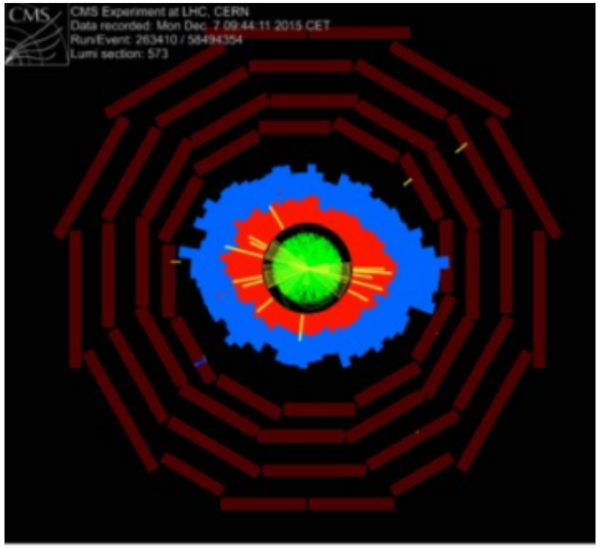
Emergence of Collectivity in Small Systems

Bridging ultra-cold atoms and heavy-ion collisions

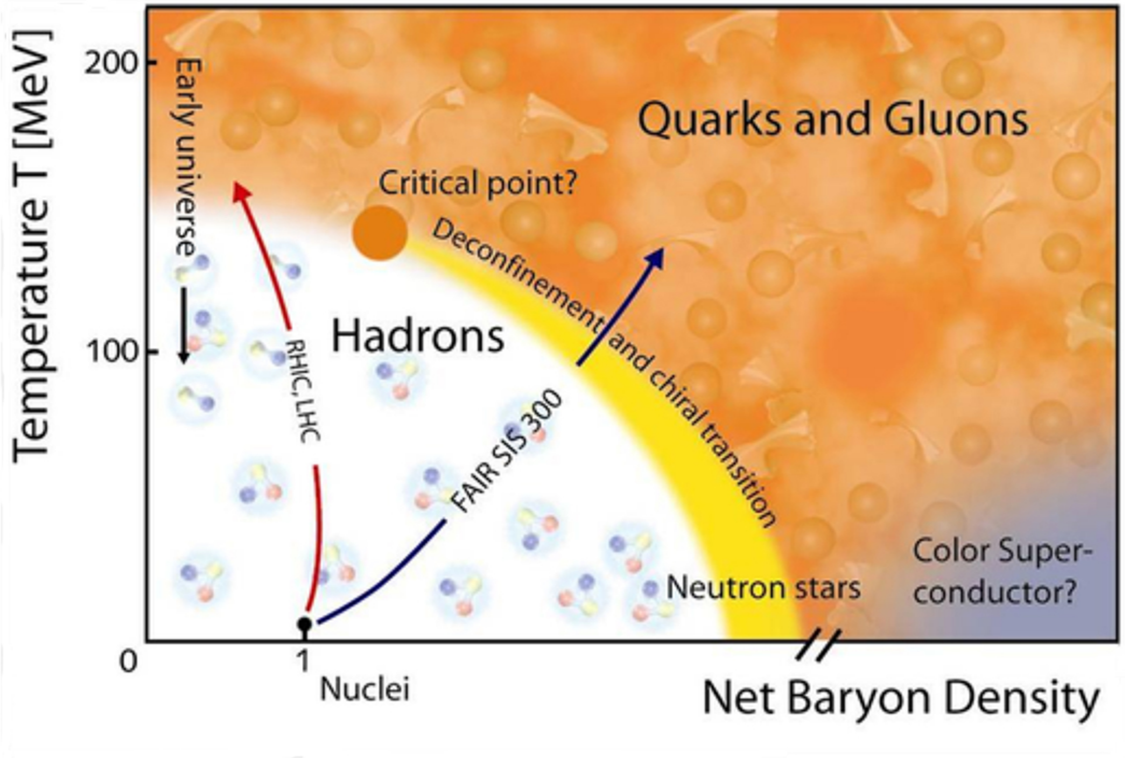
Thomas Schäfer

North Carolina State University

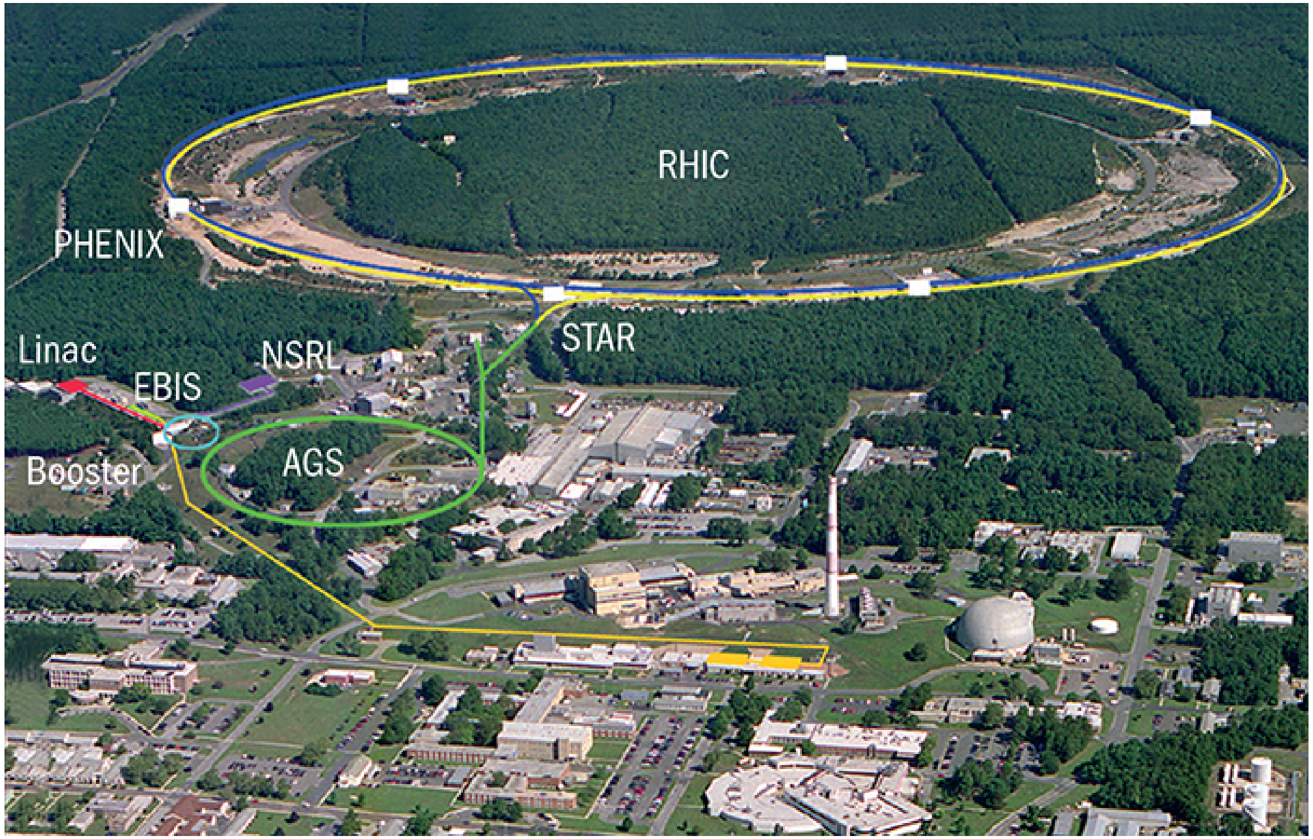


The phase diagram of QCD

$$\boxed{\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4g^2}G^a_{\mu\nu}G^a_{\mu\nu}}$$



2000: Dawn of the collider era at RHIC

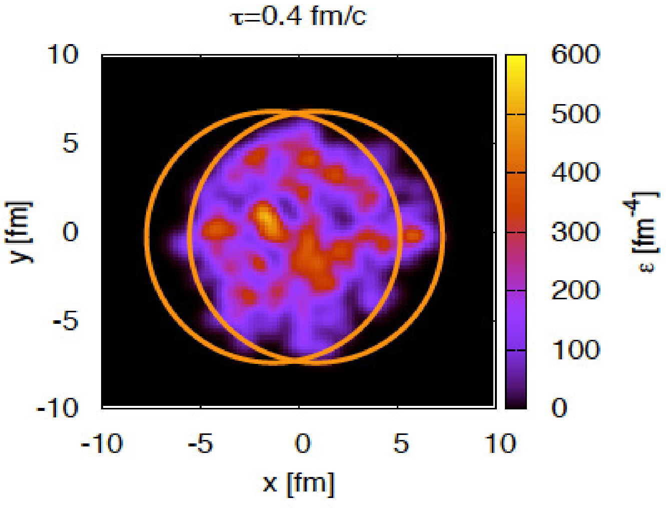


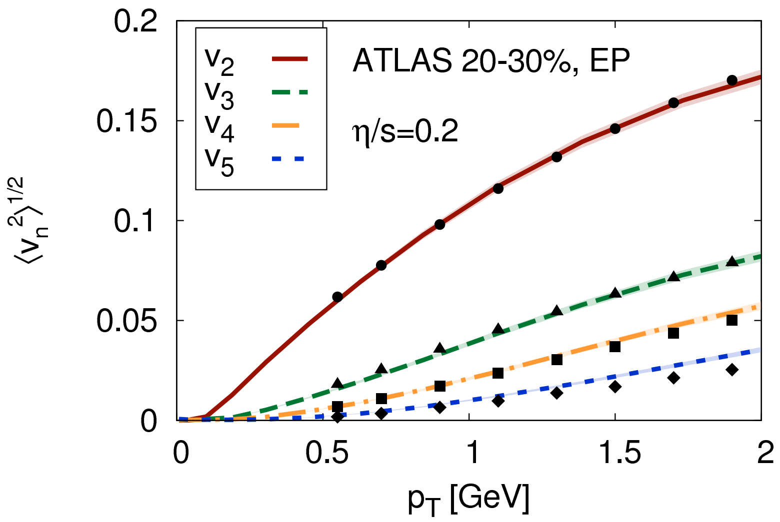
$Au + Au$ @200 $AGeV$

What did we find?

Heavy ion collisions at RHIC are described by a very simple theory:

$$\pi\alpha\nu\tau\alpha\ \rho\varepsilon\iota \quad (\textit{everything flows})$$



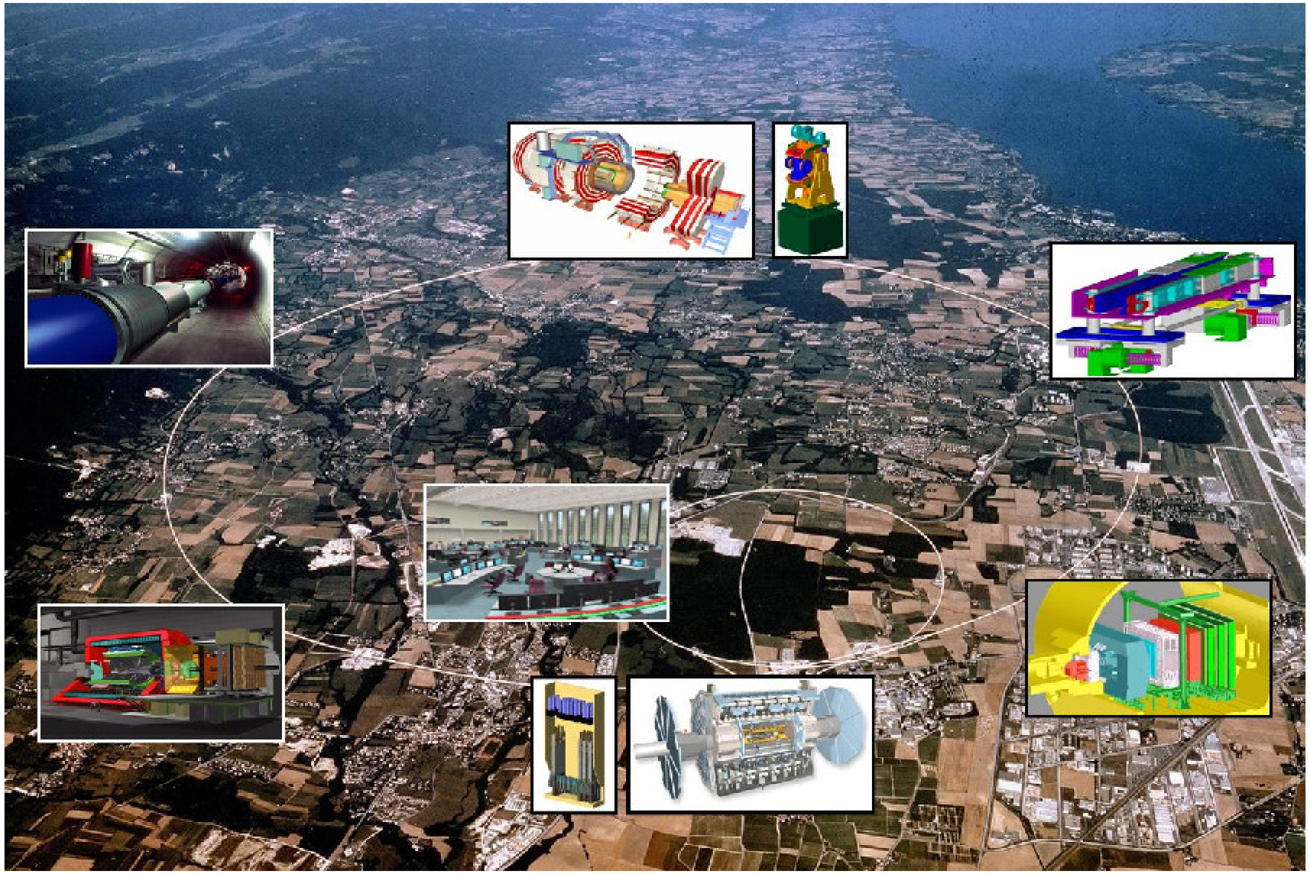


B. Schenke

C. Gale et al., arXiv:1209.6330.

Hydro converts initial state geometry, including fluctuations, to flow. Attenuation coefficient is small, $\eta/s \simeq 0.08\hbar/k_B$, indicating that the plasma is strongly coupled.

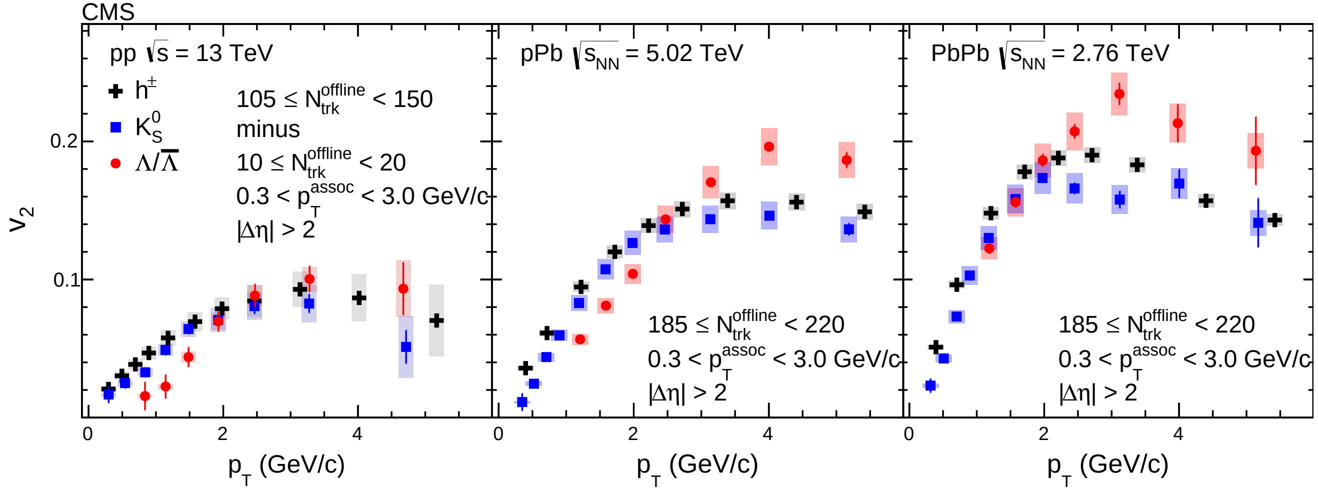
2010: The energy frontier at LHC



$Pb + Pb$ @$2.76\, ATeV$, now $5.5\, ATeV$

What did we find?

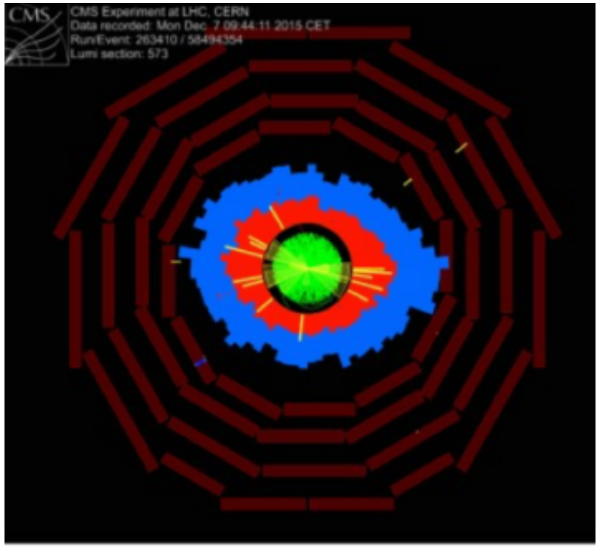
Even the smallest droplets of QGP fluid produced in (high multiplicity) pp and pA collisions exhibit collective flow.



Small viscosity $\eta/s \simeq 0.08\hbar/k_B$ implies short mean free path and rapid hydrodynamization.

Wei Li, arXiv:1704.03576, CMS arXiv:1606.06198

What is the smallest droplet of Quark Gluon Plasma?



This question is unlikely to have a sharply defined answer – the transition from small to large systems or weak to strong coupling is typically smooth. However, we can ask:

- What are the relevant degrees of freedom that carry the momentum, energy, and charges of the fluid?
- What are the main corrections that become relevant in small systems? Gradient terms, non-hydrodynamic modes, fluctuations, etc?
- What are the best tracers for infinite volume, infinite time (zero mass) phase transitions? Non-gaussian moments?

Outline

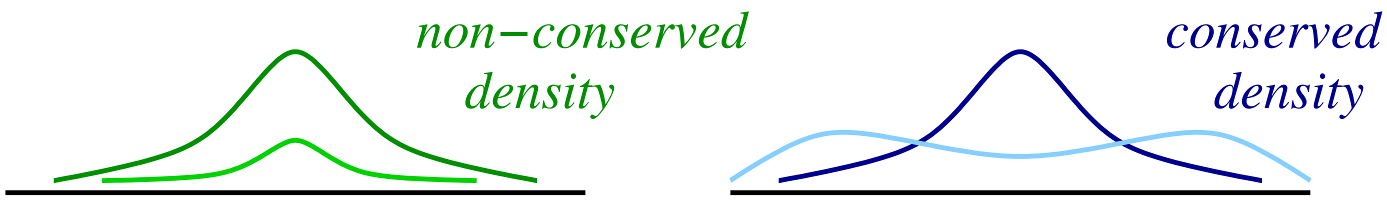
- Fluid dynamics (the basics)
- Ultra-cold atomic gases
- Heavy ion collisions and the quark gluon plasma
- Fluctuations and critical phenomena

Hydrodynamics

Hydrodynamics (undergraduate version): Newton's law for continuous, deformable media.

Fluids: Gases, liquids, plasmas, ...

Hydrodynamics (postmodern): Effective theory of non-equilibrium long-wavelength, low-frequency dynamics of any many-body system.



$\tau \sim \tau_{micro}$ $\qquad$ $\tau \sim \lambda$

$\tau \gg \tau_{micro}$: Dynamics of conserved charges.

Water: $(\rho, \epsilon, \vec{\pi})$

Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla}(\rho \vec{v}) \qquad \frac{\partial \epsilon}{\partial t} = -\vec{\nabla}\vec{\jmath}^{\,\epsilon}$$

$$\frac{\partial}{\partial t}(\rho v_i) = -\nabla_j \Pi_{ij}$$

mass $\times$ acceleration $=$ force

Constitutive relations: Stress tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3}\delta_{ij}\nabla_k v_k \right) + O(\nabla^2)$$

reactive dissipative 2nd order

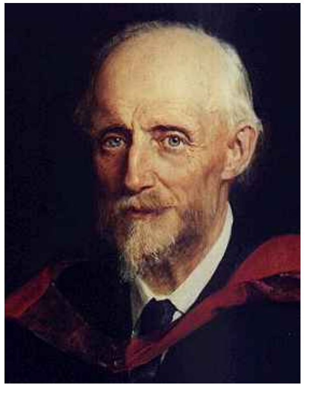
Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Also need an equation of state $P = P(\rho, \epsilon)$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

-1



$$\frac{1}{Re} = \underbrace{\frac{\eta}{\hbar n}}_{\text{fluid property}} \times \underbrace{\frac{\hbar}{m v L}}_{\text{flow property}}$$

Bath tub :	$mvL \gg \hbar$	hydro reliable
Heavy ions :	$mvL \sim \hbar$	need $\eta < \hbar n$

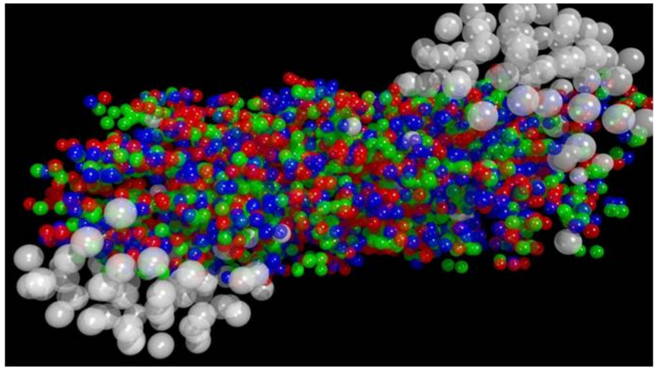
Note: Bacteria swim in the regime $Re^{-1} \gg 1$ but $Ma^2 \cdot Re^{-1} \ll 1$.

Breakdown of fluid dynamics

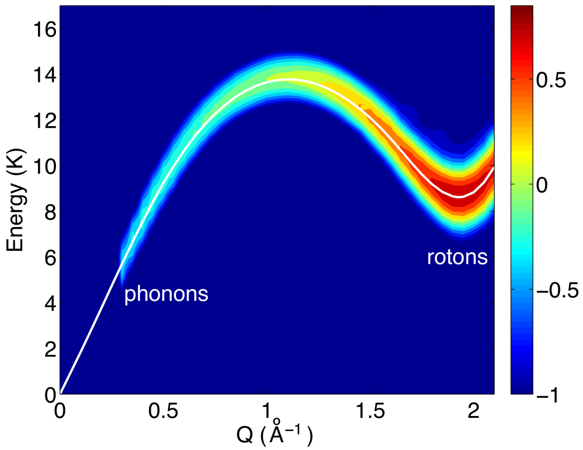
Fluid dynamics is a universal theory but the breakdown of hydro, the emergence of non-hydrodynamic modes, is not.

Two extreme cases: Non-interacting particles or strongly collective, but non-hydrodynamic ($\omega_m(q \to 0) \neq 0$) modes.

Ballistic motion



Non-hydrodynamic modes



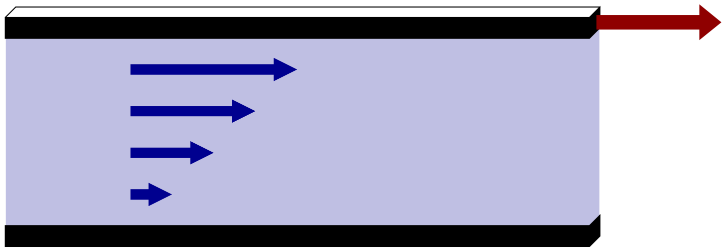
Shear viscosity and friction

Momentum conservation at $O(\nabla v)$

$$\rho \left(\frac{\partial}{\partial t}\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} \right) = -\vec{\nabla}P + \eta\nabla^2\vec{v}$$

Navier-Stokes equation

Viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

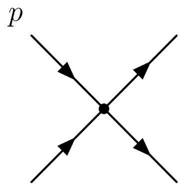
Kinetic theory

Kinetic theory: conserved quantities carried by quasi-particles.

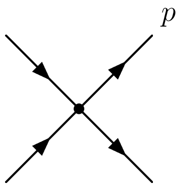
Quasi-particles described by distribution functions $f(x, p, t)$.

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = -C[f_p]$$

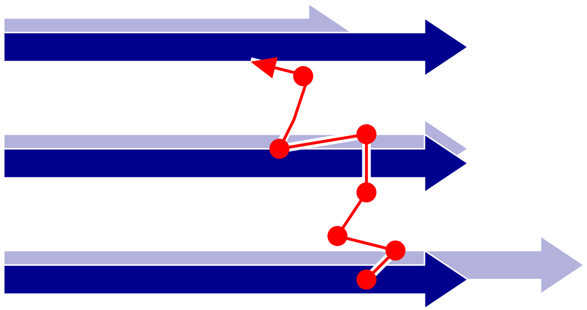
$C[f_p] \; =$



-



Shear viscosity corresponds to momentum diffusion



$$\eta \sim \frac{1}{3} n \, \bar{p} \, l_{mfp}$$

Shear viscosity: Low density limit

Weakly interacting gas, $l_{mfp} \sim \dfrac{1}{n\sigma}$

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

shear viscosity independent of density

Maxwell (1860): "Such a consequence of the mathematical theory is very startling and the only experiment I have met with on the subject does not seem to confirm it."

Shear viscosity: Additional properties

Non-interacting gas ($\sigma \to 0$): $\eta \to \infty$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

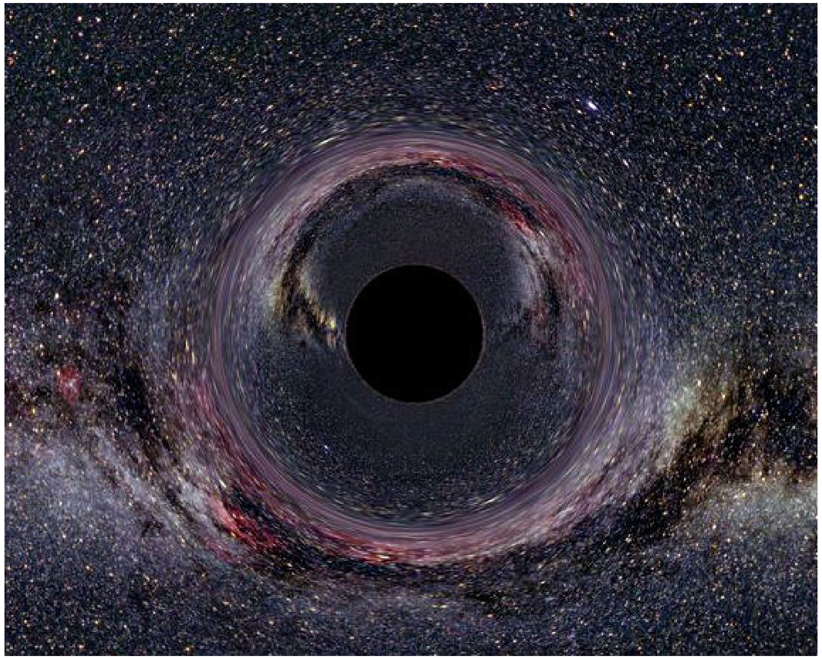
Expansion parameter: Knudsen number $Kn = \frac{l_{mfp}}{L}$

$$Re^{-1} = \frac{\eta}{\rho L v} = \frac{n\bar{p}l_{mfp}}{\rho v L} = Kn$$

Strongly interacting gas: $\frac{\eta}{n} \sim \bar{p}l_{mfp} \geq \hbar$

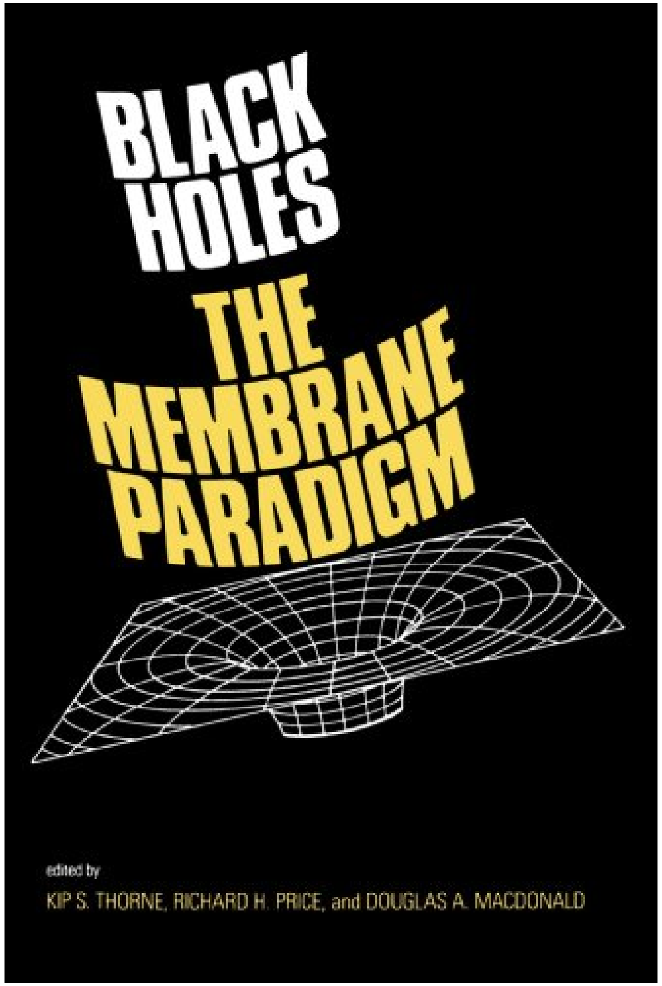
Quantum bound. But: Kinetic theory mat not be reliable!

And now for something completely different ...



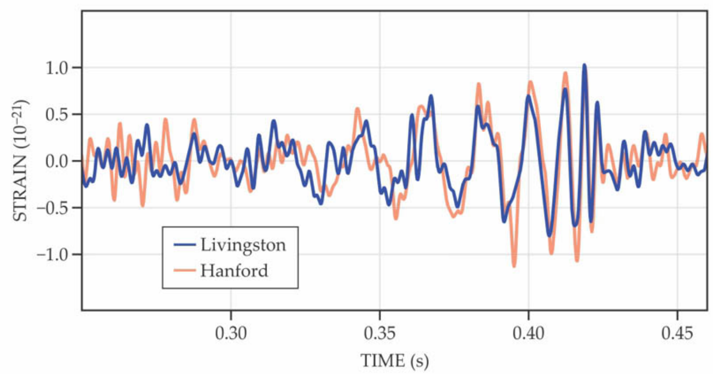
This is an irreversible process, $\Delta S > 0$.

And now for something completely different . . .



Ringdown can be described in terms of stretched horizon that behaves as a sheared fluid

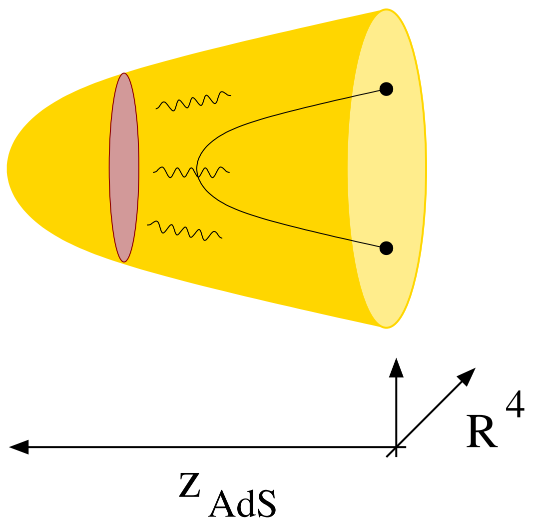
$$\eta = \frac{s}{4\pi}$$



Note: Unusual thermodynamics, e.g. $\zeta, C < 0$.

Idea can be made precise using the "AdS/CFT correspondence"

Strongly coupled thermal field theory on R^4	$\Leftrightarrow$	Weakly coupled string theory on AdS_5 black hole
CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon



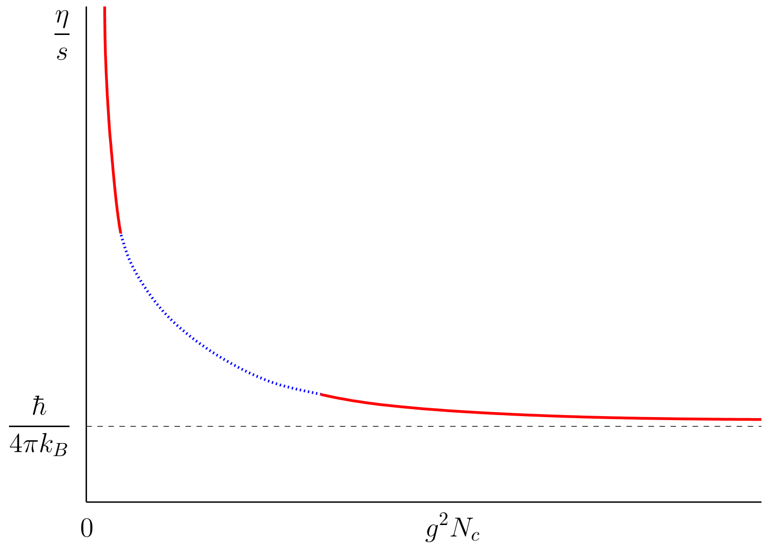
Holographic duals: Transport properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

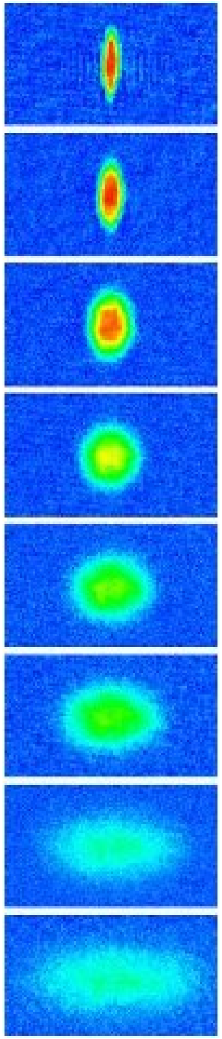


Son and Starinets (2001)

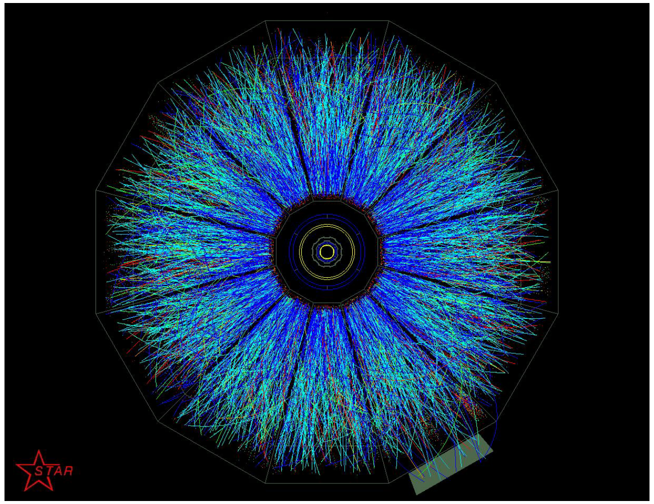
Strong coupling limit universal? Provides lower bound for all theories?

Answer appears to be no; e.g. theories with higher derivative gravity duals.

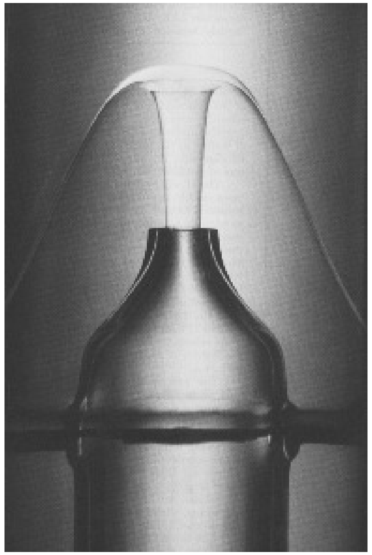
Perfect Fluids: The contenders



Trapped Atoms
(T=0.1 neV)

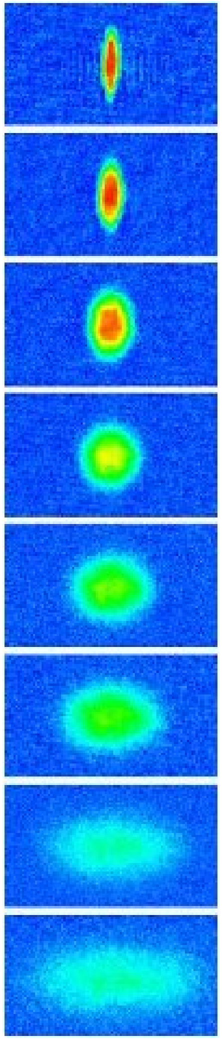


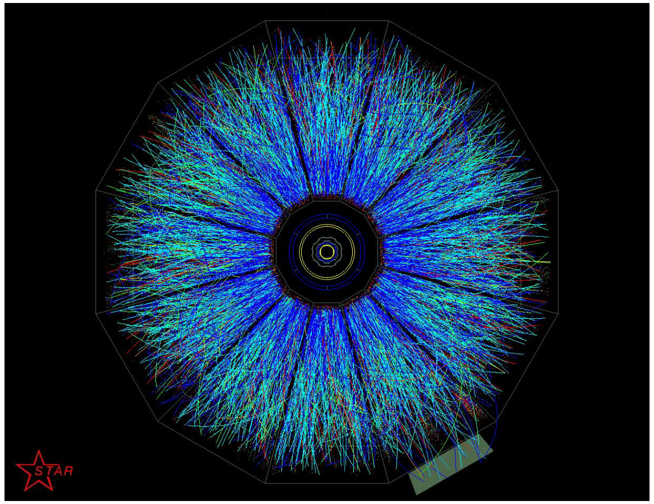
QGP (T=180 MeV)



Liquid Helium
(T=0.1 meV)

Perfect Fluids: The contenders

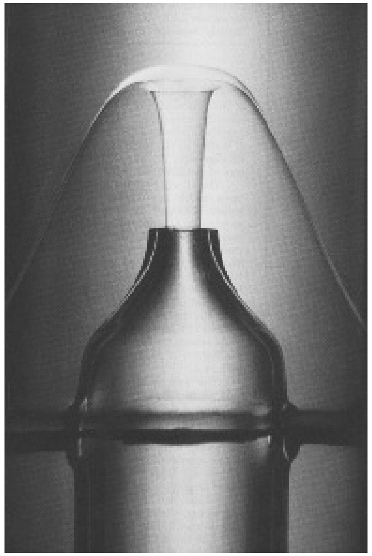




QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



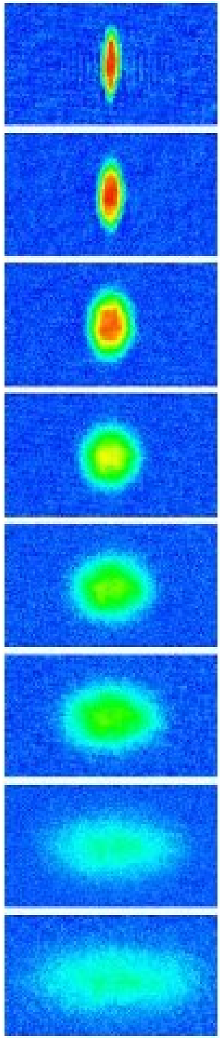
Liquid Helium

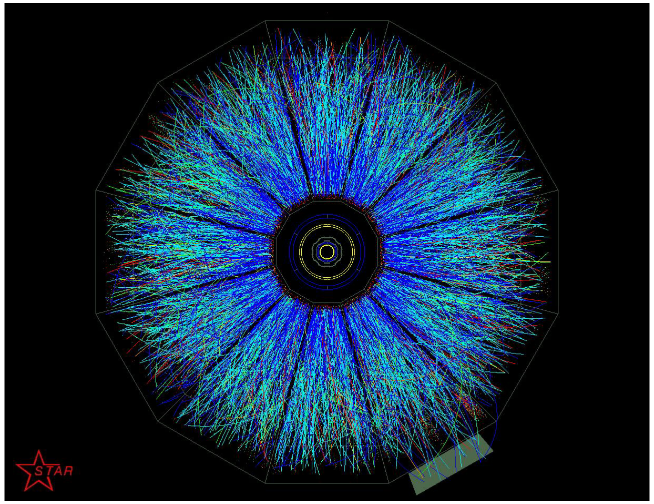
$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

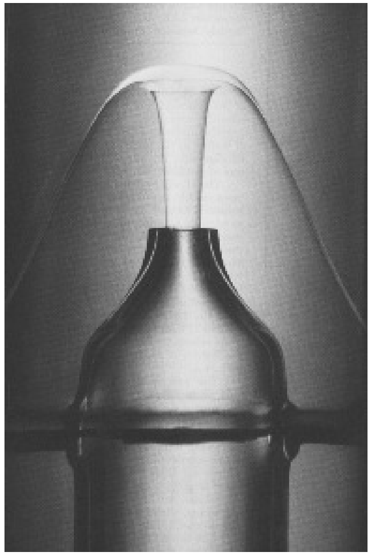
η/s

Perfect Fluids: The contenders





QGP $\eta/s \simeq 0.1$



Liquid Helium

$\eta/s \simeq 1$

η/s in units of $\hbar/k_B$

Trapped Atoms

$\eta/s \simeq 0.5$

Perfect Fluids: Not a contender

Queensland pitch-drop experiment

1927-2014 (9 drops)

$$\eta = (2.3 \pm 0.5) \cdot 10^8 \, Pa\, s$$