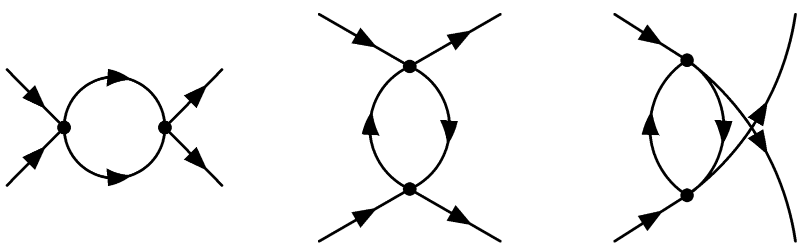
Gerry and Fermi Liquid Theory

Thomas Schaefer

North Carolina State



BCS ZS ZS'

Introduction

I learned about Fermi liquid theory (FLT) from Gerry. I was under the impression that the theory amounted to the operation

$$m \quad \longrightarrow \quad m^*$$

I only later realized that Gerry had a very sophisticated understanding of FLT (see his many-body lectures).

One of Gerry's legacies is his insistence that the nuclear many body problem should be understood in terms of quasi-particles and effective interactions.

Some of the best work along these lines was done in Stony Brook (and Darmstadt) since the late 90s, based on $V_{low\,k}$ and the renormalization group.

Many improvements could have been made, especially in Chapter XIII on effective forces in nuclei, but time is short, and I shall make them in later editions, when I am too old to ski. Of course, nobody will be interested in the subject by then.

Unified Theory of Nuclear Models and Forces
3rd edition, 1970.

Fermi liquid theory a la Landau

In a cold Fermi system the low energy excitations are spin 1/2 quasi-particles. Define a distribution function $f_p = f_p^0 + \delta f_p$. Then

$$\mathcal{E} = \mathcal{E}_0 + \int \frac{\delta \mathcal{E}}{\delta f_p} \delta f_p + \frac{1}{2} \int \int \frac{\delta^2 \mathcal{E}}{\delta f_p \delta f_{p'}} \delta f_p \delta f_{p'} + \ldots$$

$$E_p = \frac{\delta \mathcal{E}}{\delta f_p} \qquad\qquad t_{pp'} = \frac{\delta^2 \mathcal{E}}{\delta f_p \delta f_{p'}}$$

The distribution function satisfies a Boltzmann equation

$$\Big(\partial_t + v_p \cdot \nabla_r + F_p \cdot \nabla_p\Big) f_p(r,t) = C[f_p]$$

with $v_p = \nabla_p E_p$ and $F_p = -\nabla_r E_p$.

Fermi liquid theory a la Polchinski-Shankar

Free non-relativistic quasi-particles near Fermi surface

$$S = \int dt \int \frac{d^3p}{(2\pi)^3} \psi(p)^\dagger \left(i\partial_t - (\epsilon(p) - \epsilon_F)\right) \psi(p)$$

Expand momenta around Fermi momentum $\vec{p} = \vec{k} + \vec{l}$

$$\epsilon(p) - \epsilon_F = \vec{v}_F(k) \cdot \vec{l} + O(l^2)$$

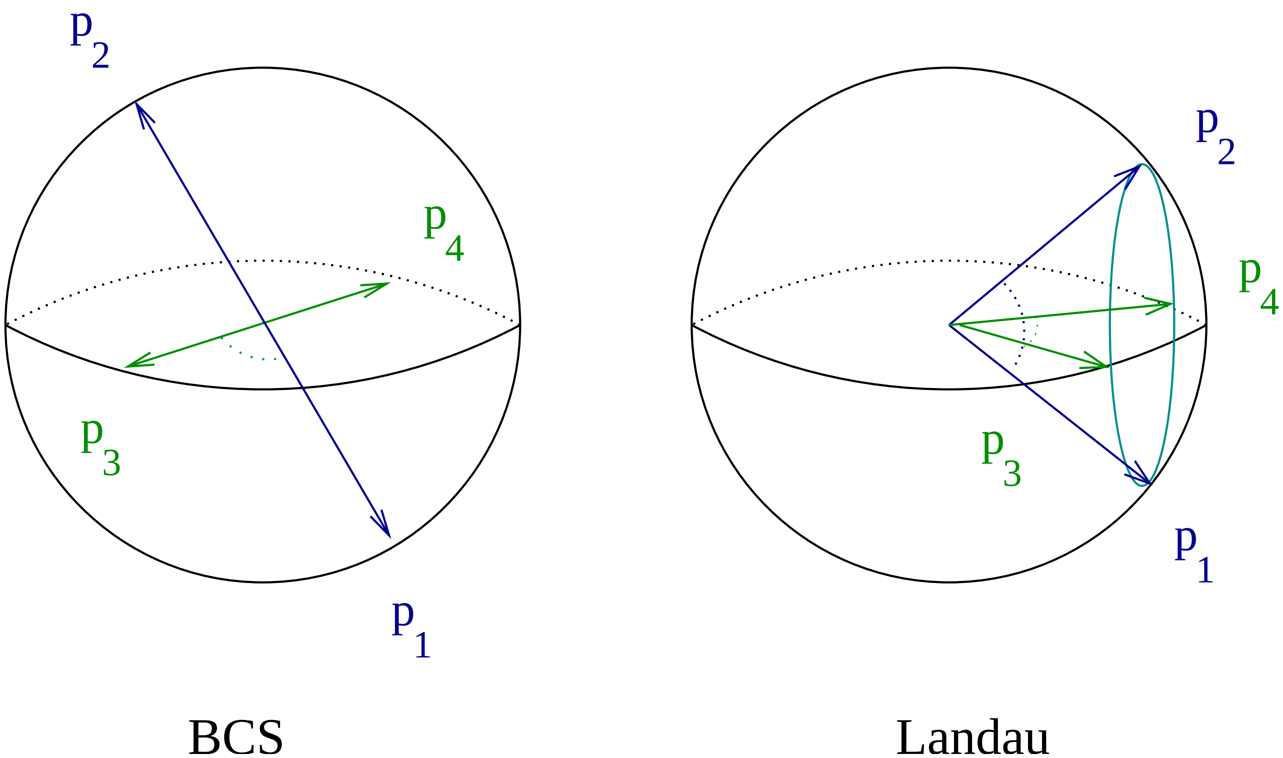
Study scaling behavior $\vec{l} \to s\vec{l}$. Scaling dimensions

$$[k] = 0, \qquad [l] = 1, \qquad [\partial_t] = 1, \qquad [d^3p] = 1, \qquad [\psi] = -\frac{1}{2}$$

Interaction

$$S_{int} = \int dt \left[\prod_{i=1}^{4} \int \frac{d^3p_i}{(2\pi)^3}\right] \psi^\dagger(p_4)\psi^\dagger(p_3)\psi(p_2)\psi(p_1)\delta^3(p_{tot})U(p_i)$$

Marginal Interactions

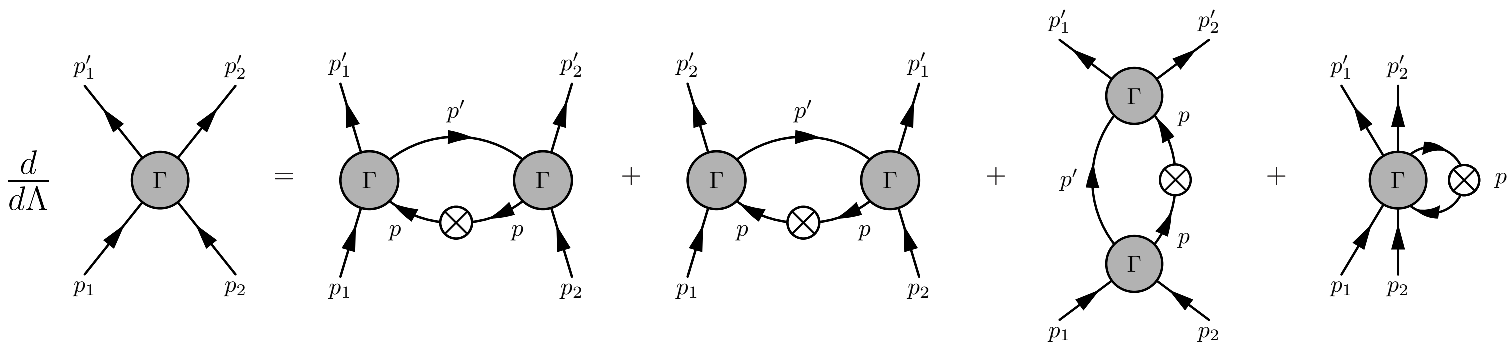


$$BCS: \qquad U(-\hat{p}_3, \hat{p}_3, -\hat{p}_1, \hat{p}_1) = V(\hat{p}_1 \cdot \hat{p}_3) = \sum_l V_l P_l(\hat{p}_1 \cdot \hat{p}_3),$$

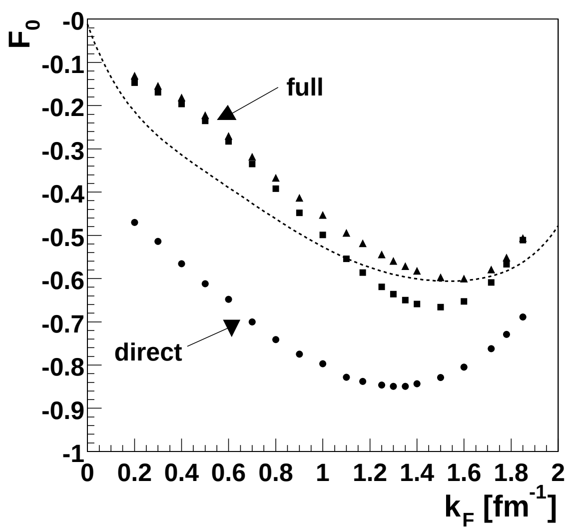
$$LFL: \qquad U(\hat{p}_4, \hat{p}_3, \hat{p}_2, \hat{p}_1)|_{\hat{p}_1 \cdot \hat{p}_2 = \hat{p}_3 \cdot \hat{p}_4} = F(\hat{p}_1 \cdot \hat{p}_2, \phi_{12,34})$$

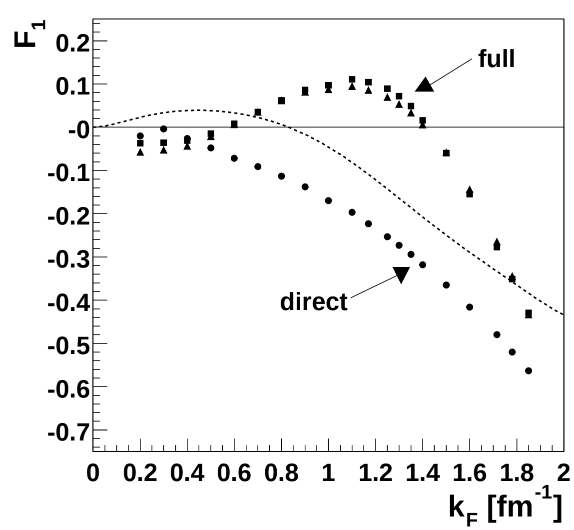
Fermi surface RG in nuclear physics

Achim and Bengt: Evolve $V_{low\ k}$ towards Fermi surface



Fermi liquid parameter $t_{pp'} = [F_l + G_l(\sigma\cdot\sigma')]P_l(\cos\theta)$



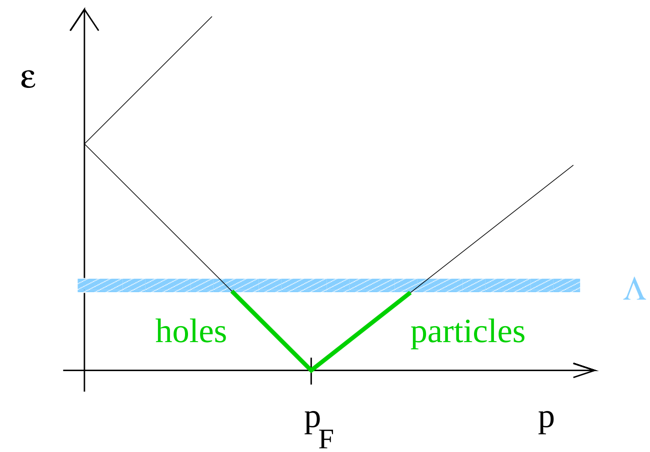


Non-FLT in QCD: High Density Effective Theory

QCD lagrangian

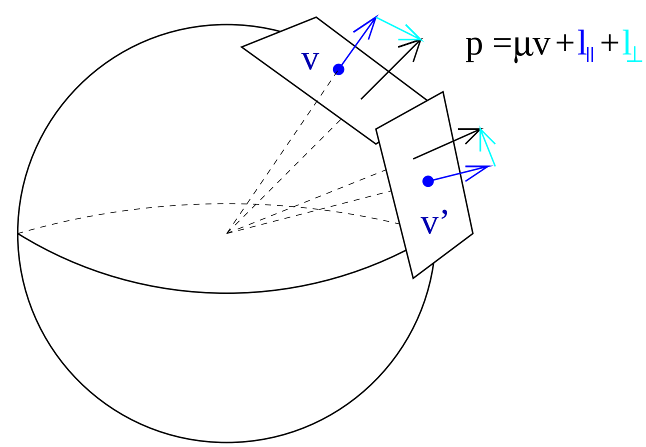
$$\mathcal{L} = \bar{\psi}\left(i\not{D} + \mu\gamma_0 - m\right)\psi - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

Quasi-particles (holes)



$$E_\pm = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$

Effective field theory on v-patches



$$\psi_{v\pm} = e^{-i\mu v\cdot x}\left(\frac{1 \pm \vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

Effective Theory for $l < m$

$$\mathcal{L} = \psi_v^\dagger \left(iv \cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v + \mathcal{L}_{4f} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{v^\alpha v^\beta}{(v \cdot D)^2} G^b_{\mu\beta}$$

Transverse gauge boson propagator

$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\frac{\pi}{2} m^2 \frac{k_0}{|\vec{k}|}},$$

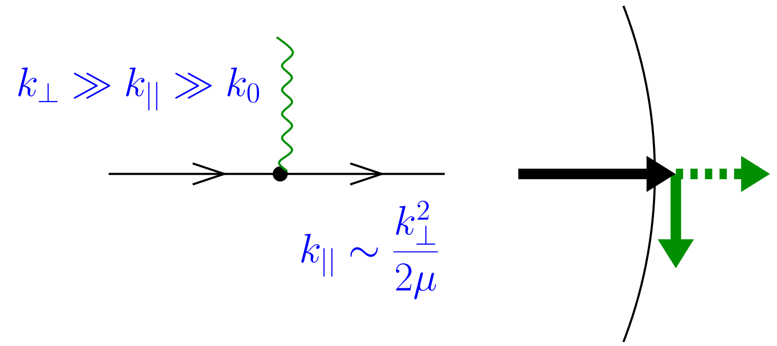
Scaling of gluon momenta

$$|\vec{k}| \sim k_0^{1/3} m^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

Non-Fermi Liquid Effective Theory

Gluons very spacelike $|\vec{k}| \gg |k_0|$. Quark kinematics?

$$k_0 \simeq k_{||} + \frac{k_\perp^2}{2\mu}$$



Scaling relations

$$k_\perp \sim m^{2/3} k_0^{1/3}, \qquad k_{||} \sim m^{4/3} k_0^{2/3}/\mu$$

Propagators

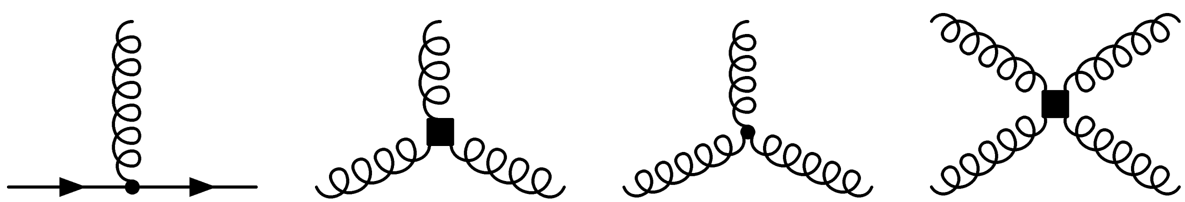
$$S_{\alpha\beta} = \frac{-i\delta_{\alpha\beta}}{p_{||} + \frac{p_\perp^2}{2\mu} - i\epsilon sgn(p_0)} \qquad D_{ij} = \frac{-i\delta_{ij}}{k_\perp^2 - i\frac{\pi}{2} m^2 \frac{k_0}{k_\perp}},$$

Non-Fermi Liquid Expansion

Scale momenta $(k_0, k_{||}, k_\perp) \to (sk_0, s^{2/3}k_{||}, s^{1/3}k_\perp)$

$$[\psi] = 5/6 \qquad [A_i] = 5/6 \qquad [S] = [D] = 0$$

Scaling behavior of vertices



$s^{1/6}$ $\qquad$ $s^{1/2}$ $\qquad$ $s^{5/6}$ $\qquad$ s

Systematic expansion in $\epsilon^{1/3} \equiv (\omega/m)^{1/3}$

Non-Fermi liquid effects

Quark self energy near "Fermi surface"

$$\Sigma(p) = \frac{g^2}{9\pi^2}\left(p_0 \log\left(\frac{2^{5/2}m}{\pi|p_0|}\right) + i\frac{\pi}{2}p_0\right) + O\left(\epsilon^{5/3}\right)$$

Luttinger: $G^{-1}|_{FS} = 0 \Rightarrow n = Vol(FSph)$

Quasi-particle velocity vanishes, IR freedom near Fermi surface

$$\frac{d\log\alpha_v}{d\Lambda} = +\frac{4\alpha_v^2}{9\pi} \qquad \alpha = \frac{g^2 v_F}{4\pi}$$

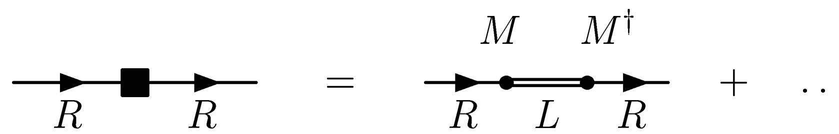
Enhanced color superconducting gap

$$\Delta = \mu\exp\left(-\frac{3\pi^2}{\sqrt{2}g} - 5\log(g) + \left[4\log(128\pi) - \frac{\pi^2+4}{8}\right] + \ldots\right)$$

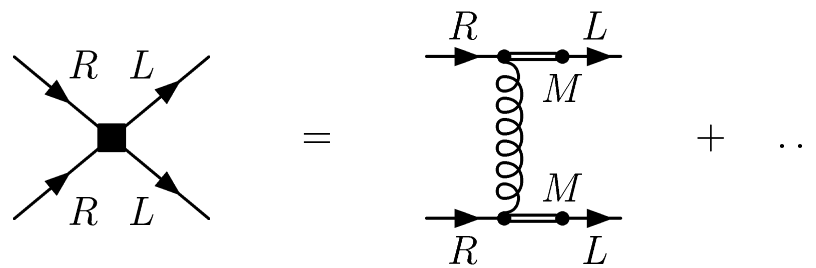
Unusual transport: $\eta \sim \mu^4 m^{2/3}/(g^4 T^{5/3}) \sim \mu^4/(g^{10/3}T)$

Remnants of Fermi liquid theory

Mass corrections to μ

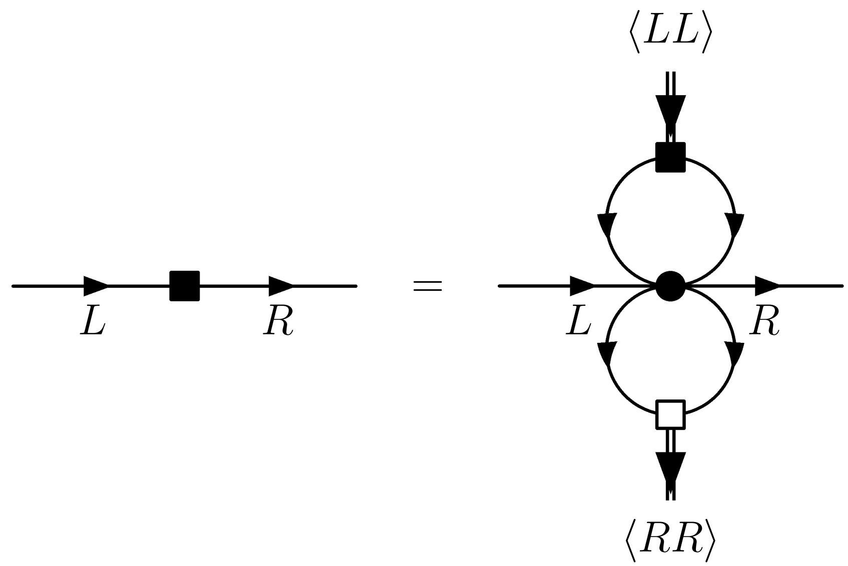


and scattering amplitudes



Instanton vertex:

χSB in CFL phase



The return of the master: FLT a la Landau

The discovery of nearly perfect fluidity in the QGP and in ultracold gases has led to (renewed) interest in transport properties as a measure of the quasi-particle interaction.

$$\left(\partial_t + v_p \cdot \nabla_r + F_p \cdot \nabla_p\right) f_p(r,t) = C[f_p]$$

Require consistency between transport and thermodynamics:

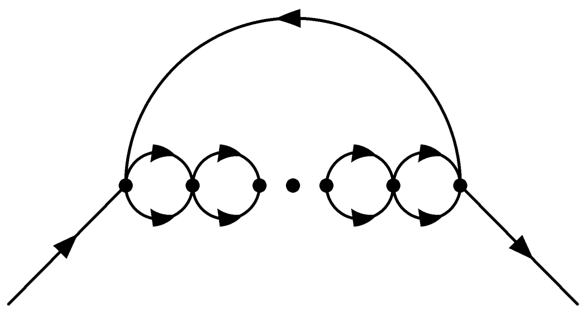
$$E_p = \frac{\delta \mathcal{E}}{\delta f_p} \qquad\qquad t_{pp'} = \frac{\delta^2 \mathcal{E}}{\delta f_p \delta f_{p'}}$$

Example I: Bulk viscosity in a dilute Fermi gas

Conformal symmetry breaking (thermodynamics)

$$1 - \frac{2\mathcal{E}}{3P} = \frac{C}{12\pi m a P} \sim \frac{1}{6\pi} n\lambda^3 \frac{\lambda}{a}$$

How does this translate into $\zeta \neq 0$? Momentum dependent $m^*(p)$.



$$Im\,\Sigma(k) \sim zT\sqrt{\frac{T}{\epsilon_k}}Erf\left(\sqrt{\frac{\epsilon_k}{T}}\right) \ll T$$

$$Re\,\Sigma(k) \sim zT\frac{\lambda}{a}\sqrt{\frac{T}{\epsilon_k}}F_D\left(\sqrt{\frac{\epsilon_k}{T}}\right)$$

Bulk viscosity

$$\boxed{\zeta = \frac{1}{24\sqrt{2}\pi}\lambda^{-3}\left(\frac{z\lambda}{a}\right)^2}$$

$$\zeta \sim \left(1 - \frac{2\mathcal{E}}{3P}\right)^2 \eta$$

Example II: Kinetics of the chiral magnetic effect

Chiral fermions modify measure

$$n = \int d^3p \left(1 + \vec{B} \cdot \vec{\Omega}_p\right) n_p \qquad \vec{\pi} = \int d^3p \left(1 + \vec{B} \cdot \vec{\Omega}_p\right) \vec{p} n_p$$

$$\mathcal{E} = \int d^3p \left(1 + \vec{B} \cdot \vec{\Omega}_p\right) \epsilon_p n_p$$

where $\Omega_p = \nabla_p \times \mathcal{A}_p = \pm\hat{p}/(2\mu^2)$ is the Berry curvature. Get energy/momentum conservation

$$\partial_0 \mathcal{E} + \vec{\nabla} \cdot \vec{\pi} = \vec{E} \cdot \vec{\jmath} \qquad \partial_0 \vec{\pi} + \vec{\nabla} \cdot \hat{\Pi} = n\vec{E} + \vec{\jmath} \times \vec{B}$$

and the anomaly

$$\partial_0 n + \vec{\nabla} \cdot \vec{\jmath} = \pm\frac{1}{4\pi^2} \vec{E} \cdot \vec{B}$$

Son & Yamamoto, Zahed, Basar et al.

Coda: Non-FLT engineering in AdS/CMT

Consider (deformations of) AdS/Reissner-Nordstrom black holes. Solve for spinor eigenmodes

$$G_R(\omega, k) = \frac{h_1}{\omega - v_F(k - k_F) + h_2 \omega^{2\nu_{k_F}}}$$

$(2\nu_{k_F} - 1) \Rightarrow$ regular/marginal/singular Fermi liquid

Study transport properties

$$\sigma(0) \sim T^{-2\nu_k} \qquad \sigma(\omega) = \frac{\sigma(0)}{1 - i\tau\omega} \quad (2\nu_K > 1)$$

QFT: Non-interacting FS interacting with strongly coupled CFT.

Faulkner, Polchinski, Iqbal, McGreevy.

Summary

Contrary to Gerry's 1970 prediction, there is a renaissance of interest in effective forces.

Much of this interest is driven by developments initiated or encouraged by Gerry: chiral forces, soft potentials, free space and Fermi surface RG.

Landau Fermi liquid theory is alive and well. Indeed we have come to (re)appreciate Landau's insight of basing the theory on kinetics rather than effective lagrangians.

I am looking forward to the next 45 years of

nuclear theory at Stony Brook!