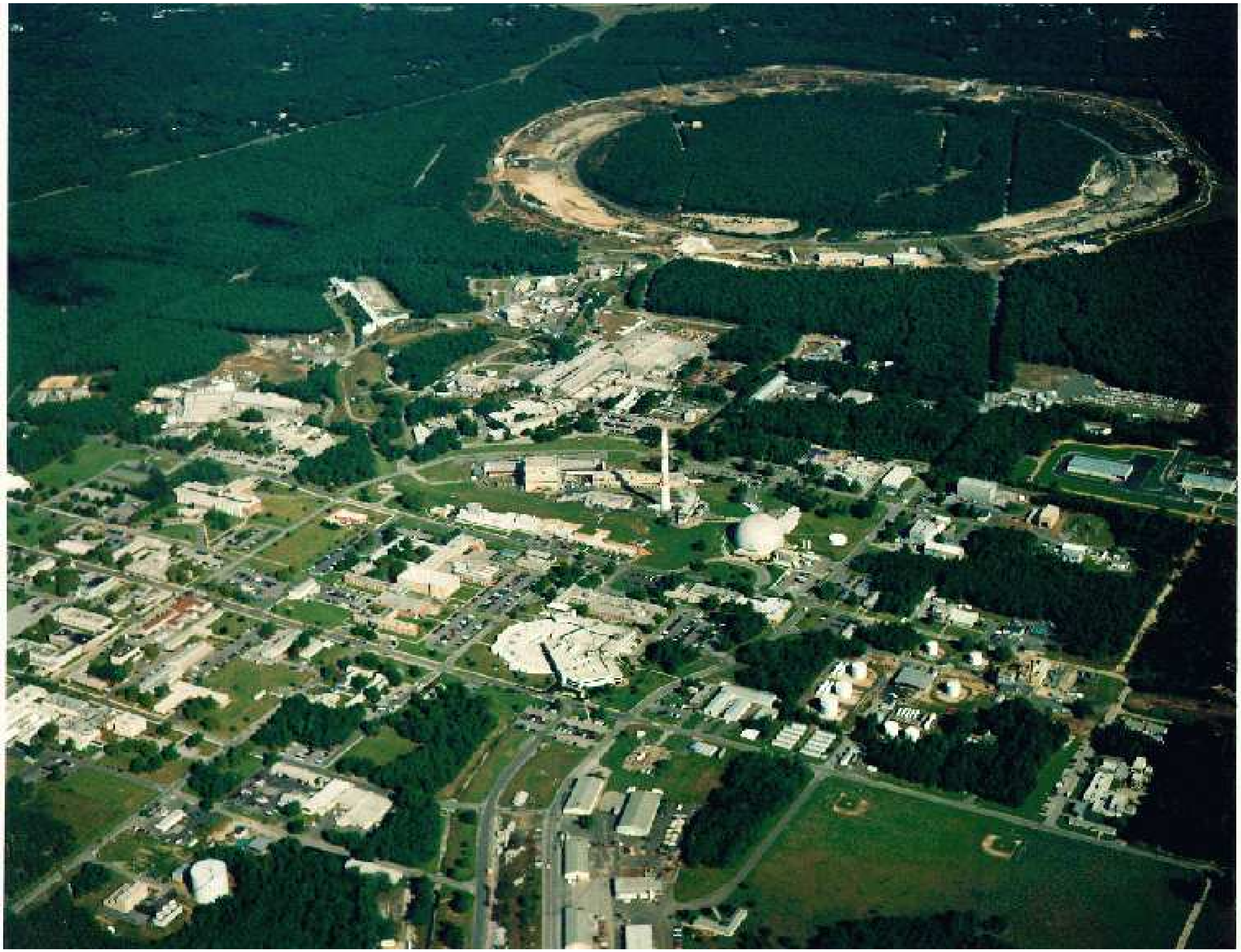
From Trapped Atoms

To Liberated Quarks

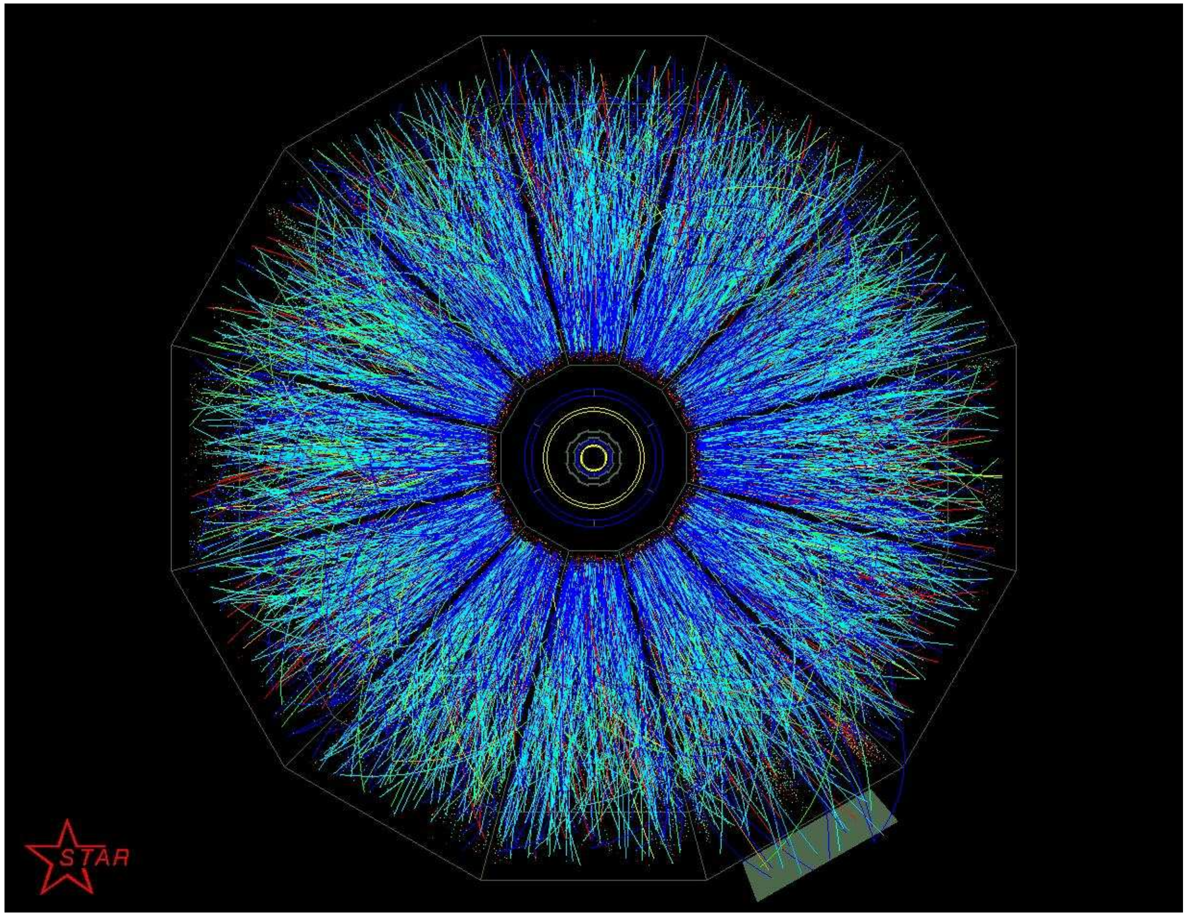
Thomas Schaefer

North Carolina State University

BNL and RHIC



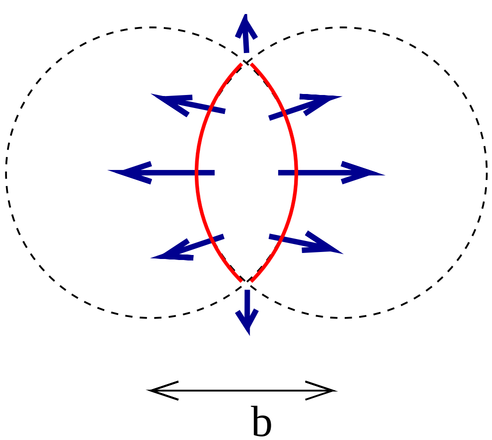
Heavy Ion Collision

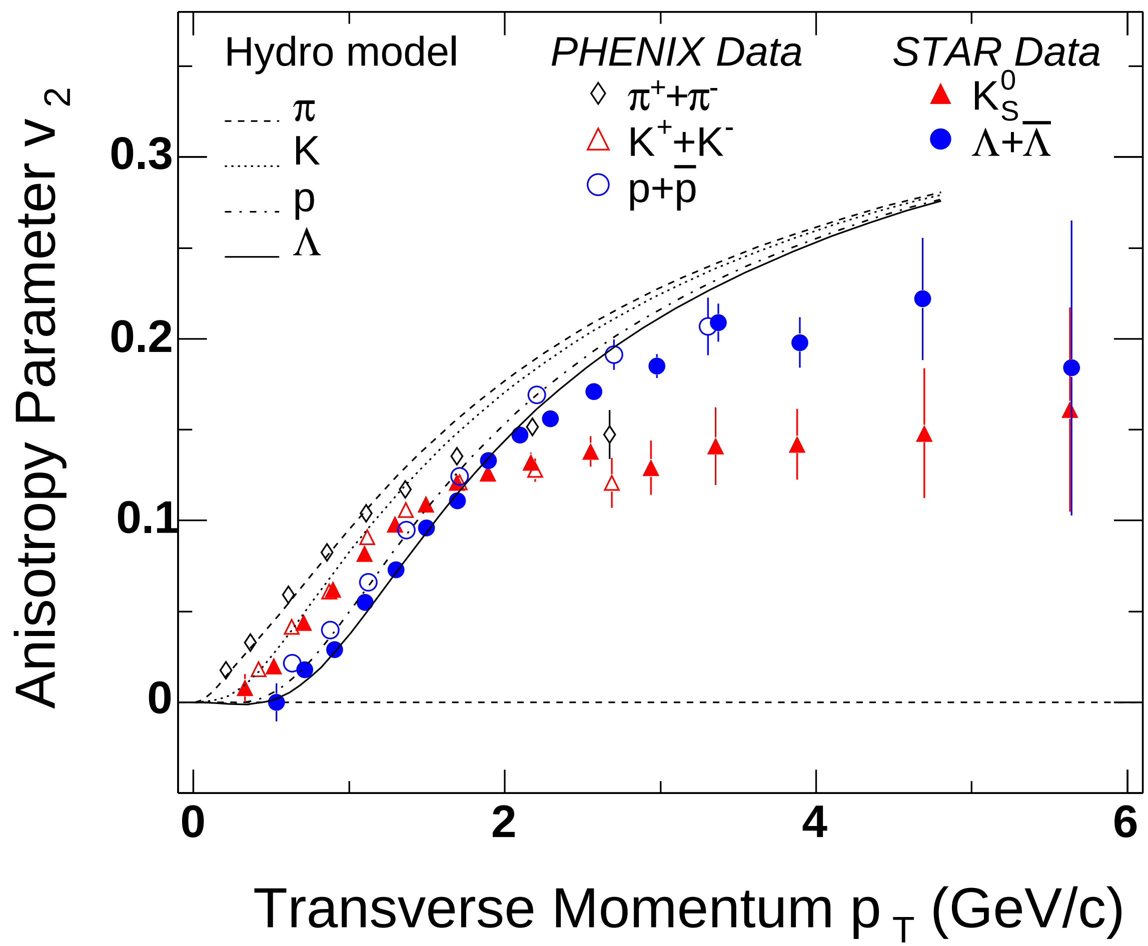


Star TPC

Elliptic Flow

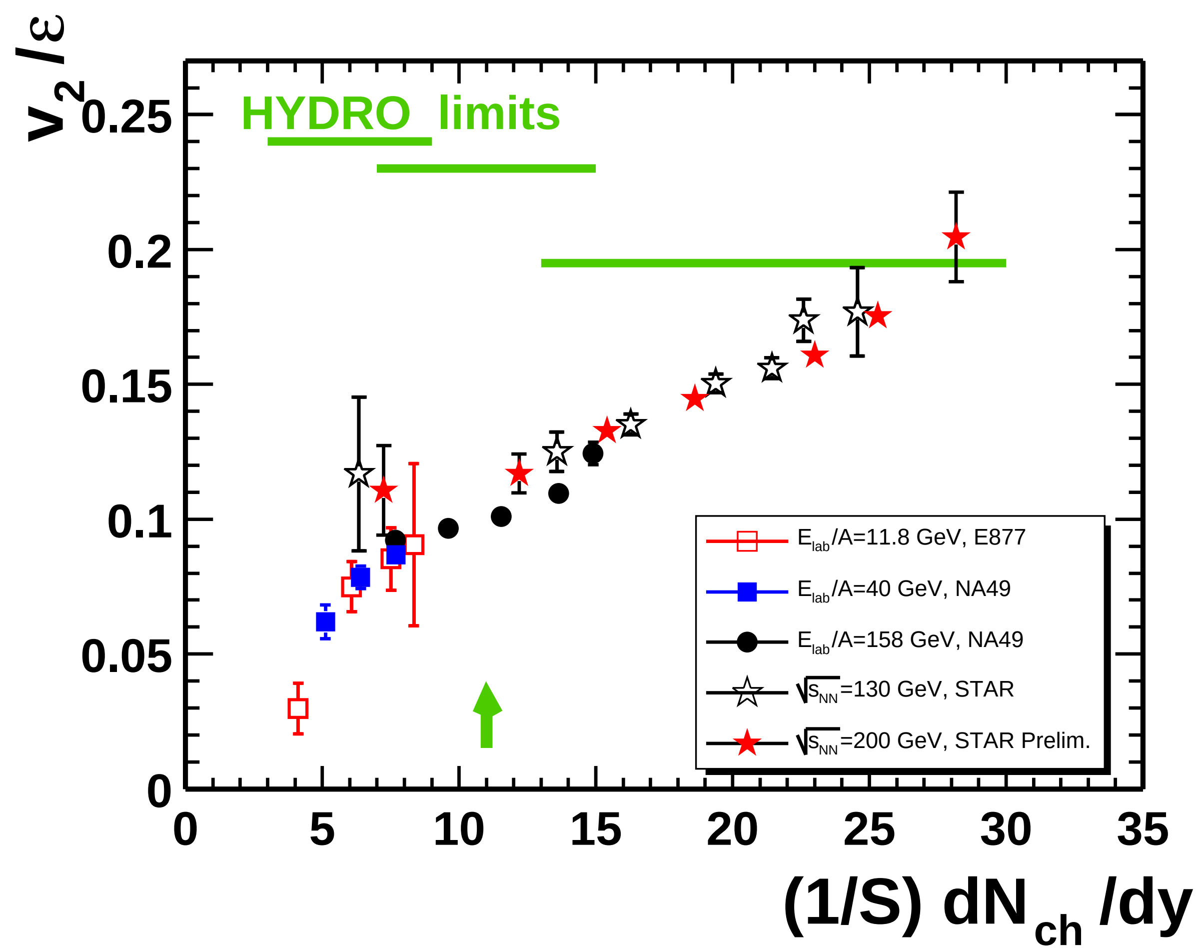
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





source: U. Heinz (2005)

Elliptic Flow II

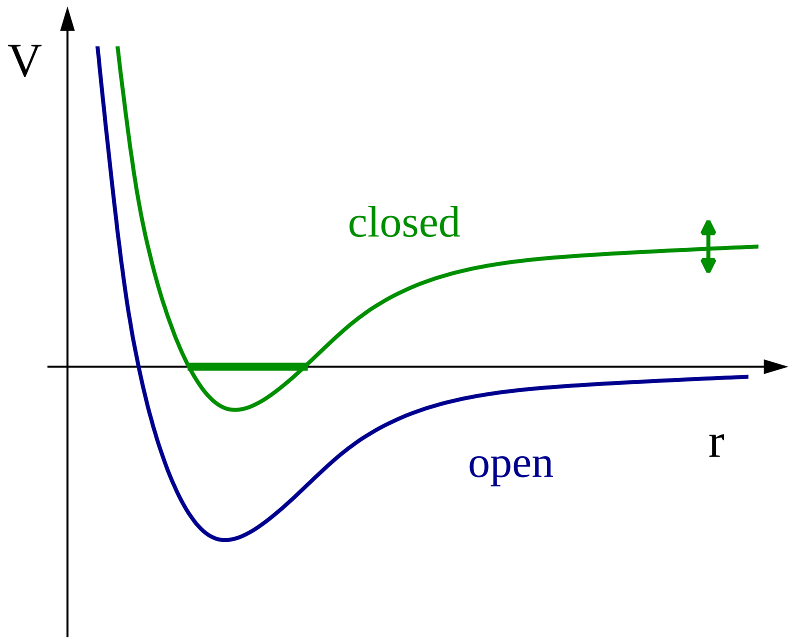


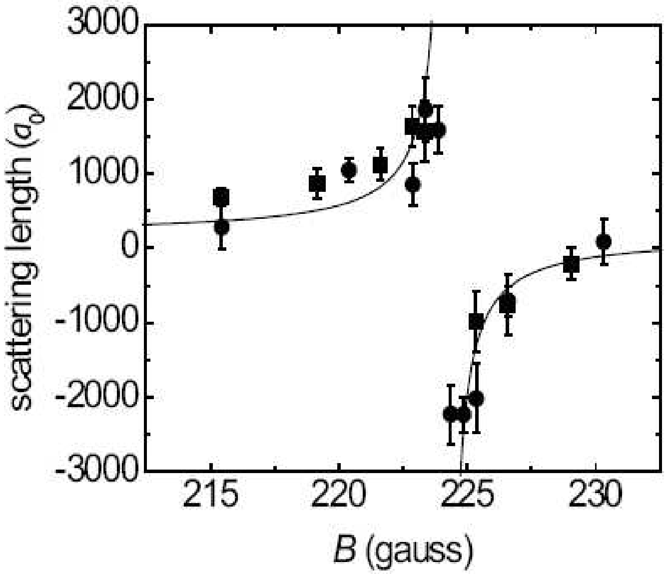
Requires "perfect" fluidity ($\eta/s < 0.1$?)

(s)QGP saturates (conjectured) universal bound $\eta/s = 1/(4\pi)$?

Designer Fluids

Atomic gas with two spin states: "↑" and "↓"





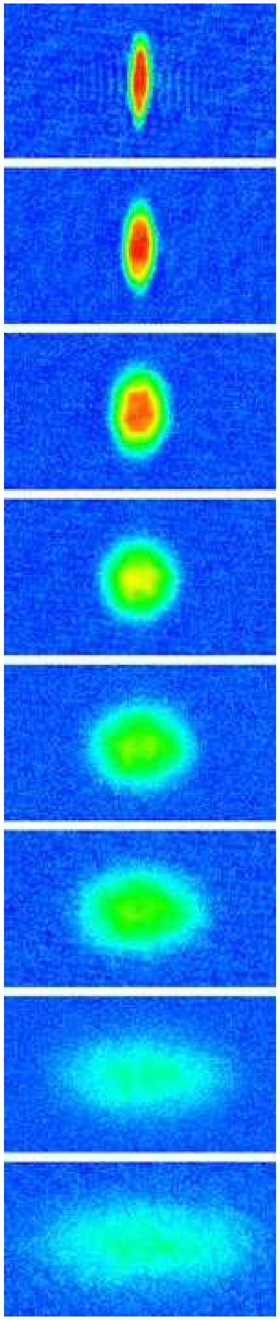
Feshbach resonance

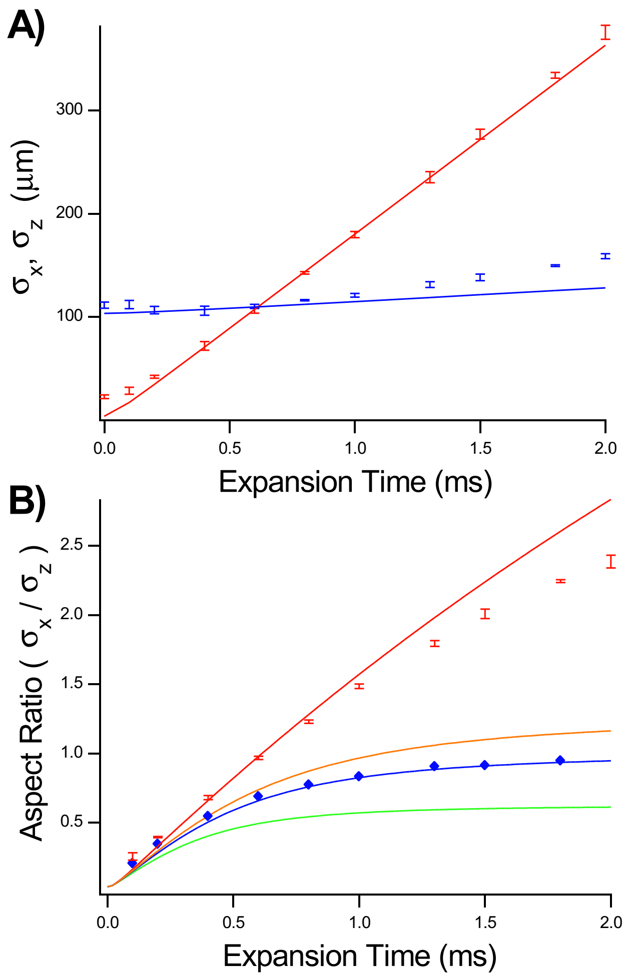
$$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

"Unitarity" limit $a \to \infty$

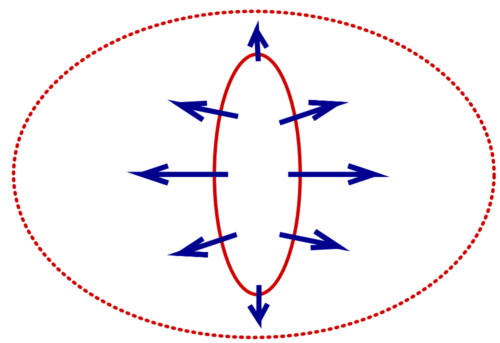
$$\sigma = \frac{4\pi}{k^2}$$

Elliptic Flow

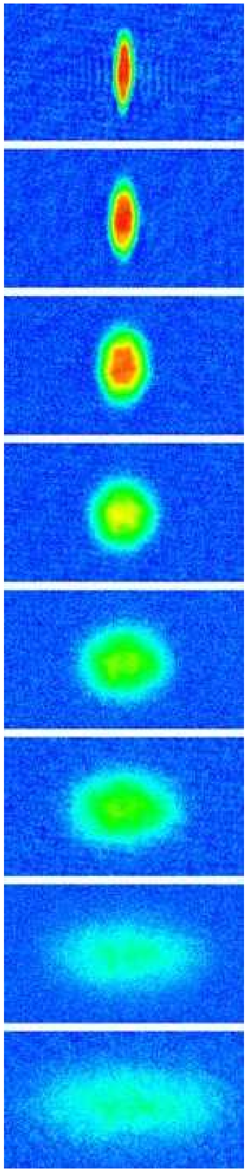


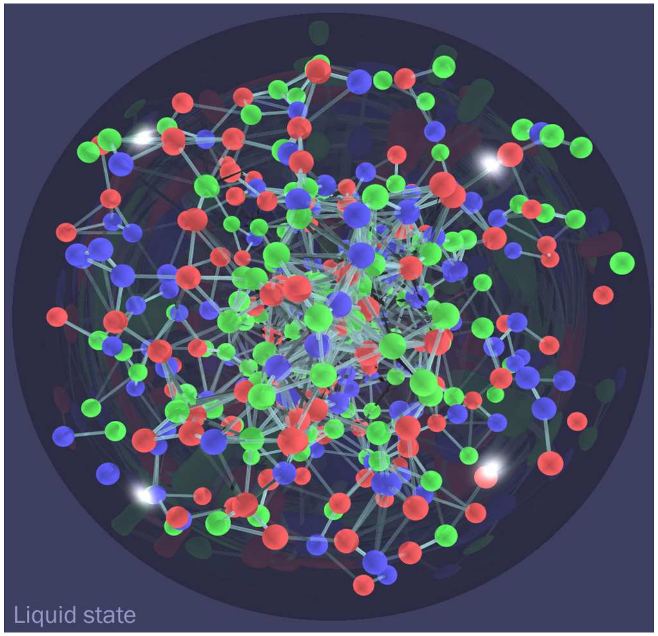


Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

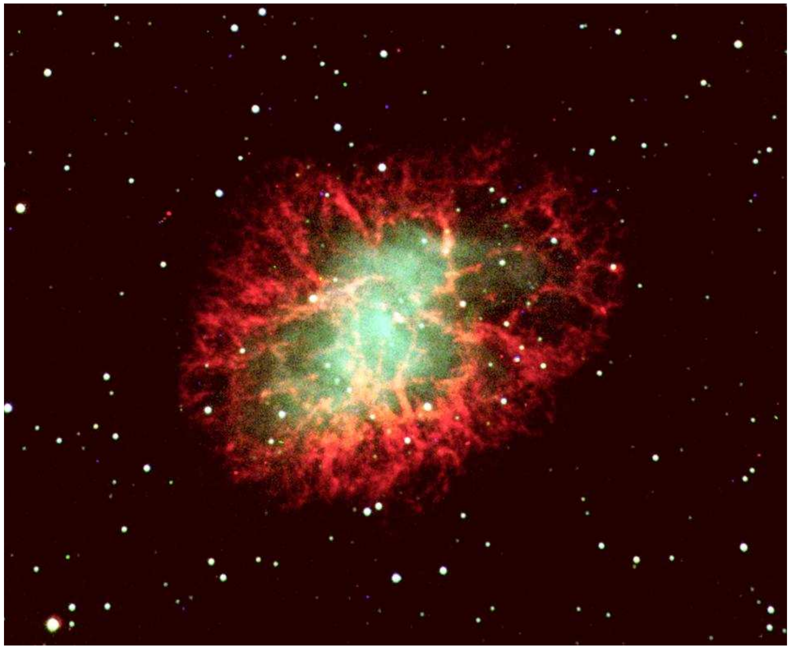


Perfect Liquids





sQGP (T=180 MeV)



Neutron Matter (T=1 MeV)

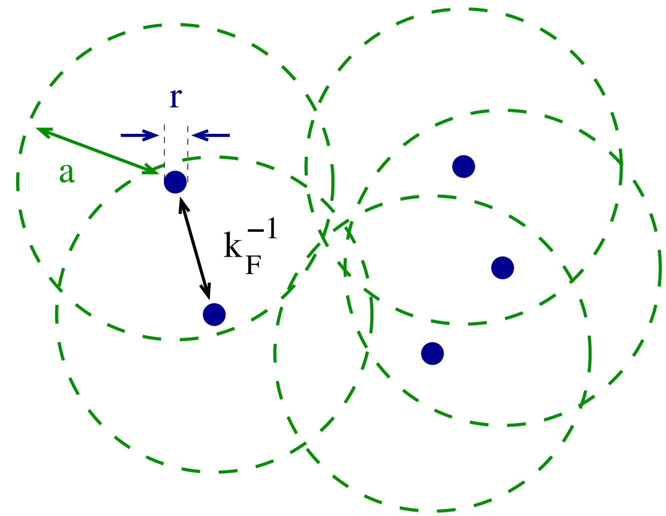
Trapped Atoms (T=1 neV)

Universality

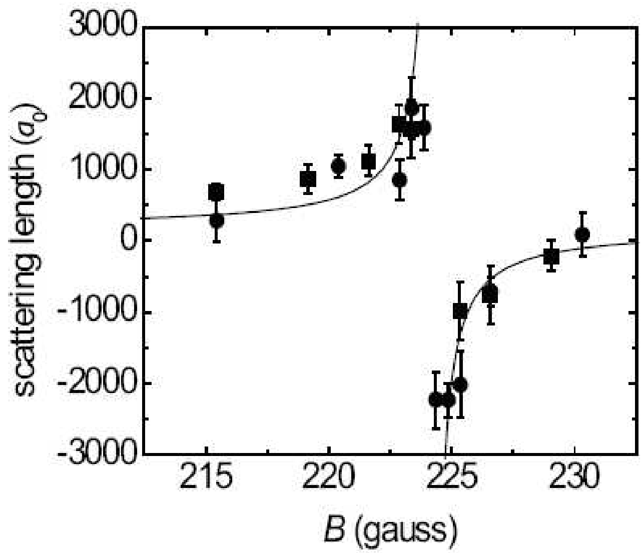
What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

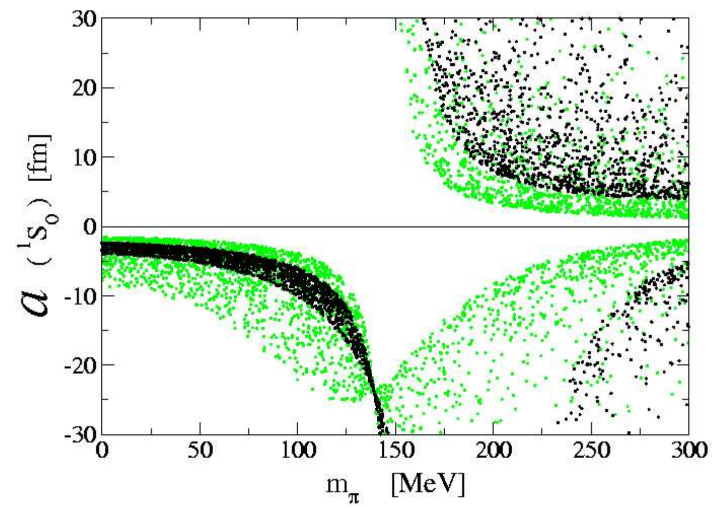
strongly correlated: $a\rho^{1/3} \gg 1$



Feshbach Resonance in ^{6}Li



Neutron Matter



Questions

Equation of State

Critical Temperature

Transport: Shear Viscosity, ...

Stressed Pairing

I. Equation of State

Universal Equation of State

Consider limiting case ("Bertsch" problem)

$$(k_F a) \to \infty \qquad\qquad (k_F r) \to 0$$

Universal equation of state

$$\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0 = \xi \frac{3}{5} \left(\frac{k_F^2}{2M}\right)$$

How to find ξ?

Numerical Simulations

Experiments with trapped fermions

Analytic Approaches

Effective Field Theory

Effective field theory for pointlike, non-relativistic neutrons

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right] + \dots$$

$$(a, r, \dots) \Rightarrow (C_0, C_2, \dots)$$

Partition Function (Hubbard-Stratonovich field s, Fermion matrix Q)

$$Z = \int Ds \exp\left[-S'\right], \qquad S' = -\log(\det(Q)) + V(s)$$

$C_0 < 0$ (attractive): $\det(Q) \geq 0$

Continuum Limit

Fix coupling constant at finite lattice spacing

$$\frac{M}{4\pi a} = \frac{1}{C_0} + \frac{1}{2}\sum_{\vec{p}} \frac{1}{E_{\vec{p}}}$$

Take lattice spacing b, b_τ to zero

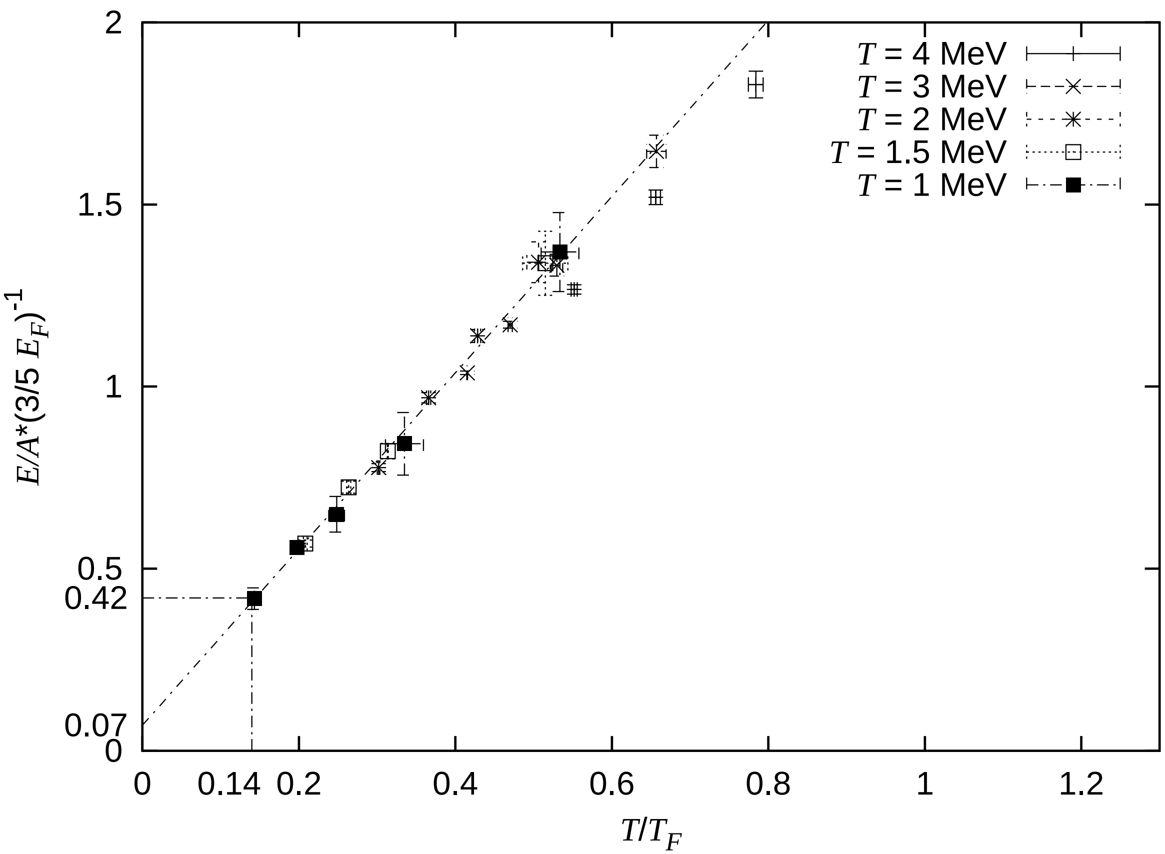
$$\mu b_\tau \to 0 \qquad n^{1/3} b \to 0 \qquad n^{1/3} a = const$$

Physical density fixed, lattice filling $\to 0$

Consider universal (unitary) limit

$$n^{1/3} a \to \infty$$

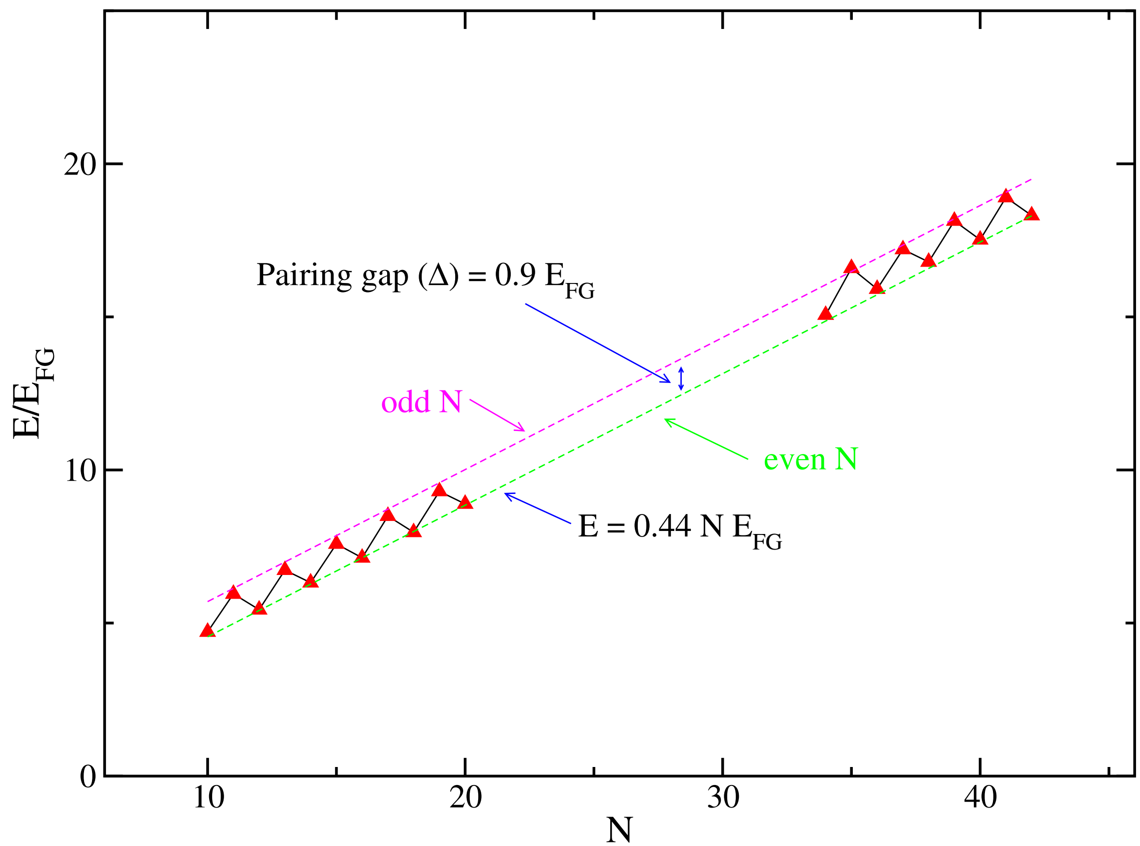
Lattice Results



Canonical $T = 0$ calculation: $\xi = 0.25(3)$ (D. Lee)

Not extrapolated to zero lattice spacing

Green Function Monte Carlo



Other lattice results: $\xi = 0.4$ (Burovski et al., Bulgac et al.)

Experiment: $\xi = 0.27^{+0.12}_{-0.09}\,[1],\ 0.51 \pm 0.04\,[2],\ 0.74 \pm 0.07\,[3]$

[1] Bartenstein et al., [2] Kinast et al., [3] Gehm et al.

Upper and lower critical dimension

Zero energy bound state for arbitrary d

$$\psi''(r) + \frac{d-1}{r}\psi'(r) = 0 \qquad (r > r_0)$$

d=2: Arbitrarily weak attractive potential has a bound state

$$\xi(d{=}2) = 1$$

d=4: Bound state wave function $\psi \sim 1/r^{d-2}$. Pairs do not overlap

$$\xi(d{=}4) = 0$$

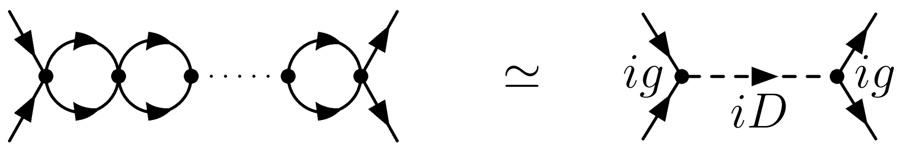
Conclude $\xi(d{=}3) \sim 1/2$?

Try expansion around $d = 4$ or $d = 2$?

Nussinov & Nussinov (2004)

Epsilon Expansion

EFT version: Compute scattering amplitude ($d = 4 - \epsilon$)



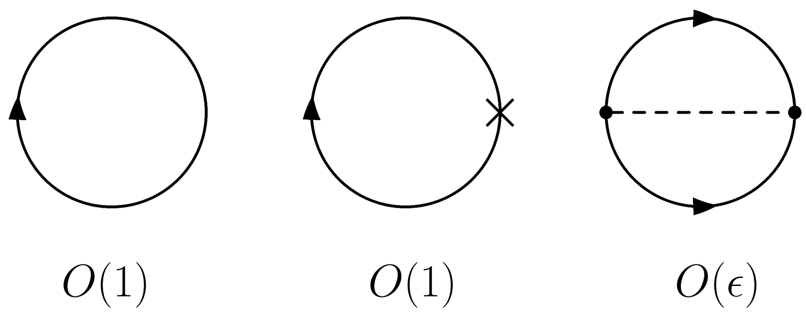
$$T = \frac{1}{\Gamma\left(1 - \frac{d}{2}\right)} \left(\frac{m}{4\pi}\right)^{-d/2} \left(-p_0 + \frac{\epsilon_p}{2}\right)^{1-d/2} \simeq \frac{8\pi^2\epsilon}{m^2} \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

$$g^2 \equiv \frac{8\pi^2\epsilon}{m^2} \qquad D(p_0, p) = \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

Weakly interacting bosons and fermions

Epsilon Expansion

Effective potential



$$\xi = \frac{1}{2}\epsilon^{3/2} + \frac{1}{16}\epsilon^{5/2}\ln\epsilon - 0.0246\,\epsilon^{5/2} + \ldots$$

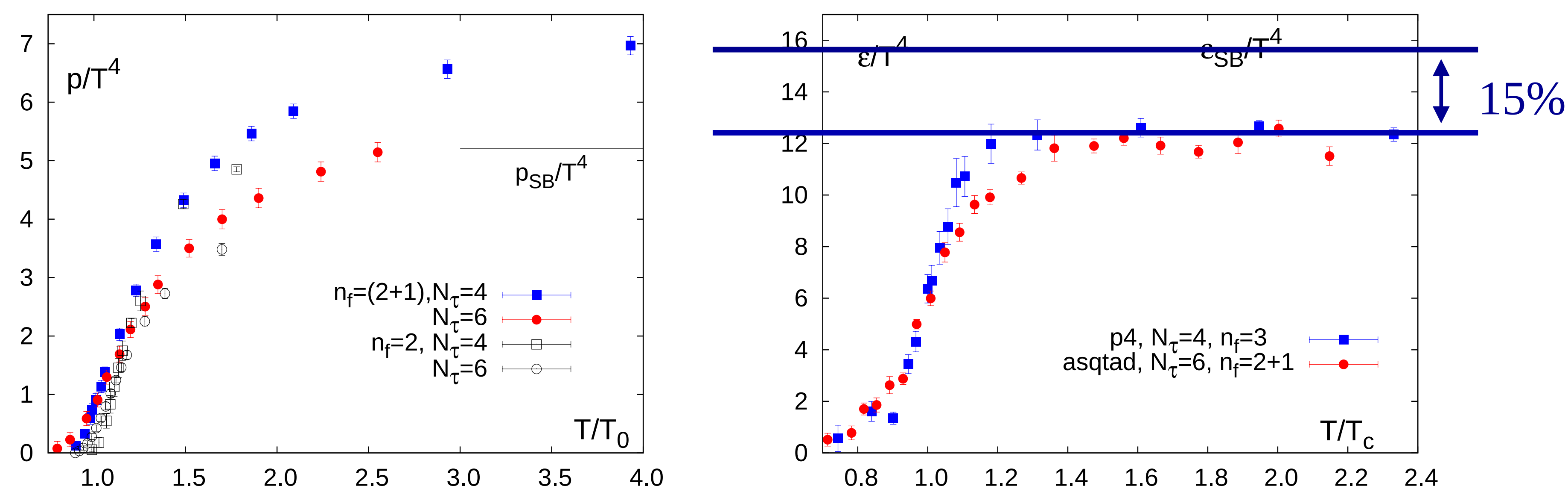
$$\xi(\epsilon=1) = 0.475$$

Problem: Higher order corrections large (~ 100 %)!

Combine $d = 4 - \epsilon$ and $d = 2 + \bar{\epsilon}$ (and Pade)

$$\xi = (0.3 - 0.35)$$

Quark Gluon Plasma Equation of State (Lattice)



Compilation by F. Karsch (SciDAC)

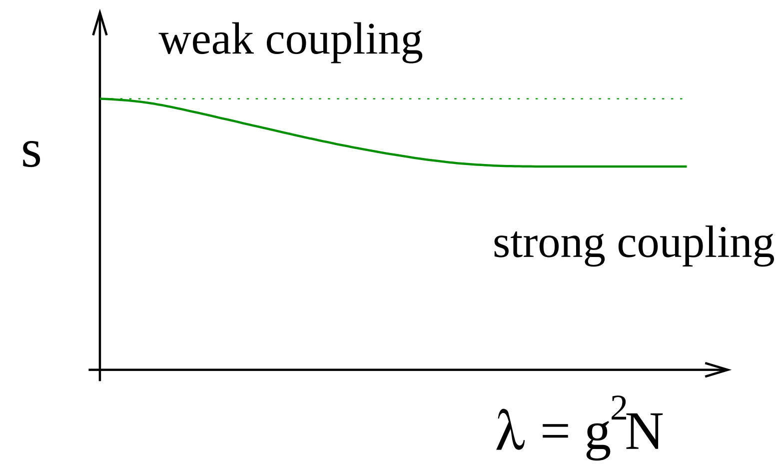
Holographic Duals at Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy =area of event horizon

$$s = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0$$

Gubser and Klebanov

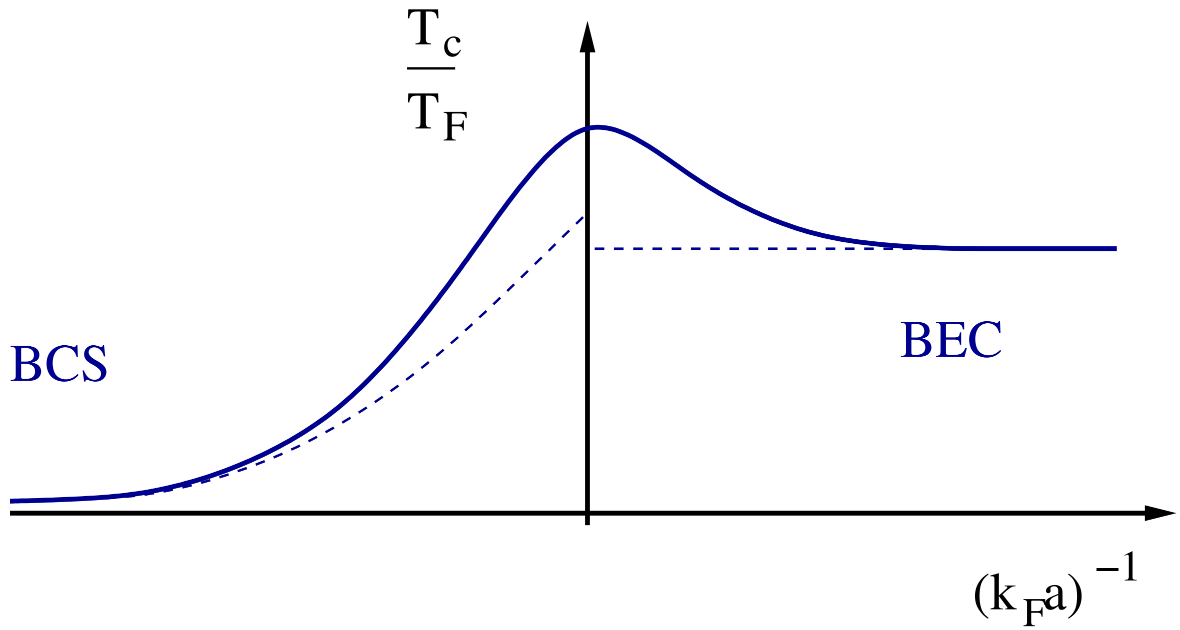


Extended to transport properties by Policastro, Son and Starinets

$$\eta = \frac{\pi}{8} N_c^2 T^3$$

II. How Large Can T_c Get?

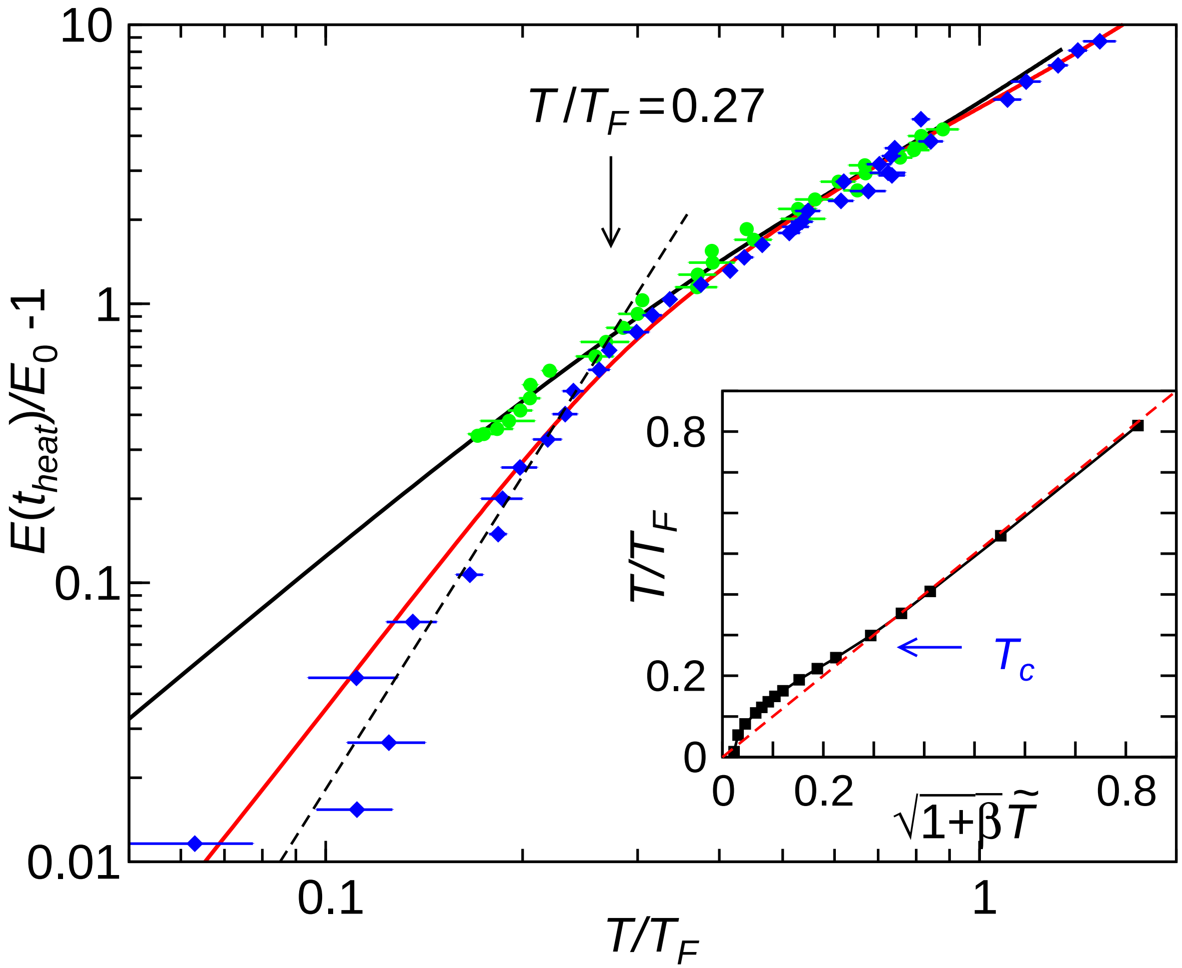
Critical Temperature: From BCS to BEC



$$T_c^{BCS} = \frac{4 \cdot 2^{1/3} e^{\gamma}}{e^{7/3}\pi} \epsilon_F \exp\left(-\frac{\pi}{|k_F a|}\right) \qquad T_c^{BEC} = 3.31\left(\frac{n^{2/3}}{m}\right)$$

$$T_c(a \to \infty) = 0.28\epsilon_F \qquad T_c = 0.21\epsilon_F + O(a_B n^{1/3})$$

Experimental Results

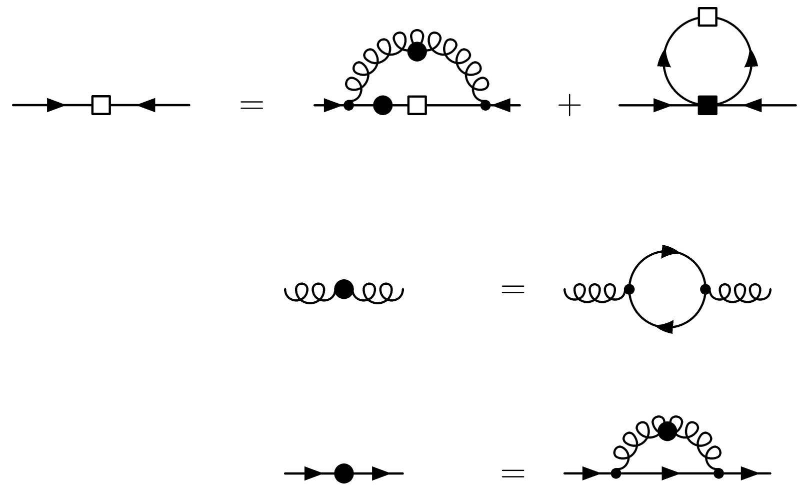


Kinast et al. (2005)

Lattice results: $T_c/T_F = 0.15$ (UMass)

Quark Matter: Color Superconductivity

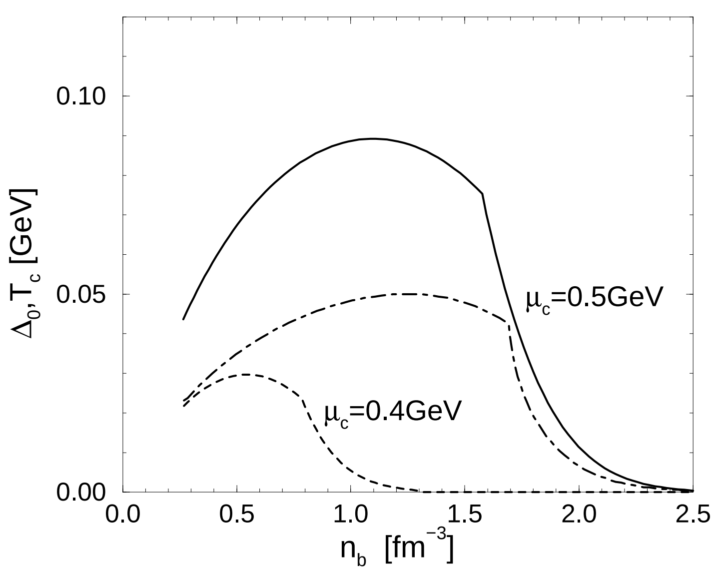
Weak coupling result



$$\frac{T_c}{T_F} = \frac{be^{\gamma}}{\pi} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

$$b = 512\pi^4 g^{-5} (2/N_f)^{\frac{5}{2}} e^{-\frac{\pi^2+4}{8}}$$

Maximum $T_c/T_F = 0.025$. Strong coupling?

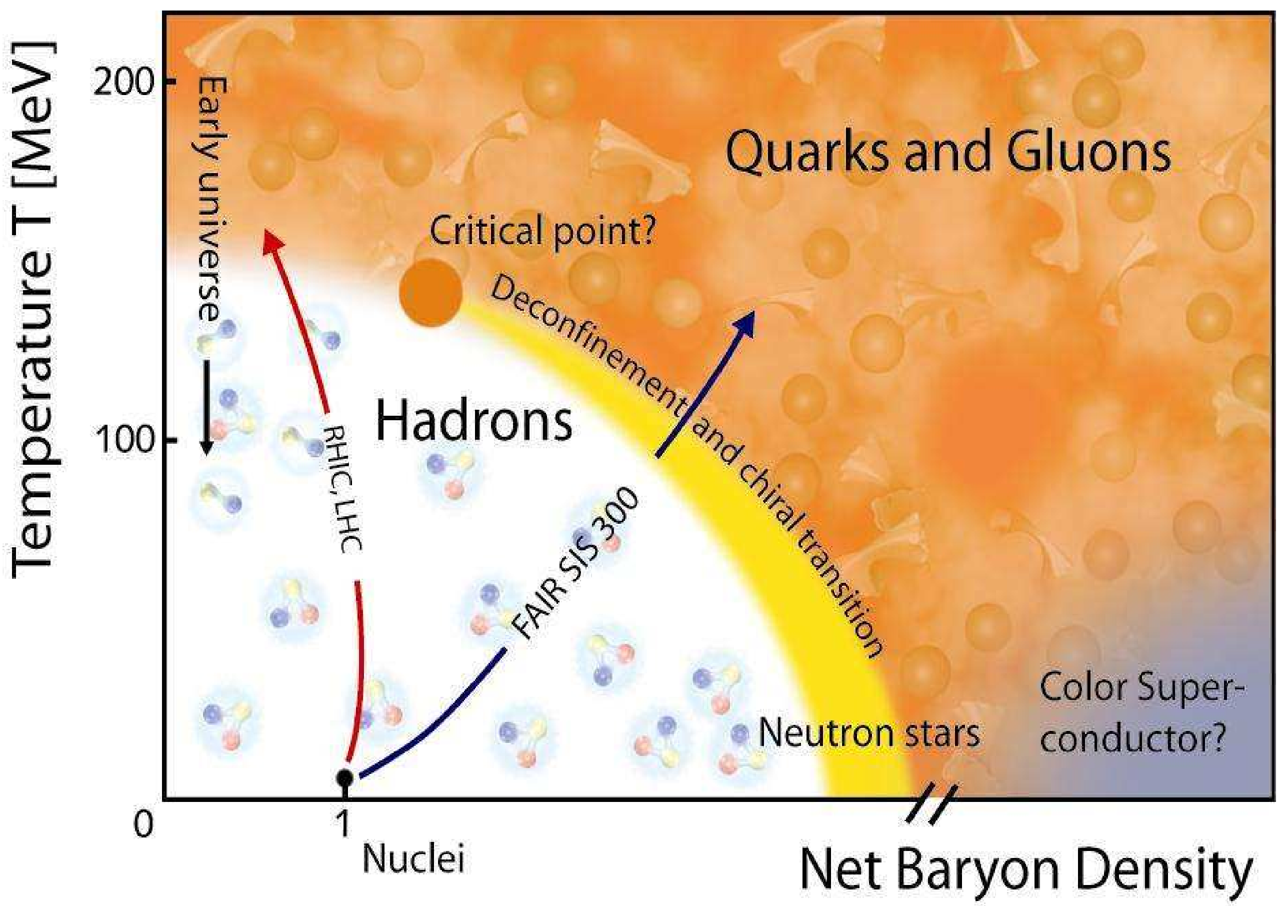


Find $T_c/T_F \simeq 0.2$

Note: Transition to χSB

Consider $N_c = 2$ QCD?

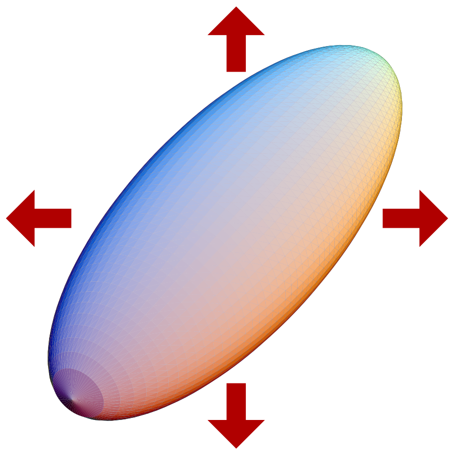
Importance of T_c/T_F: Heavy Ion Collisions at Fair



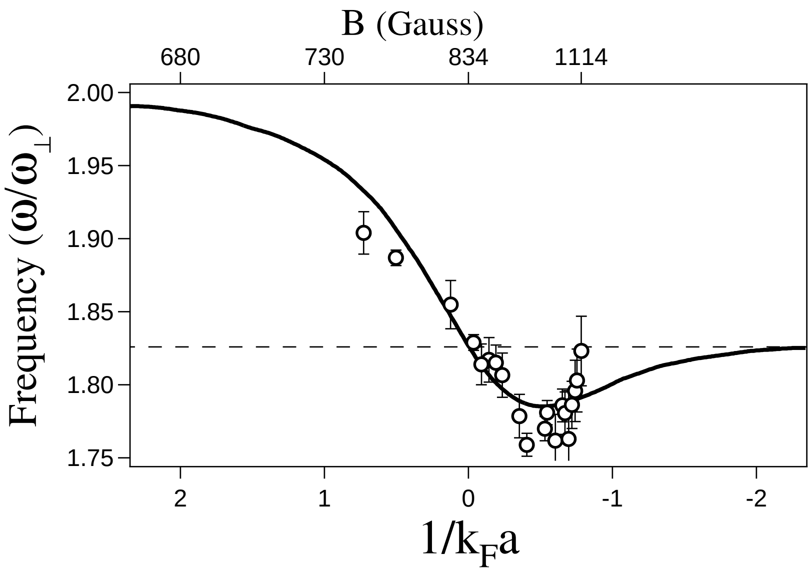
III. Transport Properties

Collective Modes

Radial breathing mode



Kinast et al. (2005)

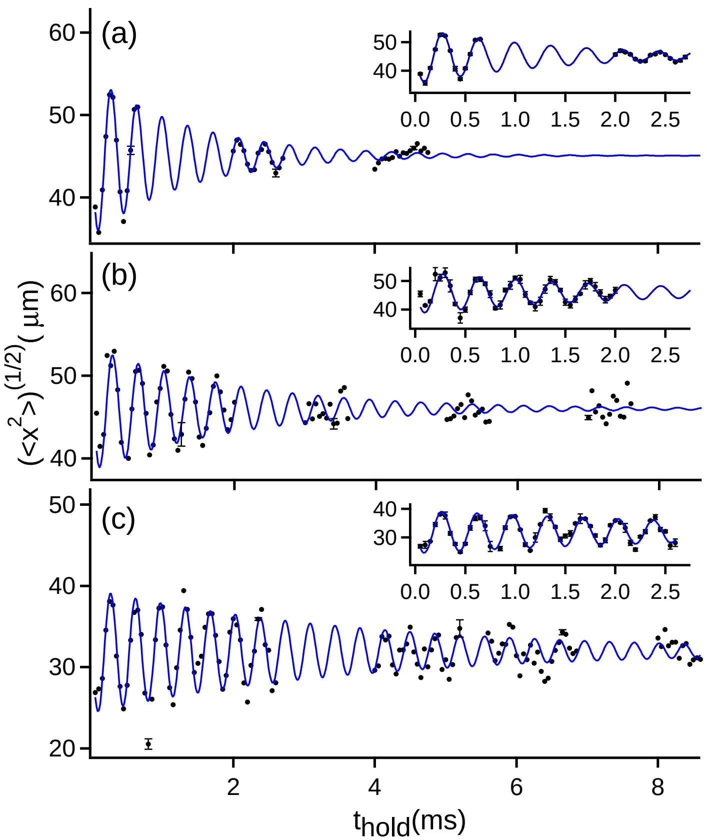


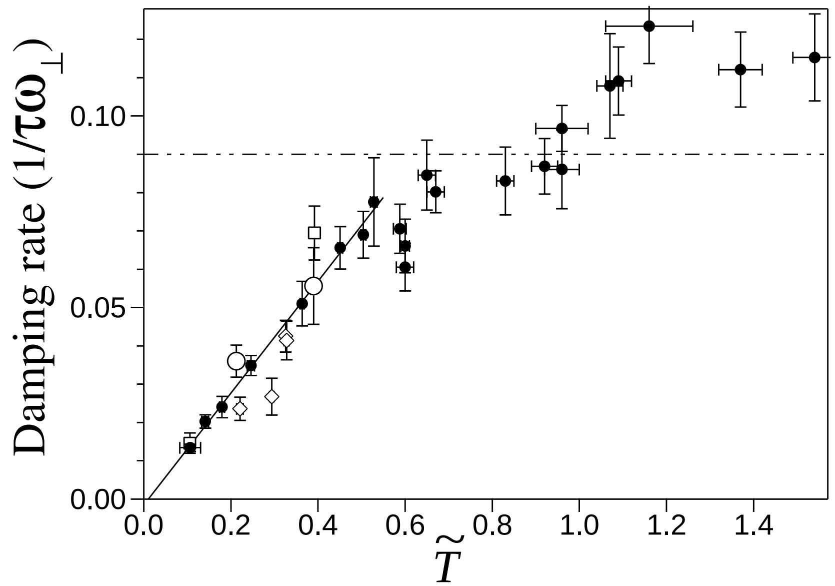
Ideal fluid hydrodynamics, equation of state $P \sim n^{5/3}$

$$\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) = 0$$
$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\frac{1}{mn}\vec{\nabla}\left(P + nV\right)$$

$$\omega = \sqrt{\frac{10}{3}}\omega_\perp$$

Damping of Collective Excitations





$T/T_F = (0.5, 0.33, 0.17)$

$\tau\omega$: decay time $\times$ trap frequency

Kinast et al. (2005)

Viscous Hydrodynamics

Energy dissipated due to viscous effects is

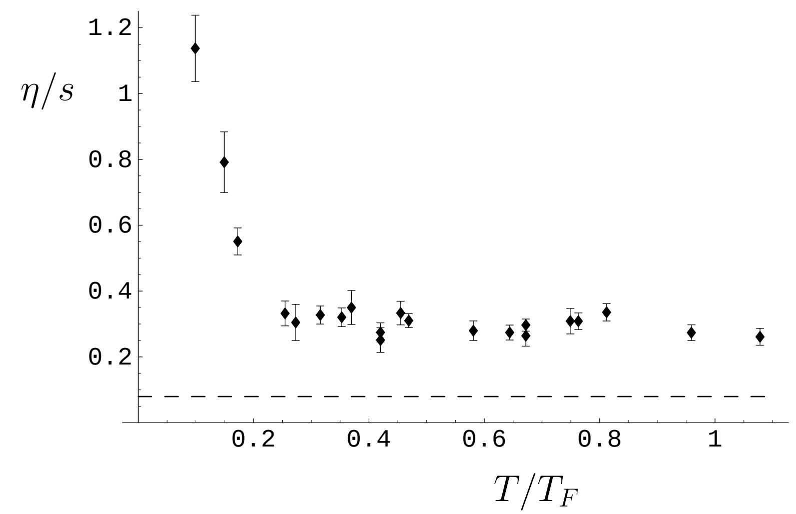
$$\dot{E} = -\frac{\eta}{2}\int d^3x\, \left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \zeta \int d^3x\, \left(\partial_i v_i\right)^2 ,$$

η, ζ: shear, bulk viscosity

Shear viscosity to entropy ratio ($\zeta = 0$)

$$\frac{\eta}{s} \sim c_i \times \frac{\Gamma}{\omega_\perp} \times \frac{\mu}{\omega_\perp} \times \frac{N}{S}$$

c_i determined by Hydro solution



Bruun, Smith, Gelman et al.

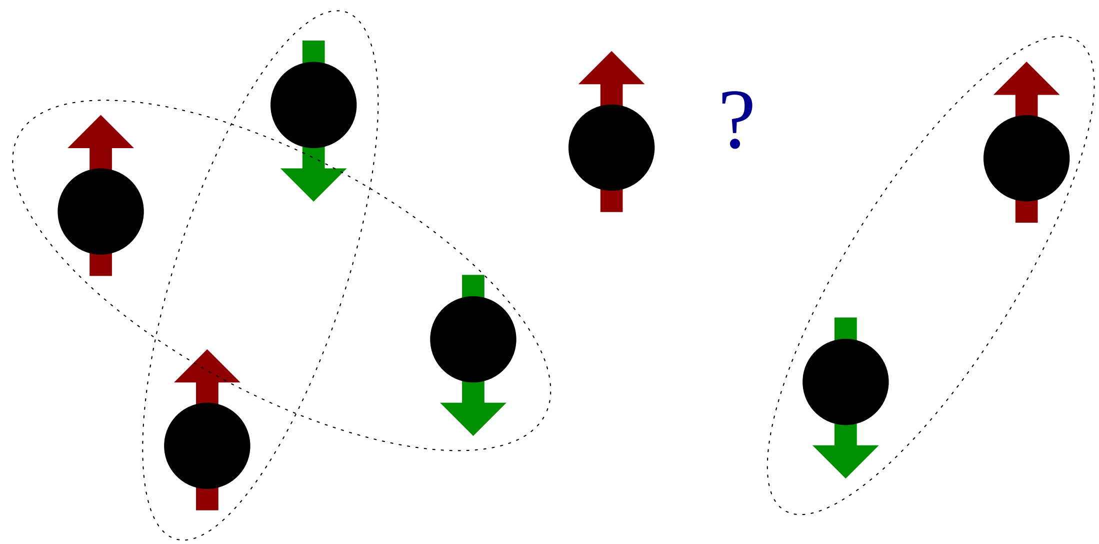
Problems: Scaling with N; T dependence below T_c

IV. Stressed Pairing

Polarized Fermions: From BEC to BCS

Response of paired state to pair breaking stress (e.g. Zeeman field)

$$\mathcal{L}_{ext} = \delta\mu\psi^\dagger\sigma_3\psi$$



BEC limit: Tightly bound bosons, no polarization for $\delta\mu < \Delta$

$\delta\mu > \Delta$: Mixture of Fermi and Bose liquid, no phase separation

BCS limit: No homogeneous mixed phase

$\delta\mu > \delta\mu_{c1}$: LOFF pairing $\Delta(x) = e^{iqx}\Delta$

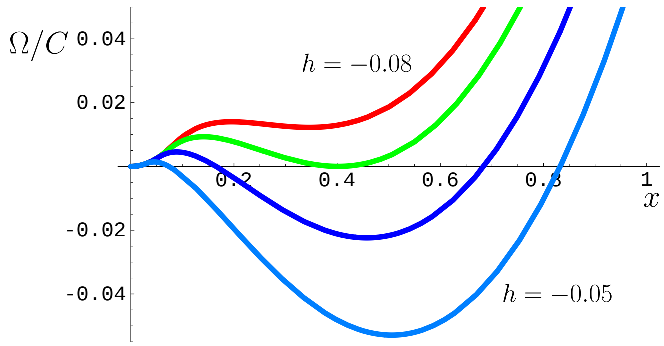
Inhomogeneous pairing

Onset? Consider EFT for gapless fermions interacting with GB's

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 - \epsilon(-i\vec{\partial}) - (\vec{\partial}\varphi) \cdot \frac{\overleftrightarrow{\partial}}{2m} \right) \psi + \frac{f_t^2}{2} \dot{\varphi}^2 - \frac{f^2}{2} (\vec{\partial}\varphi)^2$$

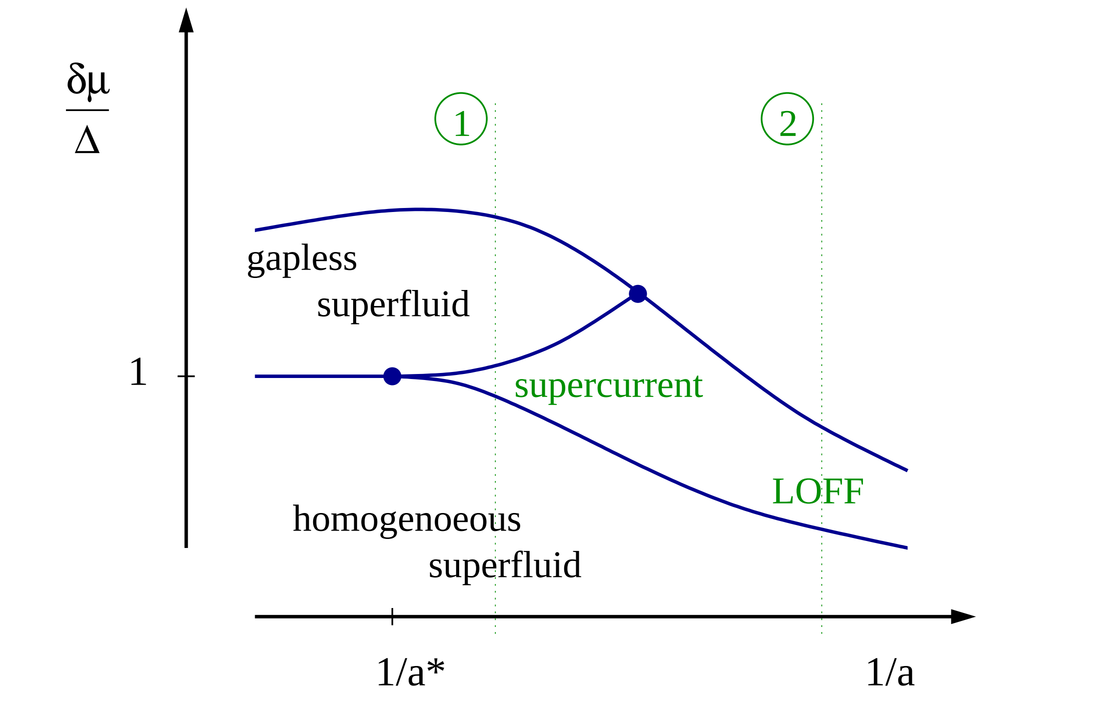
Free energy of state with non-zero current $v_s = \partial\varphi / m$

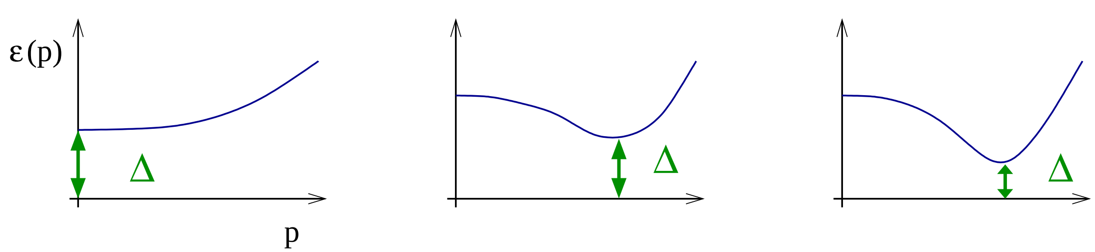
$$F(v_s) = \frac{1}{2} n m v_s^2 + \int \frac{d^3p}{(2\pi)^3} \epsilon_v(\vec{p}) \Theta\left(-\epsilon_v(\vec{p})\right)$$

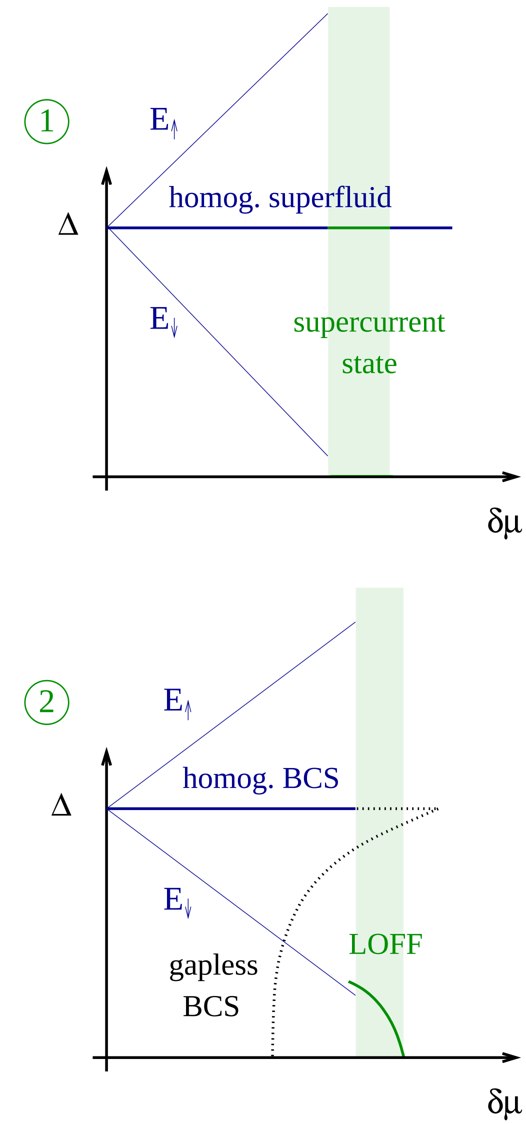


Unstable for BCS-type dispersion relation $\quad x \sim \frac{\jmath}{\Delta} \quad h \sim \frac{\delta\mu - \delta\mu_c}{\Delta}$

Minimal Phase Diagram

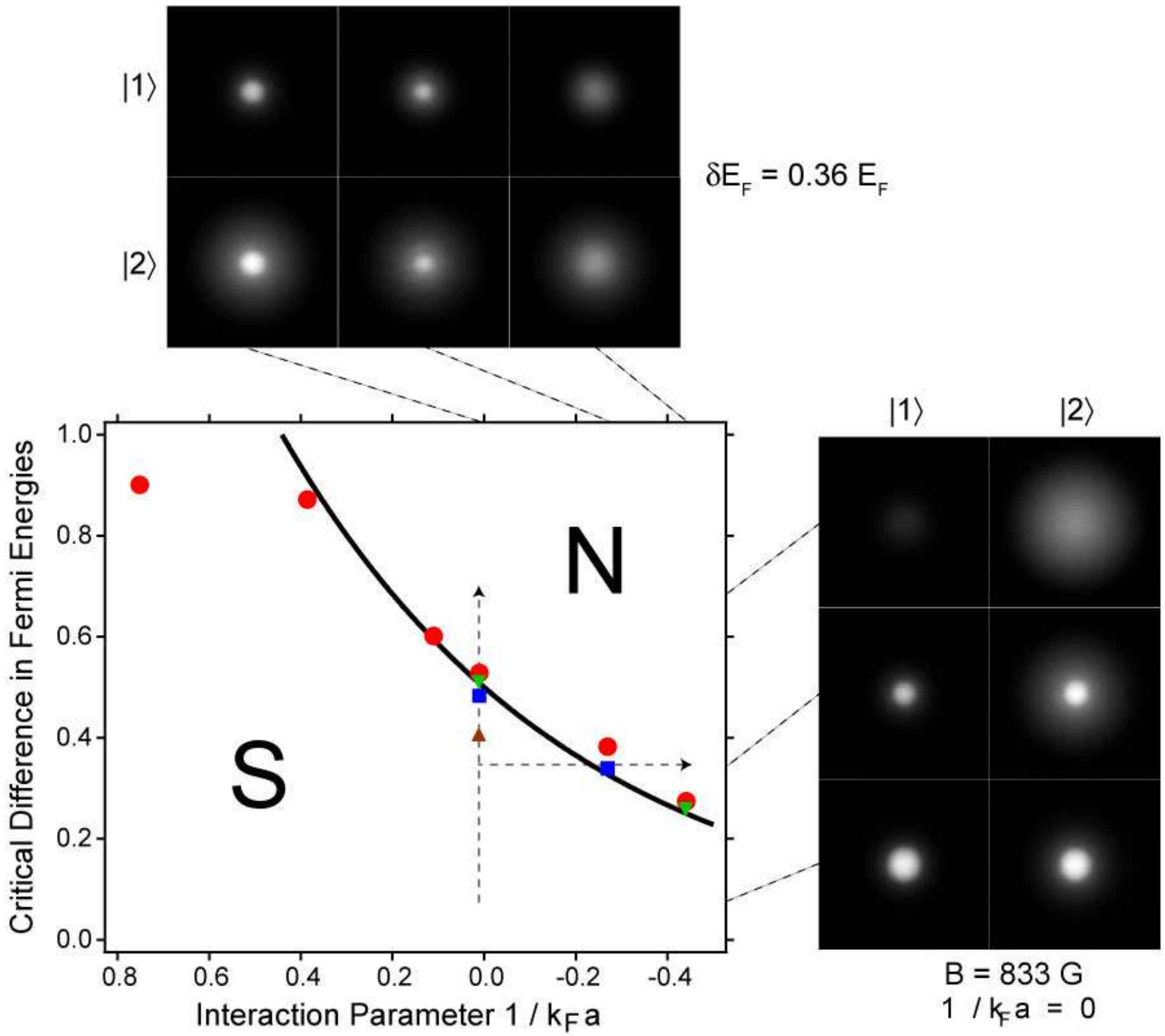






Son & Stephanov (2005)

Experimental Situation



Zwierlein et al.(MIT group)

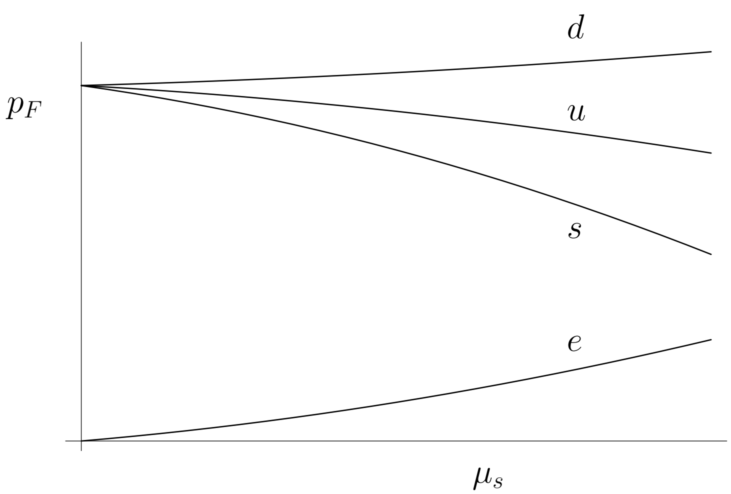
Color Superconductivity in QCD: Response to $m_s \neq 0$

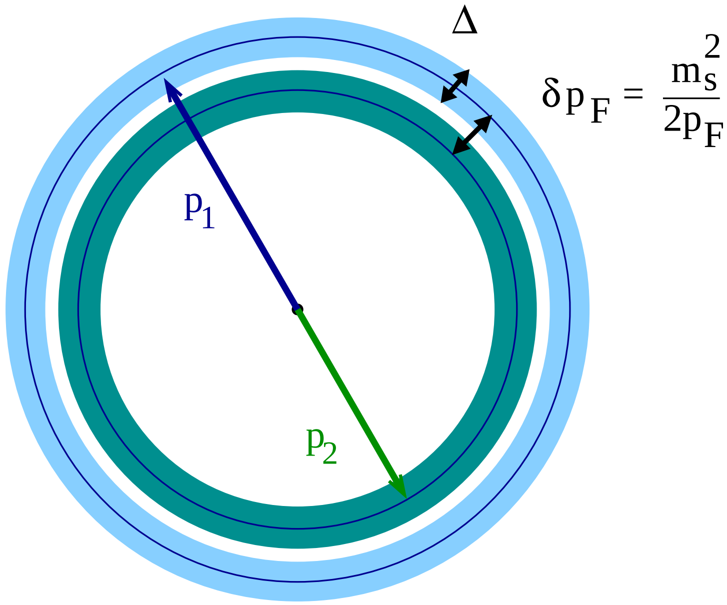
QCD with three degenerate flavors: CFL pairing

$$\langle q_i^a q_j^b \rangle = (\delta_i^a \delta_j^b - \delta_j^a \delta_i^b)\phi$$

$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

Pair breaking stress due to $\mu_s = m_s^2/(2p_F) \neq 0$

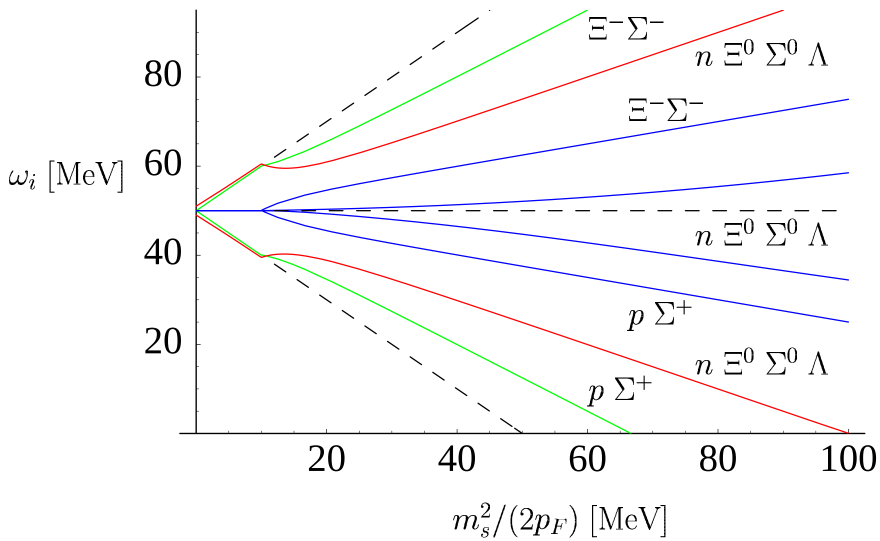


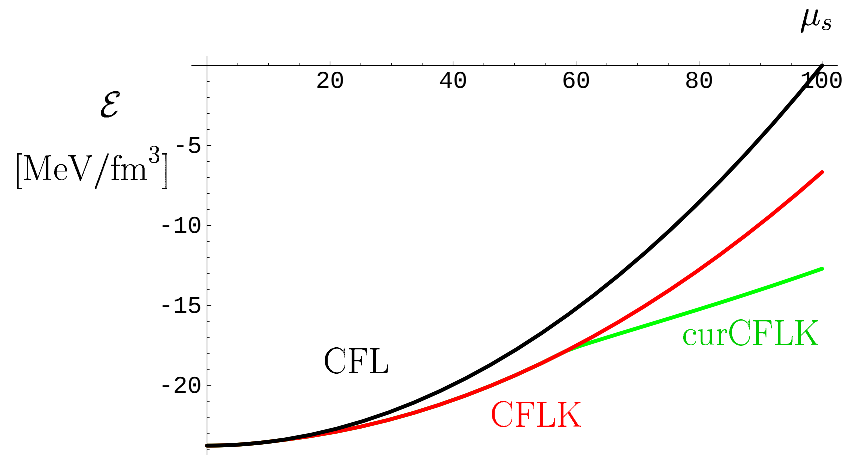


kinematics + electric neutrality + weak equilibrium

Phase Structure of CFL Quark Matter

How does CFL ($\langle ud \rangle = \langle ds \rangle = \langle su \rangle$) pairing responds to m_s?





Excitation energy of fermions

Gapless modes appear at

$$\mu_s(crit) \sim 0.75\Delta$$

Energy density of superfluid phases

$$\mu_s(K-cond) \sim m_u^{2/3}\Delta^{4/3}/\mu$$

$$\mu_s(GB-cur) \sim 0.75\Delta$$

Figures: Kryjevski & Schäfer (2004)

Schäfer (2005)

Trapped atoms provide interesting model system

equation of state of strongly correlated systems (neutron matter, sQGP)

viscosity of strongly correlated systems (sQGP?)

superfluidity at strong coupling (T_c/T_F, response to pair breaking fields, precursor phenomena)