The Phases of QCD

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Motivation

Different phases of QCD occur in the universe

Neutron Stars, Big Bang

Exploring the phase diagram is important to understanding the phase that we happen to live in

Structure of hadrons is determined by the structure of the vacuum

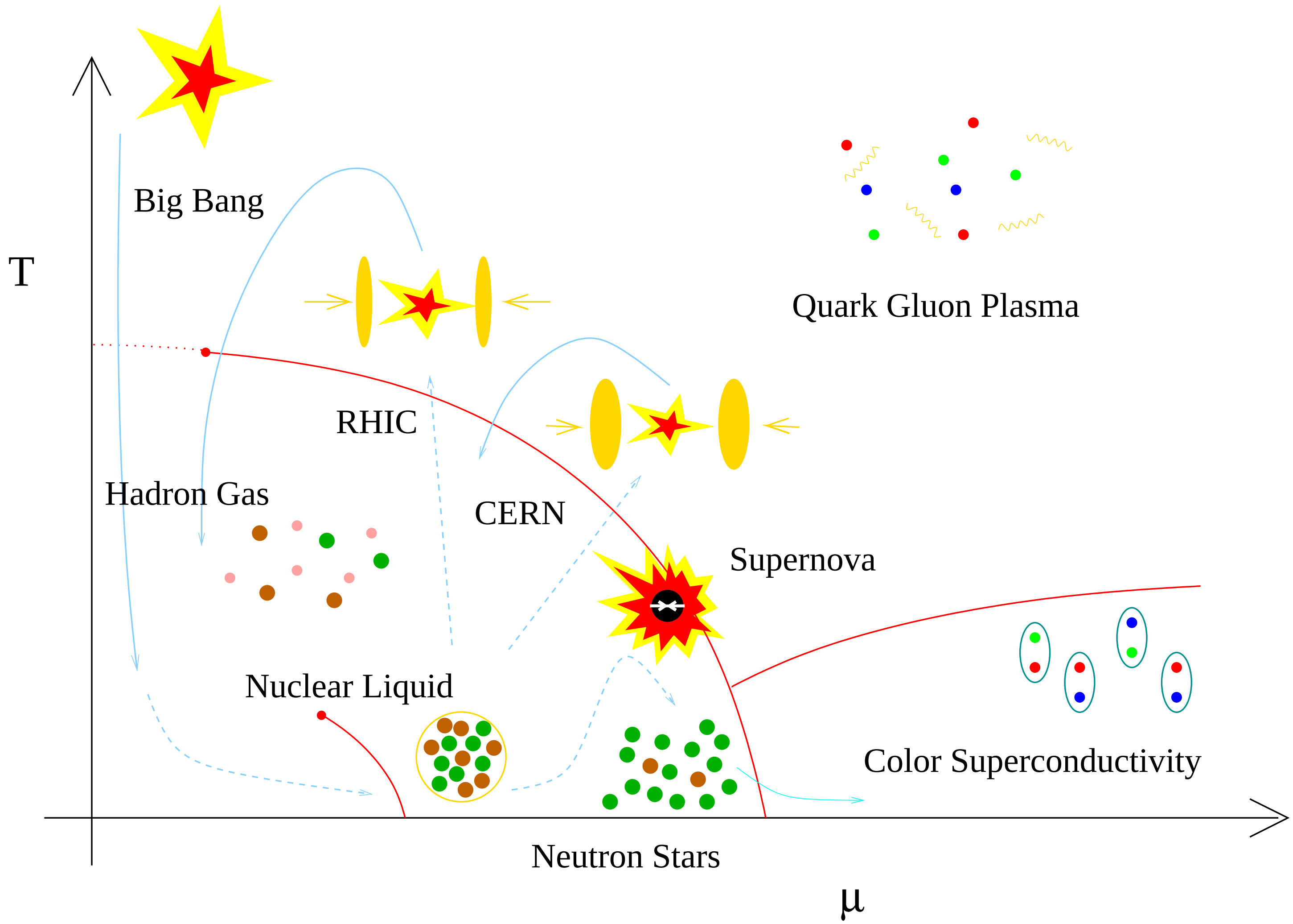
Need to understand how vacuum can be modified

QCD simplifies in extreme environments

Study QCD matter in a regime where quarks and gluons

are the correct degrees of freedom

QCD Phase Diagram



Plan

1. QCD and Symmetries
2. The High Temperature Phase: Theory
3. Exploring QCD at High Temperature: Experiment
4. QCD at Low Density: Nuclear Matter
5. QCD at High Density: Quark Matter
6. Matter at Finite Density: From the Lab to the Stars

Quantumchromodynamics

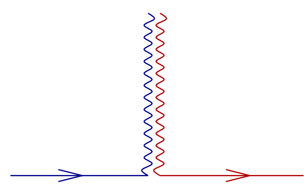
Elementary fields: **Quarks** **Gluons**

$$(q_\alpha)^a_f \quad \begin{cases} \text{color} & a = 1,\ldots,3 \\ \text{spin} & \alpha = 1,2 \\ \text{flavor} & f = u,d,s,c,b,t \end{cases}$$

$$A^a_\mu \quad \begin{cases} \text{color} & a = 1,\ldots,8 \\ \text{spin} & \epsilon^\pm_\mu \end{cases}$$

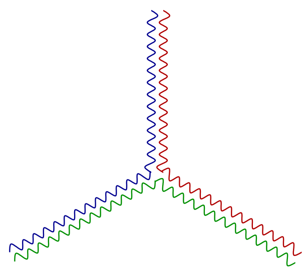
Dynamics: non-abelian gauge theory

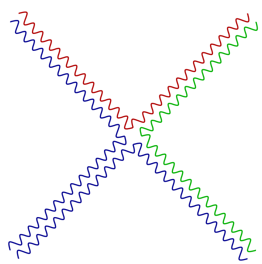
$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$



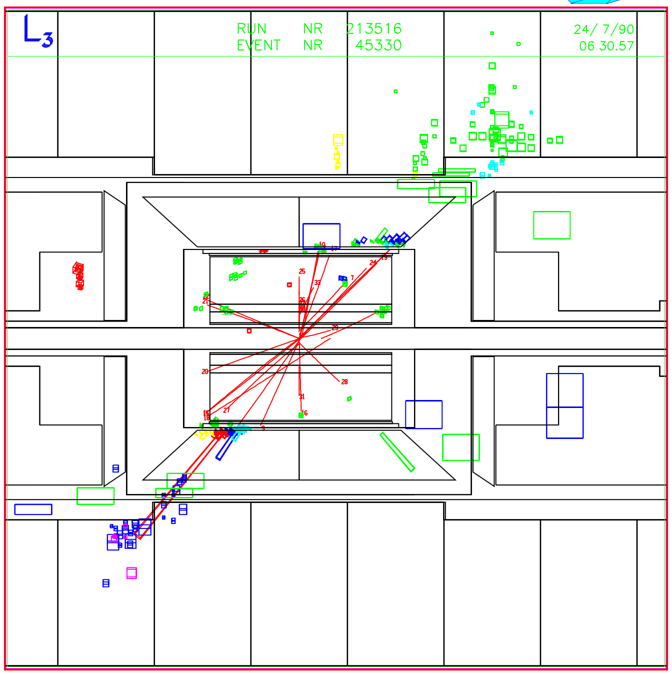
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

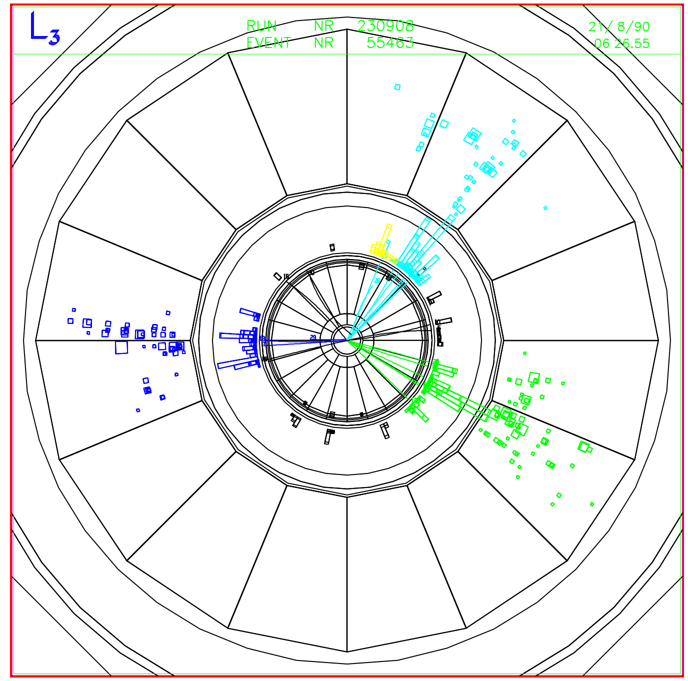
$$i\not{D}q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu t^a\right) q$$

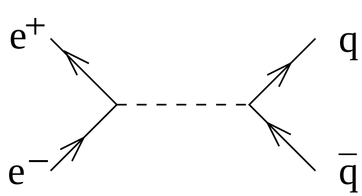


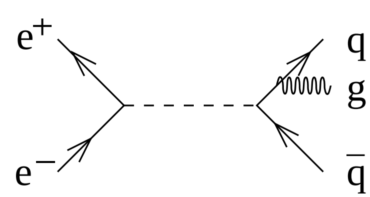


“Seeing” Quarks and Gluons

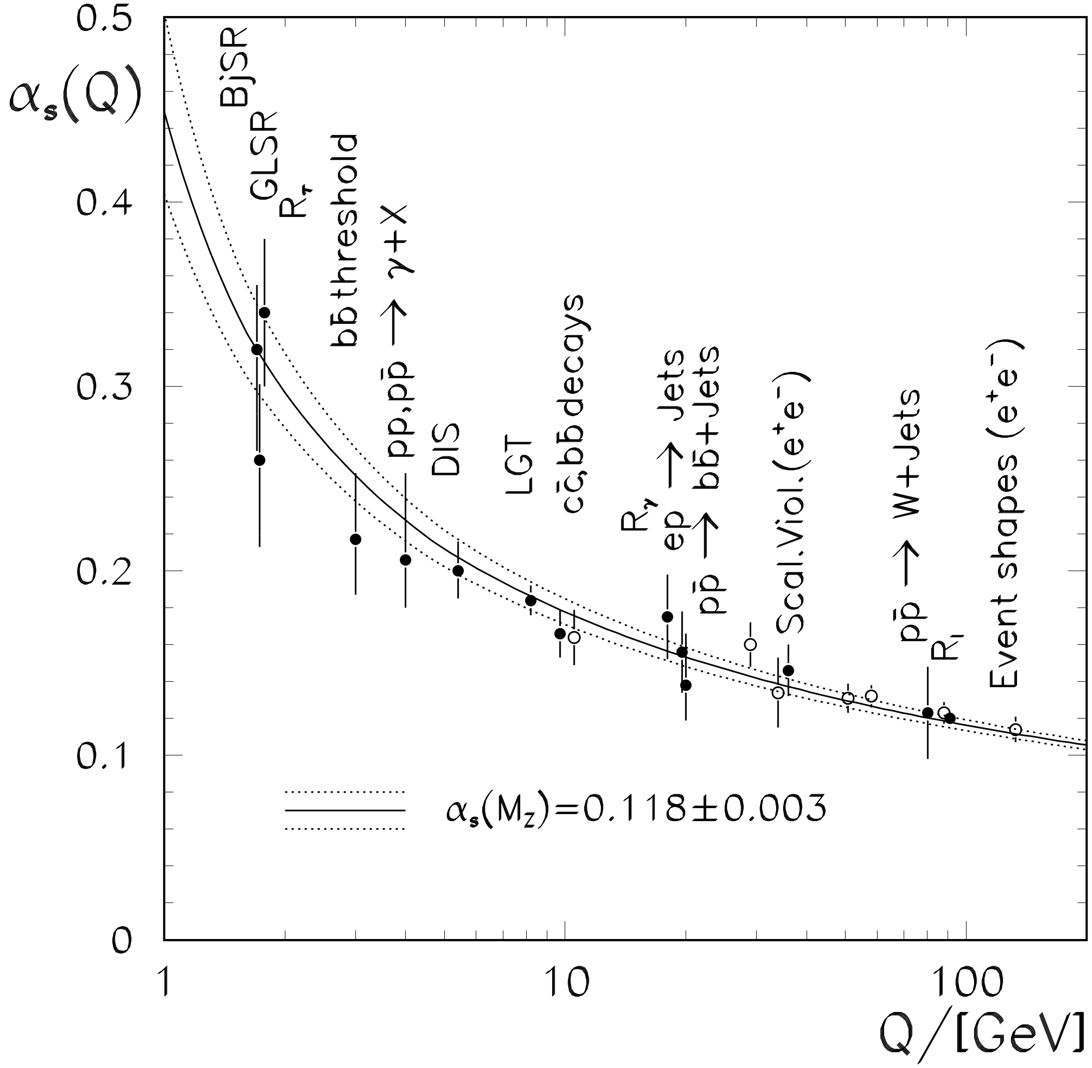








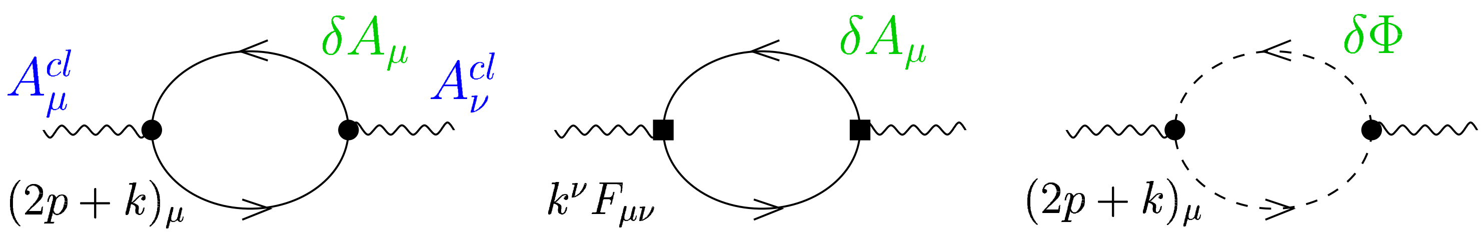
Running Coupling Constant



Asymptotic Freedom

Study modification of classical field by quantum fluctuations

$$A_\mu = A_\mu^{cl} + \delta A_\mu \qquad \frac{1}{g^2}F^2 \to \left(\frac{1}{g^2} + c\log\left(\frac{k^2}{\mu^2}\right)\right)F^2$$



dielectric $\epsilon > 1$ paramagnetic $\mu > 1$ dielectric $\epsilon > 1$

$$\mu\epsilon = 1 \;\Rightarrow\; \epsilon < 1$$

$$\beta(g) = \frac{g^3}{(4\pi)^2}\left\{\left[\frac{1}{3} - 4\right]N_c + \frac{2}{3}N_f\right\}$$

About Units

QCD Lite* is a parameter free theory

The lagrangian has a coupling constant, g, but no scale.

After renormalization g becomes scale dependent

g is traded for a scale parameter Λ

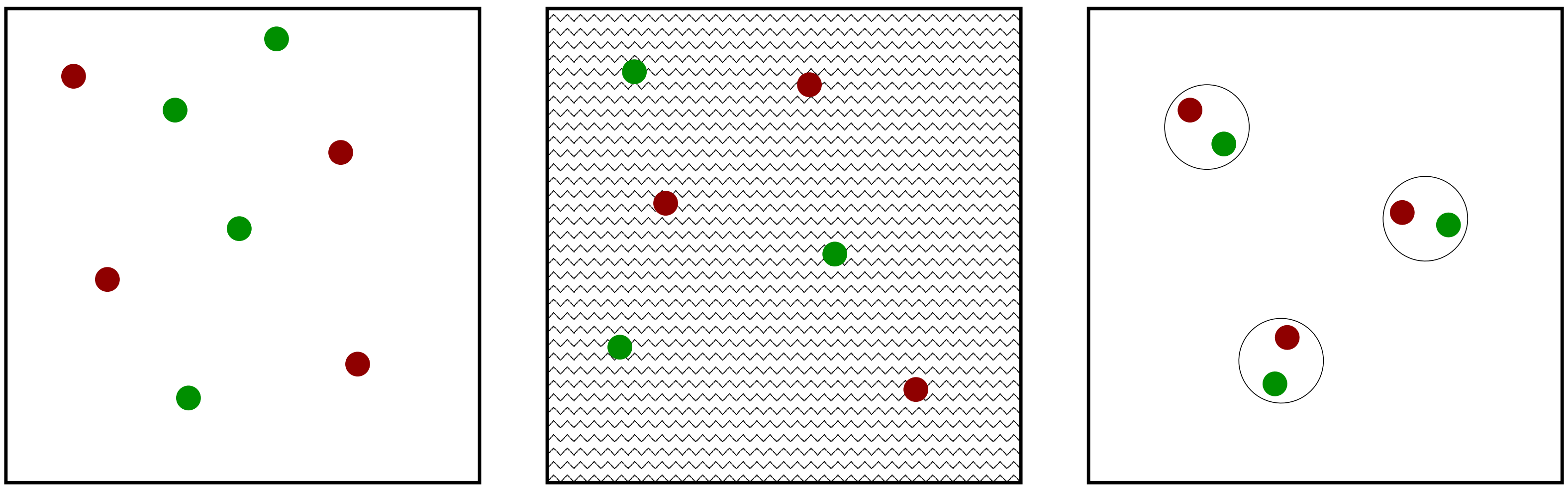
Λ is the only scale, the QCD "standard kilogram"

$$\Lambda_{QCD} \simeq 200\,\mathrm{MeV} \simeq 1\,\mathrm{fm}^{-1}$$

*QCD Lite is QCD in the limit $m_q \to 0$, $m_Q \to \infty$

Phases of Gauge Theories

Coulomb | Higgs | Confinement



$$V(r) \sim \frac{e^2}{r} \qquad V(r) \sim \frac{e^{-mr}}{r} \qquad V(r) \sim kr$$

Standard Model: $U(1) \times SU(2) \times SU(3)$

Phases of Matter

phase	order param	broken symmetry	rigidity phenomenon	Goldstone boson
crystal	ρ_k	translations	rigid	phonon
magnet	$\vec{M}$	rotations	magnetization	magnon
superfluid	$\langle \Phi \rangle$	particle number	supercurrent	phonon
supercond.	$\langle \psi\psi \rangle$	gauge symmetry	supercurrent	none (Higgs)

Gauge Symmetry

Local gauge symmetry $U(x) \in SU(3)_c$

$$\begin{aligned} \psi &\to U\psi & D_\mu\psi \to U D_\mu \psi \\ A_\mu &\to U A_\mu U^\dagger + iU\partial_\mu U^\dagger & F_{\mu\nu} \to U F_{\mu\nu} U^\dagger \end{aligned}$$

Gauge "symmetries" cannot be broken

Gauge "symmetries" can be realized in different modes

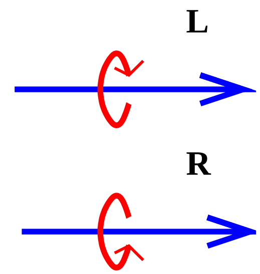
	Coulomb	Higgs	confined
d.o.f:	2 (massless)	3 (massive)	3 (massive)

Distinction between Higgs and confinement phase not always sharp

Chiral Symmetry

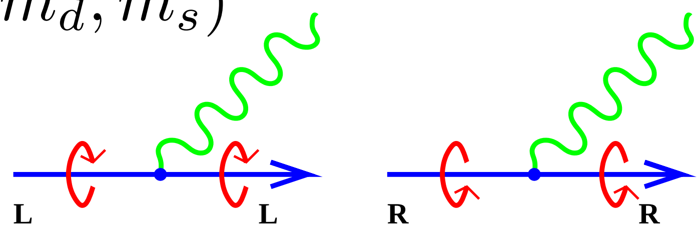
Define left and right handed fields

$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$

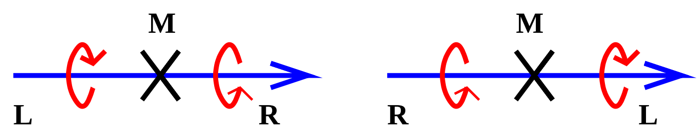


Fermionic lagrangian, $M = \mathrm{diag}(m_u, m_d, m_s)$

$$\mathcal{L} \;=\; \bar{\psi}_L(i\not{D})\psi_L + \bar{\psi}_R(i\not{D})\psi_R$$



$$+\,\bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$



$M = 0$: Chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

$$\psi_L \to L\psi_L, \qquad\qquad \psi_R \to R\psi_R$$

Chiral Symmetry Breaking

Chiral symmetry implies massless, degenerate fermions

$$m_N^{(1/2)^+} = 935\,\mathrm{MeV} \qquad\qquad m_{N^*}(1/2)^- = 1535\,\mathrm{MeV}$$

Chiral symmetry is spontaneously broken

$$\langle \bar\psi_L^f \psi_R^g + \bar\psi_L^f \psi_R^g \rangle \simeq -(230\,\mathrm{MeV})^3\ \delta^{fg}$$

$$SU(3)_L \times SU(3)_R \to SU(3)_V \qquad\qquad (G \to H)$$

Consequences: dynamical mass generation $m_Q = 300\,\mathrm{MeV} \gg m_q$

$$m_N = 890\,\mathrm{MeV} + 45\,\mathrm{MeV} \qquad\qquad (\mathrm{QCD},\ 95\%) + (\mathrm{Higgs},\ 5\%)$$

Goldstone Bosons: Consider broken generator Q_5^a

$$[H, Q_5^a] = 0 \qquad Q_5^a|0\rangle = |\pi^a\rangle \qquad H|\pi^a\rangle = HQ_5^a|0\rangle = Q_5^a H|0\rangle = 0$$

Low energy effective theory for the Goldstone modes

Step 1: Parameterize G/H =pseudoscalar GB's

$$U(x): \qquad U \to LUR^\dagger \qquad (L, R) \in SU(3)_L \times SU(3)_R$$

Vacuum $U^{fg} = \delta^{fg}$. Massless fluctuations (G/H)

$$U(x) = \exp(i\phi^a\lambda^a/f_\pi) \qquad \phi^a = (\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta)$$

Step 2: Write most general G invariant effective lagrangian

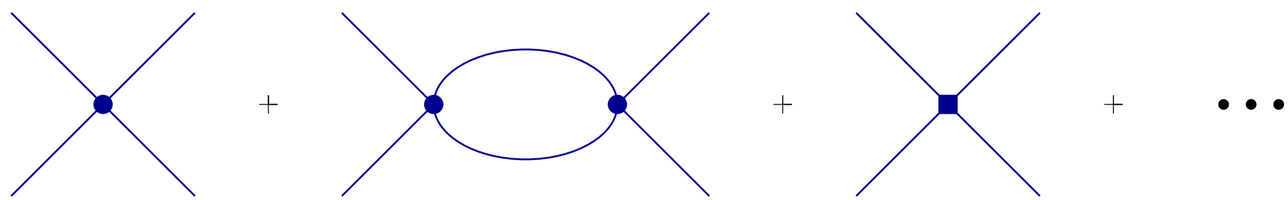
$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \ldots$$

Non-linear sigma model

Expand lagrangian ($SU(2)$ sector)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi^a)^2 + \frac{1}{6f_\pi^2}\left[(\phi^a \partial_\mu \phi^a)^2 - (\phi^a)^2(\partial_\mu \phi^b)^2\right] + O\left(\frac{\partial^4}{f_\pi^4}\right)$$

Step 3: Low energy expansion (power counting)

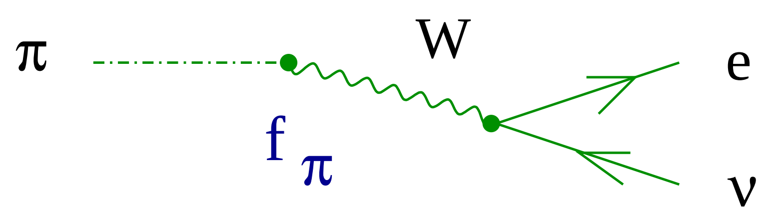


$$T_{\pi\pi} \sim k_{1,2}^2/f_\pi^2 + k_{1,2}^4/f_\pi^4 + \dots$$

Relation to f_π: Couple weak gauge fields

$$\partial_\mu U \to (\partial_\mu + igW_\mu^\pm \tau^\mp)U$$

$$\mathcal{L} = gf_\pi W_\mu^\pm \partial^\mu \pi^\mp$$



Quark Masses

Non-zero quark masses: $\mathcal{L} = \bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_R$

$$M \to L M R^\dagger \qquad\qquad \text{spurion field } M$$

Chiral lagrangian at leading order in M

$$\mathcal{L} = B\mathrm{Tr}[MU] + h.c.$$

Mass matrix $M = \mathrm{diag}(m_u, m_d m_s)$. Minimize effective potential

$$U_{vac} = 1, \qquad E_{vac} = -B\mathrm{Tr}[M] \qquad \langle\bar{\psi}\psi\rangle = -B$$

Expand around U_{vac}: pion mass

$$m_\pi^2 f_\pi^2 = (m_u + m_d)\langle\bar{\psi}\psi\rangle$$

Chiral expansion

$$\mathcal{L} = f_\pi^4 \left(\frac{\partial U}{\Lambda_\chi}\right)^m \left(\frac{m_\pi}{\Lambda_\chi}\right)^n \qquad \Lambda_\chi = 4\pi f_\pi$$

Symmetries of the QCD Vacuum: Summary

Local $SU(3)$ gauge symmetry

confined: $V(r) \sim kr$

Chiral $SU(3)_L \times SU(3)_R$ symmetry

spontaneously broken to $SU(3)_V$

Axial $U(1)_A$ symmetry

$$\text{anomalous}: \qquad \partial_\mu A^0_\mu = \frac{N_f}{16\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

Vectorial $U(1)_B$ symmetry

unbroken: $B = \int d^3x\, \psi^\dagger \psi$ conserved

Notes

QCD with general N_f, N_c (with or without SUSY)

find theories without confinement and/or chiral symmetry breaking

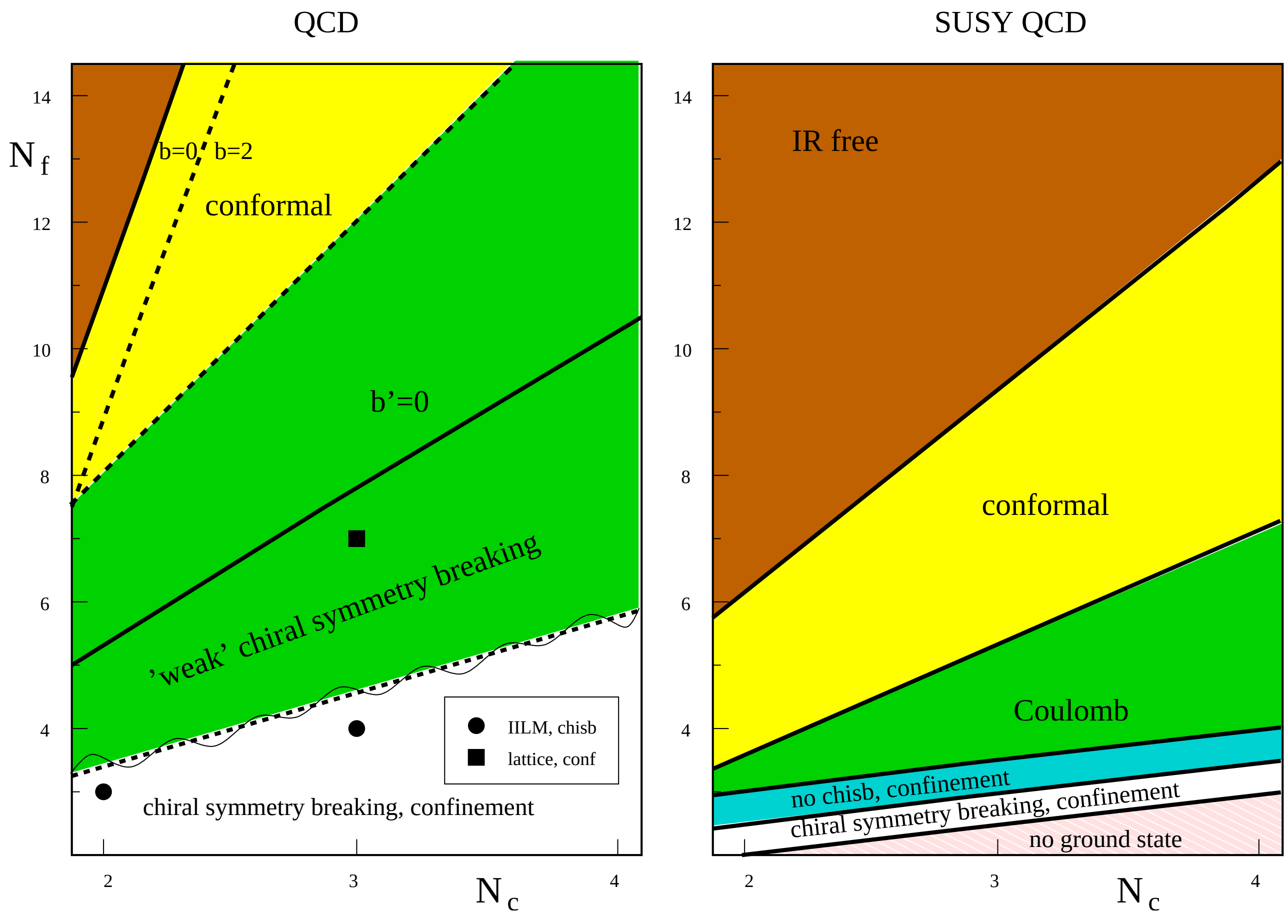
QCD with $N_f = N_c = 3$

confinement implies chiral symmetry breaking

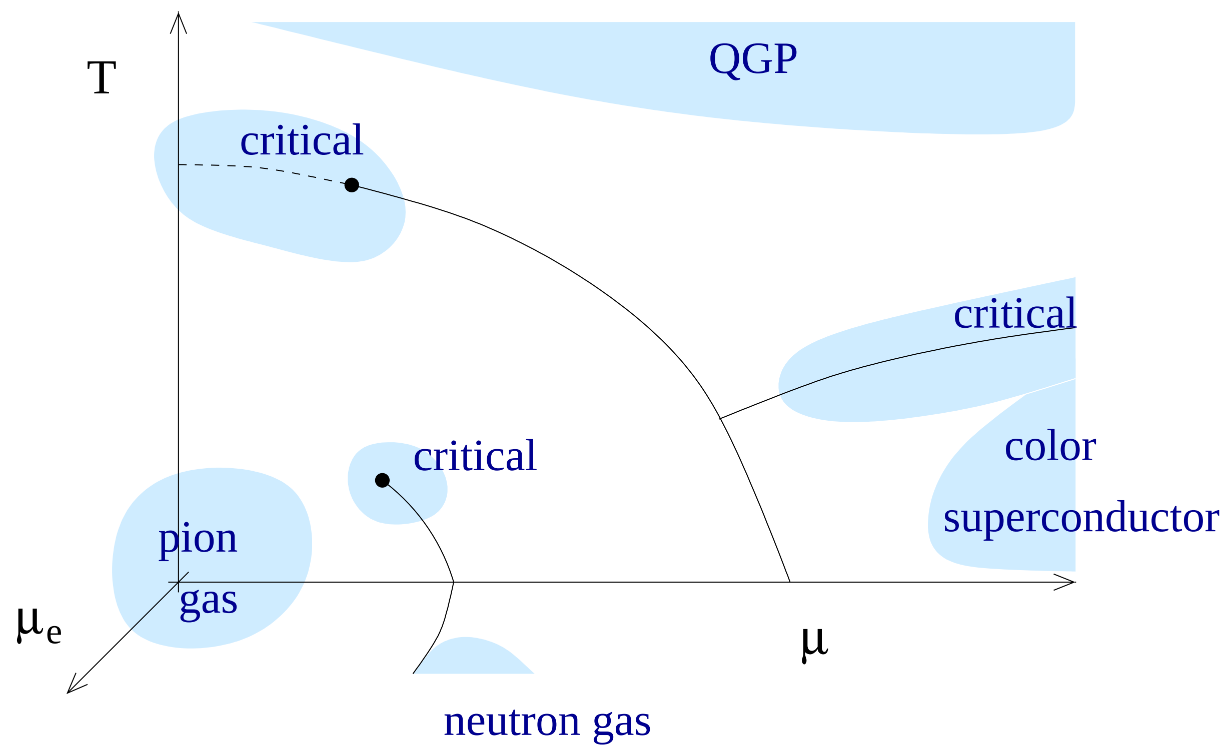
symmetry breaking pattern $SU(3)_L \times SU(3)_R \to SU(3)_V$ unique*

order parameter $\langle \bar{\psi}\psi \rangle \neq 0$

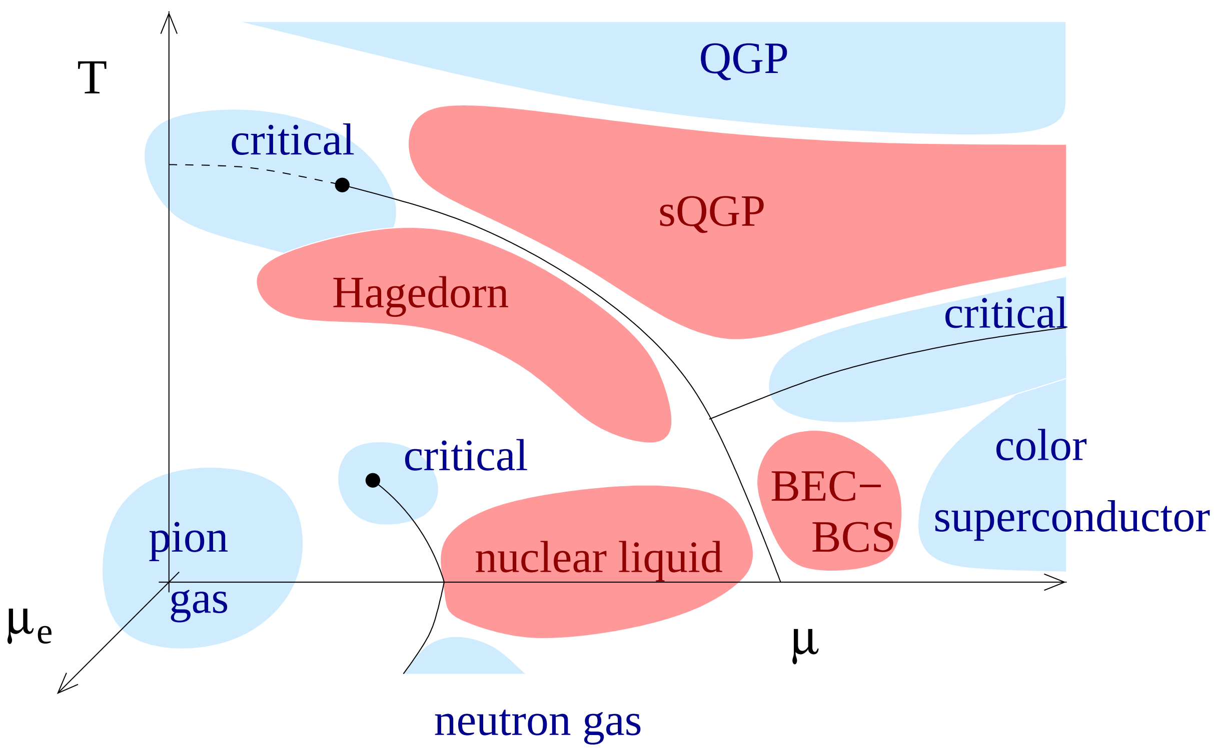
QCD Phase Diagram: N_c and N_f



Approaching the Phase Diagram: Symmetries and Weak Coupling Arguments



Approaching the Phase Diagram: Strongly Correlated Phases



Approaching the Phase Diagram: Experiments and Numerical Simulations