# Nearly Perfect Fluidity in Cold Atomic Gases

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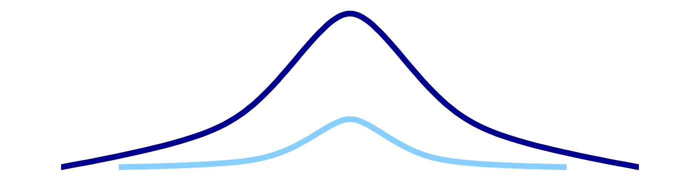
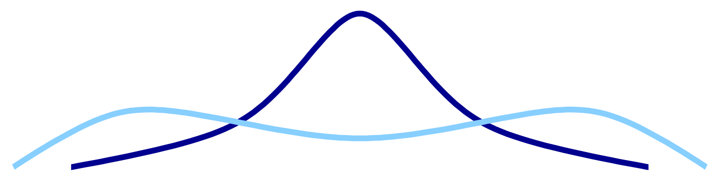
# Fluids: Gases, liquids, plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



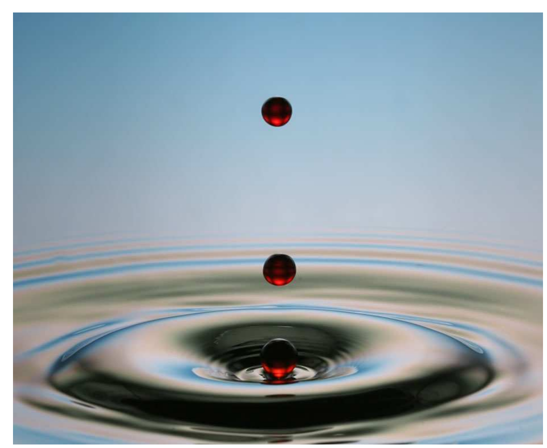
$\tau \sim \tau_{micro}$



$\tau \sim \lambda$

Historically: Water

$(\rho, \epsilon, \vec{\pi})$



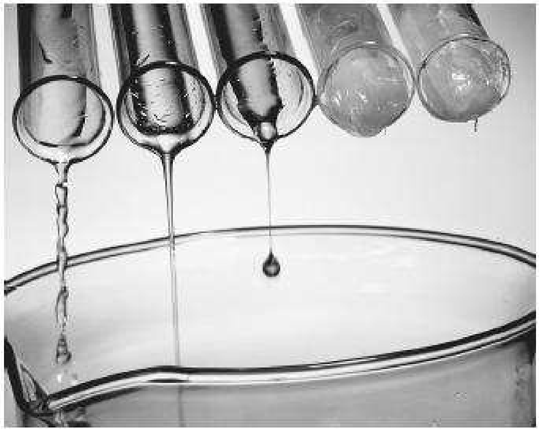
# Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$



Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left( \partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

reactive dissipative 2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

# Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$Re^{-1} = \underbrace{\frac{\eta}{\hbar n}}_{\text{fluid property}} \times \underbrace{\frac{\hbar}{mvL}}_{\text{flow property}}$$

Consider $mvL \sim \hbar$: Hydrodynamics requires $\eta/(\hbar n) < 1$

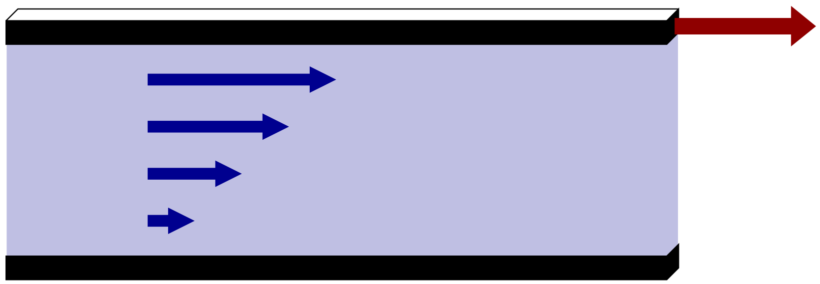
Kinetic theory estimate: $\eta \sim npl_{mfp}$

$$Re^{-1} = \frac{v}{c_s} Kn = Ma \cdot Kn \qquad Kn = \frac{l_{mfp}}{L}$$

expansion parameter $Kn \ll 1$

# Shear viscosity

Viscosity determines shear stress ("friction") in fluid flow

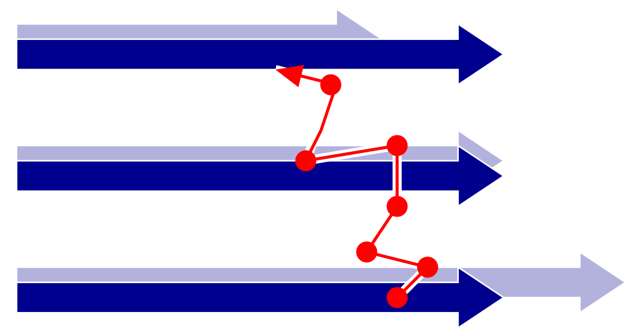


$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Kinetic theory: conserved quantities carried by quasi-particles

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$



Dilute, weakly interacting gas: $l_{mfp} \sim 1/(n\sigma)$

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

independent of density!

# Shear viscosity

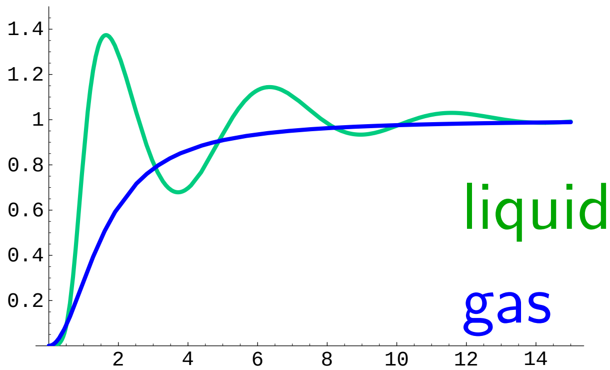
non-interacting gas ($\sigma \to 0$): $\eta \to \infty$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

strongly interacting gas: $\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$

but: kinetic theory not reliable!

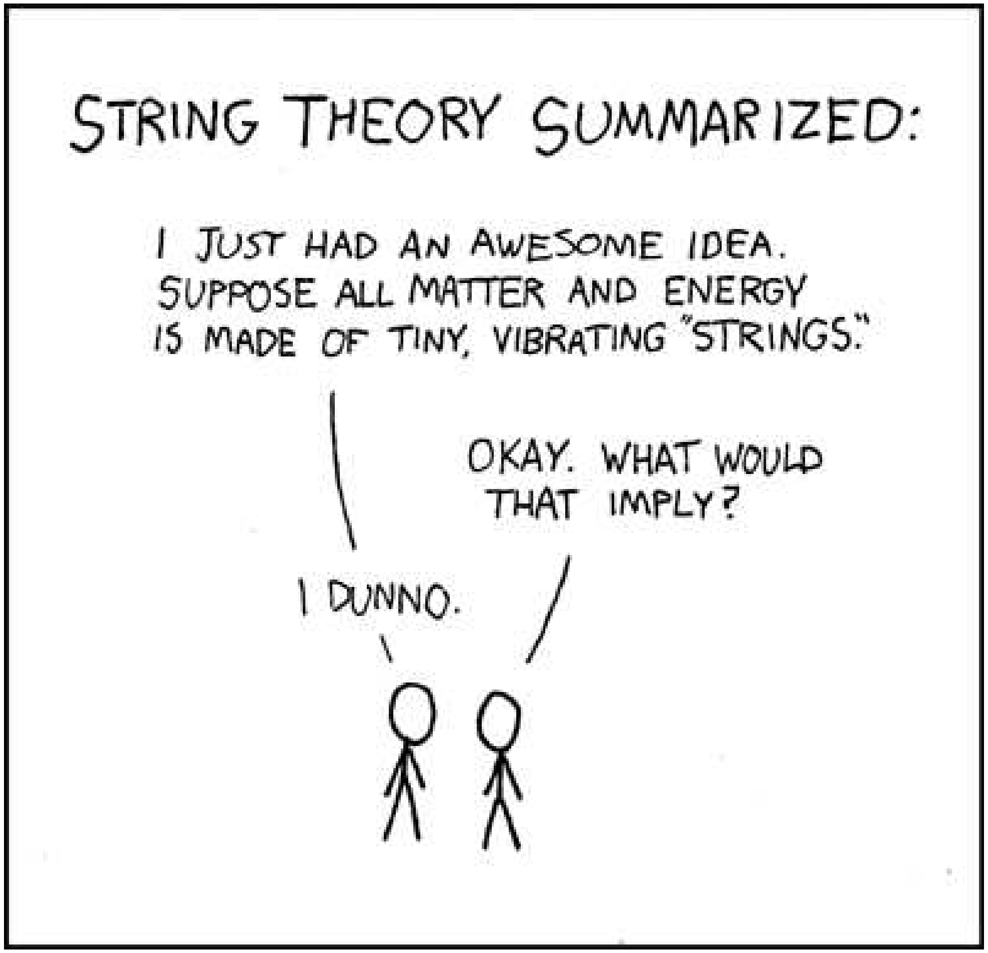
what happens if the gas condenses into a liquid?




Eyring, Frenkel:

$$\eta \simeq hn \exp(E/T) \geq hn$$

# And now for something completely different . . .



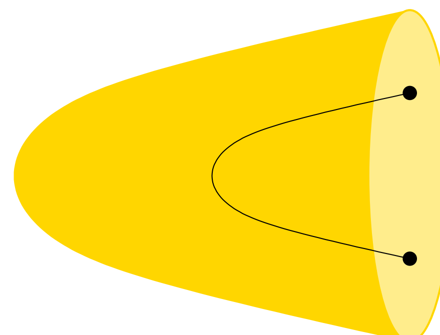
# Gauge theory at strong coupling: Holographic duality

The AdS/CFT duality relates

| | | |
|---|---|---|
| large $N_c$ (conformal) gauge theory in 4 dimensions | $\Leftrightarrow$ | string theory on 5 dimensional Anti-de Sitter space $\times S_5$ |
| correlation fcts of gauge invariant operators | $\Leftrightarrow$ | boundary correlation fcts of AdS fields |

$$\langle \exp \int dx\ \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



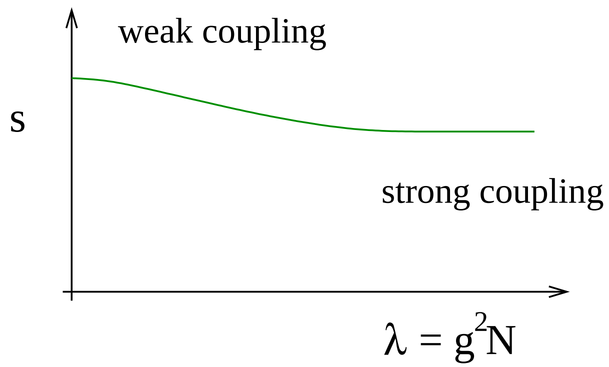
The correspondence is simplest at strong coupling $g^2 N_c$

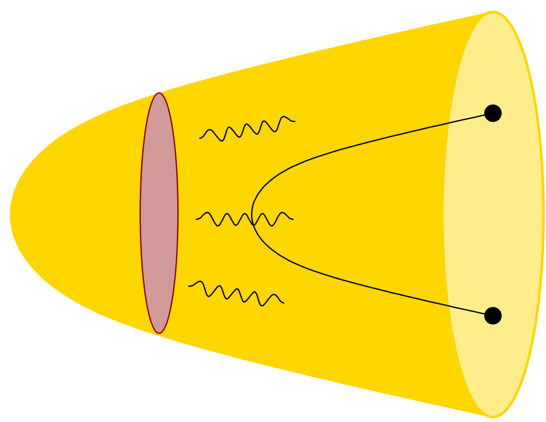
strongly coupled gauge theory $\Leftrightarrow$ classical string theory

# Holographic duals at finite temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

| | | |
|---|---|---|
| CFT temperature | $\Leftrightarrow$ | Hawking temperature of black hole |
| CFT entropy | $\Leftrightarrow$ | Hawking-Bekenstein entropy $\sim$ area of event horizon |



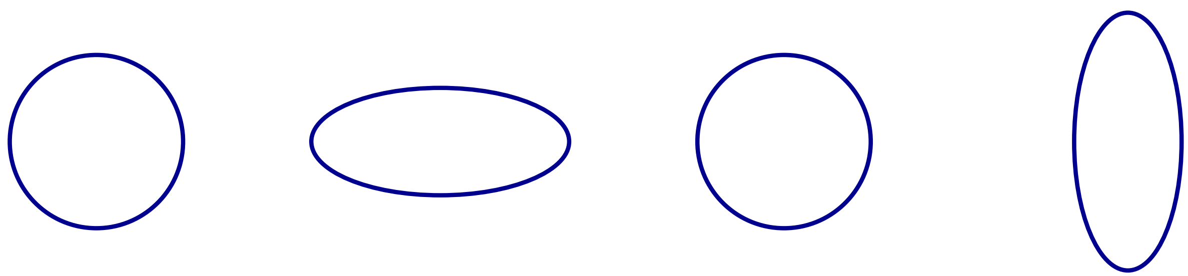
$$s(\lambda \to \infty) = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s(\lambda = 0)$$

Gubser and Klebanov

# Holographic duals: Transport properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \qquad g_{\mu\nu} = g^0_{\mu\nu} + \gamma_{\mu\nu}$$



CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy

$\sim$ area of event horizon

shear viscosity $\Leftrightarrow$ Graviton absorption cross section

$\sim$ area of event horizon

# Holographic duals: Transport properties

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CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy

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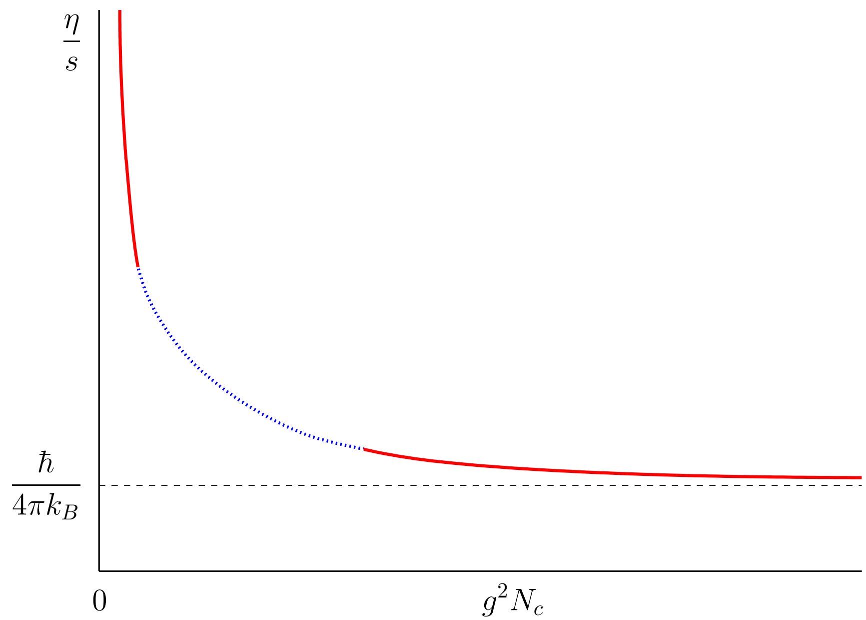
Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Son and Starinets (2001)




Strong coupling limit universal? Provides lower bound for all theories?

# Comment: Why $\eta$? Why $s$? Why $\eta/s$?

Everything is a fluid.

(Hydrodynamics is a general theory of long time behavior.)

At leading order, only need equation of state. But: EOS does not discriminate weakly and strongly interacting fluids.

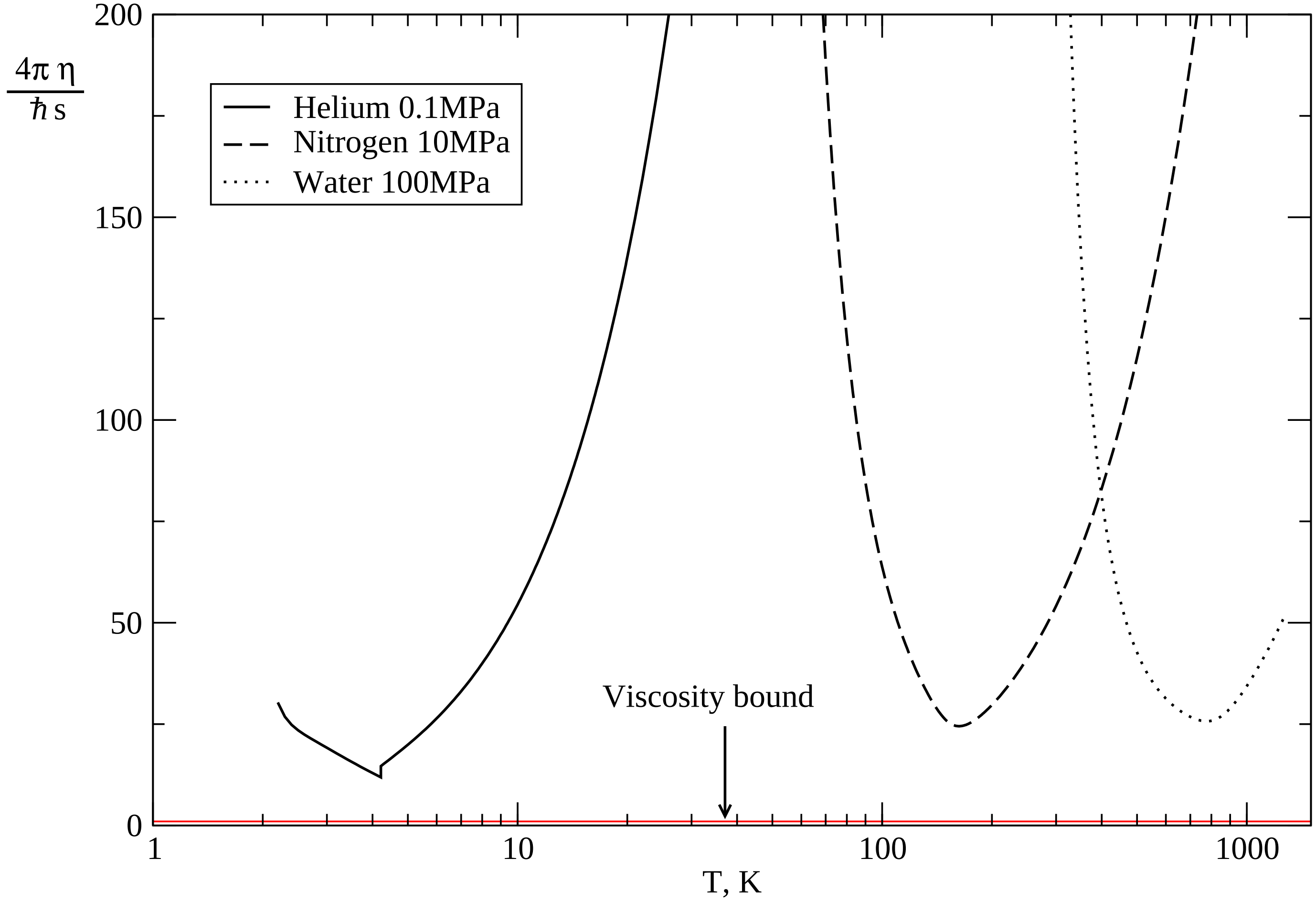
Every fluid has at least $T_{ij}$.

$\eta$ is the most basic transport coefficient.

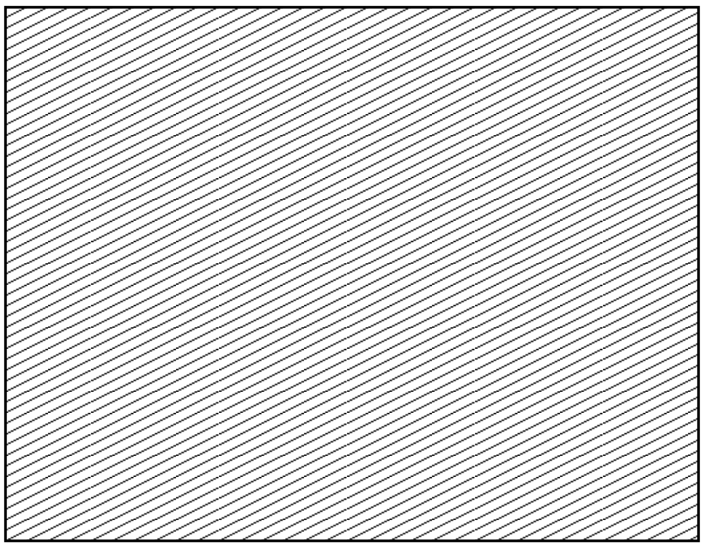
$s$ is the most basic density.

$\eta/s$ is the most universal measure of dissipation.

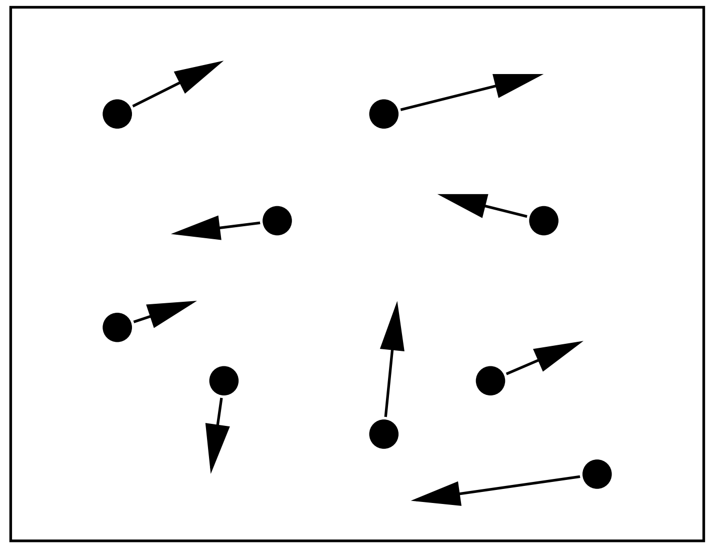
# Viscosity bound: Common fluids



# Kinetics vs no-kinetics



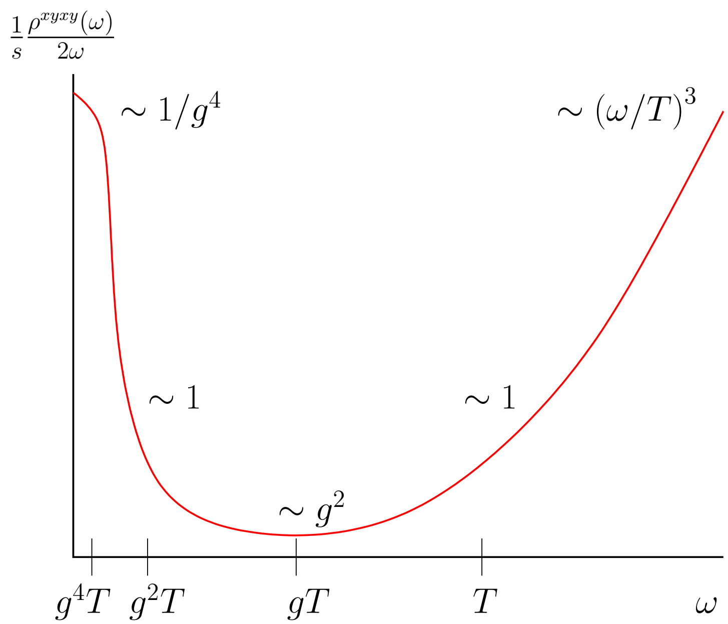
AdS/CFT low viscosity goo

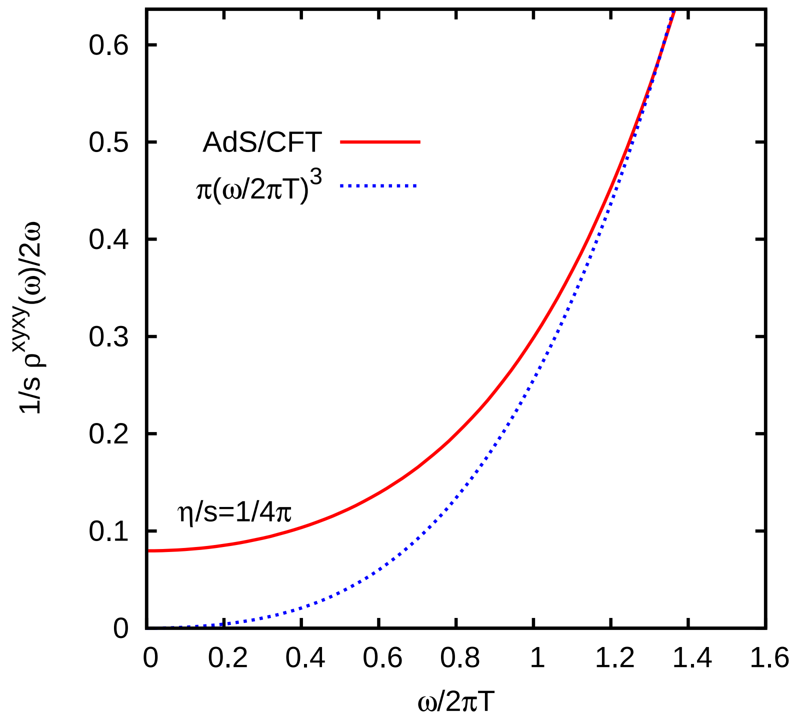


kinetic liquid

# Kinetics vs no-kinetics

Spectral function $\rho(\omega) = \mathrm{Im} G_R(\omega, 0)$ associated with $T_{xy}$




weak coupling QCD　　　　strong coupling AdS/CFT

transport peak vs no transport peak

## Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

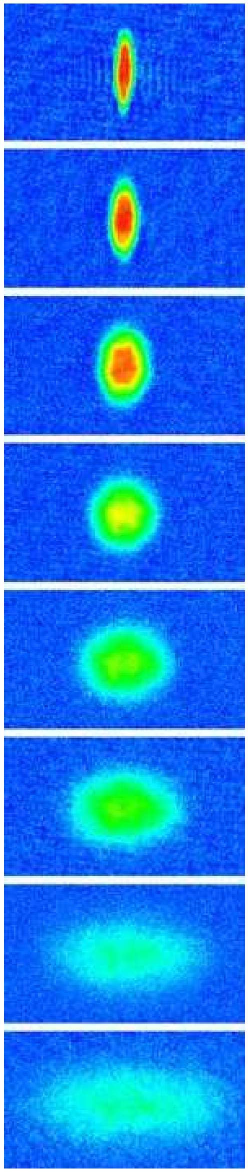
Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

Model system has conformal invariance (essential?)

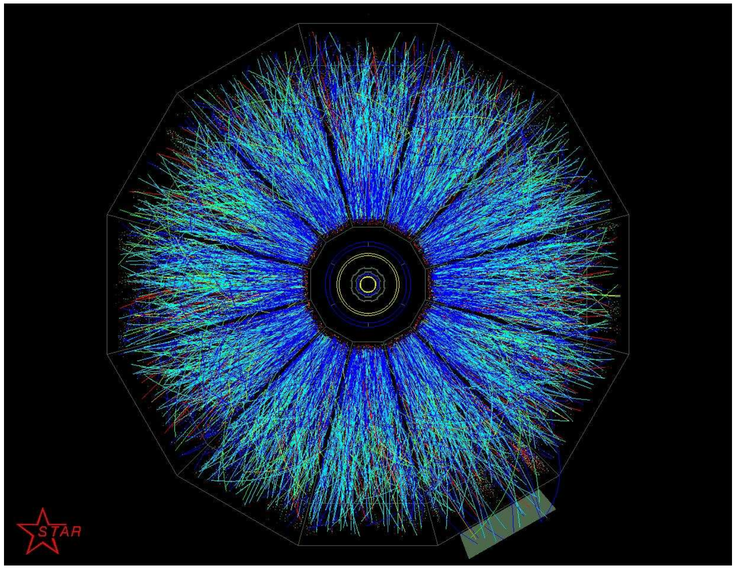
(Almost) scale invariant systems

# Perfect Fluids: The contenders

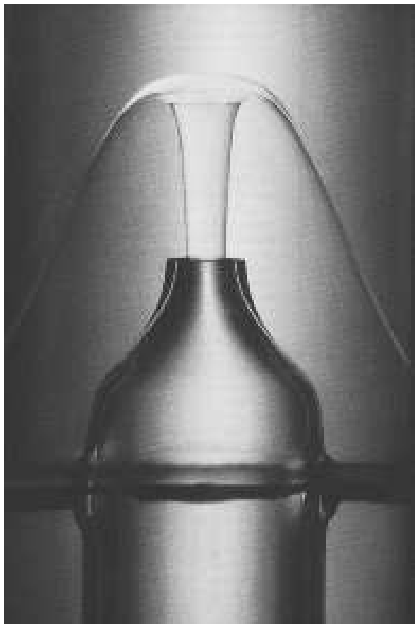


Trapped Atoms

(T=0.1 neV)



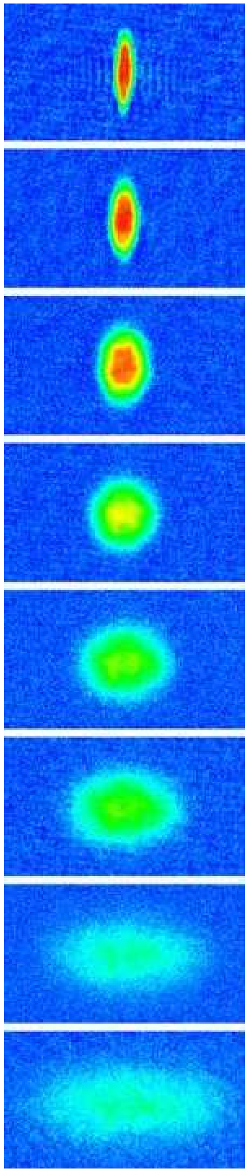

QGP (T=180 MeV)

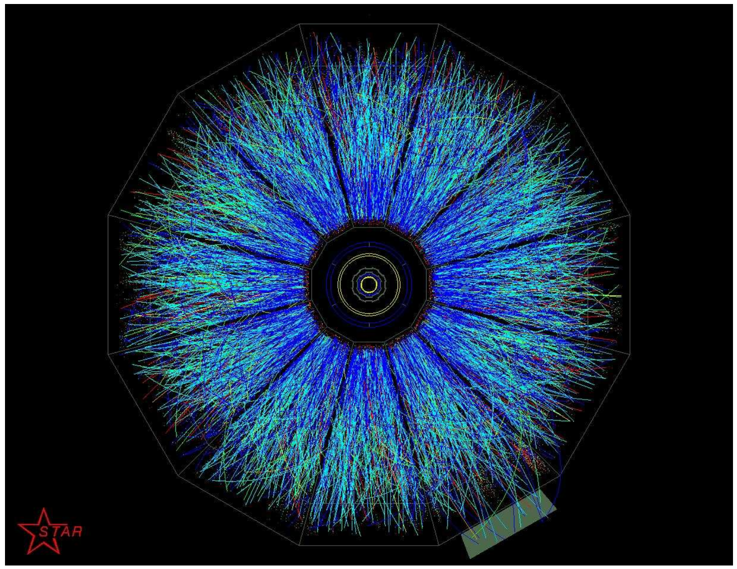


Liquid Helium

(T=0.1 meV)

# Perfect Fluids: The contenders






QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$

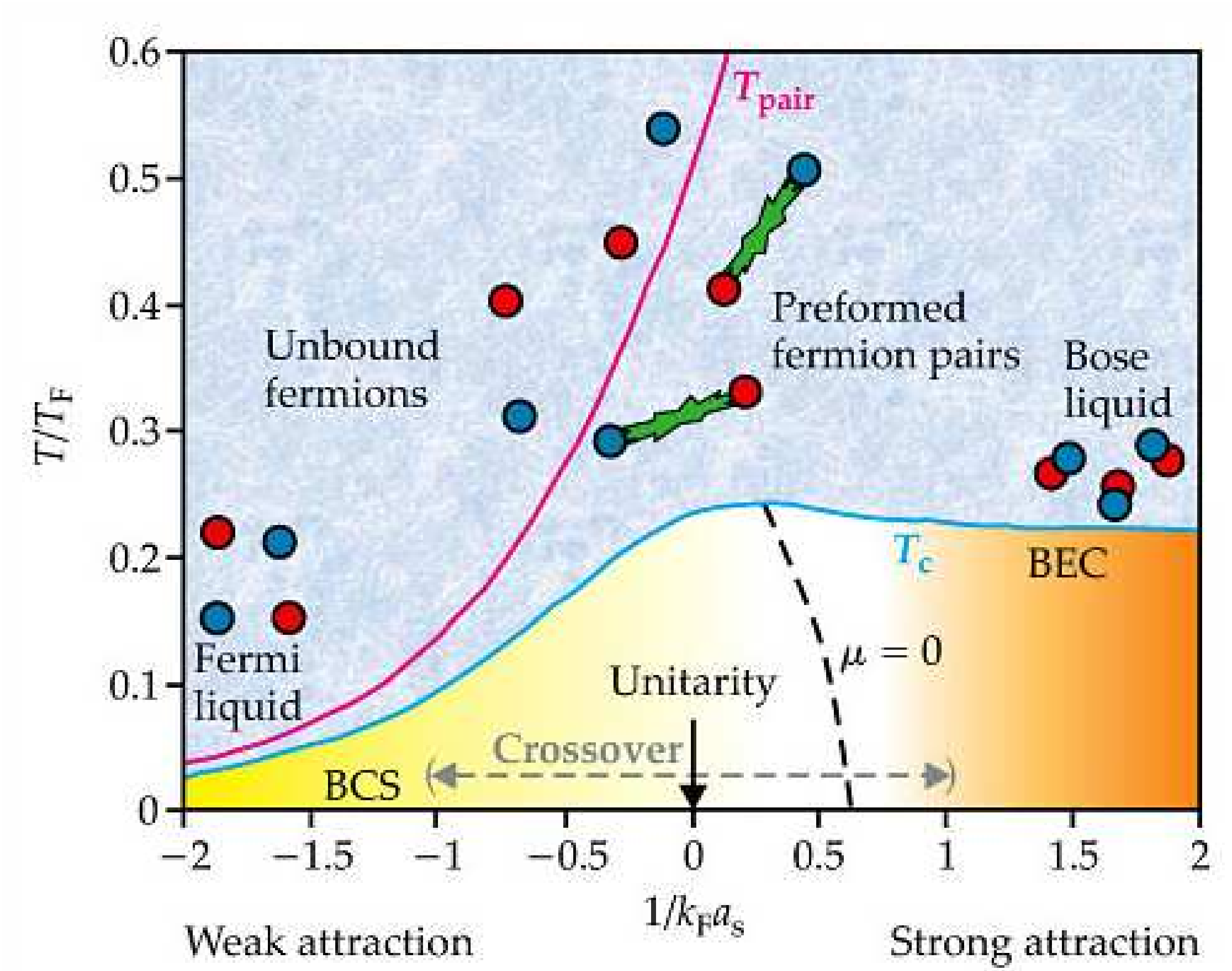
Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

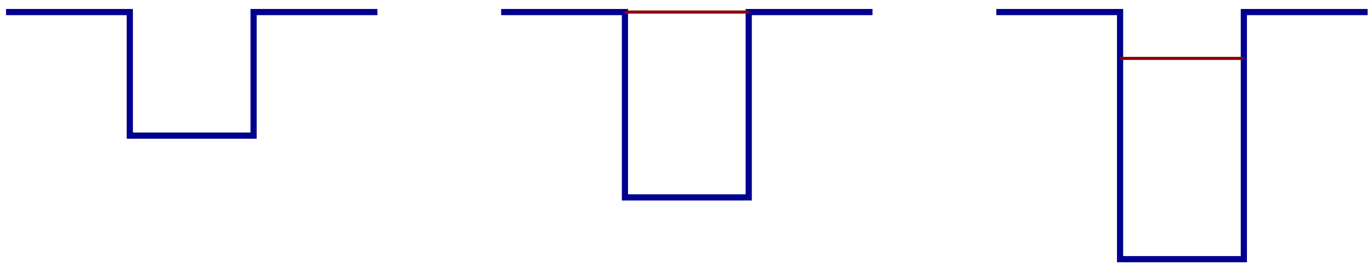
$\eta/s$

# Dilute Fermi gas: BCS-BEC crossover



# Unitarity limit

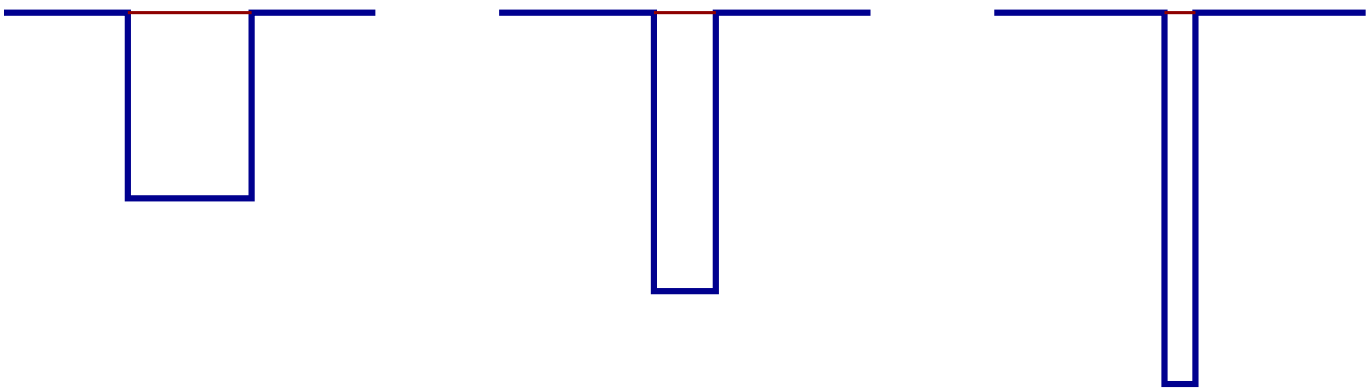
Consider simple square well potential



$a < 0$ $\quad$ $a = \infty,\ \epsilon_B = 0$ $\quad$ $a > 0,\ \epsilon_B > 0$

# Unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$

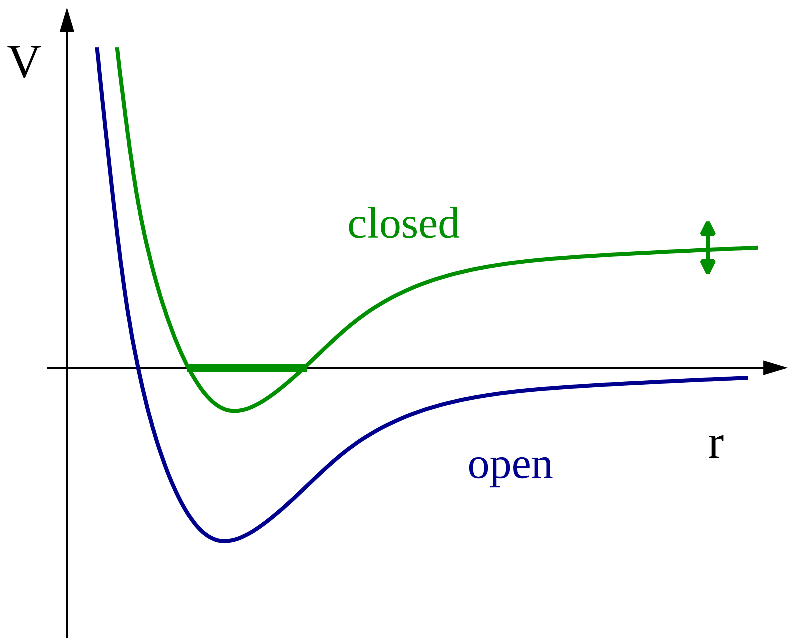


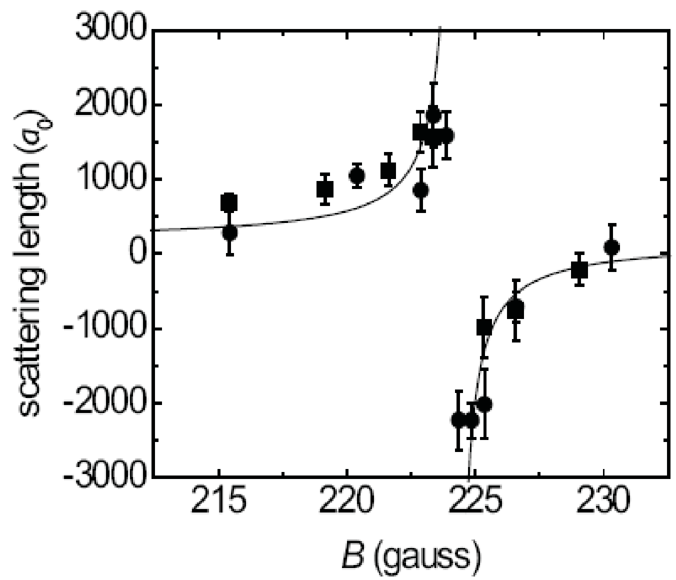
Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{ar}} \exp(-r/a)$$

# Feshbach resonances

Atomic gas with two spin states: "$\uparrow$" and "$\downarrow$"




Feshbach resonance

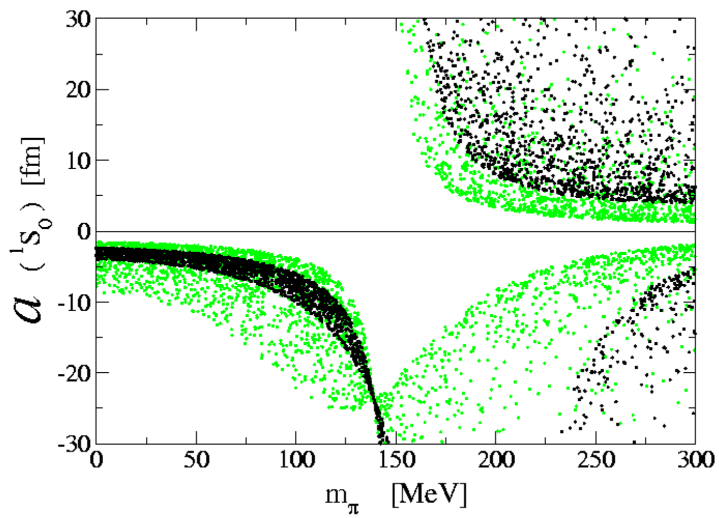
$$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

"Unitarity" limit $a \rightarrow \infty$

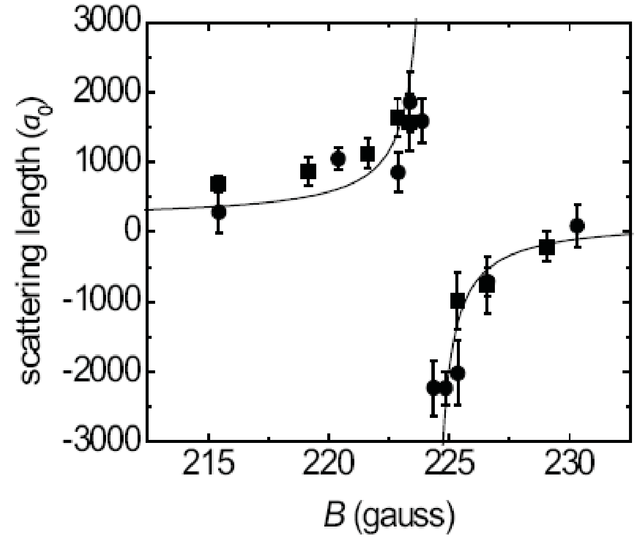
$$\sigma = \frac{4\pi}{k^2}$$

# Universality

## Neutron Matter



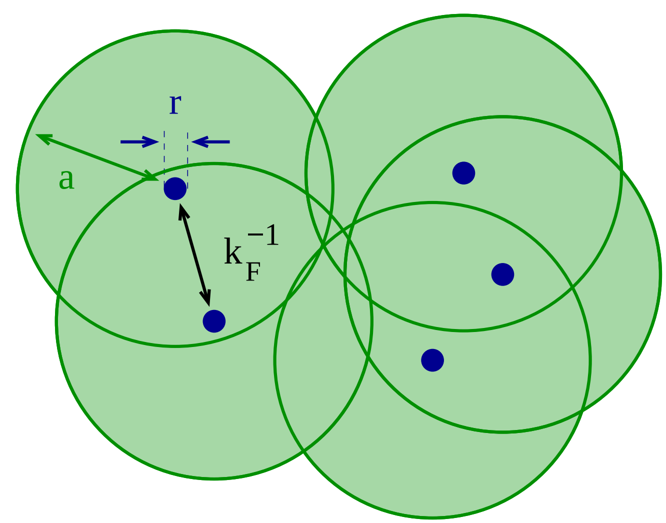

## Feshbach Resonance in $^6$Li




What do these systems have in common?

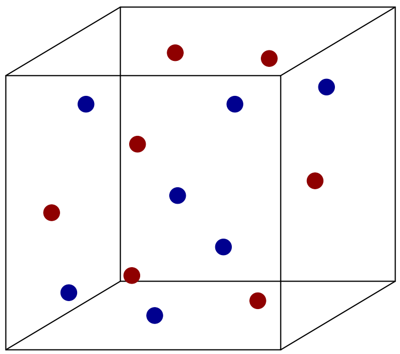
dilute: $r\rho^{1/3} \ll 1$

strongly correlated: $a\rho^{1/3} \gg 1$



# Universality: Many body physics

Free fermi gas at zero temperature



$$\frac{E}{N} = \frac{3}{5}\frac{k_F^2}{2m} \qquad \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

Consider unitarity limit ($a \longrightarrow \infty$, $r \longrightarrow 0$)

$$\frac{E}{N} = \xi \, \frac{3}{5}\frac{k_F^2}{2m} \qquad k_F \equiv (3\pi^2 N/V)^{1/3}$$

Prize problem (Bertsch, 1998): Determine $\xi$

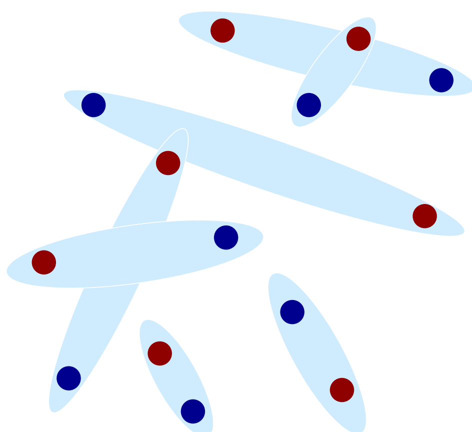
# Equation of state, pairing

Fermi gas at non-zero temperature

$$P(\mu,T) = P_0(\mu,T) f\left(\frac{\mu}{T}\right) \qquad P_0(\mu,T) = -\frac{k_B T}{\lambda_{deB}^3} f_{5/2}\left((-e^{\mu/(k_B T)}\right)$$

Universal function $f(z)$

Pairing: $\langle \psi_\downarrow \psi_\uparrow \rangle \neq 0$



$$\Delta = \alpha\mu \;\; k_B T_c = \beta\mu$$

Universal coefficients $\alpha, \beta$

Scale invariant system with conserved charge: $T_c \sim \mu$

# Dilute Fermi gas: field theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \Big( i\partial_0 + \frac{\nabla^2}{2M} \Big) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ ($C_0 \to \infty$)

This limit is smooth: HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$

$$\mathcal{L} = \Psi^\dagger \left[ i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left( \Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$

Low $T$ ($T < T_c \sim \mu$): Pairing and superfluidity

# Many body methods

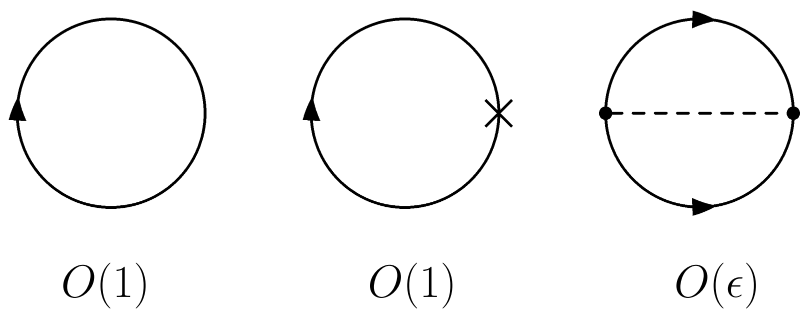
Large N: $\psi_\alpha \to \psi_\alpha^A$ $(A = 1, \ldots, N)$

$$\xi = 0.59 + O(1/N)$$

Bruckner theory (ladder diagrams, hole line expansion)

$$\xi = 0.24 + O(1/d)$$

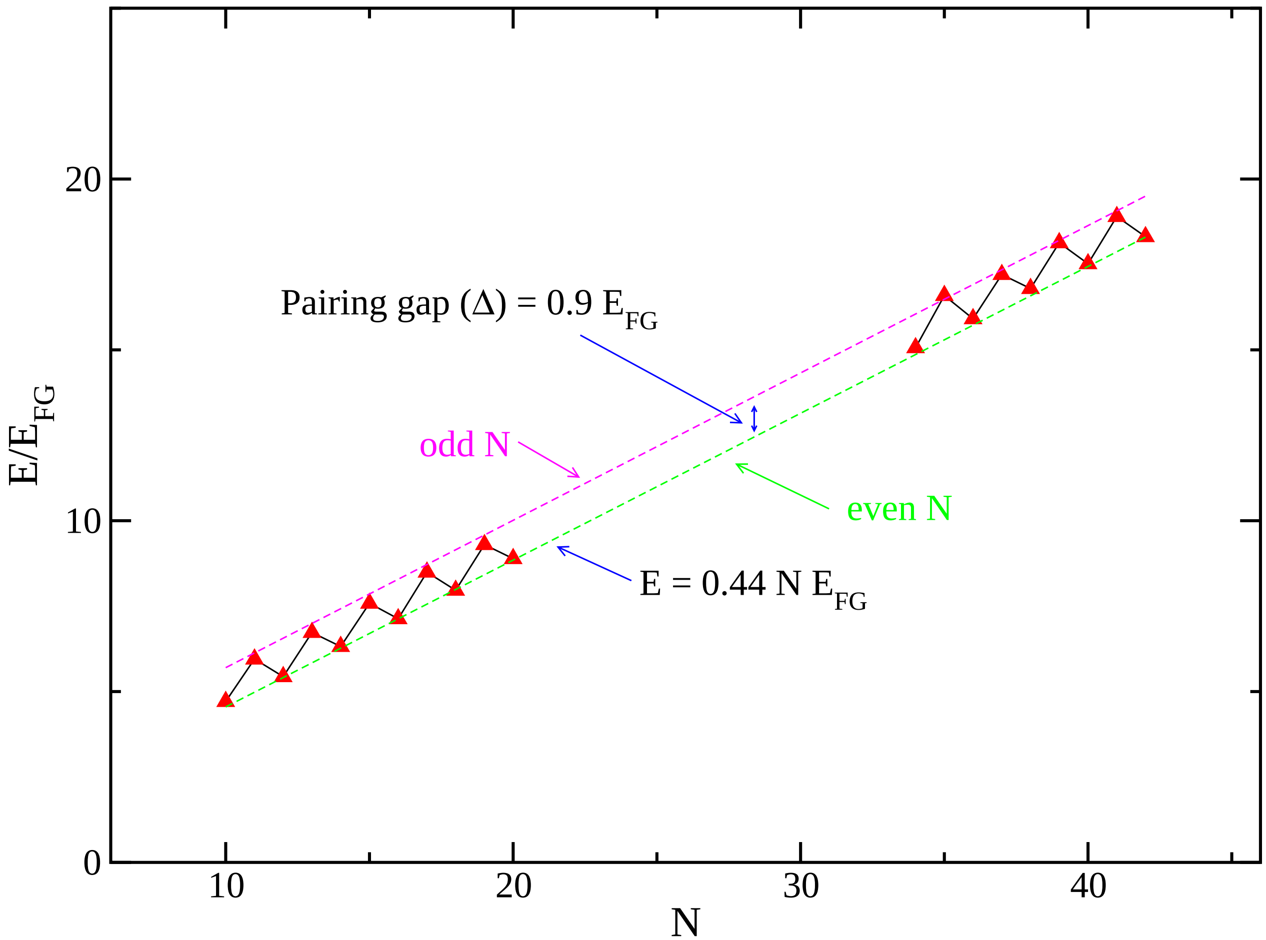
Epsilon expansion: $d = 4 - \epsilon$ ($d = 4$ non-interacting Bose gas)




$$\xi = \frac{1}{2}\epsilon^{3/2} + \frac{1}{16}\epsilon^{5/2}\ln\epsilon - 0.0246\,\epsilon^{5/2} + \ldots$$

$$\xi(\epsilon = 1) = 0.475$$

# Green function MC

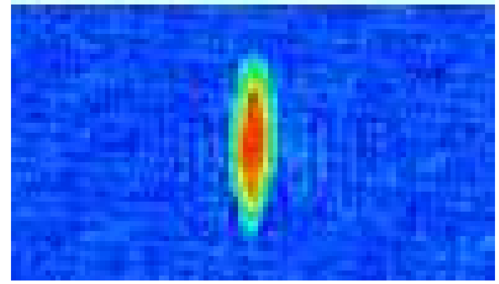



$\xi = 0.40\text{-}0.44$ (Carlson et al.)

# Experiment: Equation of state

Harmonic trap: $\xi$ determined by cloud size (Virial theorem)

$$\langle \mathcal{E} \rangle = \frac{3}{2} m\omega_x^2 \langle x^2 \rangle$$

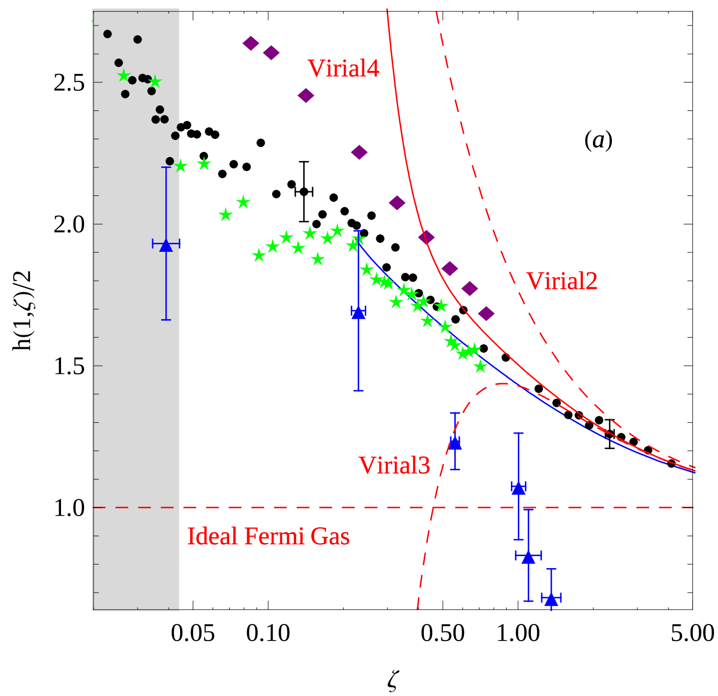


$\xi = 0.38(2)$ (Luo, Thomas)

Harmonic trap: $f(z)$ determined by twice integrated column density (Gibbs-Duhem)

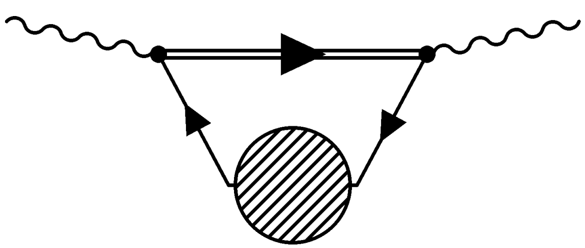
$$P(\mu(x), T) = \frac{m\omega_\perp^2}{\pi} \tilde{n}(x)$$

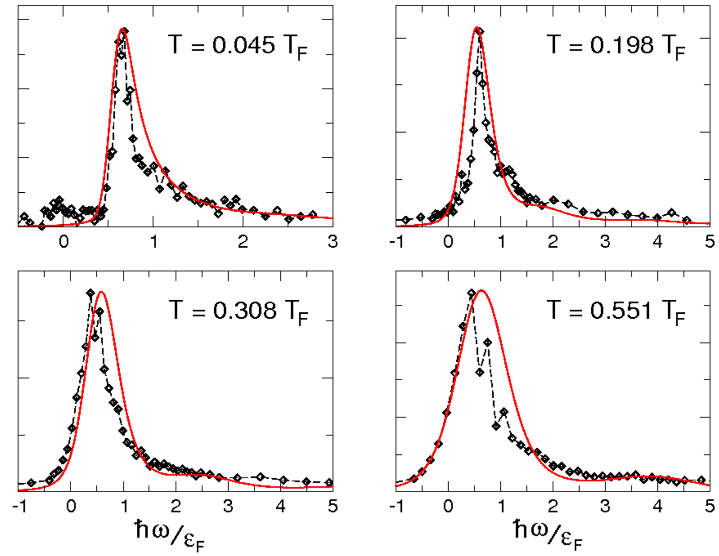
Nascimbene et al, Science (2010).



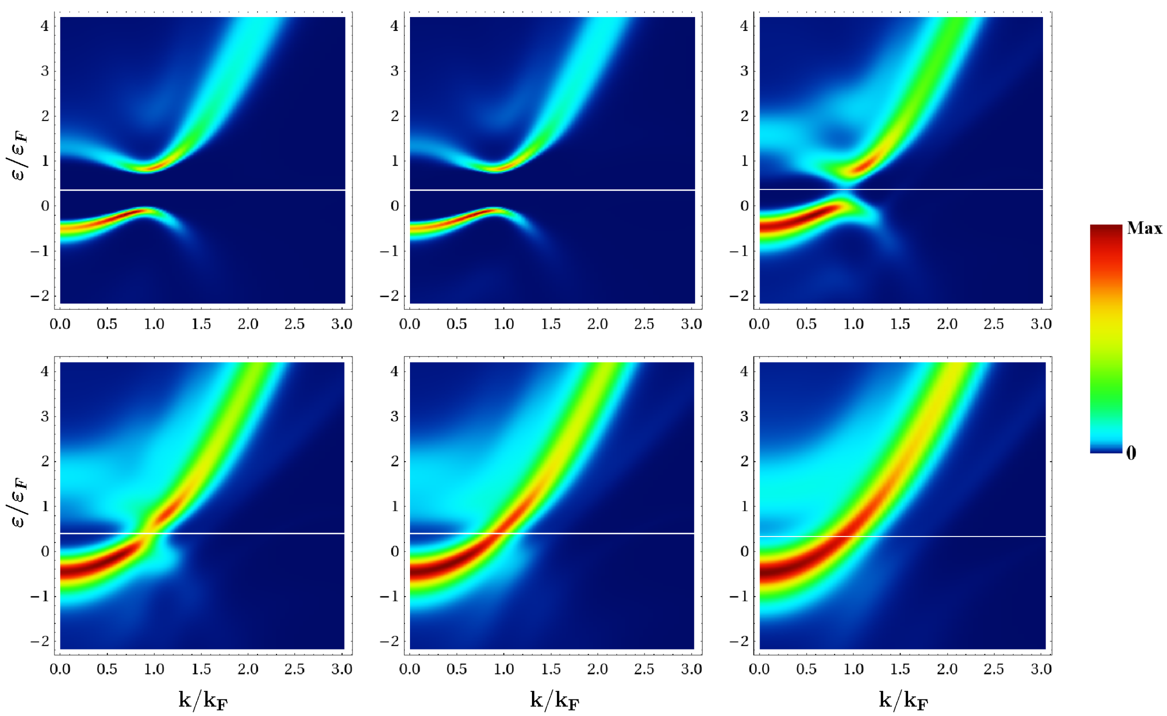
# RF spectroscopy

$$\Gamma(\omega) \sim Im \int dt\, d^3x\, e^{i\omega t} \langle \psi_1^\dagger \psi_3(x,t) \psi_3^\dagger \psi_1 \rangle$$






Schirotzek et al.(2008)




$T/T_F$

0.01,0.06,0.14

0.16,0.18,0.30

Zwerger et al. (2010)

## The Fermi gas in equilibrium: where are we?

Thermodynamics well under control (numerically and experimentally)

Theoretical approaches (BCS/BEC crossover, T-matrix, ERG, . . .) “work”

Evidence for quasi-particles at large $q$ and $T$