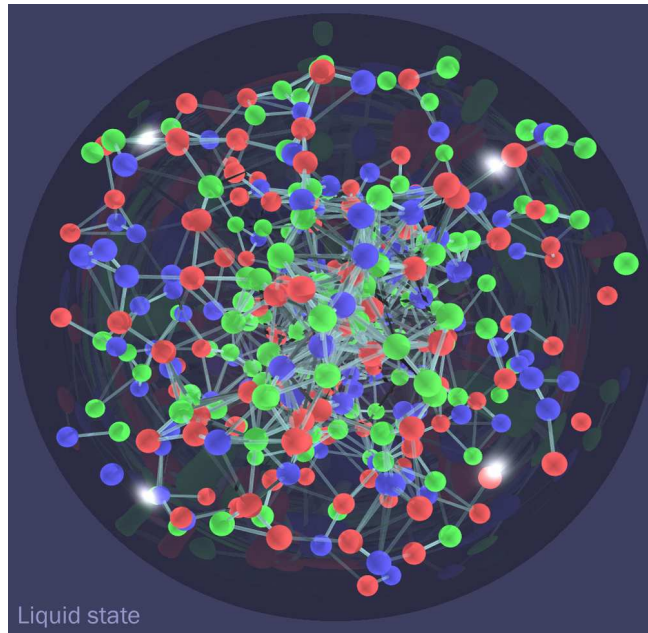
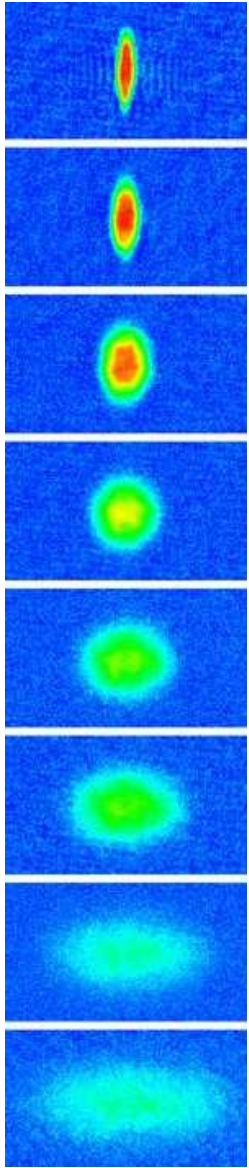


From Trapped Atoms To Liberated Quarks

Thomas Schaefer

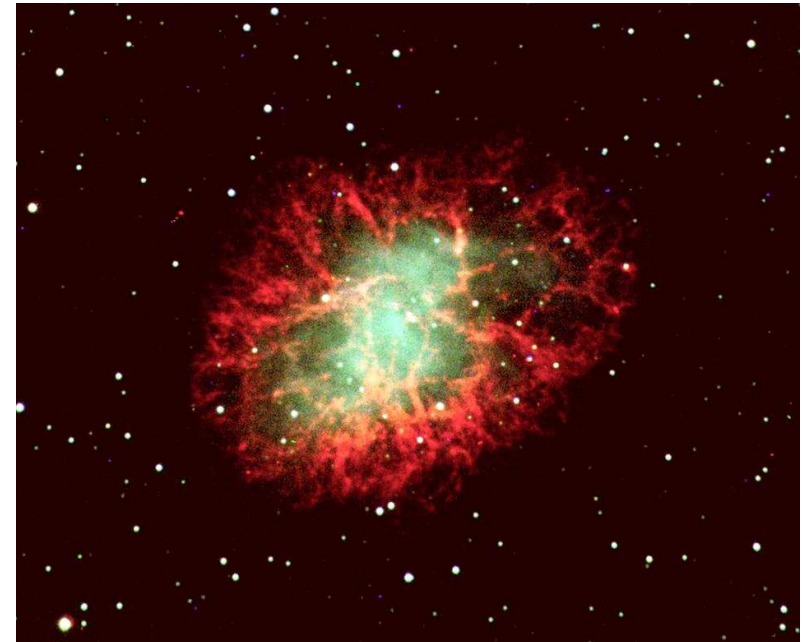
North Carolina State University

Perfect Liquids



sQGP ($T=180$ MeV)

Trapped Atoms ($T=1$ neV)



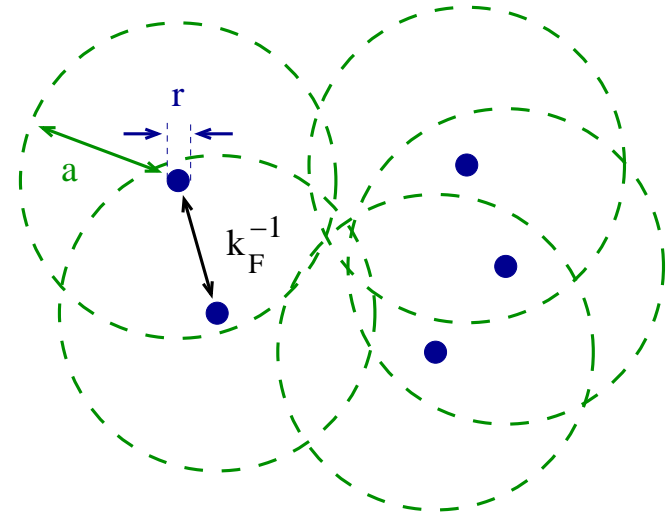
Neutron Matter ($T=1$ MeV)

Universality

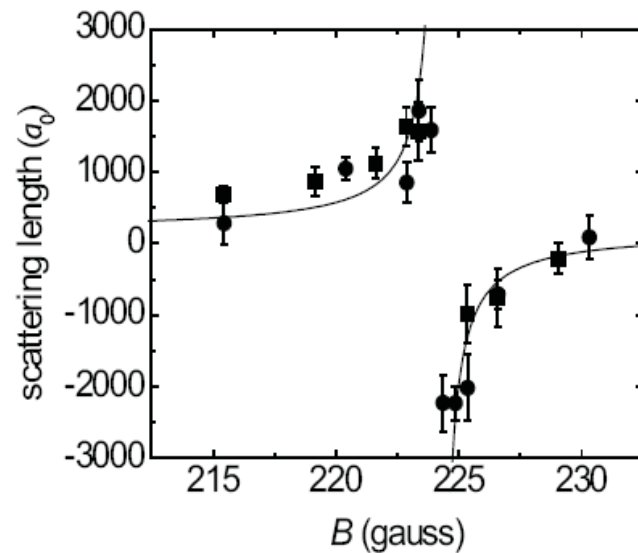
What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

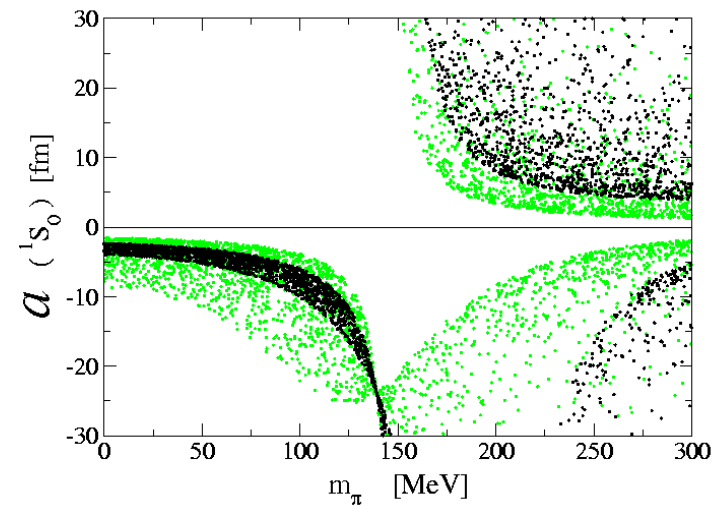
strongly correlated: $a\rho^{1/3} \gg 1$



Feshbach Resonance in ^6Li



Neutron Matter



Questions

Equation of State

Critical Temperature

Transport: Shear Viscosity, ...

Stressed Pairing

I. Equation of State

Universal Equation of State

Consider limiting case (“Bertsch” problem)

$$(k_F a) \rightarrow \infty \qquad (k_F r) \rightarrow 0$$

Universal equation of state

$$\frac{E}{A} = \xi \left(\frac{E}{A} \right)_0 = \xi \frac{3}{5} \left(\frac{k_F^2}{2M} \right)$$

How to find ξ ?

Numerical Simulations

Experiments with trapped fermions

Analytic Approaches

Effective Field Theory

Effective field theory for pointlike, non-relativistic neutrons

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right] + \dots$$

$$(a, r, \dots) \Rightarrow (C_0, C_2, \dots)$$

Partition Function (Hubbard-Stratonovich field s , Fermion matrix Q)

$$Z = \int Ds \exp[-S'], \quad S' = -\log(\det(Q)) + V(s)$$

$$C_0 < 0 \text{ (attractive): } \det(Q) \geq 0$$

Continuum Limit

Fix coupling constant at finite lattice spacing

$$\frac{M}{4\pi a} = \frac{1}{C_0} + \frac{1}{2} \sum_{\vec{p}} \frac{1}{E_{\vec{p}}}$$

Take lattice spacing b, b_τ to zero

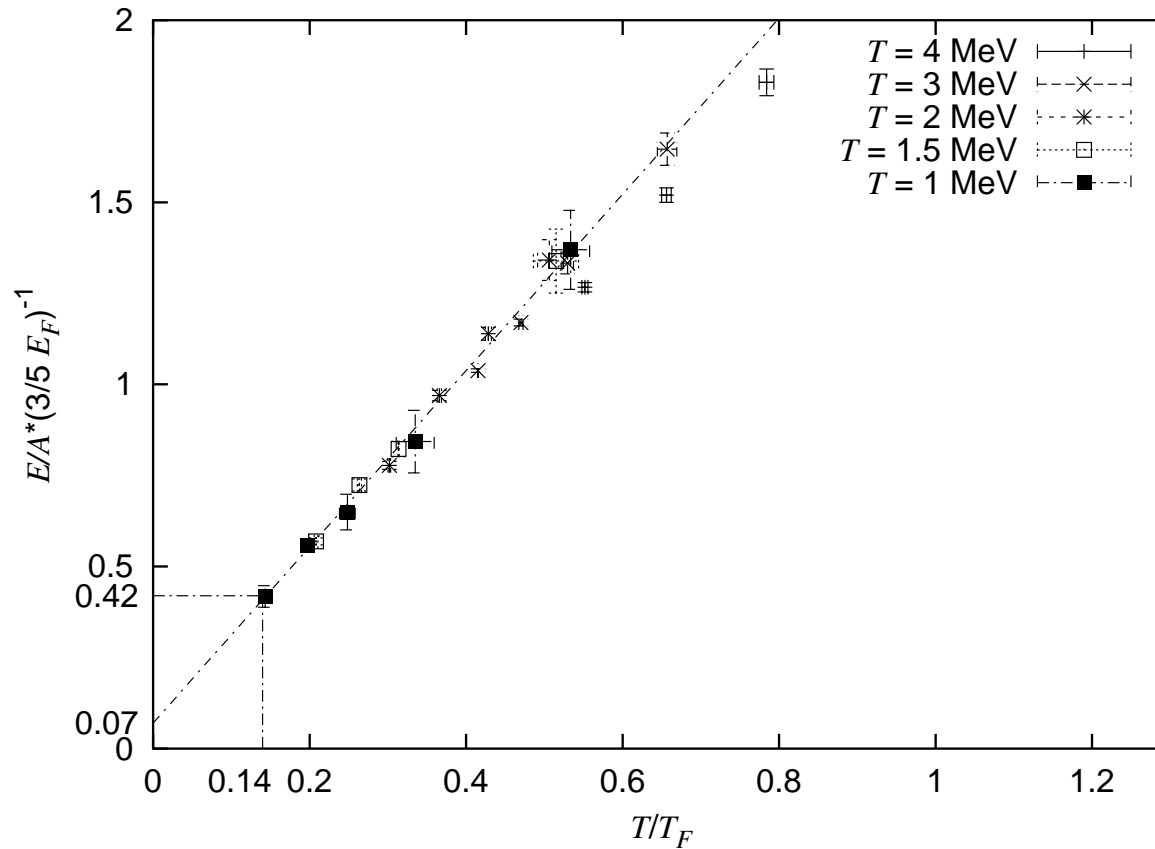
$$\mu b_\tau \rightarrow 0 \quad n^{1/3} b \rightarrow 0 \quad n^{1/3} a = \text{const}$$

Physical density fixed, lattice filling $\rightarrow 0$

Consider universal (unitary) limit

$$n^{1/3} a \rightarrow \infty$$

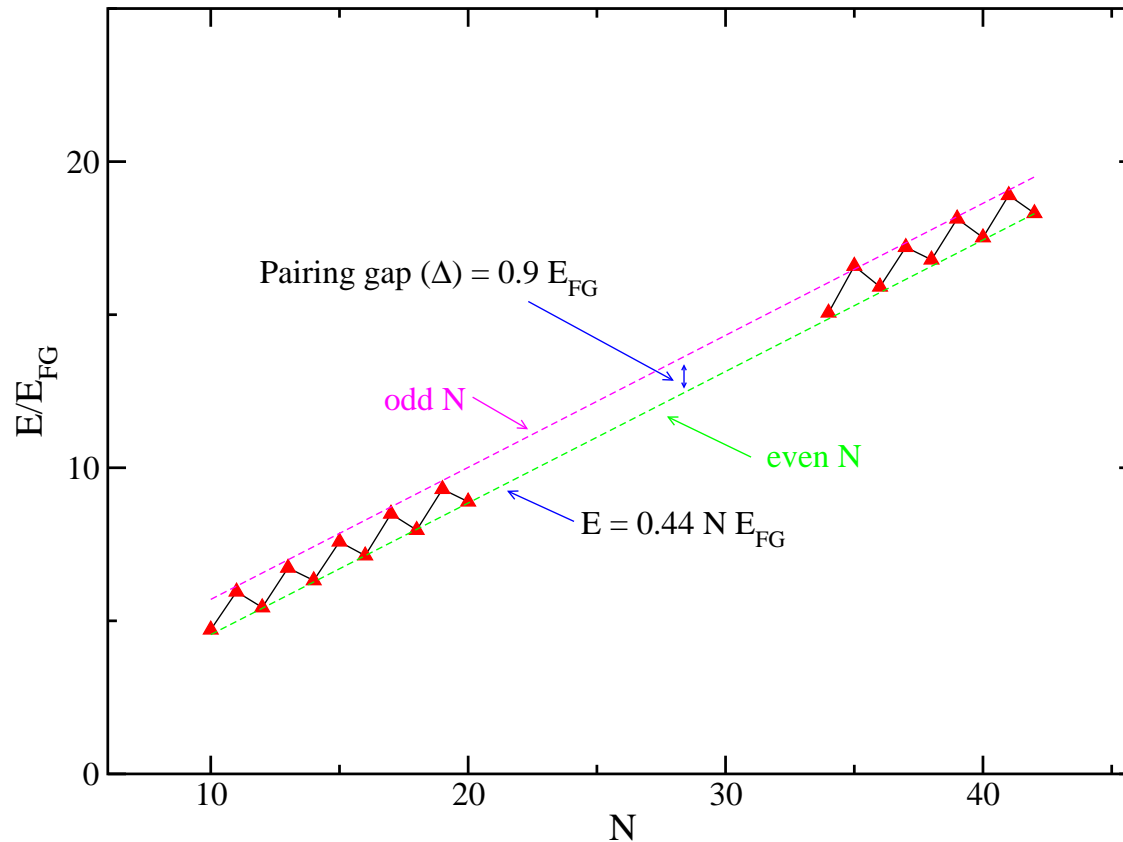
Lattice Results



Canonical $T = 0$ calculation: $\xi = 0.25(3)$ (D. Lee)

Not extrapolated to zero lattice spacing

Green Function Monte Carlo



Other lattice results: $\xi = 0.4$ (Burovski et al., Bulgac et al.)

Experiment: $\xi = 0.27^{+0.12}_{-0.09}$ [1], 0.51 ± 0.04 [2], 0.74 ± 0.07 [3]

[1] Bartenstein et al., [2] Kinast et al., [3] Gehm et al.

Large N approximation(s)

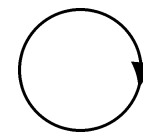
Large N gives mean field dynamics. What mean field?

Determined by symmetries of the interaction

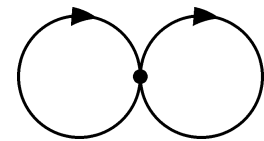
SU(2N) symmetric: $\rho = \langle \psi^\dagger \psi \rangle$ (Hartree-Fock + RPA)

$$\mathcal{L} = C_0 (\psi_f^\dagger \psi_f)^2$$

$$\xi \rightarrow \infty$$



N

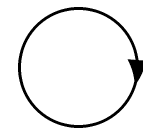


$N(C_0 N)$

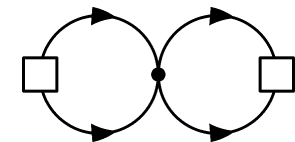
Sp(2N) symmetric: $\Phi = \langle \psi_f \mathcal{J}^{fg} \psi_g \rangle$ (BCS + fluctuations)

$$\mathcal{L} = C_0 |\psi_f \mathcal{J}^{fg} \psi_g|^2$$

$$\xi = 0.591 - 0.312/N + \dots = 0.279$$



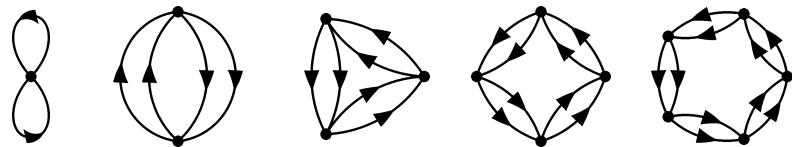
N



$N(C_0 N)$

Dimensional Expansions

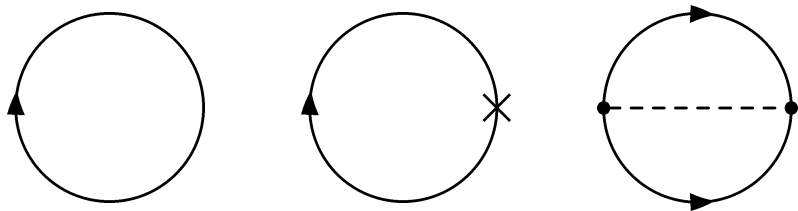
Large d expansion: Bethe-Brueckner (hole line) expansion



$$\xi = \frac{1}{2} + O(1/d)$$

$$(C_0/d)^k \cdot 1/d$$

$\epsilon = 4 - d$ expansion: Weakly interacting bosons (dimers) and fermions



$$\xi = \frac{1}{2}\epsilon^{3/2} + \frac{1}{16}\epsilon^{5/2} \ln \epsilon + \dots$$

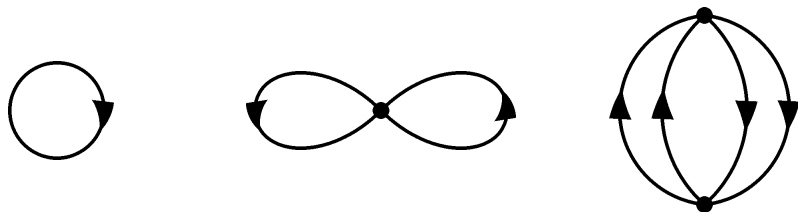
$$\xi(\epsilon=1) = 0.475 - 0.455 N^2 LO$$

$O(1)$

$O(1)$

$O(\epsilon)$

$\bar{\epsilon} = d - 2$ expansion: Weakly interacting fermions



$$\xi = 1 - \bar{\epsilon} + O(\bar{\epsilon}^2)$$

$$\xi(\bar{\epsilon}=1) = 0$$

$O(1)$

$O(\bar{\epsilon})$

$O(\bar{\epsilon}^2)$

Holographic Duals at Finite Temperature

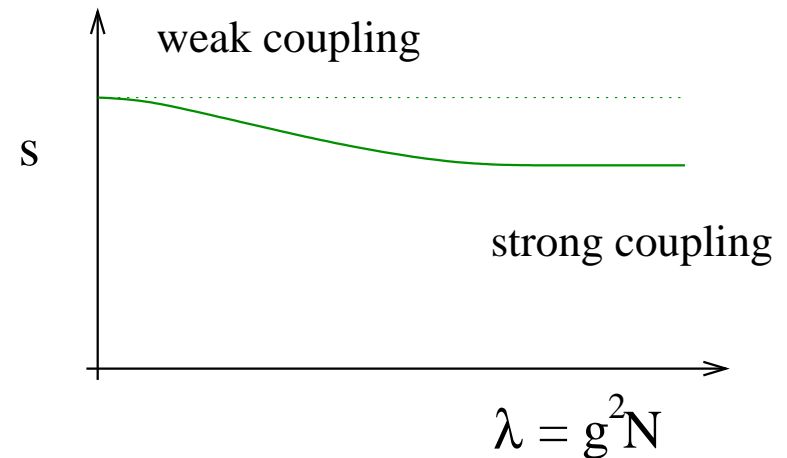
Thermal (conformal) field theory \equiv AdS_5 black hole

CFT temperature \Leftrightarrow Hawking temperature of
black hole

CFT entropy \Leftrightarrow Hawking-Bekenstein entropy
= area of event horizon

$$s = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0$$

Gubser and Klebanov

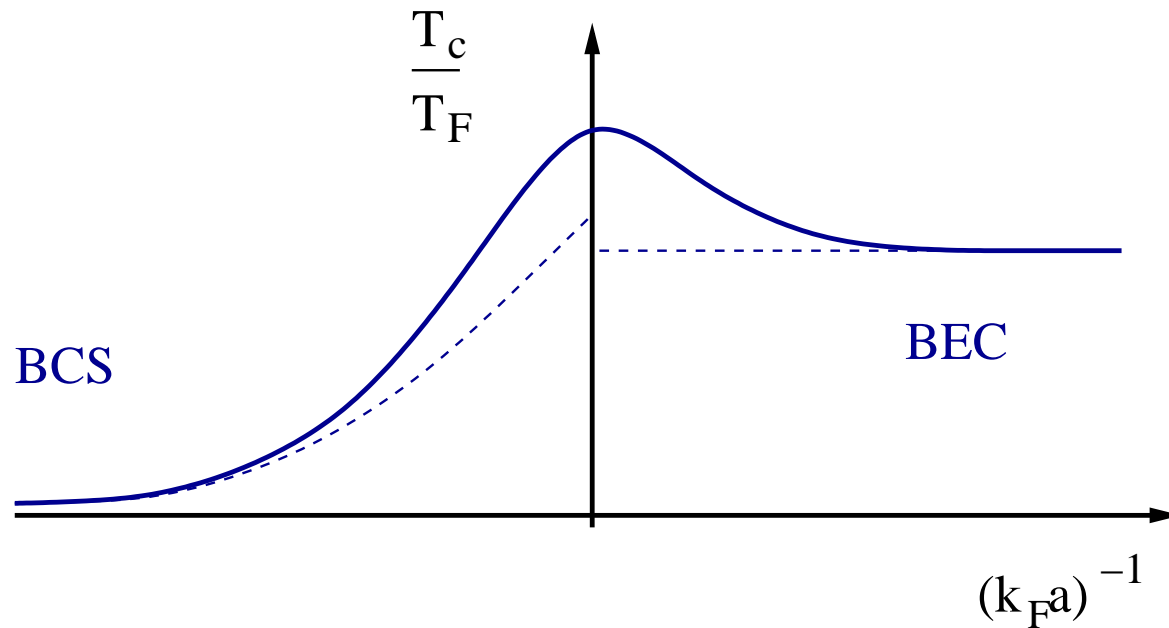


Extended to transport properties by Policastro, Son and Starinets

$$\eta = \frac{\pi}{8} N_c^2 T^3$$

II. How Large Can T_c Get?

Critical Temperature: From BCS to BEC



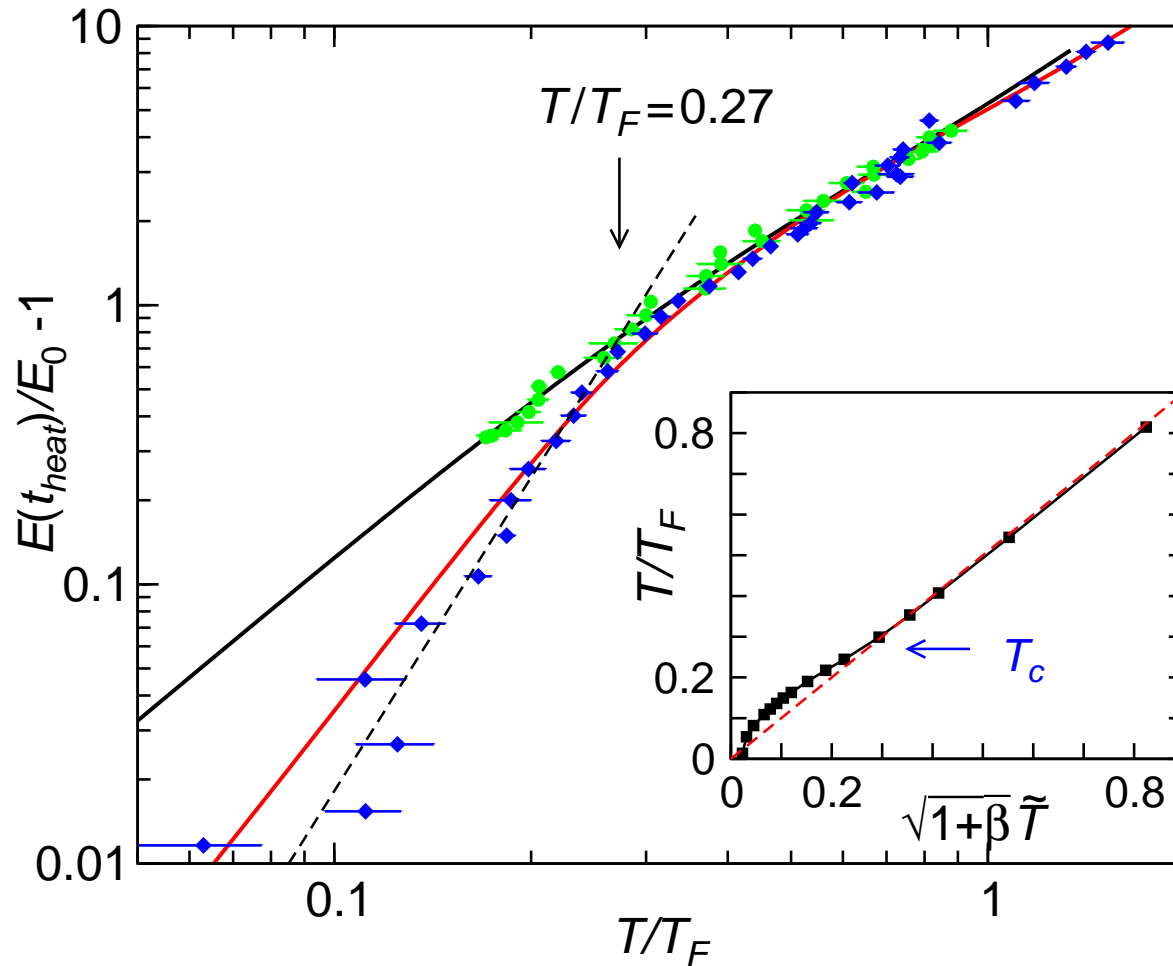
$$T_c^{BCS} = \frac{4 \cdot 2^{1/3} e^\gamma}{e^{7/3} \pi} \epsilon_F \exp\left(-\frac{\pi}{|k_F a|}\right)$$

$$T_c^{BEC} = 3.31 \left(\frac{n^{2/3}}{m}\right)$$

$$T_c(a \rightarrow \infty) = 0.28 \epsilon_F$$

$$T_c = 0.21 \epsilon_F + O(a_B n^{1/3})$$

Experimental Results



Kinast et al. (2005)

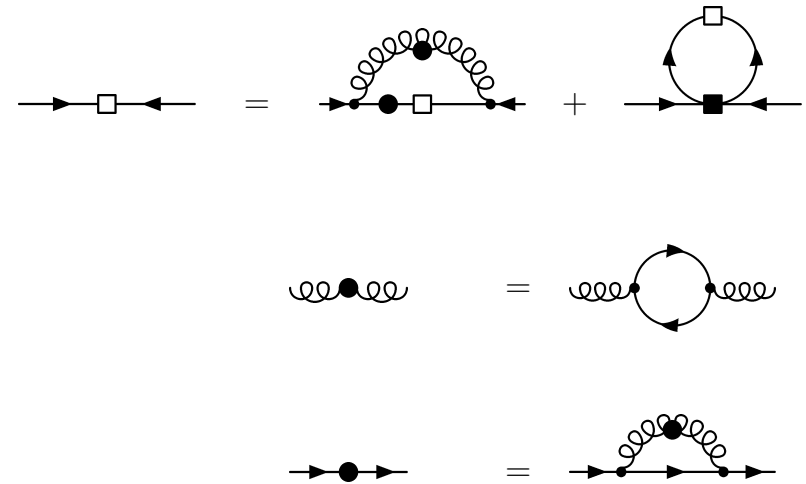
Lattice results: $T_c/T_F = 0.15$ (UMass)

Quark Matter: Color Superconductivity

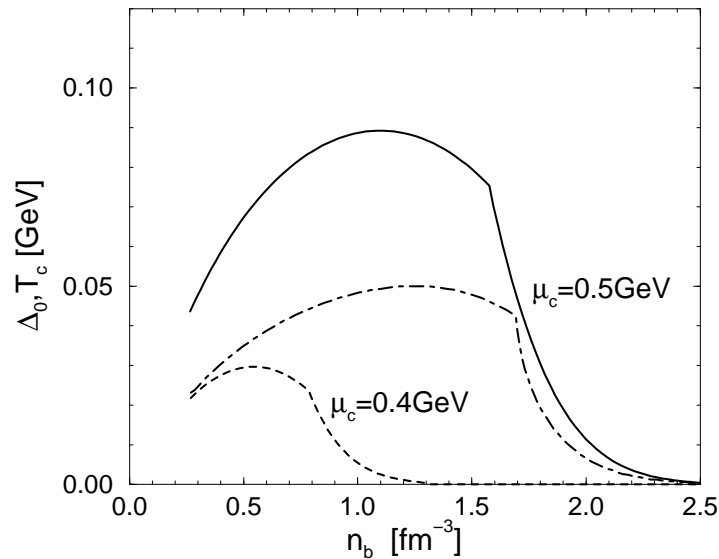
Weak coupling result

$$\frac{T_c}{T_F} = \frac{be^\gamma}{\pi} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

$$b = 512\pi^4 g^{-5} (2/N_f)^{\frac{5}{2}} e^{-\frac{\pi^2+4}{8}}$$



Maximum $T_c/T_F = 0.025$. Strong coupling?

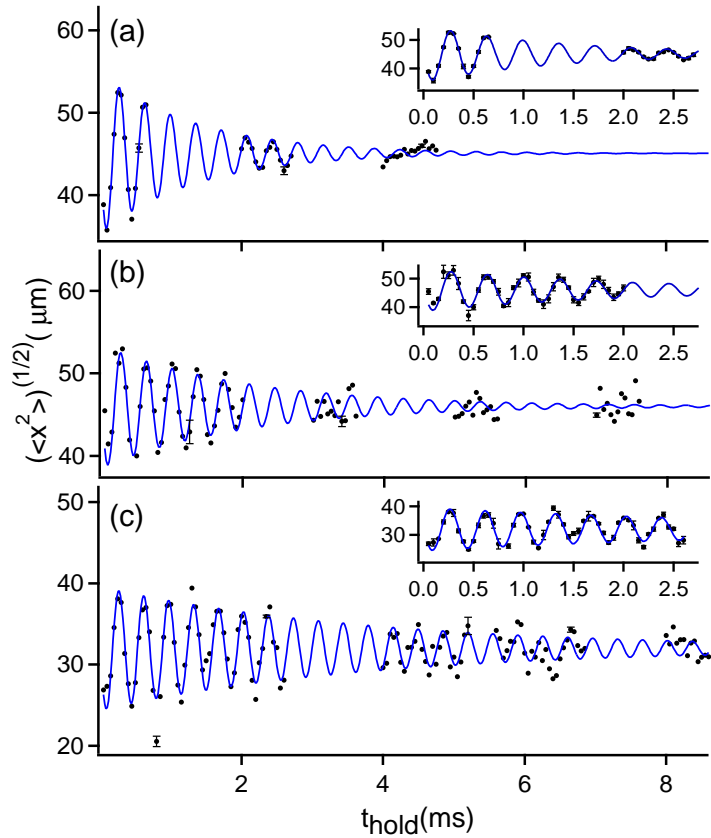


Find $T_c/T_F \simeq 0.2$

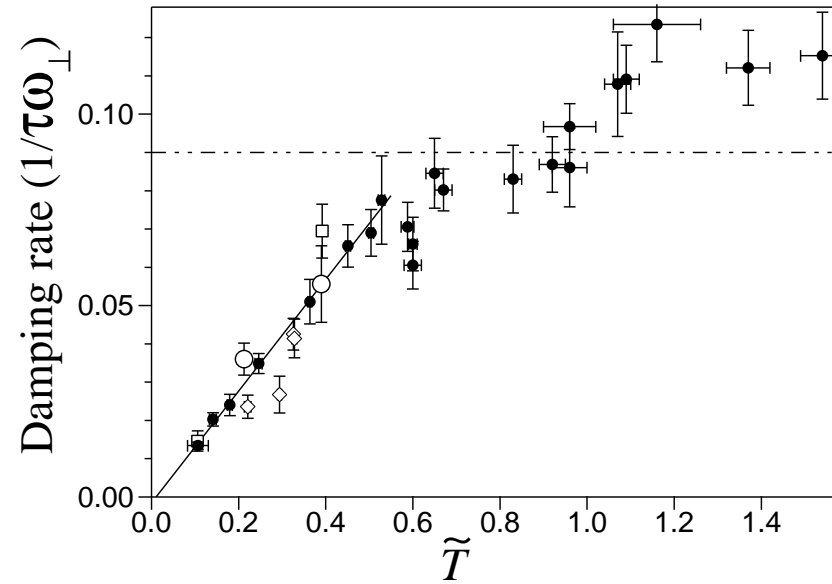
Note: Transition to χSB
Consider $N_c = 2$ QCD?

III. Transport Properties

Damping of Collective Excitations



$$T/T_F = (0.5, 0.33, 0.17)$$



$\tau\omega$: decay time \times trap frequency

Kinast et al. (2005)

Viscous Hydrodynamics

Energy dissipated due to viscous effects is

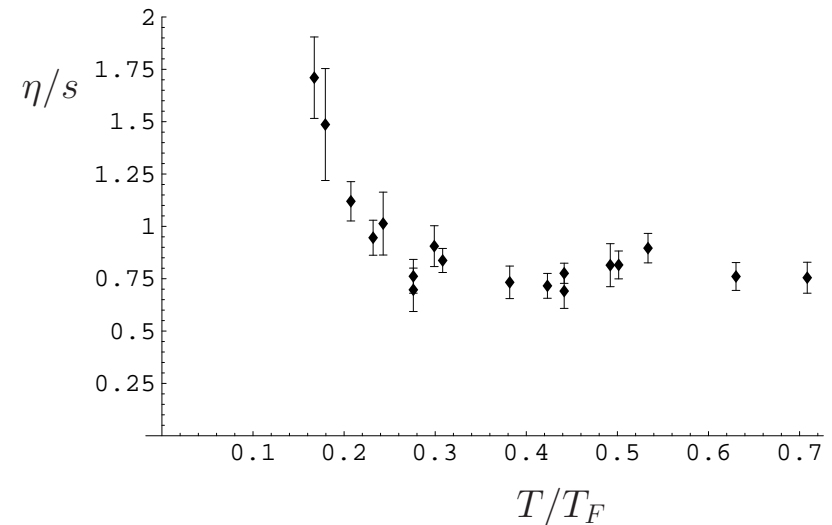
$$\dot{E} = -\frac{\eta}{2} \int d^3x \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right)^2 - \zeta \int d^3x (\partial_i v_i)^2,$$

η, ζ : shear, bulk viscosity

Shear viscosity to entropy ratio

$$\frac{\eta}{s} \sim c_i \times \frac{\Gamma}{\omega_{\perp}} \times \frac{\mu}{\omega_{\perp}} \times \frac{N}{S}$$

c_i determined by Hydro solution



Problems: Scaling with N ; T dependence below T_c

IV. Stressed Pairing

Polarized Fermions: From BEC to BCS

BEC limit: Tightly bound bosons, no polarization for $\delta\mu < \Delta$

$\delta\mu > \Delta$: Mixture of Fermi and Bose liquid, no phase separation

BCS limit: No homogeneous mixed phase

$\delta\mu > \delta\mu_{c1}$: LOFF pairing $\Delta(x) = e^{iqx} \Delta$

Onset? Consider EFT for gapless fermions interacting with GB's

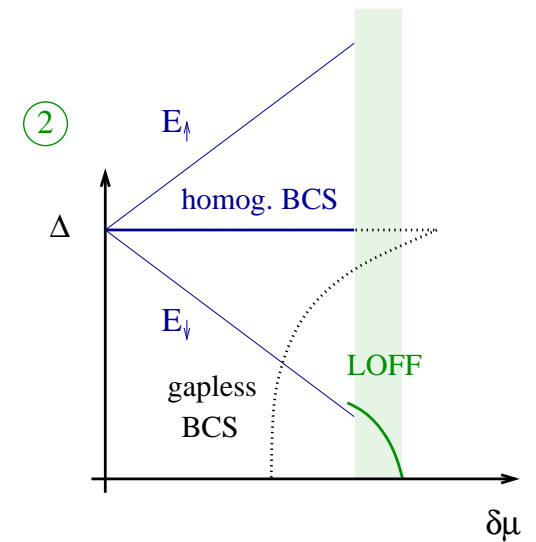
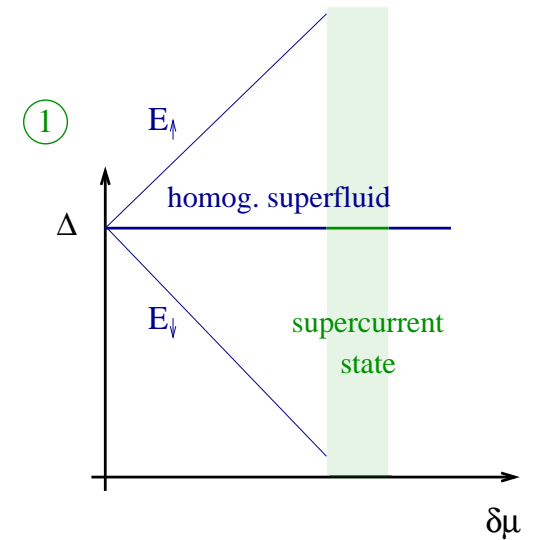
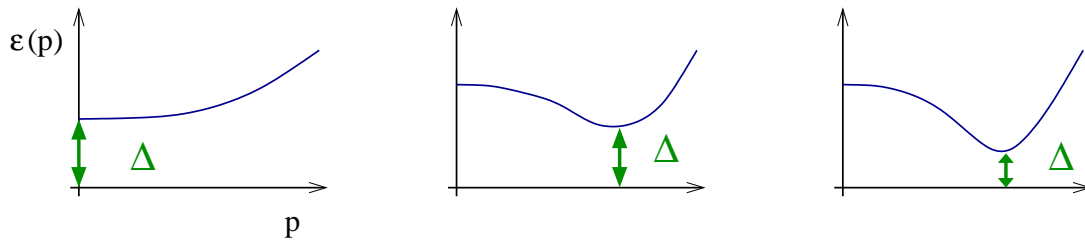
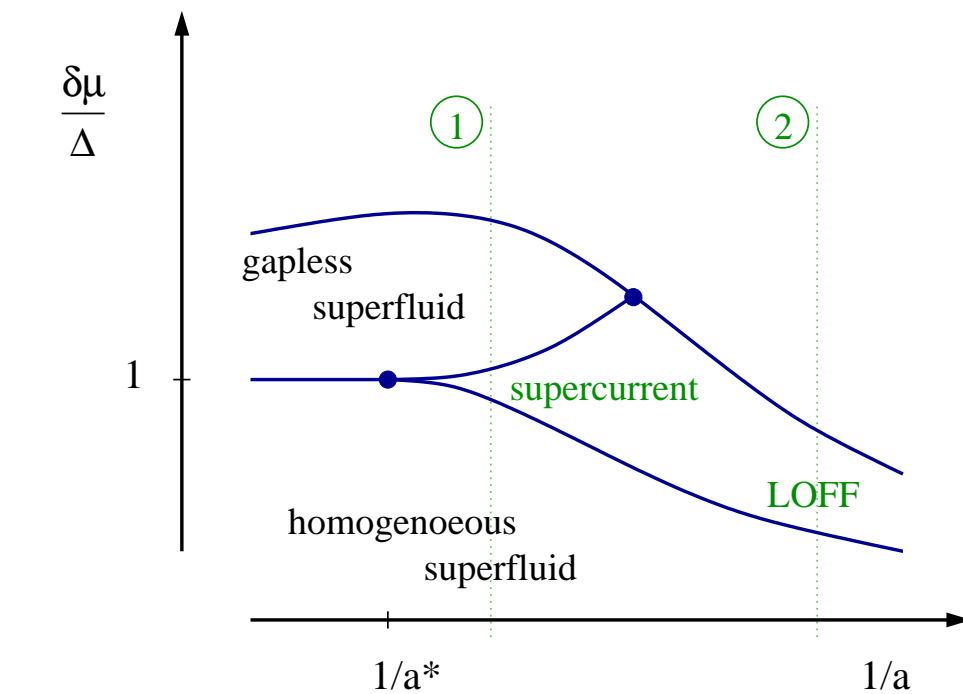
$$\mathcal{L} = \psi^\dagger \left(i\partial_0 - \epsilon(-i\vec{\partial}) - (\vec{\partial}\varphi) \cdot \frac{\overleftrightarrow{\partial}}{2m} \right) \psi + \frac{f_t^2}{2} \dot{\varphi}^2 - \frac{f^2}{2} (\vec{\partial}\varphi)^2$$

Free energy of state with non-zero current $v_s = \partial\varphi/m$

$$F(v_s) = \frac{1}{2} nmv_s^2 + \int \frac{d^3p}{(2\pi)^3} \epsilon_v(\vec{p}) \Theta(-\epsilon_v(\vec{p}))$$

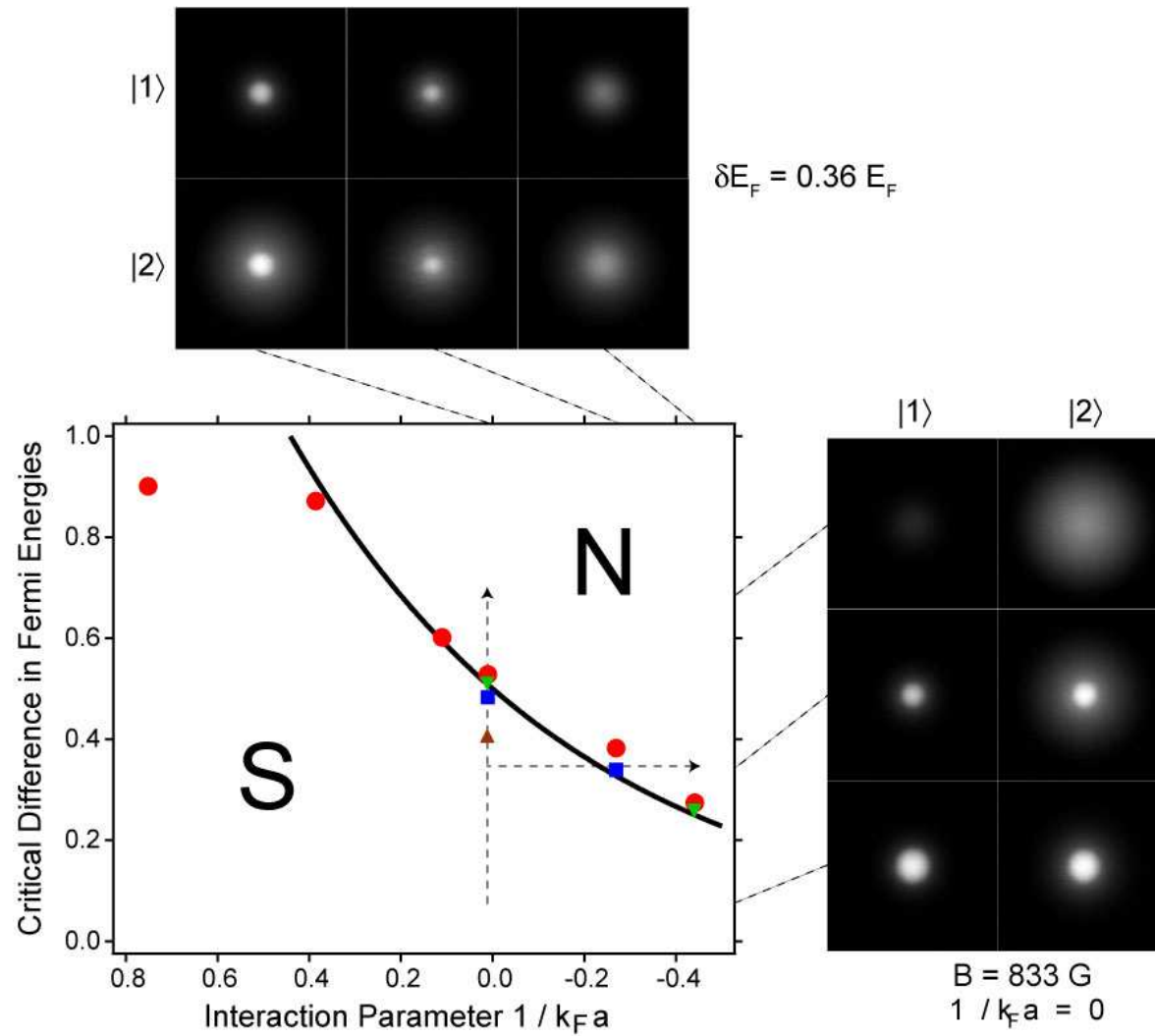
Unstable for BCS-type dispersion relation

Minimal Phase Diagram



Son & Stephanov (2005)

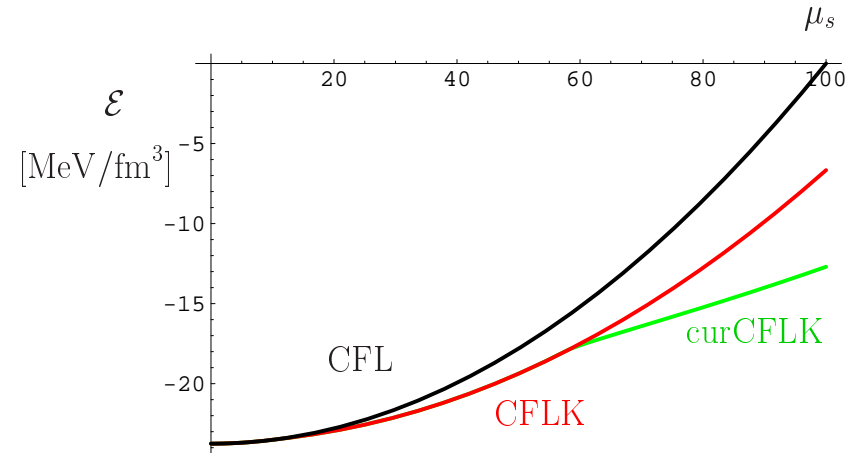
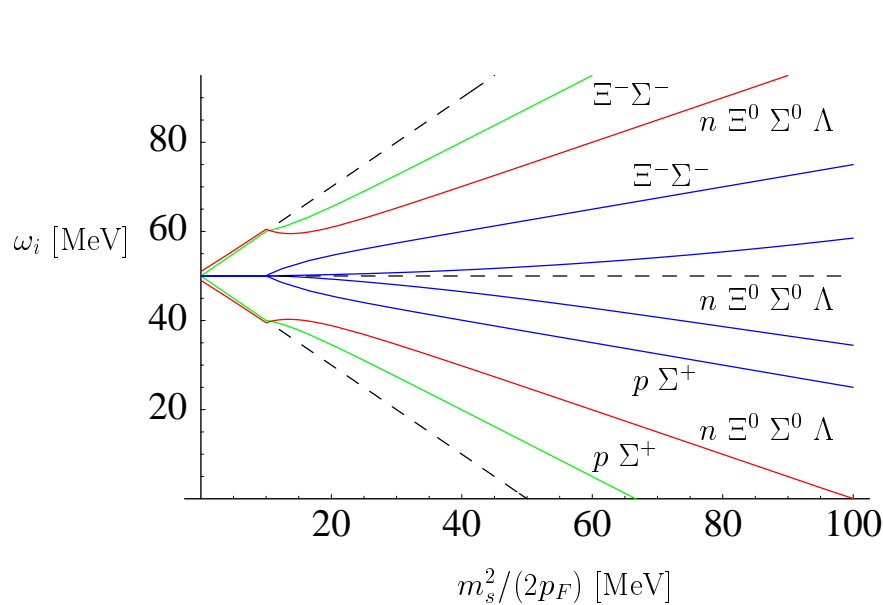
Experimental Situation



Zwierlein et al. (MIT group)

Phase Structure of CFL Quark Matter

How does CFL ($\langle ud \rangle = \langle ds \rangle = \langle su \rangle$) pairing responds to m_s ?



Excitation energy of fermions

Gapless modes appear at

$$\mu_s(\text{crit}) \sim 0.75\Delta$$

Energy density of superfluid phases

$$\mu_s(K - \text{cond}) \sim m_u^{2/3} \Delta^{4/3} / \mu$$

$$\mu_s(GB - \text{cur}) \sim 0.75\Delta$$

Figures: Kryjevski & Schäfer (2004)

Schäfer (2005)

Trapped atoms provide interesting model system

equation of state of strongly correlated systems
(neutron matter, sQGP)

viscosity of strongly correlated systems (sQGP?)

superfluidity at strong coupling (T_c/T_F , response to
pair breaking fields, precursor phenomena)