Tunneling in QCD

and the Structure of the Proton

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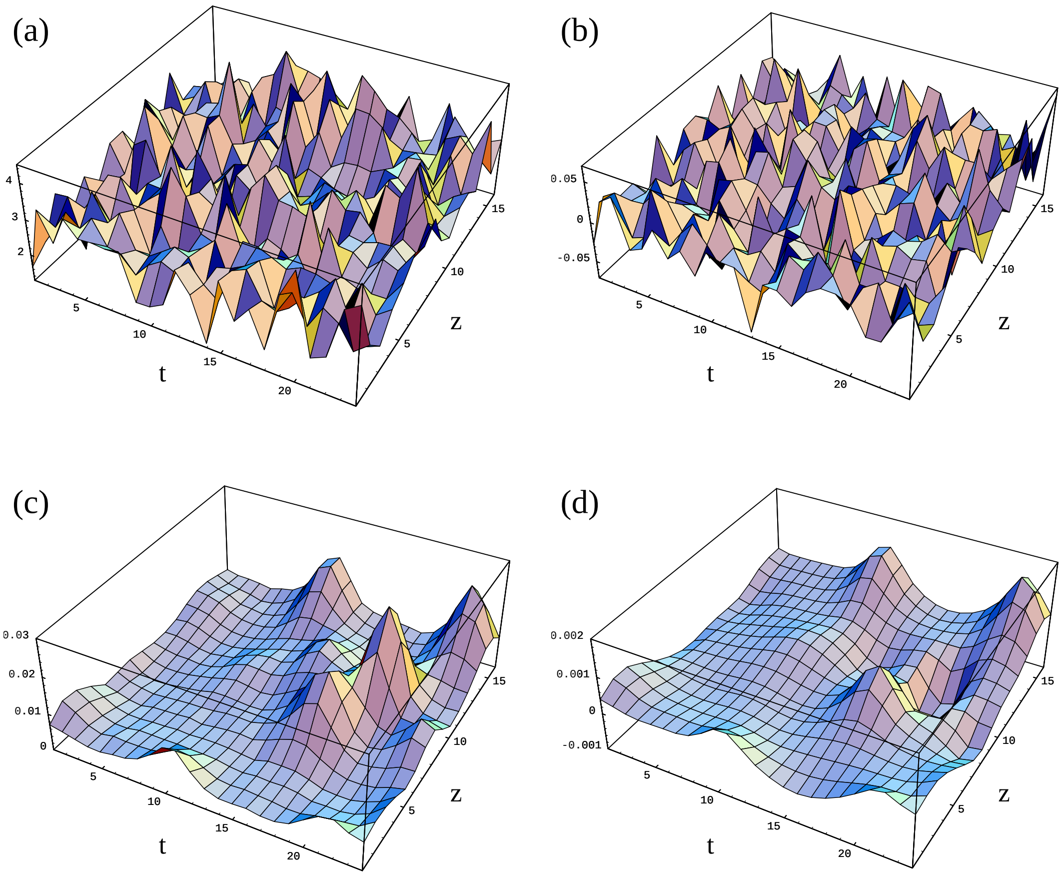


Quantum Chromo Dynamics

- elementary fields

$$\text{quarks} \qquad (q_\alpha)^a_f \qquad \begin{cases} \text{color} & a = 1, \ldots, 3 \\ \text{spin} & \alpha = 1, 2 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$$

$$\text{gluons} \qquad A^a_\mu \qquad \begin{cases} \text{color} & a = 1, \ldots, 8 \\ \text{polarization} & \epsilon^\pm_\mu \end{cases}$$

- dynamics: gauge theory

$$\mathcal{L} = \sum_f^{N_f} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

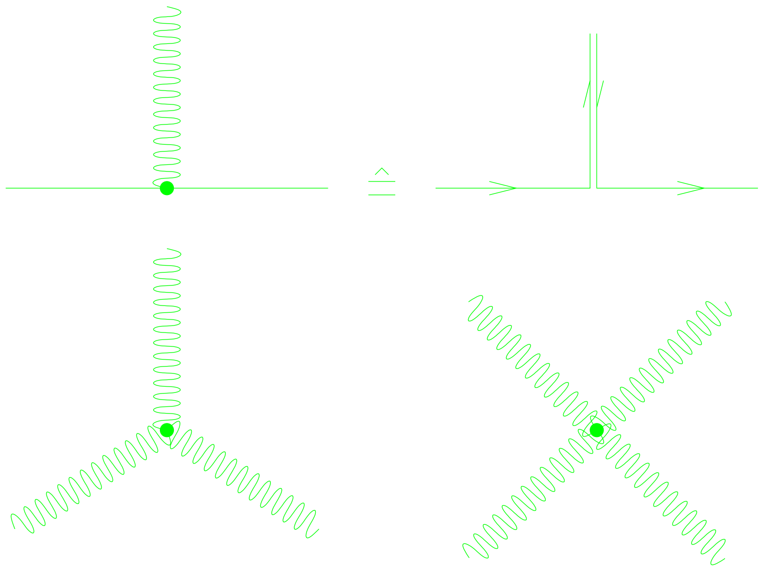
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

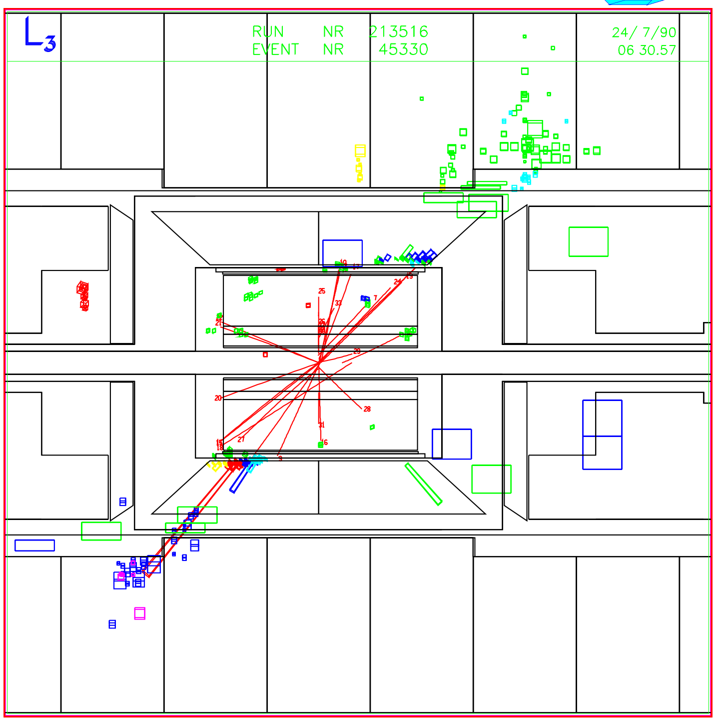
$$i\not{D} q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu \frac{\lambda^a}{2} \right) q$$

- quantum mechanics: path integral (partition function)

$$Z = \int DA_\mu \ \mathrm{Det}(i\not{D} - m) \ \exp(i \int d^4x \mathcal{L}_g)$$

- dynamics in diagrams (Feynman)

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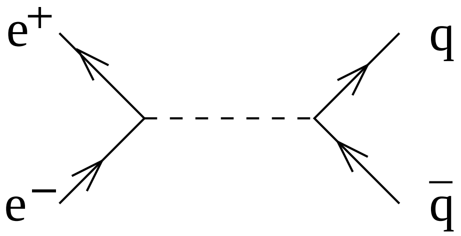
e+
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q
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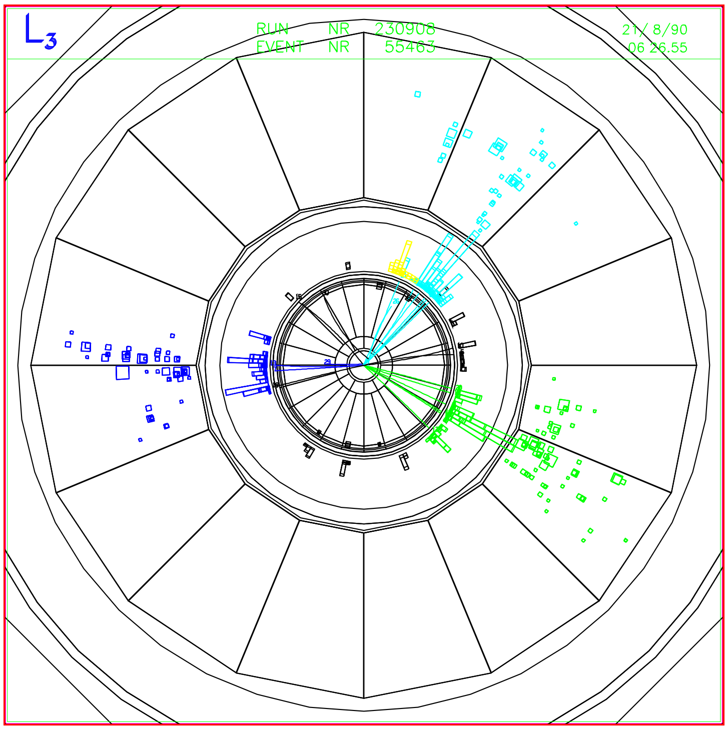
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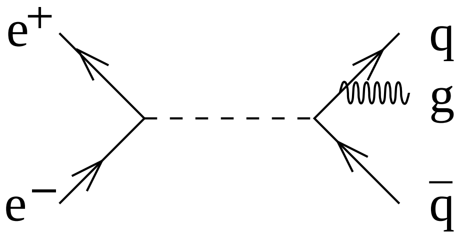
e+
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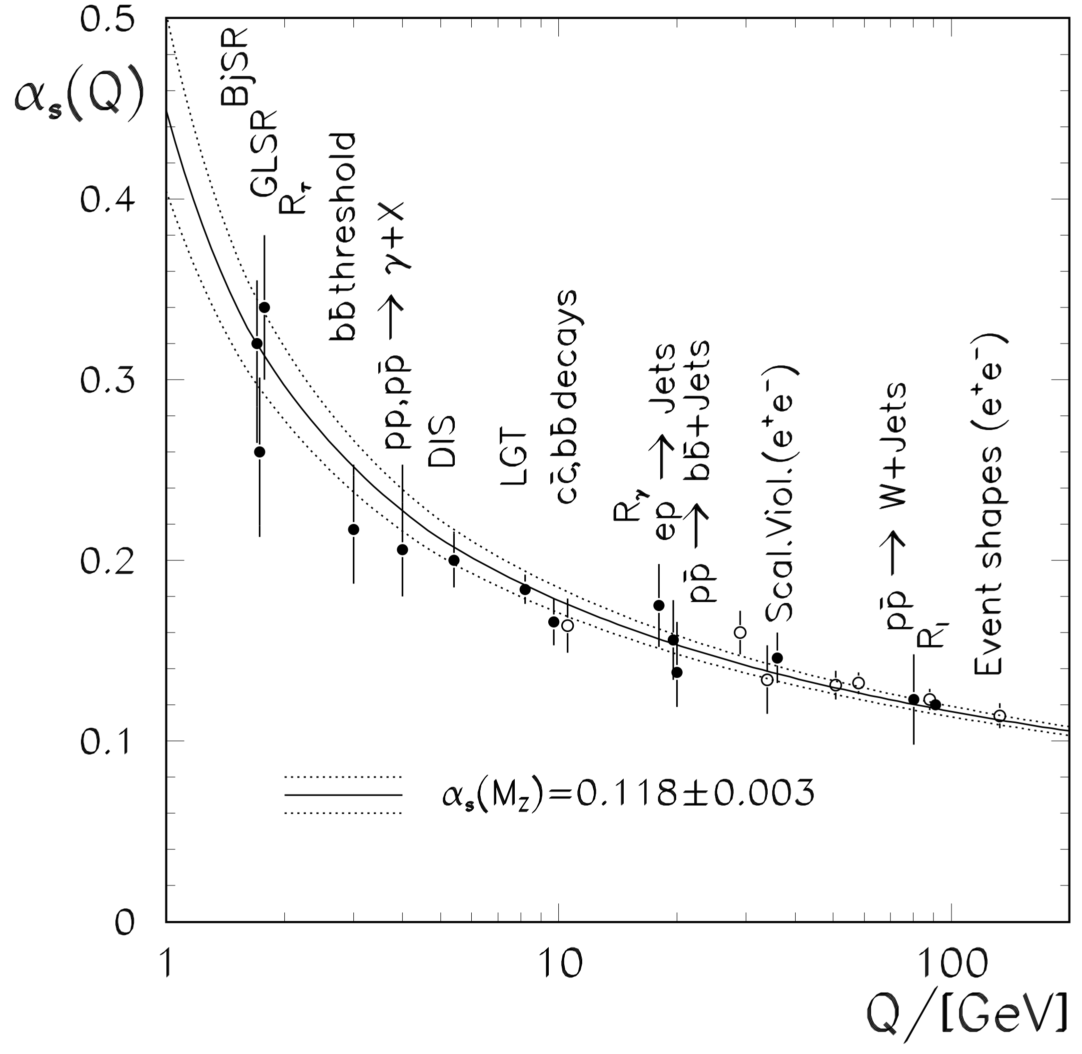


$\alpha_s(Q)$
0.5
0.4
0.3
0.2
0.1
0
1
10
100
Q/[GeV]
BjSR
GLSR
R_τ
$b\bar{b}$ threshold
$pp, p\bar{p} \rightarrow \gamma + X$
DIS
LGT
$c\bar{c}, b\bar{b}$ decays
R_γ
$ep \rightarrow$ Jets
$p\bar{p} \rightarrow b\bar{b}$+Jets
Scal.Viol.(e^+e^-)
$p\bar{p} \rightarrow$ W+Jets
R_l
Event shapes (e^+e^-)
$\alpha_s(M_Z)=0.118\pm0.003$

About units

- QCD is essentially parameter free: The lagrangian has a coupling constant, g, but no scale (if $m_q \simeq 0$).
- After renormalization g becomes scale dependent (g is traded for a scale parameter Λ)
- QCD has an "urkilogramm"

$$\Lambda_{QCD} \simeq 200\,\mathrm{MeV} \simeq 1\,\mathrm{fm}^{-1}$$

  $\Lambda^{-1} \simeq 1\,\mathrm{fm}$ is the distance at which the coupling becomes strong

- In practice, perturbation theory breaks down at much shorter distances

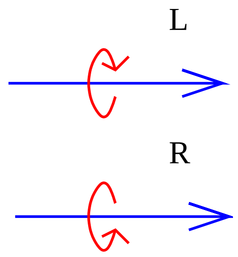
$$x \sim \Lambda_{\chi}^{-1} \sim 1\,\mathrm{GeV}^{-1} \sim 0.2\,\mathrm{fm}$$

- Note: $m_\rho \sim m_p \sim 1\,\mathrm{GeV}$, $r_p \sim 1\,\mathrm{fm}$.

Chiral Symmetry

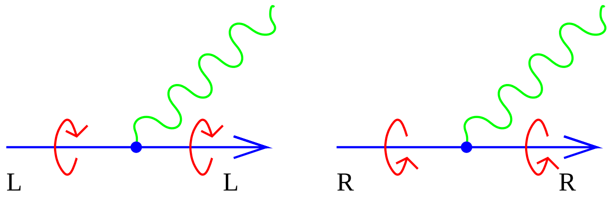
- define L/R fields

$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

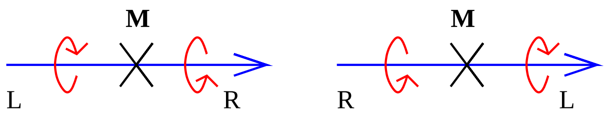


- fermionic lagrangian

$$\mathcal{L} = \bar{\psi}_L(i\not{D})\psi_L + \bar{\psi}_R(i\not{D})\psi_R$$



$$+ \bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$



$$M = \mathrm{diag}(m_u, m_d, m_s)$$

- $m_u = m_d = m_s$: Flavor symmetry $V \in SU(3)$

$$\psi_L \to V\psi_L, \qquad \psi_R \to V\psi_R$$

- $m_u = m_d = m_s = 0$: Chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

$$\psi_L \to L\psi_L, \qquad \psi_R \to R\psi_R$$

The Fate of Chiral Symmetry

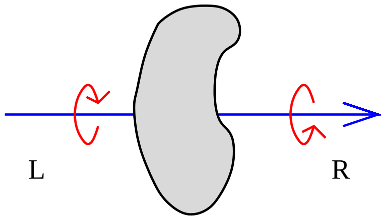
- Chiral symmetry implies parity doubling, e.g.

$$\begin{array}{lcc} & \rho(LL+RR) \quad \leftrightarrow & a_1(LL-RR) \\ \text{expt} & m_\rho = 770\,\text{MeV} & m_{a_1} = 1230\,\text{MeV} \end{array}$$

too large in order to be explained by quark masses $m_u, m_d \sim 10\,\text{MeV}$

- chiral symmetry is spontaneously broken

$$\langle \bar\psi_L \psi_R + \bar\psi_L \psi_R \rangle \simeq -(230\,\text{MeV})^3$$

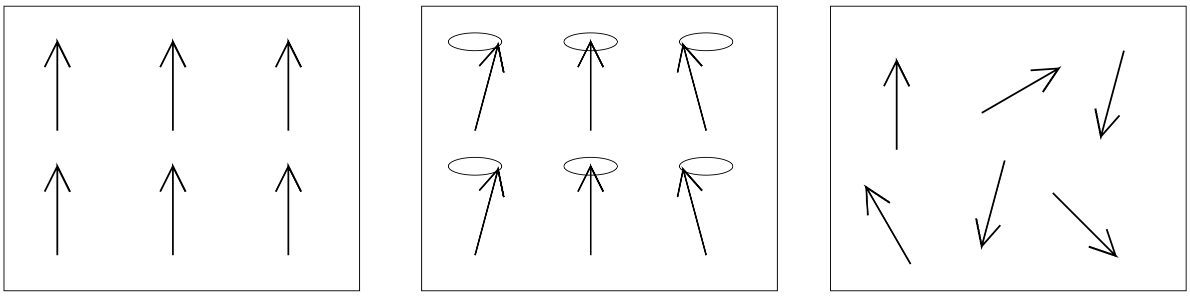


- consequences: dynamical mass generation $m_Q = 300\,\text{MeV} \gg m_q$

$$\begin{array}{cccc} m_p = & 890\,\text{MeV} & + & 45\,\text{MeV} \\ & (\text{QCD},\ 95\%) & & (\text{Higgs},\ 5\%) \end{array}$$

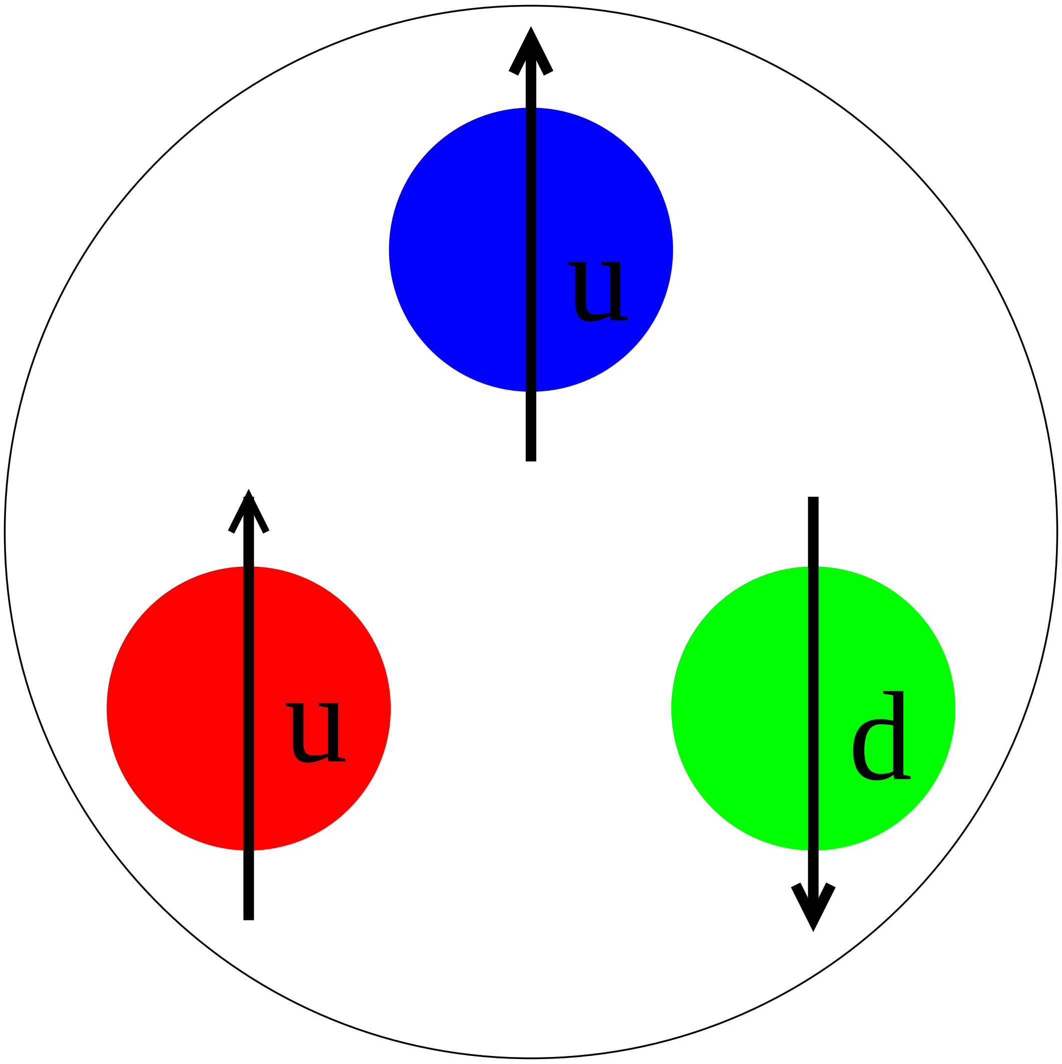
- existence of Goldstone modes

$$\langle \bar\psi_L^f \psi_R^g \rangle = B\,(\sigma + i\vec\tau\vec\pi)^{fg}$$



ordered	spin wave	disordered
chiral symmetry broken	pion	chiral symmetry restored

What is a proton?



Mass $M_p = 935$ MeV

Spin $S = 1/2$, Isospin $I = 1/2$

$\mu_p/\mu_n = -3/2$

The Structure of the Proton

- The mass of the Proton (from DIS, trace anomaly)

$$\begin{aligned} E_q &= \langle p|\int d^3x\,(-i\vec{\alpha}\cdot\vec{D})|p\rangle & \simeq\ 310\,\mathrm{MeV} \\ E_g &= \langle p|\int d^3x\,\tfrac{1}{2}\,(E^2+B^2)\,|p\rangle + \ldots & \simeq\ 545\,\mathrm{MeV} \\ E_m &= \langle p|\int d^3x\,(m_u\bar{u}u + m_d\bar{d}d)|p\rangle & \simeq\ 45\,\mathrm{MeV} \end{aligned}$$

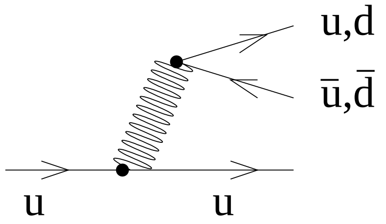
- Gluon field strength is large

$$\langle p|E^2|p\rangle \simeq 1700\,\mathrm{MeV} \qquad \langle p|B^2|p\rangle \simeq -1050\,\mathrm{MeV}$$

- number of quark-anti-quark pairs is large

$$\langle p|\bar{u}u + \bar{d}d|p\rangle = \frac{\Sigma_{\pi N}}{\overline{m}} \simeq 6$$

... and not flavor symmetric



$$\frac{\bar{d}(x)}{\bar{u}(x)} \simeq 2 \qquad \text{(NuSea, \ldots)}$$

$$x \sim 0.1$$

- quark contribution to proton spin is small

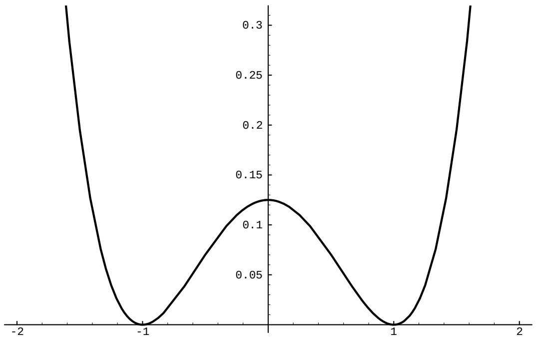
$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = (0.2 \pm 0.1) \qquad \text{(SMC, SLAC, Hermes)}$$

- ... and strange quarks are polarized

$$\Delta s = -0.12$$

Instantons in Quantum Mechanics

- Consider particle moving in a potential



$$H = T + V$$
$$V = \lambda(x^2 - \eta^2)^2$$

classically degenerate states
are mixed by tunneling

$$\boxed{E_{1,2} \simeq E_0 \pm \Delta E}$$

- Use classical $\leftrightarrow$ quantum map

$$\begin{array}{ccc} e^{iS} & \leftrightarrow & e^{-\beta H} \\ \text{quantum fluctuations} & \leftrightarrow & \text{thermal fluctuations} \end{array}$$

- formal trick: continuation to euclidean time

$$t \to it = \tau, \qquad e^{iS} \to e^{-S_E}$$

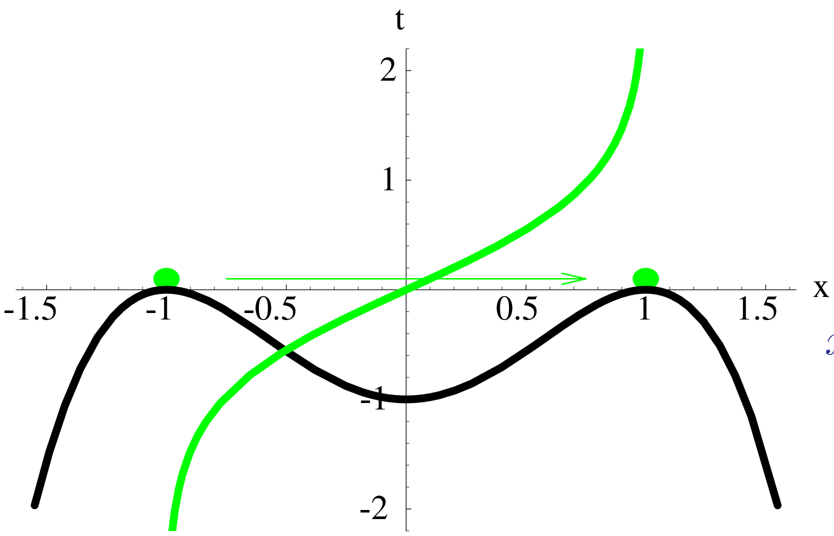
- partition function

$$Z = \mathrm{tr}(e^{-\beta H}) = \int Dx(\tau)\, e^{-S_E}$$
$$S_E = \int_0^\beta d\tau\, \left(\frac{1}{2}m\dot{x}^2 + V(x)\right)$$

- Tunneling is (classically) forbidden in Minkowski space, but allowed in Euclidean space

$$\Rightarrow \text{semi} - \text{classical (WKB) approximation}$$

- classical path $\frac{\delta S}{\delta x(\tau)} = 0 \Rightarrow$ equation of motion $m\ddot{x} = V'$



$$\frac{1}{2}m\dot{x}^2 - V(x) = 0$$

$$x_{cl}(\tau) = \eta \tanh\left[\frac{\omega}{2}(\tau - \tau_0)\right]$$

This solution has euclidean action

$$\boxed{S = \frac{\omega^3}{12\lambda}}$$

- $S \gg 1$ ($S \gg \hbar$): semi-classical expansion

$$\langle -\eta | e^{-H\tau} | \eta \rangle = e^{-S_0} \int Dx(\tau) \, \exp\left(-\frac{1}{2}\delta x \left. \frac{\delta^2 S}{\delta x^2} \right|_{x_{cl}} \delta x + \ldots \right)$$

- Compare to

$$\begin{aligned} \langle -\eta | e^{-H\tau_m} | \eta \rangle &= \psi_0^\dagger(-\eta)\psi_0(\eta) e^{-\tau_m E_0} + \psi_1^\dagger(-\eta)\psi_1(\eta) e^{-\tau_m E_1} + \ldots \\ &= 2\psi_0^\dagger(-\eta)\psi_0(\eta) e^{-\tau_m \bar{E}} \sinh(\tau_m \Delta E) \end{aligned}$$

from which we finally read off the level spacing

$$\boxed{\Delta E = \sqrt{\frac{6S_0}{\pi}} \omega \exp\left(-S_0\right)}$$

Lattice Quantum Mechanics

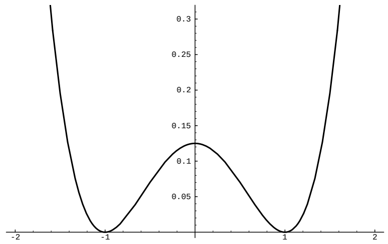
- Partition Function (euclidean space)

$$Z = \mathrm{tr}(e^{-\beta H}) = \int Dx(\tau)\, e^{-S} \qquad S = \int d\tau \left(\frac{1}{2} m \dot{x}^2 + V(x)\right)$$

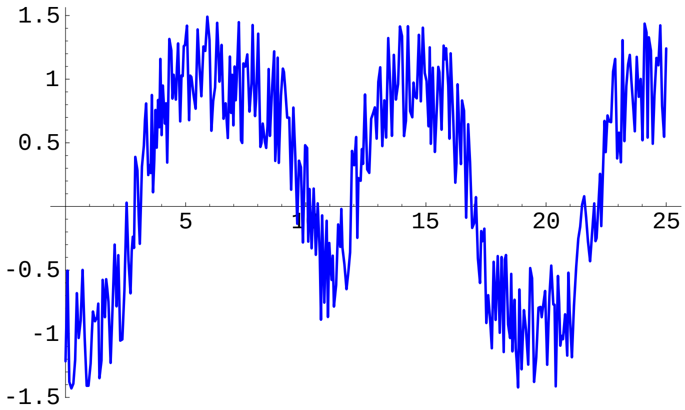
- Lattice version: discretize time $x(\tau = na) = x_n$

$$S = \sum_n \left[\frac{m}{2a}(x_n - x_{n-1})^2 + aV(x_n)\right]$$

- Consider double well potential

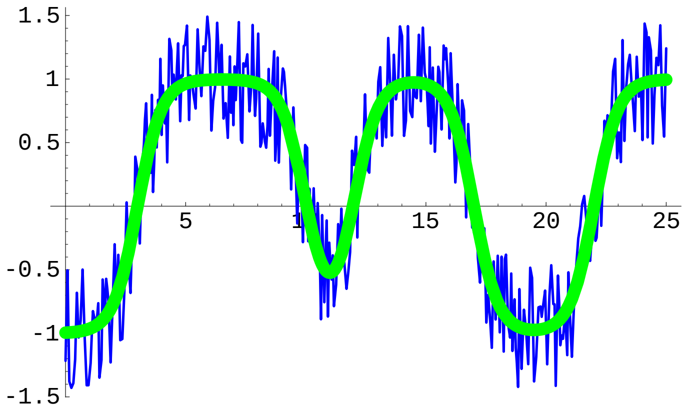


- Typical lattice configuration



$$\{x_i\}_n \to \{x_i\}_{n+1} \qquad p(n \to n+1) = \exp(-\Delta S)$$

• Effective description: Tunneling events (instantons)



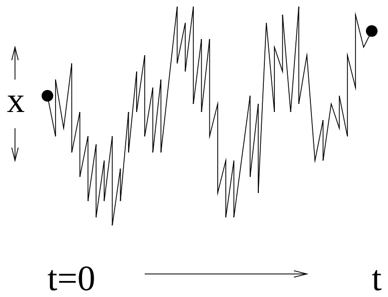
• Can be extracted using cooling (or more sophisticated techniques)

$$\{x_i\}_n \to \{x_i\}_{n+1} \qquad S_n > S_{n+1} \;\; (\hbar \to 0)$$

Correlation Functions

- consider coordinate correlation function

$$\Pi(\tau) = \langle x(\tau) x(0) \rangle$$



- spectral representation

$$\Pi(\tau) = \sum_n |\langle 0|x|n\rangle|^2 e^{-\Delta E_n \tau}$$

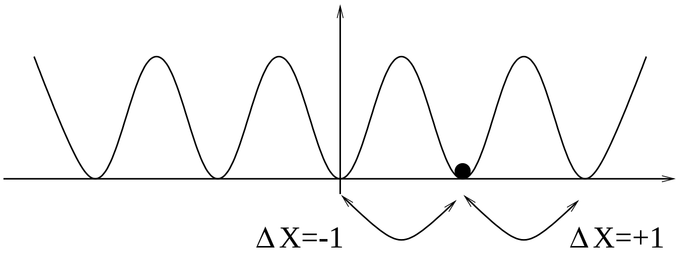
dipole matrix element $\times$ exponential decay

- energy gap

$$\Delta E_1 = -\frac{d}{d\tau} \log(\Pi(\tau))$$

Tunneling in QCD

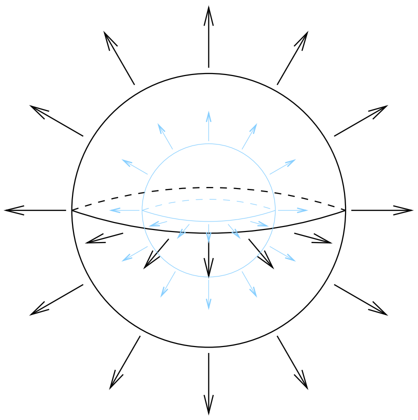
- Classical potential is periodic in variable X



$$X = \int d^3x\, K_0(x,t)$$

$$\partial^\mu K_\mu = \frac{1}{32\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$$

- classical minima correspond to pure gauge configurations ($A_0 = 0$)



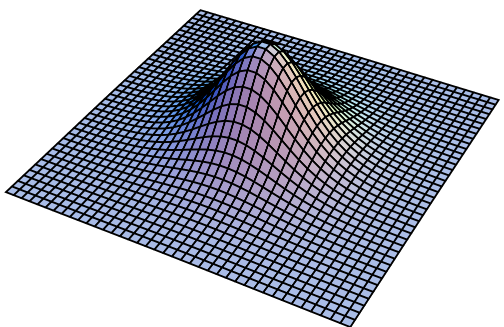
$$A_i(x) = iU^\dagger(x)\partial_i U(x) \qquad\qquad E^2 = B^2 = 0$$

Instanton solution in QCD

- Tunneling processes change X by $\Delta X = Q_{top} = \pm 1$. The action of a solution with topological charge Q_{top} is

$$S = \frac{8\pi^2}{g^2} |Q_{top}|$$

- gauge potential and field strength



$$A^a_\mu(x) = 2\frac{\eta_{a\mu\nu} x_\nu}{x^2+\rho^2},$$

$$(G^a_{\mu\nu})^2 = \frac{192\rho^4}{(x^2+\rho^2)^4}.$$

Phenomenology

- tunneling lowers the ground state energy

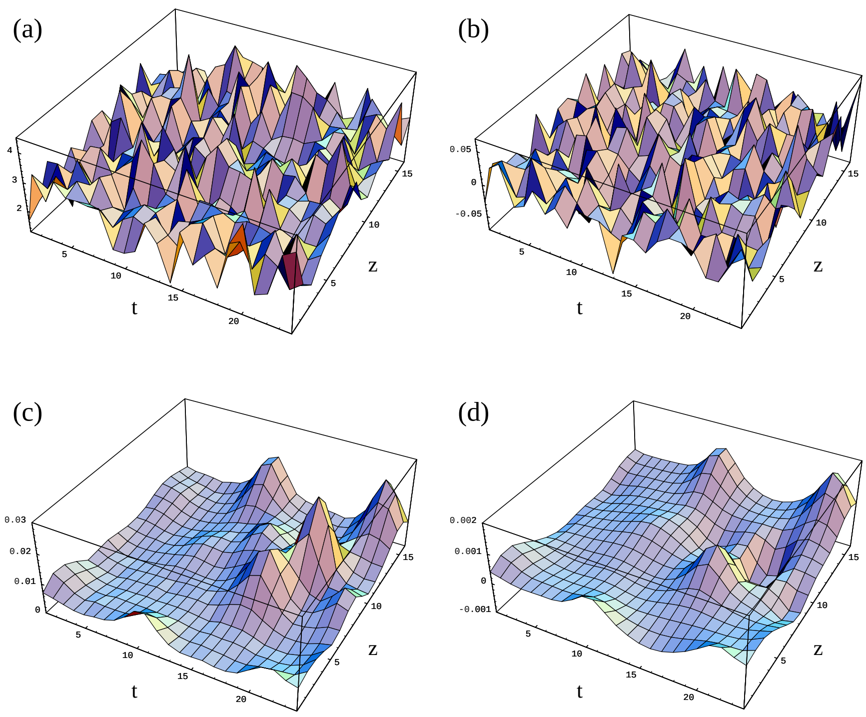
$$\epsilon = -\frac{b}{4}\left(\frac{N}{V}\right), \qquad b = \frac{11}{3}N_c - \frac{2}{3}N_f$$

- QCD vacuum energy $\epsilon \simeq -0.5\mathrm{GeV}/\mathrm{fm}^3$. Expect

$$\left(\frac{N}{V}\right) \simeq 1\,\mathrm{fm}^{-4}$$

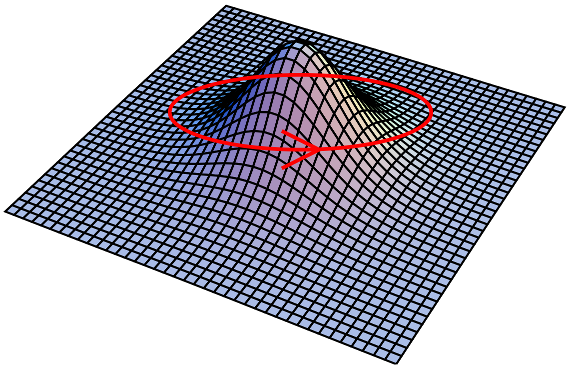
- typical instanton size $\rho \simeq 0.3\,\mathrm{fm}$

$$\left(\frac{N}{V}\right)\rho^4 = 0.01, \quad \Rightarrow \text{inhomogeneous vacuum!}$$



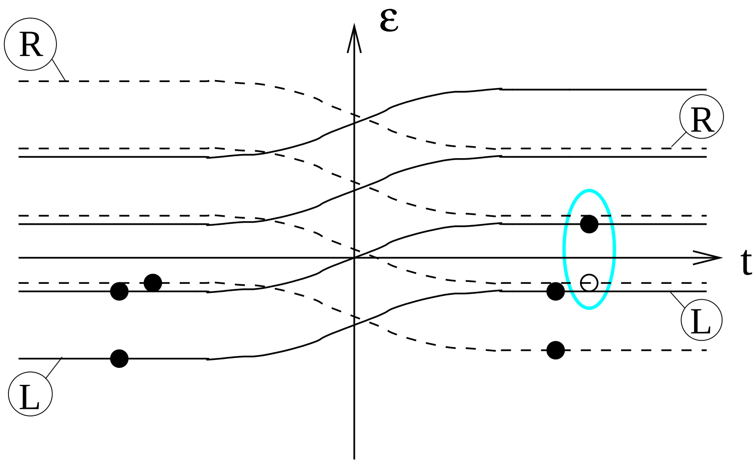
Instantons and Light Quarks

- (Anti)Instantons: Dirac operator has a left (right) handed zero mode.



$$\gamma \cdot (\partial + A_{I,A})\, \psi^0_{L,R} = 0$$

- 4-D zero mode $\rightarrow$ level crossing in spectrum of H



$$\psi_i(\vec{x}, t) =$$

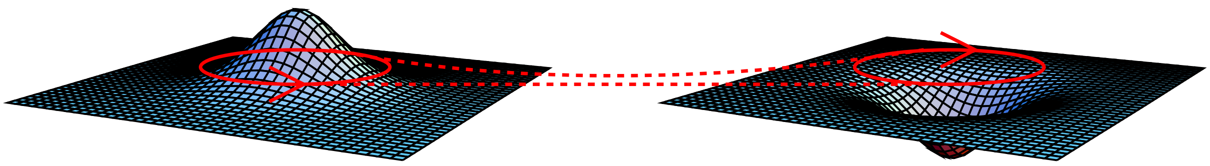
$$\psi_i(\vec{x}, t = -\infty) \exp\left(- \int_{-\infty}^{t} dt'\, \epsilon_i(t') \right).$$

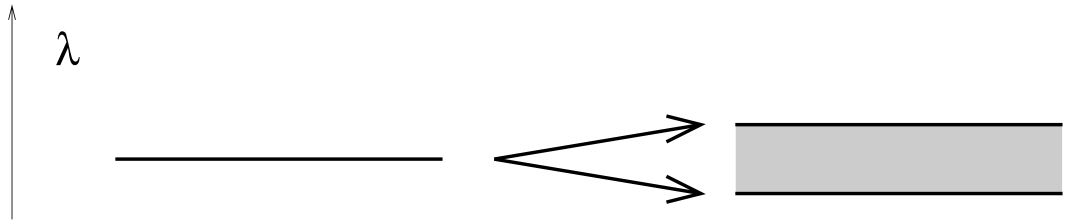
- level crossing $\rightarrow$ axial charge violation (axial anomaly)

- Banks-Casher: Chiral symmetry breaking related to density $\rho(0)$ of "almost" zero modes

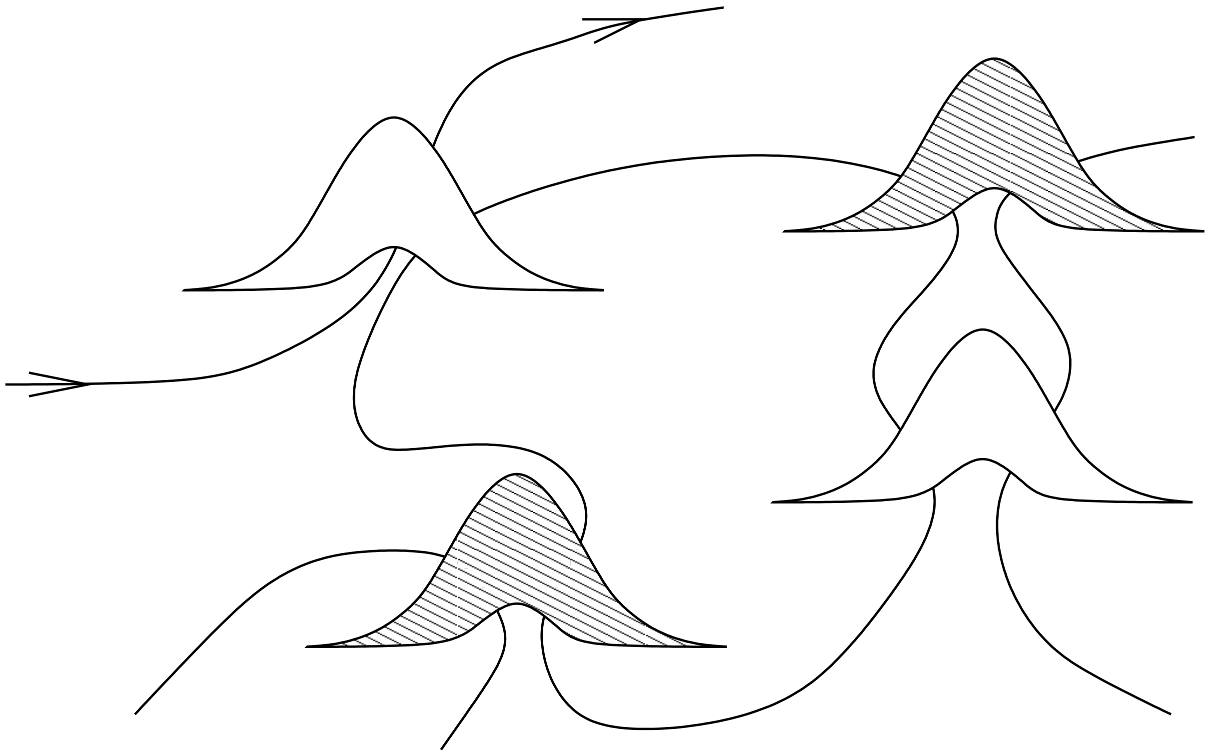
$$\boxed{\langle\bar{q}q\rangle = -\pi\rho(\lambda=0)}$$

- Instanton-Anti-Instanton configurations: zero modes mix

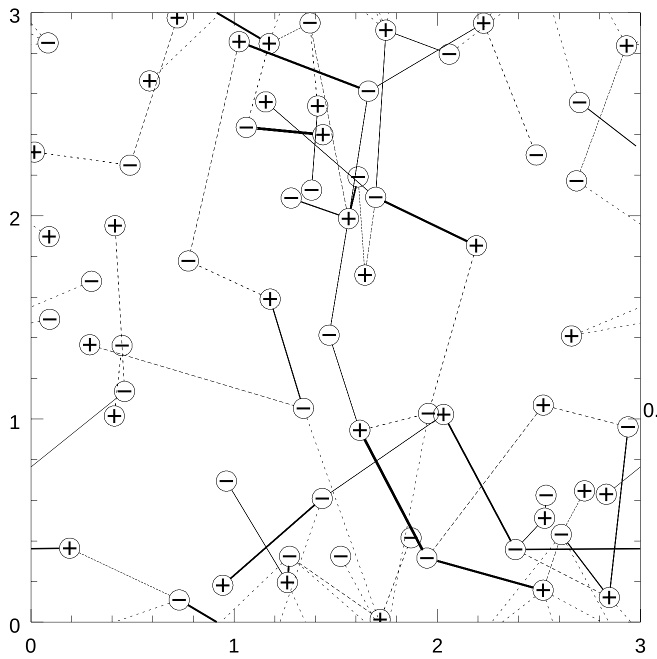


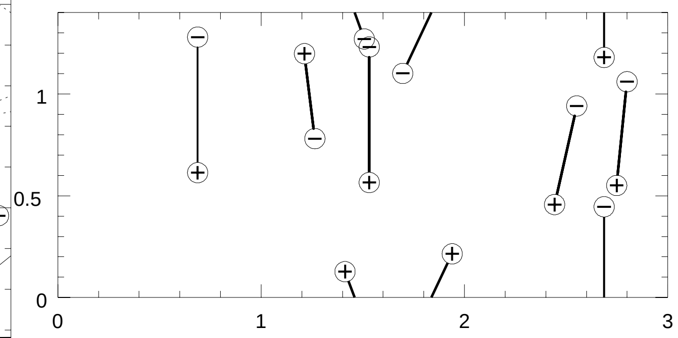


- quark condensate = delocalized zero modes

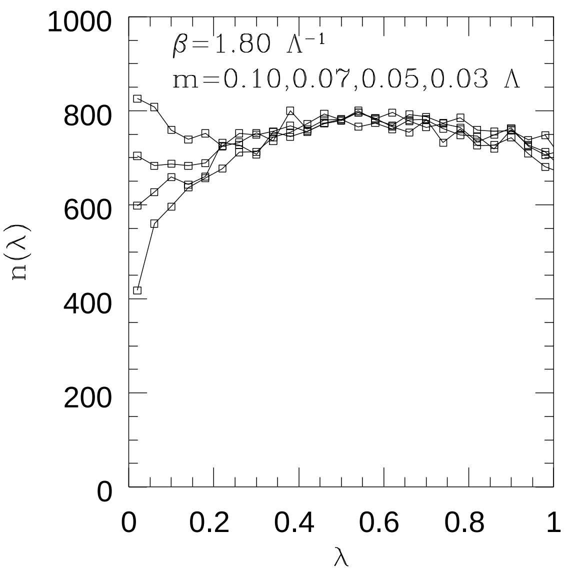


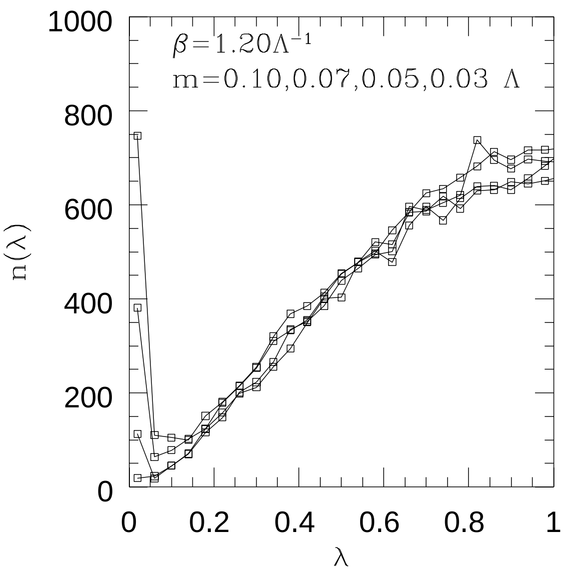
• numerical simulations





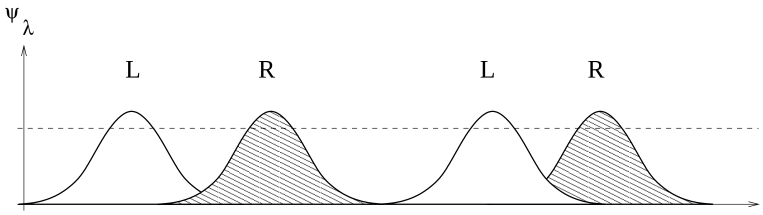
• dirac spectra

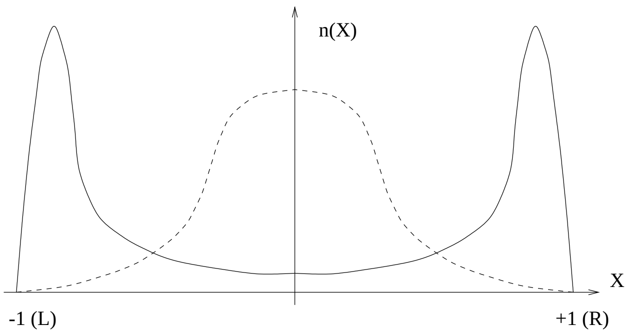




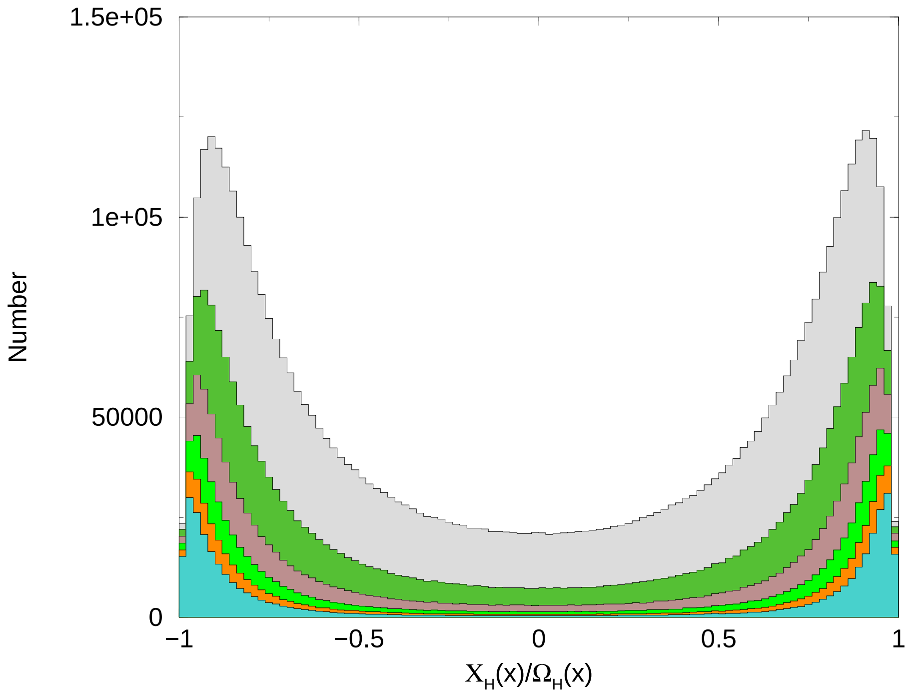
Chiral Symmetry Breaking on the Lattice

- Chirality distribution function





- Chirality distribution measured using overlap fermions



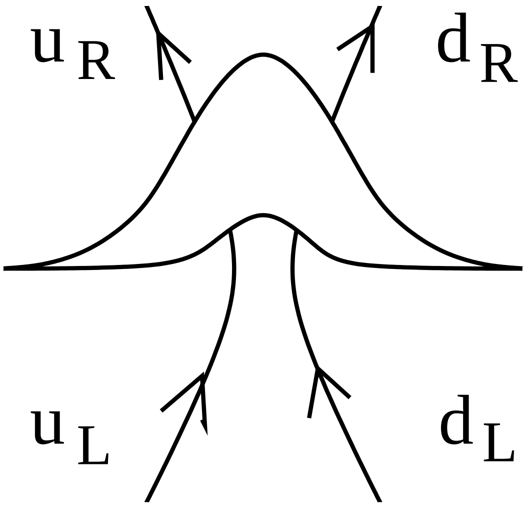
T. Blum et al., [hep-lat/0105006]

Quarks and Tunneling

- The three rules of tunneling

1. there has to be one representative of every quark flavor
2. all quarks have to be left handed when they enter (tunnel)
... or right handed (anti-tunnel)
3. all quarks have to flip their chirality while they tunnel

- The two facets of tunneling: instanton vertex ...

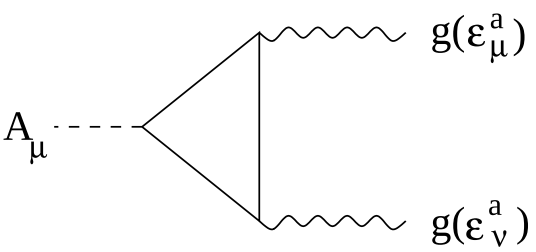


$$\mathcal{L} = G \det_f(\bar\psi_L \psi_R) + (L \leftrightarrow R)$$

$$G = \int d\rho\, n(\rho)$$

$\sigma, \vec\pi$	attractive
$\eta', \vec a_0$	repulsive
$\vec\rho, \vec a_1$	no interaction

... vs axial anomaly



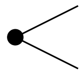
$$\partial_\mu j_5^\mu = \frac{N_f}{16\pi^2} G^a_{\mu\nu} \tilde G^a_{\mu\nu}$$

$$\Delta Q_5 = 2 N_f Q_{top}$$

Correlation Functions in QCD

- pion source function

$$j(x) = \bar{u}(x)\gamma_5 d(x)$$

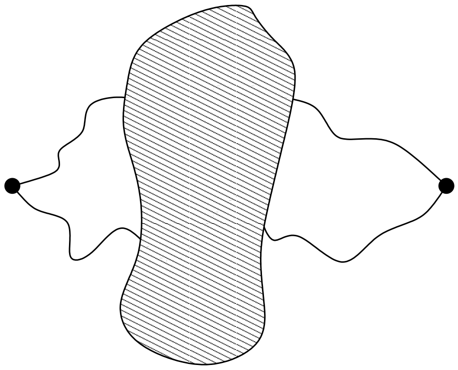


- correlation function

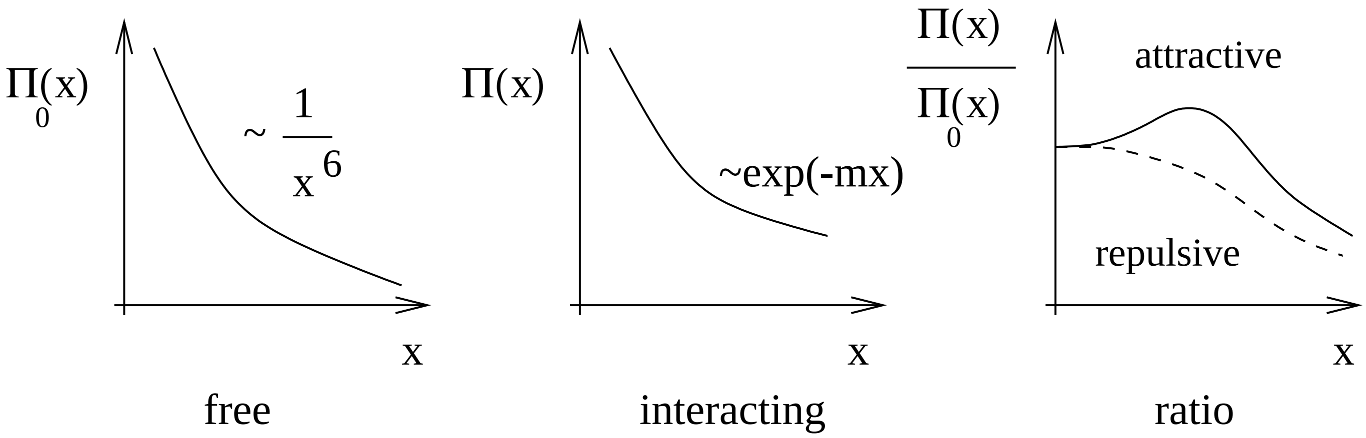
$$\Pi(x) = \langle j(0) j(x) \rangle$$

$$= P(\text{pion propagates from 0 to } x)$$

$$\sim \exp(-m_\pi x)$$

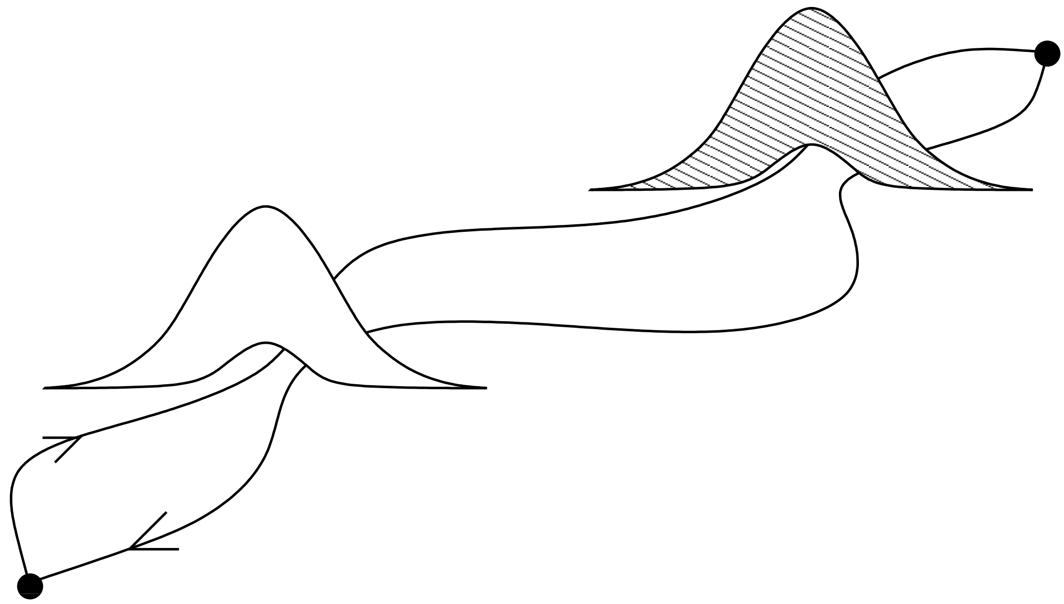


- normalize to non-interacting quarks

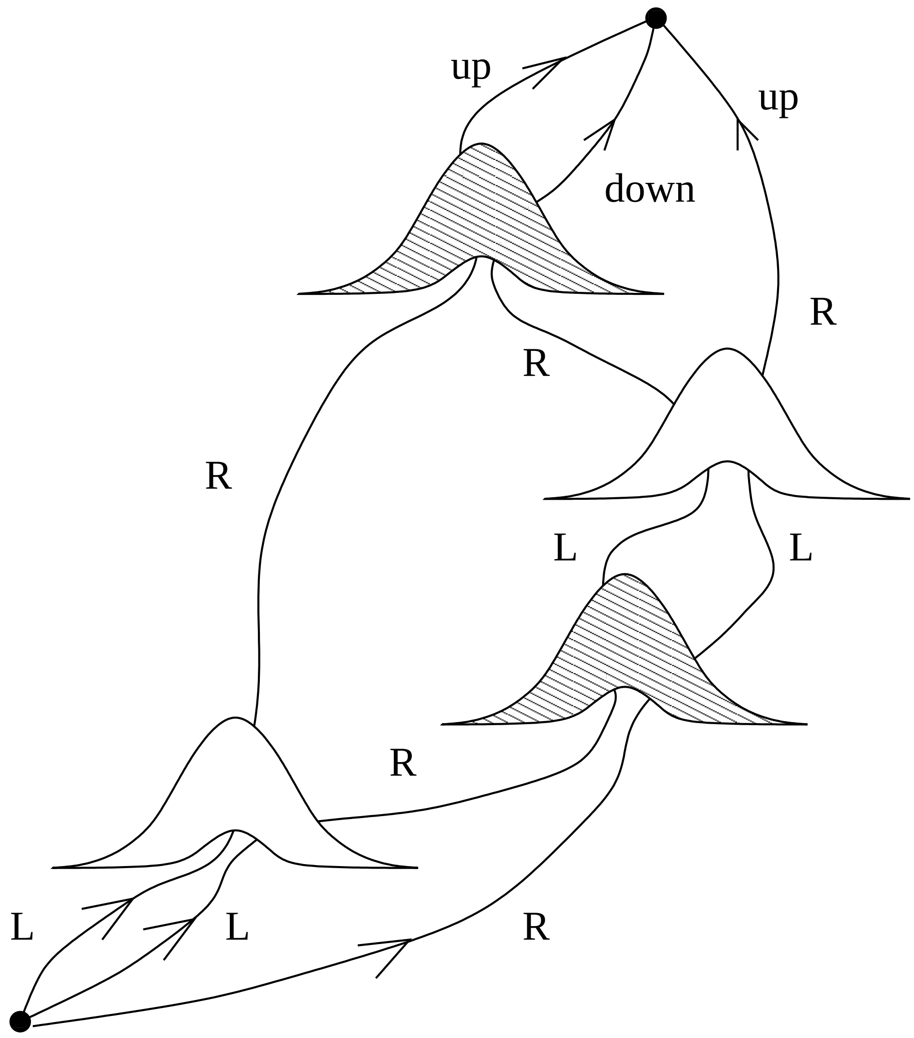


Correlation Functions

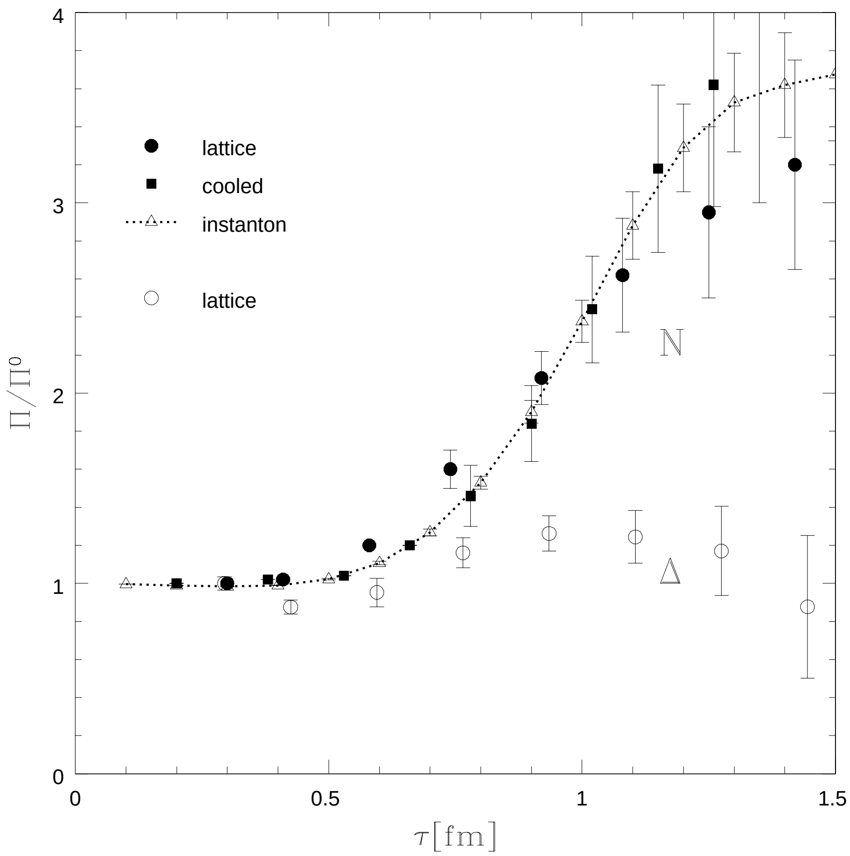
- pion propagator



- nucleon propagator



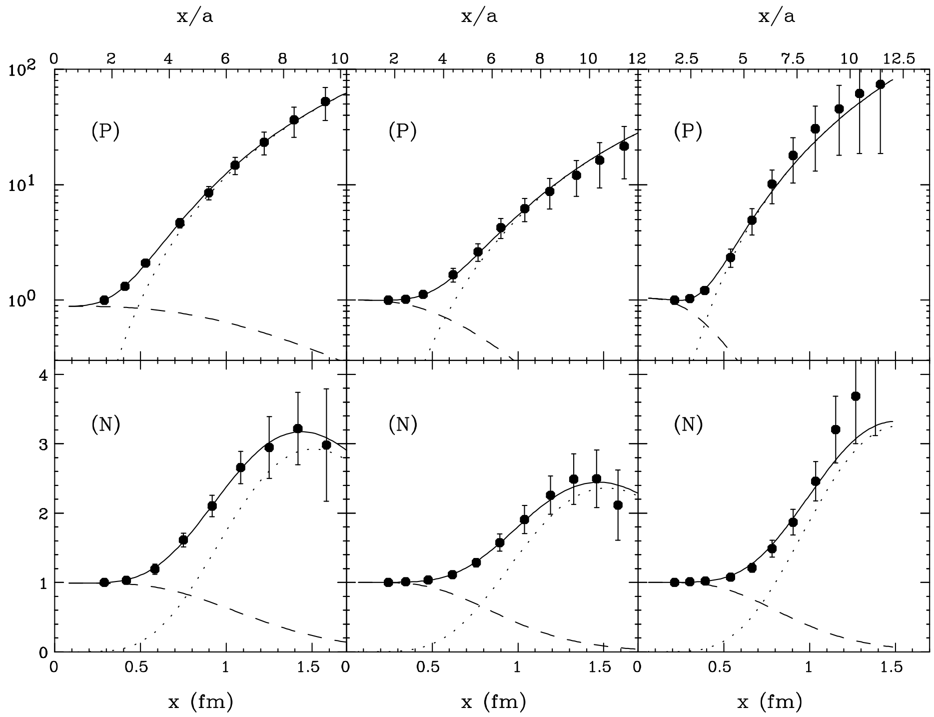
• baryons: nucleon and delta



$$m_p = 960\,\mathrm{MeV} \qquad \lambda_1 = 0.035\,\mathrm{GeV}^3$$

$$\langle 0|\eta_1|N(p)\rangle = \lambda_1 u(p)$$
$$\eta_1 = \epsilon^{abc}(u^a C\gamma_\mu u^b)\gamma^\mu d^c$$

- cooled correlation functions: pion and proton



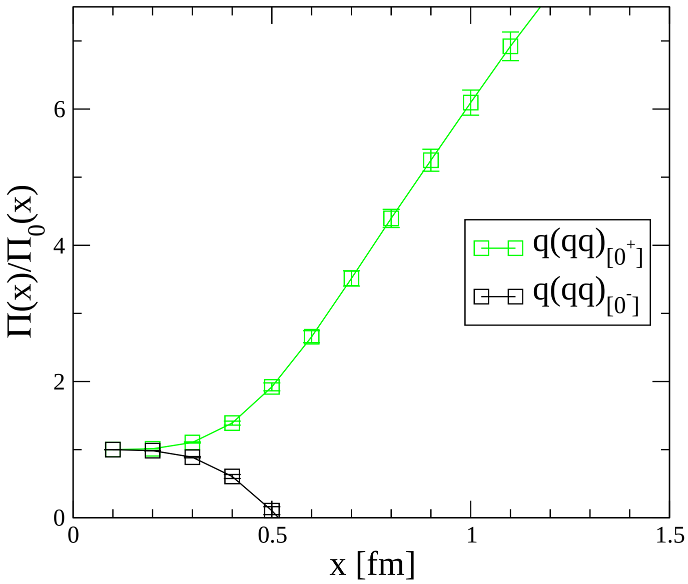
$$m_p(\text{full}) = 950\,\text{MeV} \qquad m_p^*(\text{cool}) = 950\,\text{MeV}$$
$$\lambda_1(\text{full}) = 0.293\,\text{GeV}^3 \qquad \lambda_1(\text{cool}) = 0.297\,\text{GeV}^3$$

* m_p used to set the scale ($\Delta m_\rho / m_\rho = 2\%$)

M.C. Chu et al., Phys.Rev.D49:6039-6050,1994, [hep-lat/9312071]

- nucleon currents constructed from scalar and pseudoscalar diquarks

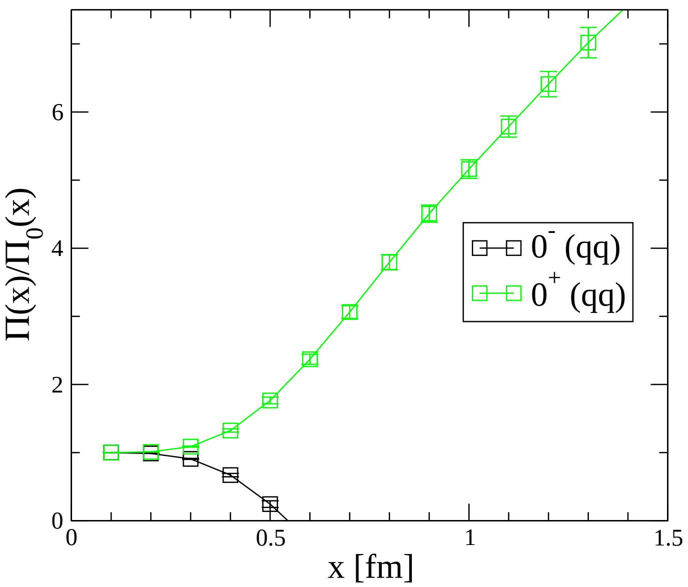
$$\eta_{SD} = \epsilon^{abc}(u^a C\gamma_5 d^b)u^c, \qquad \eta_{PD} = \epsilon^{abc}(u^a C d^b)\gamma_5 u^c$$



$\lambda_{SD} = 0.15\,\mathrm{GeV}^3$

$\lambda_{PD} < 0.02\,\mathrm{GeV}^3$

- compare: scalar and pseudoscalar diquark correlators



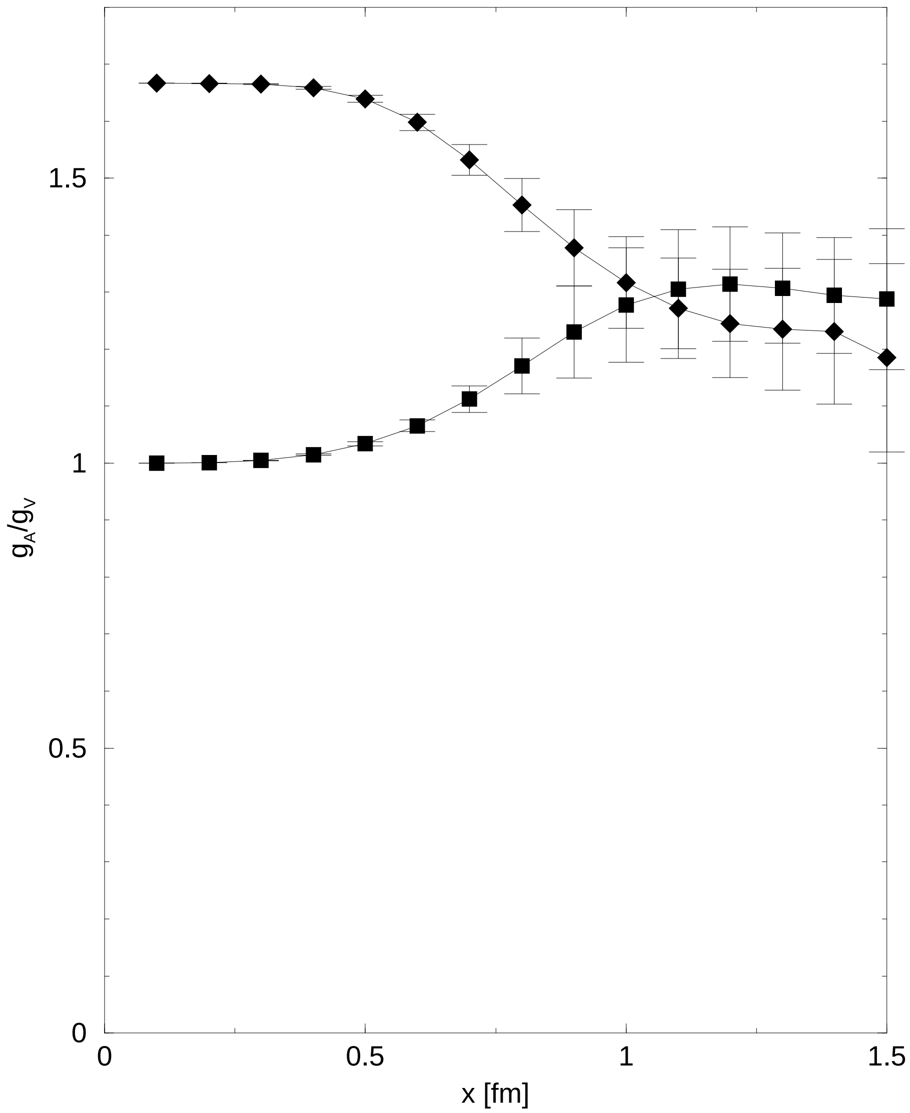
$\lambda_S = 0.23\,\mathrm{GeV}^2$

$\lambda_P < 0.02\,\mathrm{GeV}^3$

- axial coupling constant

$$g_A = \Delta u - \Delta d$$

controls $n \to p + e^- + \nu$ ($g_A^{exp} = 1.25$)

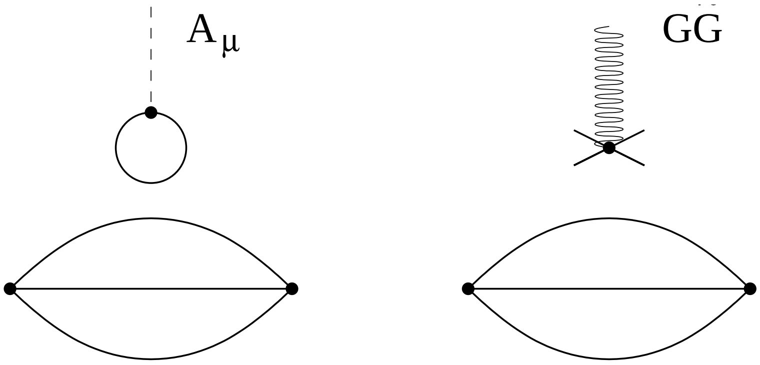


- Also: gluon content of the proton

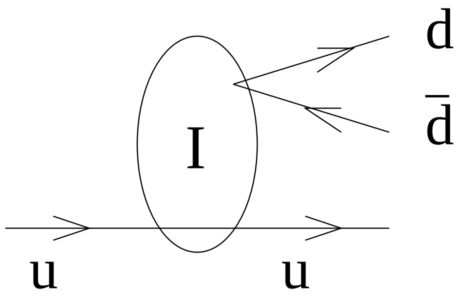
$$\langle p | \frac{\alpha_s}{4\pi} F^2 | p \rangle = -740\,\mathrm{MeV}$$

Outlook

- spin content $\Delta\Sigma$ and coupling to $G\tilde{G}$



- violation of Gottfried sum rule $(\bar{d}/\bar{u})$



- sigma term $\langle N|m_u\bar{u}u + m_d\bar{d}d|N\rangle$
- strangeness content $2\langle N|\bar{s}s|N\rangle/\langle N|\bar{u}u + \bar{d}d|N\rangle$
- charm content, polarized glue, polarized sea, ...
- correlation with gauge field topology and low-lying eigenvalues of the Dirac operator