The “Big” Picture

Thomas Schaefer

North Carolina State University

“Big” Questions

What is QCD?

What is a Phase of QCD?

What is a Plasma?

What is a (perfect) Liquid?

What is a wQGP/sQGP?

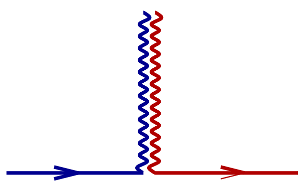
What is QCD (Quantum Chromo Dynamics)?

Elementary fields: **Quarks** **Gluons**

$$(q_\alpha)^a_f \begin{cases} \text{color} & a = 1,\ldots,3 \\ \text{spin} & \alpha = 1,2 \\ \text{flavor} & f = u,d,s,c,b,t \end{cases} \qquad A^a_\mu \begin{cases} \text{color} & a = 1,\ldots,8 \\ \text{spin} & \epsilon^\pm_\mu \end{cases}$$

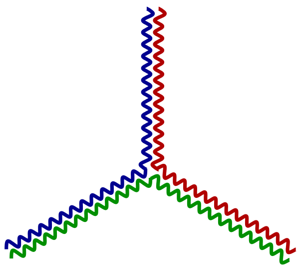
Dynamics: Generalized Maxwell (Yang-Mills) + Dirac theory

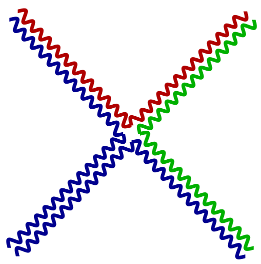
$$\boxed{\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}}$$



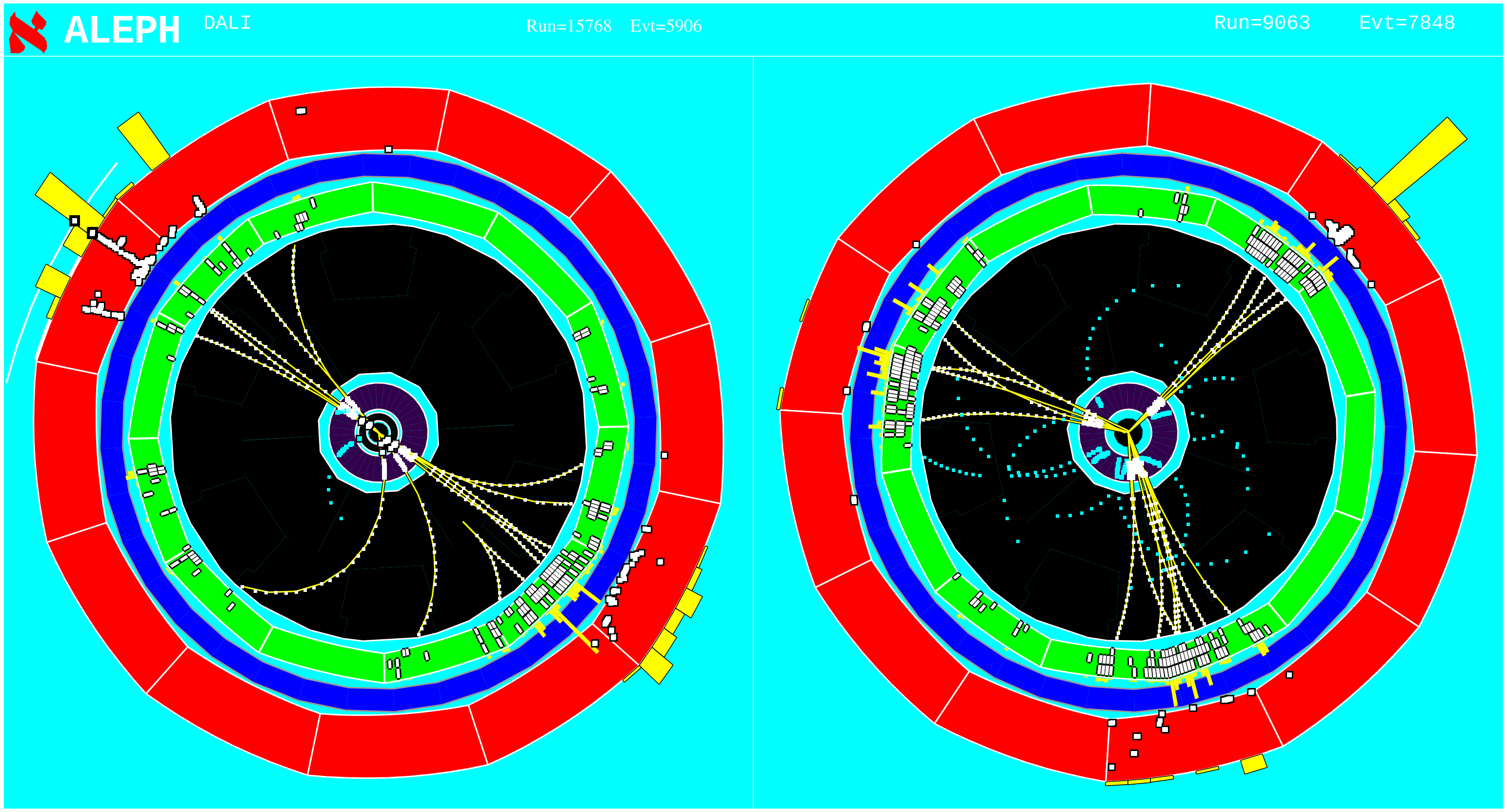
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

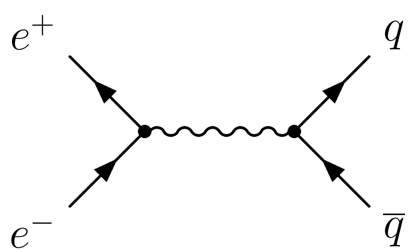
$$i\not{D}q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu t^a\right) q$$

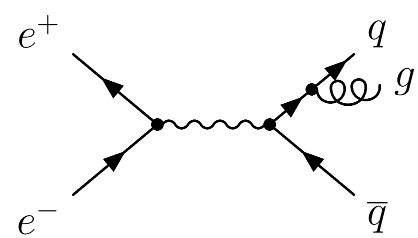




“Seeing” Quarks and Gluons



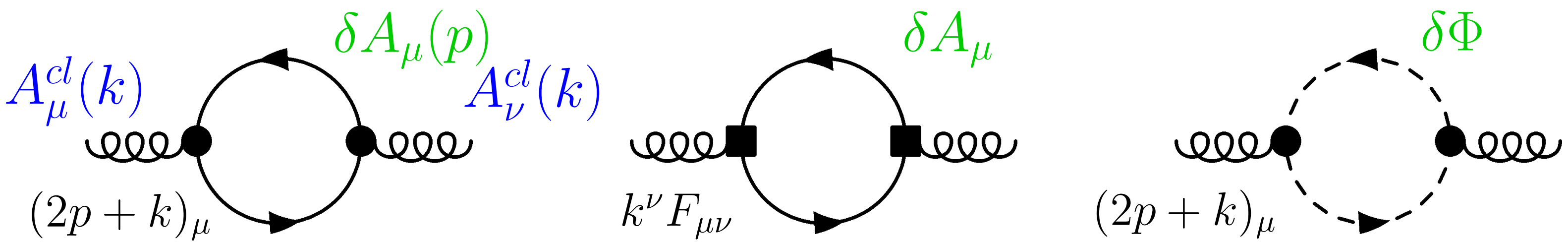




Asymptotic Freedom

Classical field A_μ^{cl}. Modification due to quantum fluctuations:

$$A_\mu = A_\mu^{cl} + \delta A_\mu \qquad \frac{1}{g^2} F_{cl}^2 \to \left(\frac{1}{g^2} + c \log\left(\frac{k^2}{\mu^2}\right)\right) F_{cl}^2$$

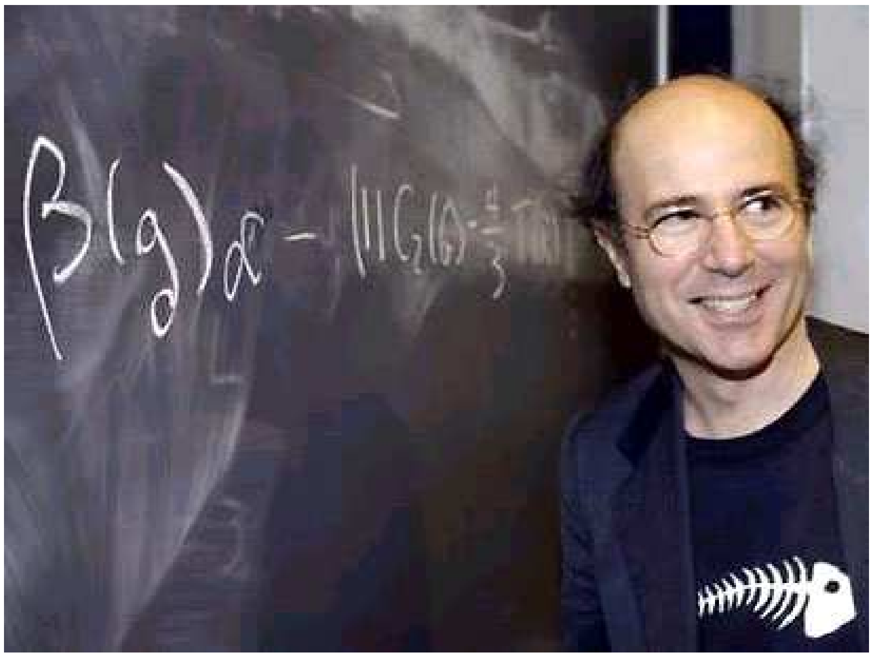


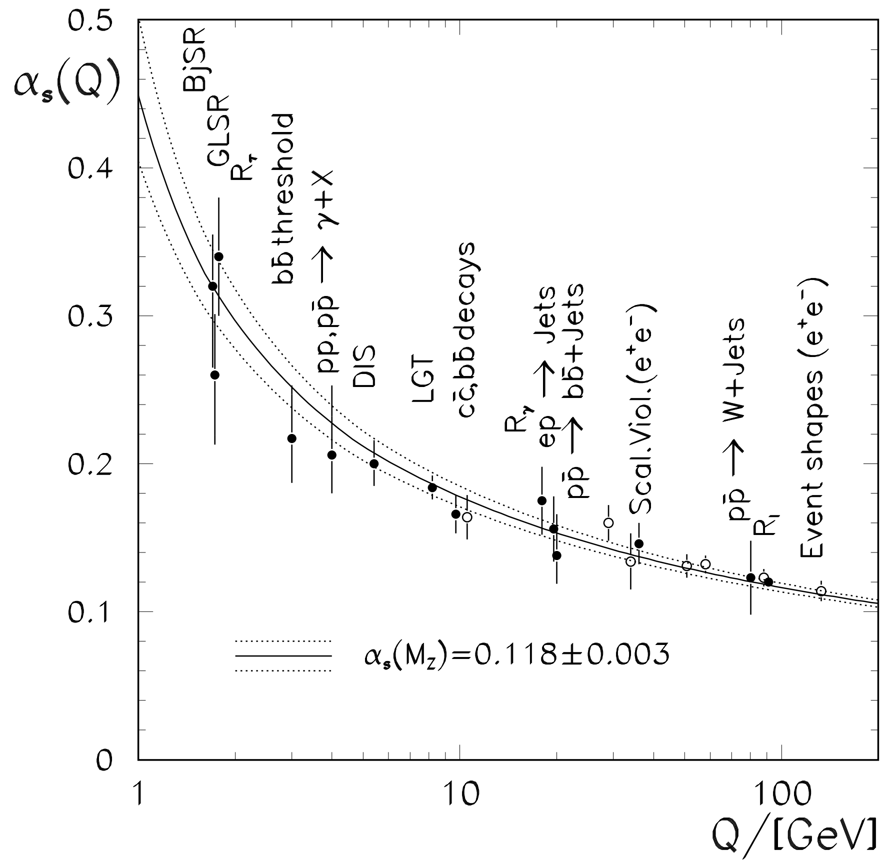
dielectric $\epsilon > 1$ paramagnetic $\mu > 1$ dielectric $\epsilon > 1$

$$\mu\epsilon = 1 \;\Rightarrow\; \epsilon < 1$$

$$\beta(g) = \frac{\partial g}{\partial \log(\mu)} = \frac{g^3}{(4\pi)^2}\left\{\left[\frac{1}{3} - 4\right] N_c + \frac{2}{3} N_f\right\} < 0$$

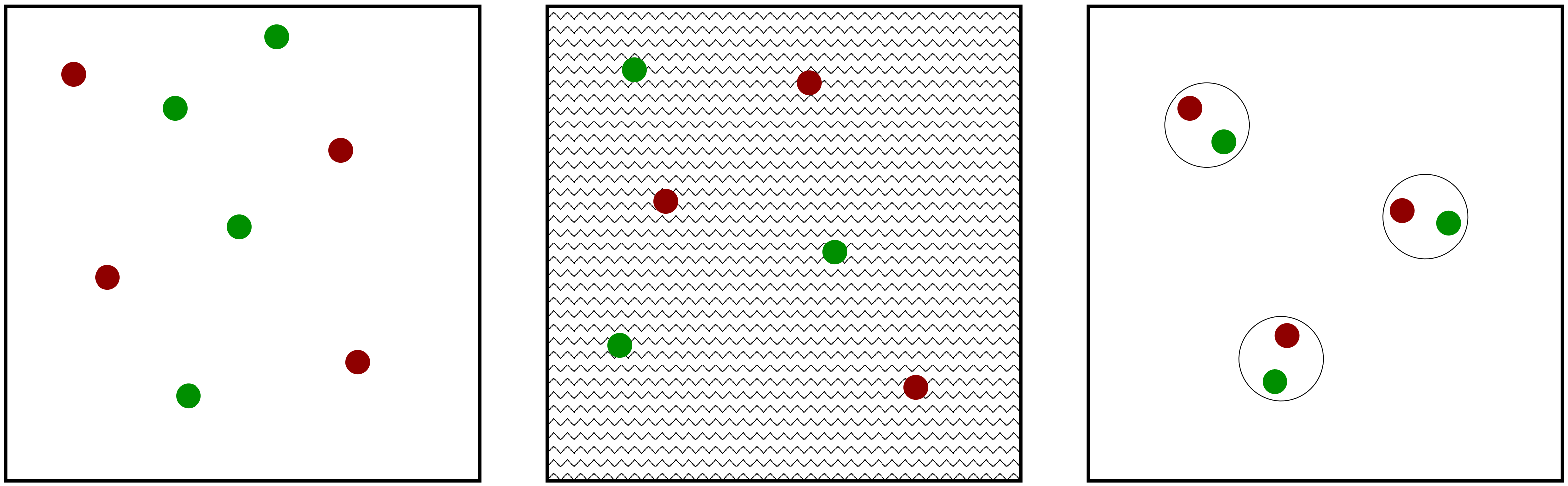
Running Coupling Constant





What is a Phase of QCD? Phases of Gauge Theories

Coulomb | Higgs | Confinement

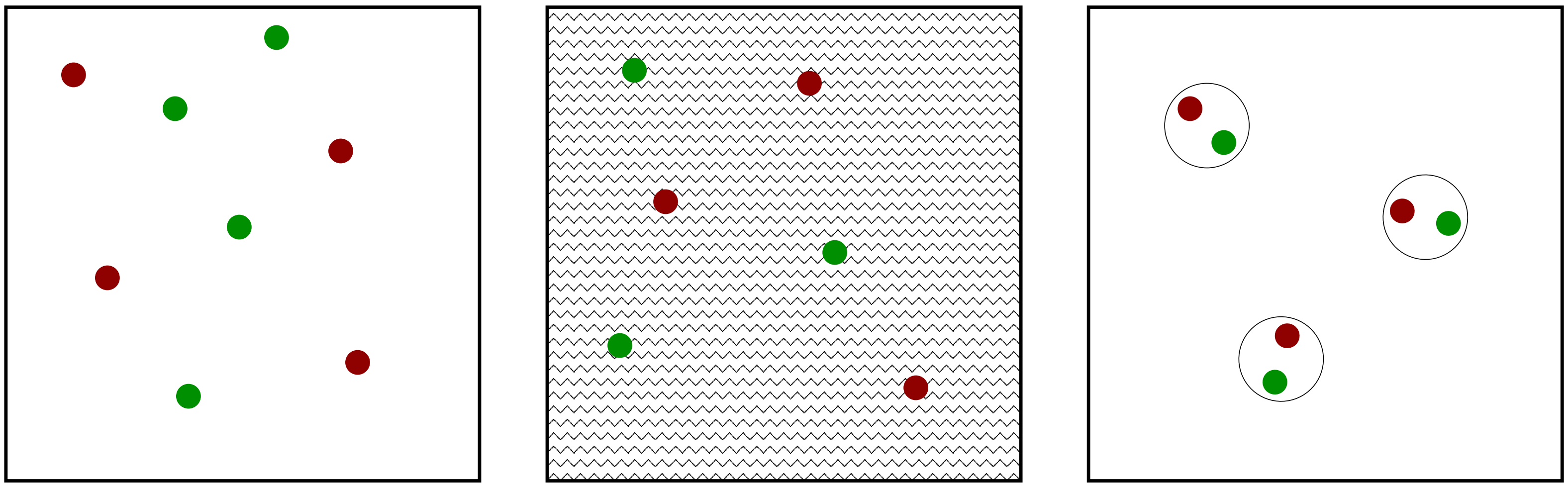


$$V(r) \sim -\frac{e^2}{r} \qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

Standard Model: $U(1) \times SU(2) \times SU(3)$

What is a Phase of QCD? Phases of Gauge Theories

Coulomb | Higgs | Confinement



$$V(r) \sim -\frac{e^2}{r} \qquad V(r) \sim -\frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

QCD: High T phase | High μ phase | Low T, μ phase

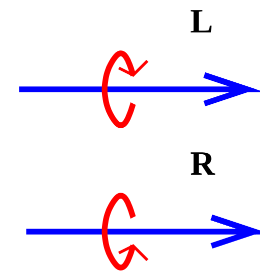
Phases of Matter: Symmetries

phase	order param	broken symmetry	rigidity phenomenon	Goldstone boson
crystal	ρ_k	translations	rigid	phonon
magnet	$\vec{M}$	rotations	hysteresis	magnon
superfluid	$\langle \Phi \rangle$	particle number	supercurrent	phonon
supercond.	$\langle \psi\psi \rangle$	gauge symmetry	supercurrent	none (Higgs)
χsb	$\langle \bar{\psi}\psi \rangle$	chiral symmetry	axial current	pion

Chiral Symmetry

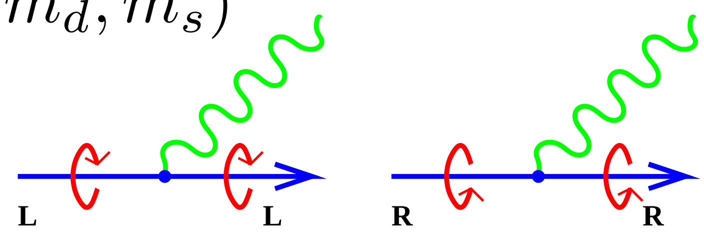
Define left and right handed fields

$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

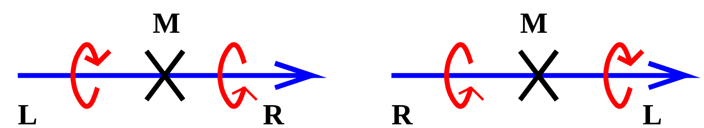


Fermionic lagrangian, $M = \mathrm{diag}(m_u, m_d, m_s)$

$$\mathcal{L} \;=\; \bar{\psi}_L(i\not{D})\psi_L + \bar{\psi}_R(i\not{D})\psi_R$$



$$+\, \bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$



$M = 0$: Chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

$$\psi_L \to L\psi_L, \qquad\qquad \psi_R \to R\psi_R$$

Chiral Symmetry Breaking

Chiral symmetry is spontaneously broken

$$\langle \bar{\psi}_L^f \psi_R^g + \bar{\psi}_L^f \psi_R^g \rangle \simeq -(230\,\mathrm{MeV})^3\ \delta^{fg}$$

$$SU(3)_L \times SU(3)_R \to SU(3)_V \qquad\qquad (G \to H)$$

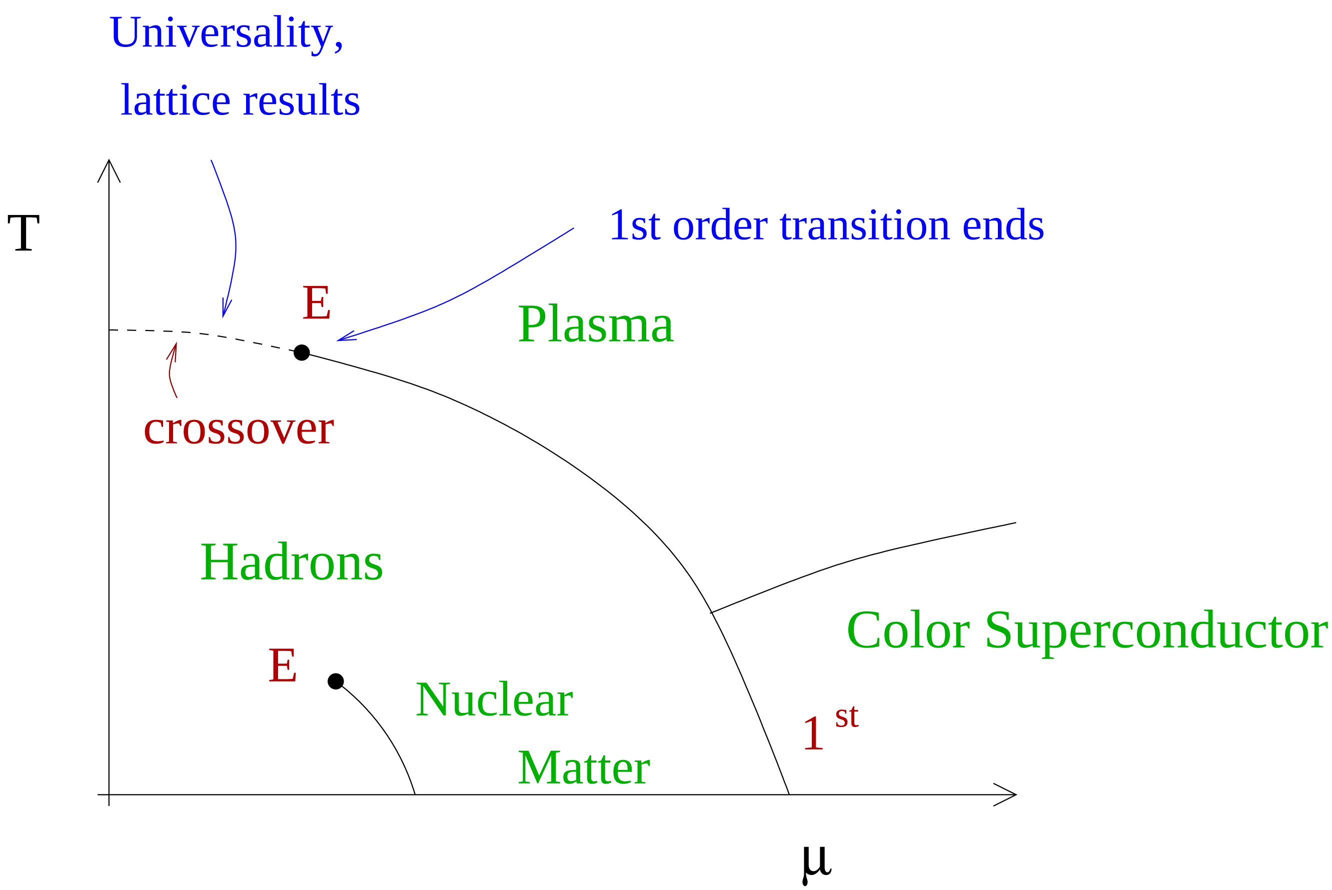
Consequences: dynamical mass generation $m_Q = 300\,\mathrm{MeV} \gg m_q$

$$m_N = 890\,\mathrm{MeV} + 45\,\mathrm{MeV} \qquad\qquad (\mathrm{QCD},\ 95\%) + (\mathrm{Higgs},\ 5\%)$$

Goldstone Bosons: Consider broken generator Q_5^a

$$[H, Q_5^a] = 0 \qquad Q_5^a|0\rangle = |\pi^a\rangle \qquad\qquad H|\pi^a\rangle = HQ_5^a|0\rangle = Q_5^a H|0\rangle = 0$$

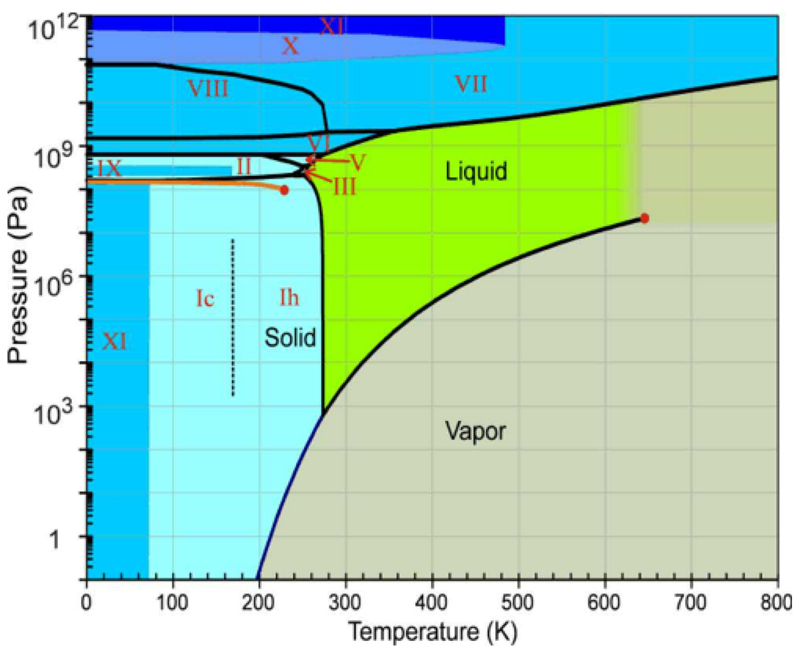
Phase Diagram: Minimal Version



critical endpoint (E) persists even if $m \neq 0$

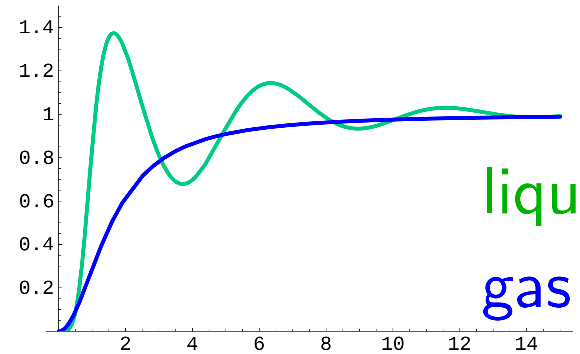
Transitions without change of symmetry: Liquid-Gas

Phase diagram of water

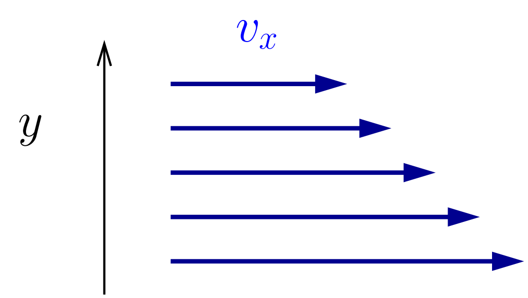


Characteristics of a liquid

Pair correlation function



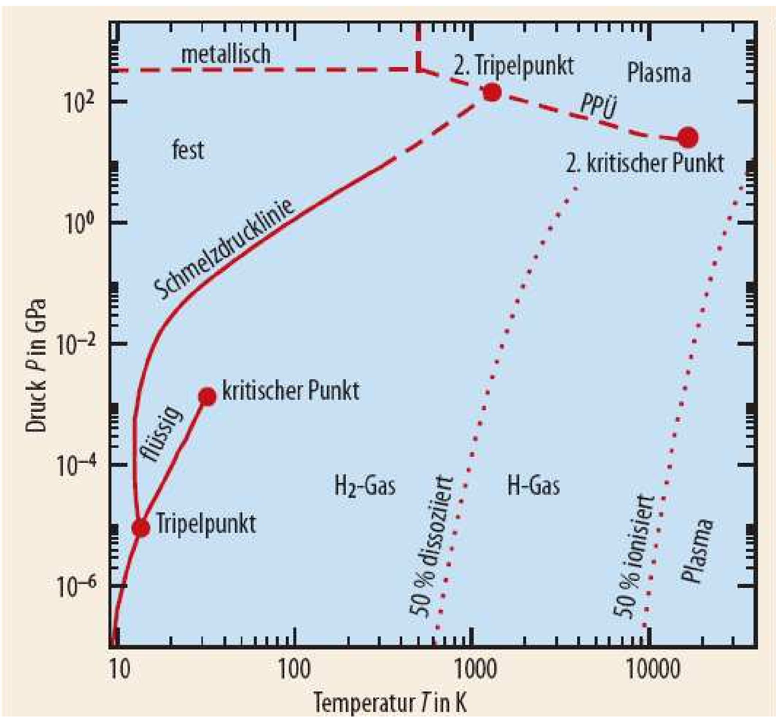
Good fluid: low viscosity



$$F_x = \eta A \frac{\partial v_x}{\partial y}$$

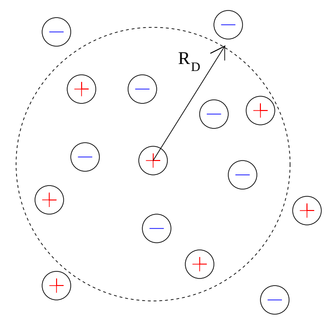
Transitions without change of symmetry: Gas-Plasma

Phase diagram of hydrogen



Plasma Effects

Debye screening



$$V(r) = -\frac{e}{r} e^{-m_D r}$$

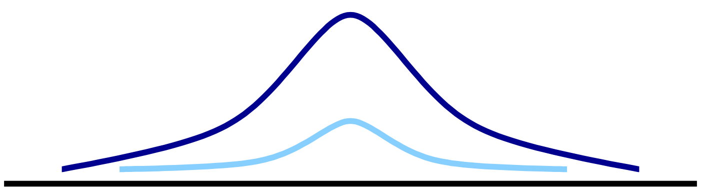
$$m_D^2 = \frac{4\pi e^2 n}{kT}$$

Plasma oscillations

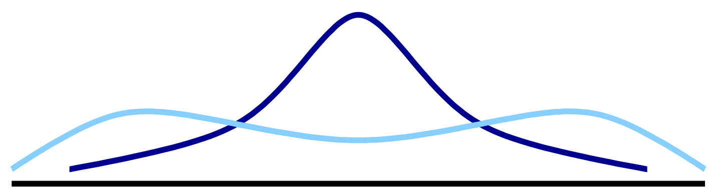
$$\omega_{pl} = \frac{4\pi e^2 n}{m}$$

Fluids: Gases, Liquids, Plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



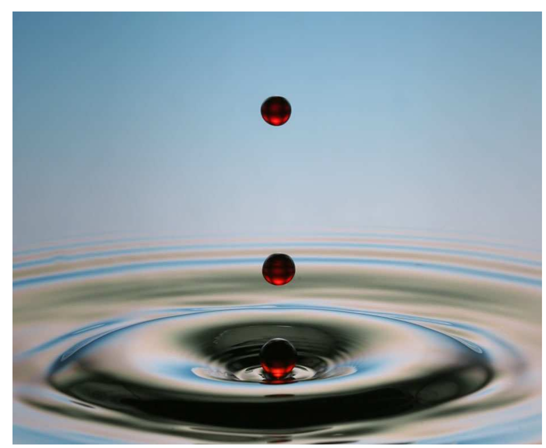
$\tau \sim \tau_{micro}$



$\tau \sim \lambda^{-1}$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Example: Simple Fluid

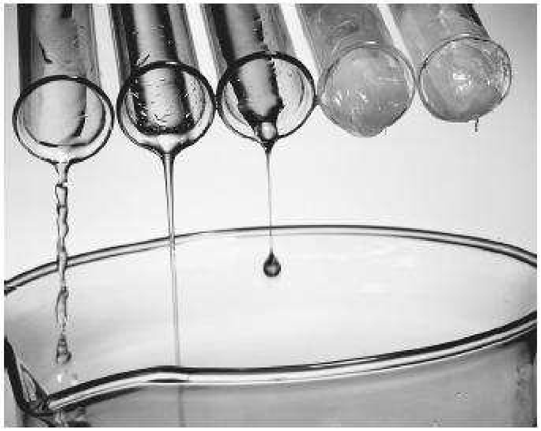
Conservation laws: mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\,\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$

[Euler/Navier-Stokes equation]



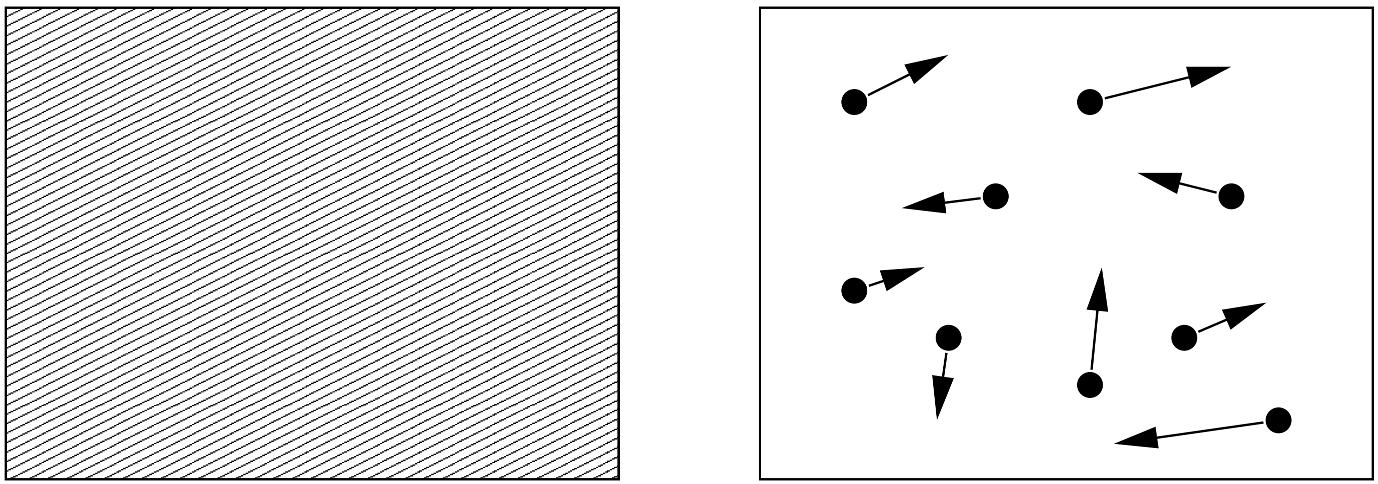
Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + \ldots$$

reactive dissipative

Weakly Coupled Fluids: Kinetics

Weakly coupled fluid $\equiv$ Collection of Quasi-Particles



$l_{mfp} \gg l_{pp}$ and $E \gg \Gamma$

Introduce distribution function $f_p(x,t)$

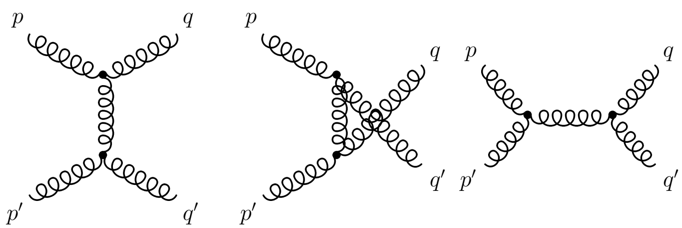
$$N = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E_p} f_p \qquad T_{ij} = \int \frac{d^3p}{(2\pi)^3} \frac{p_i p_j}{2E_p} f_p$$

Transport from Kinetics

Boltzmann equation

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = C[f_p]$$

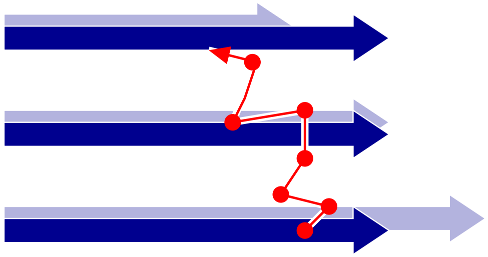
Collision term $C[f_p] = C_{gain} - C_{loss}$



Linearized theory (Chapman-Enskog): $f_p = f_p^0(1 + \chi_p/T)$

suitable for transport coefficients

shear viscosity $\chi_p = g_p p_x p_y \partial_x v_y$



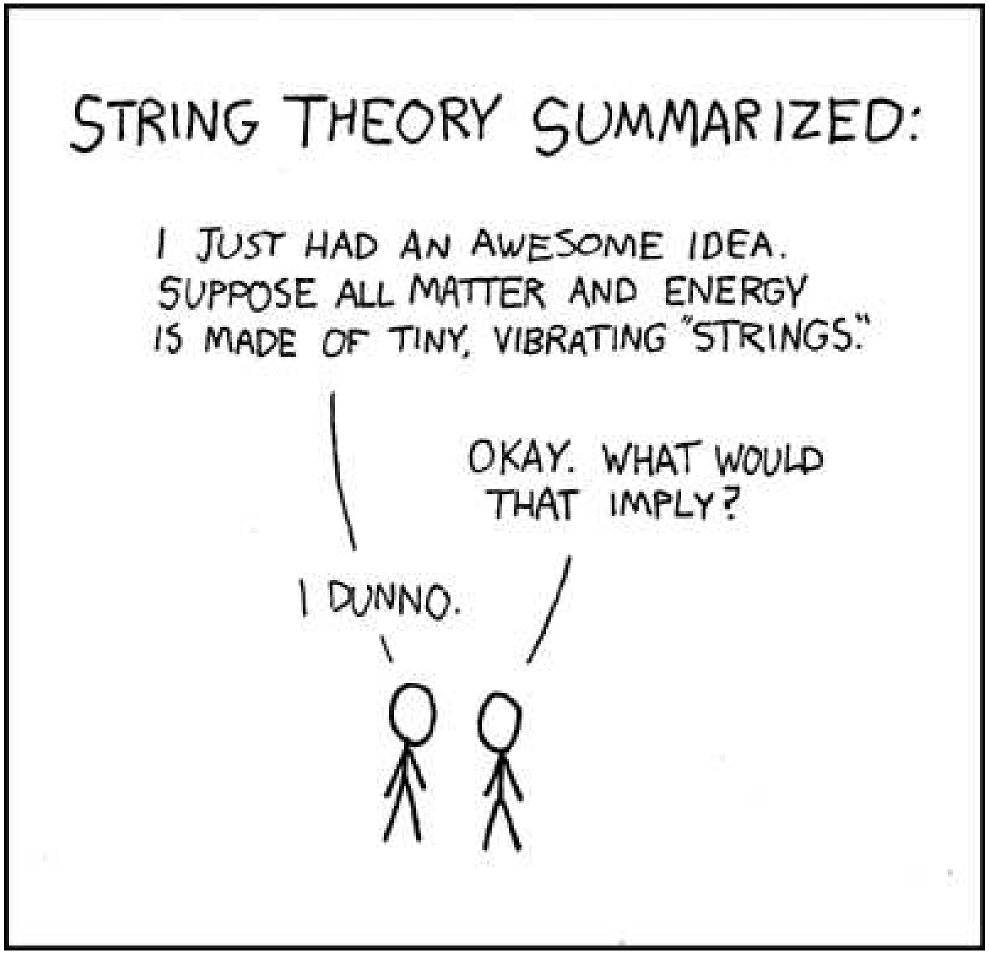
Effective Theories for Fluids (Here: Weak Coupling QCD)

$$\mathcal{L} = \bar{q}_f (i \not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p = C[f_p] \qquad (\omega < T)$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0 \qquad (\omega < g^4 T)$$

And now for something completely different . . .



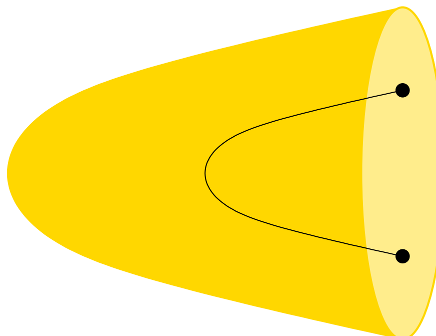
Gauge Theory at Strong Coupling: Holographic Duals

The AdS/CFT duality relates

large N_c (Conformal) gauge theory in 4 dimensions	$\Leftrightarrow$	string theory on 5 dimensional Anti-de Sitter space $\times S_5$
correlation fcts of gauge invariant operators	$\Leftrightarrow$	boundary correlation fcts of AdS fields

$$\langle \exp \int dx \; \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



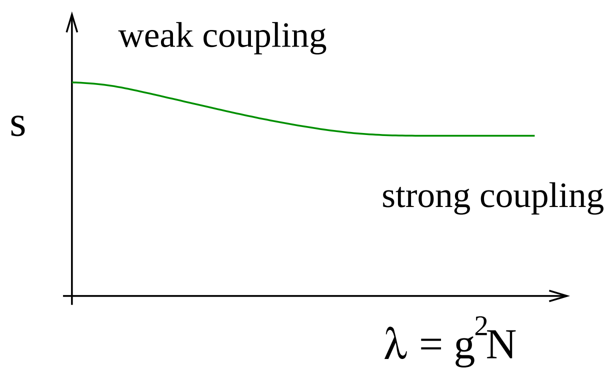
The correspondence is simplest at strong coupling $g^2 N_c$

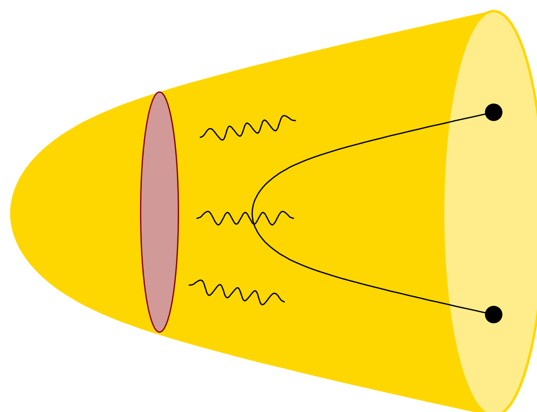
strongly coupled gauge theory $\Leftrightarrow$ classical string theory

Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv$ AdS_5 black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon





$$s(\lambda \to \infty) = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s(\lambda = 0)$$

Gubser and Klebanov

Holographic Duals: Transport Properties

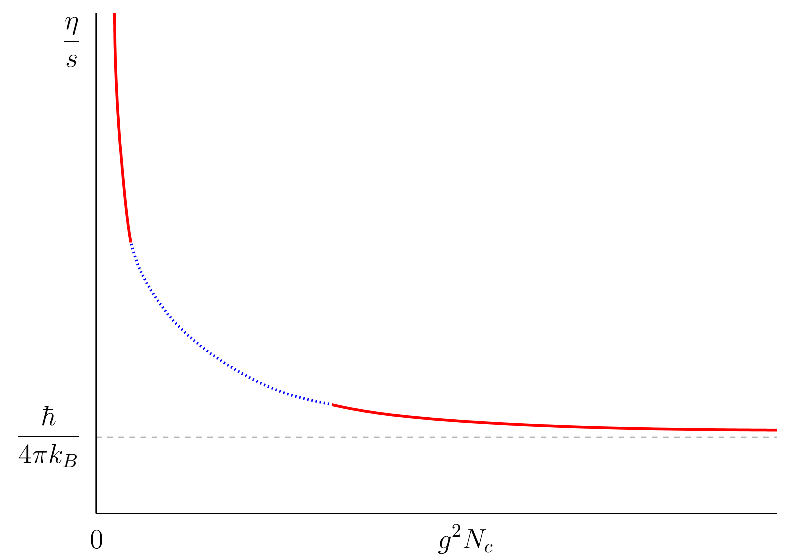
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

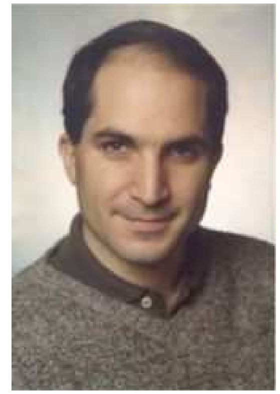
$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets



Strong coupling limit universal? Provides lower bound for all theories?

Effective Theories (Strong coupling)

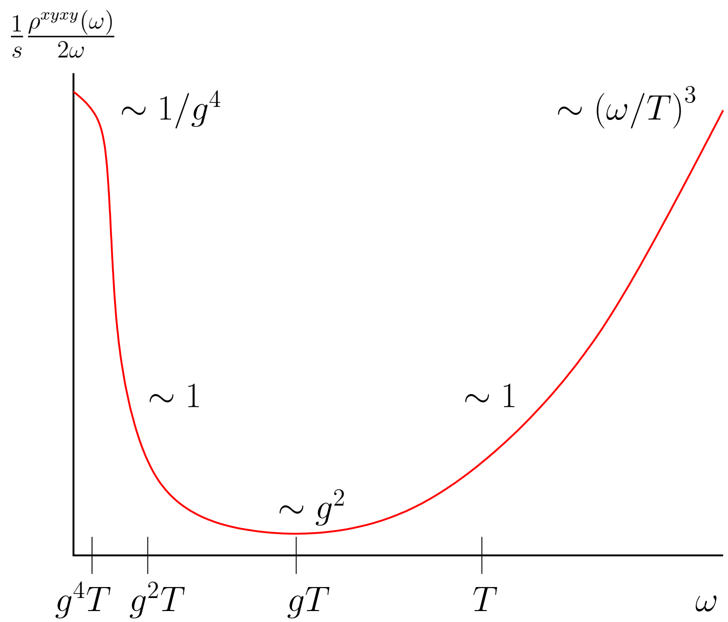


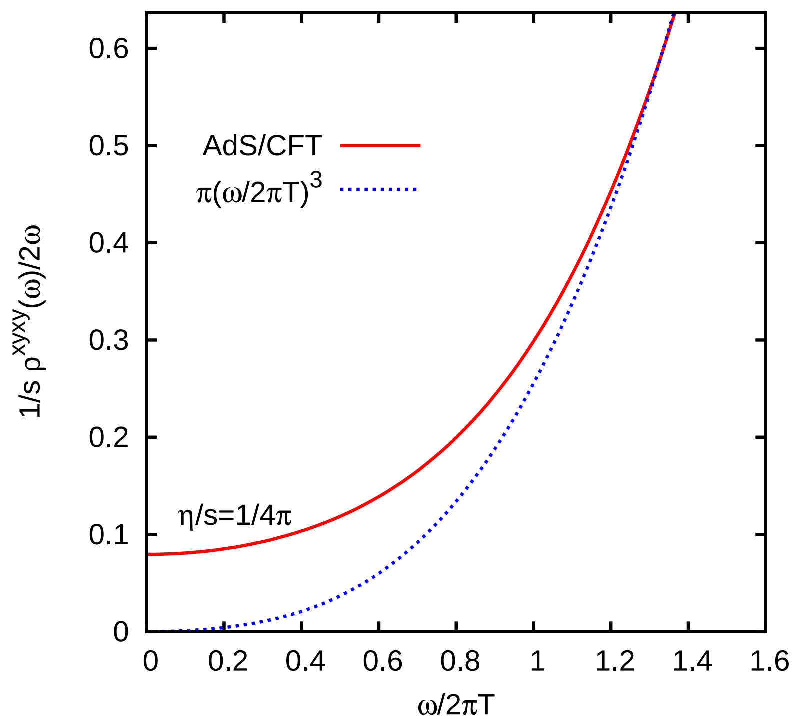
$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \Leftrightarrow S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

Kinetics vs No-Kinetics

Spectral function $\rho(\omega) = \mathrm{Im} G_R(\omega, 0)$ associated with T_{xy}





weak coupling QCD — strong coupling AdS/CFT

transport peak vs no transport peak

Summary (Theory)

Lattice QCD: single chiral and deconfinement crossover transition

$T_c \sim 185$ MeV, $\epsilon_{cr} \sim 1.5\,\mathrm{GeV/fm}^3$

Weakly coupled Quark Gluon Plasma

Quark and gluon quasi-particles, $\gamma \ll \omega$

Thermodynamics: Stefan-Boltzmann gas

Transport: long equilibration times, $\eta/s \simeq 1/\alpha_s^2 \gg 1$

Strongly coupled plasma

No quasi-particles, no kinetics, only hydrodynamics

Thermodynamics: Stefan-Boltzmann law

Transport: fast equilibration, $\eta/s \simeq 1/(4\pi) < 1$