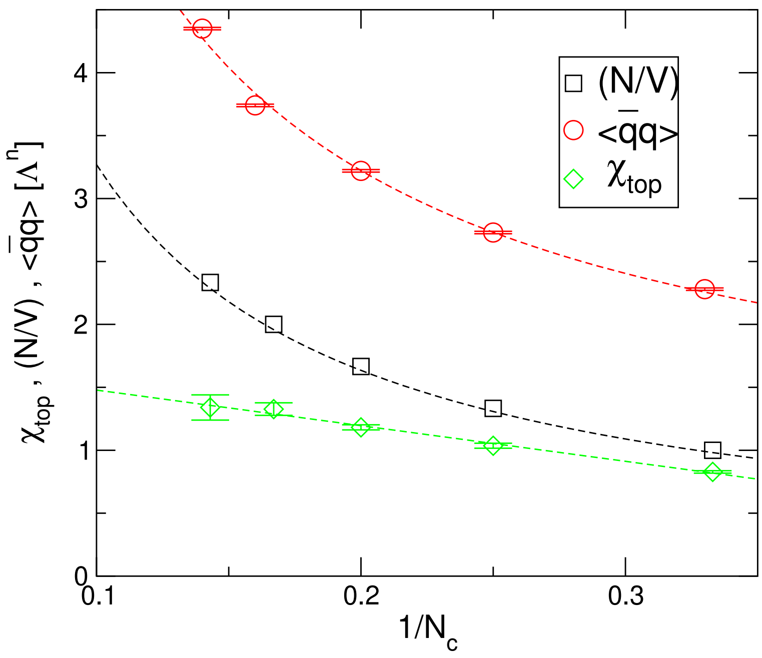
- conclude

$$\boxed{\left(\frac{N}{V}\right) = O(N_c)}$$

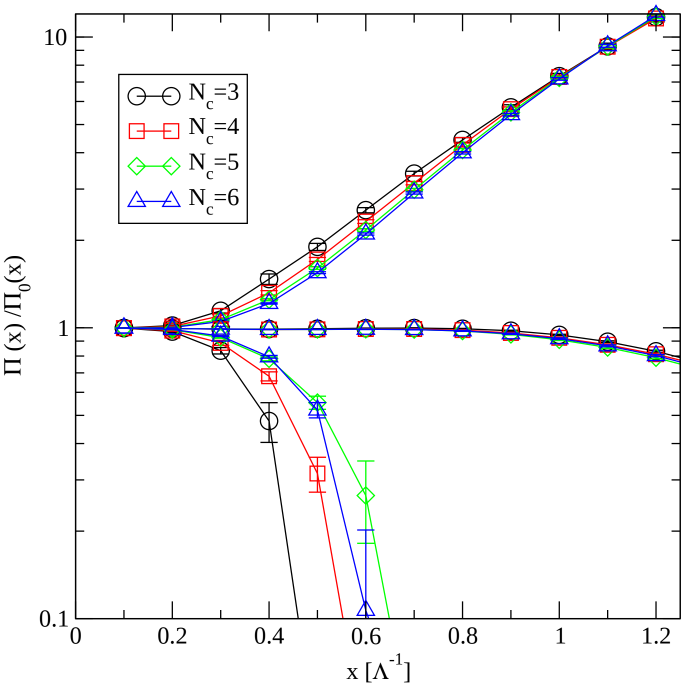
- instanton size distribution



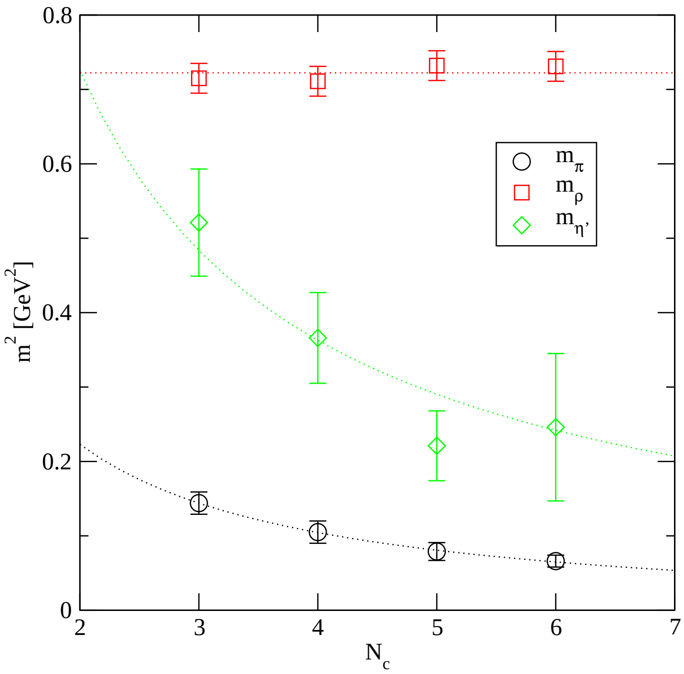
- instanton density, quark condensate, topological susceptibility



- meson correlation functions (π, ρ, η')



- meson masses: $m_\pi^2, m_\rho^2 \sim 1,\ m_{\eta'}^2 \sim 1/N_c$



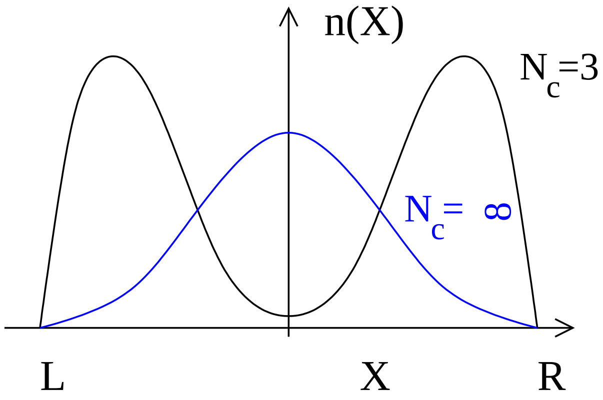
<u>**Summary**</u>

- instanton liquid can have a smooth large N_c limit

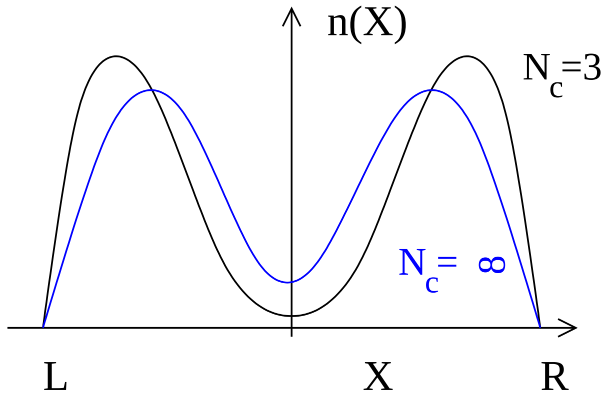
$$\left(\frac{N}{V}\right) = O(N_c), \qquad \chi_{top} = O(1), \qquad m^2_{\eta'} = O(1/N_c)$$

- how can we check this?

chirality distribution



SU(2) projected



- can we identify the instanton contribution to the η' mass by its scaling behavior (Witten: $1/N_c$ vs $\exp(-N_c)$)?

$$\textit{consider}\ N_c = 2,\ \mu \neq 0 \quad \textit{or}\ N_c = 3,\ \mu_I \neq 0$$