Strongly interacting quantum fluids: Experimental status

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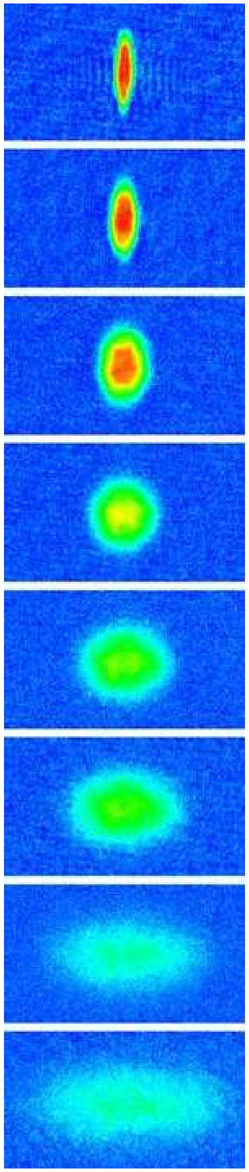
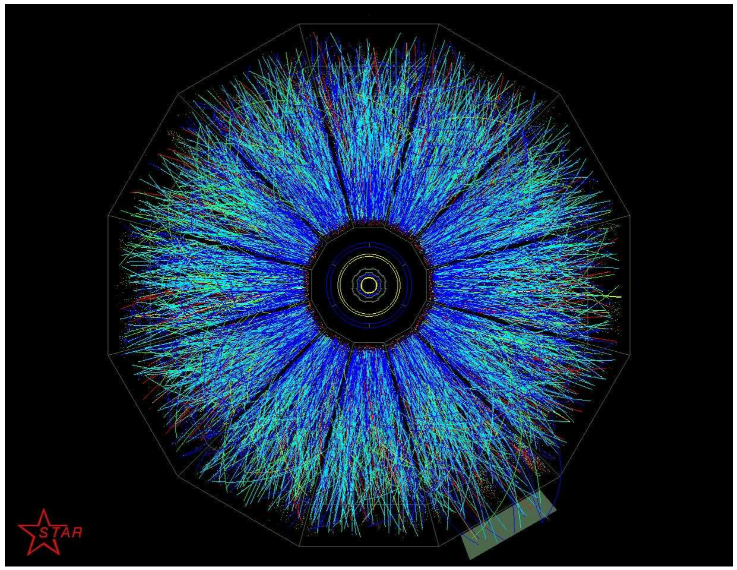
Perfect fluids: The contenders





QGP (T=180 MeV)

Liquid Helium

(T=0.1 meV)

Trapped Atoms

(T=0.1 neV)

I. Experiment (liquid helium)

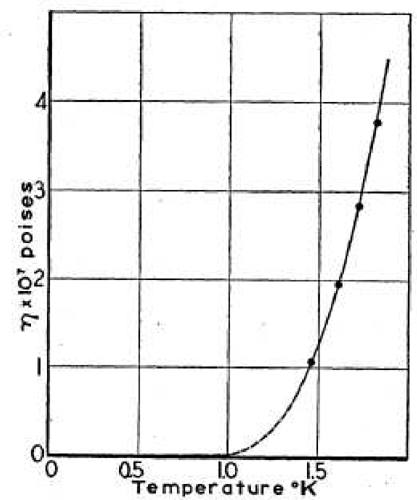
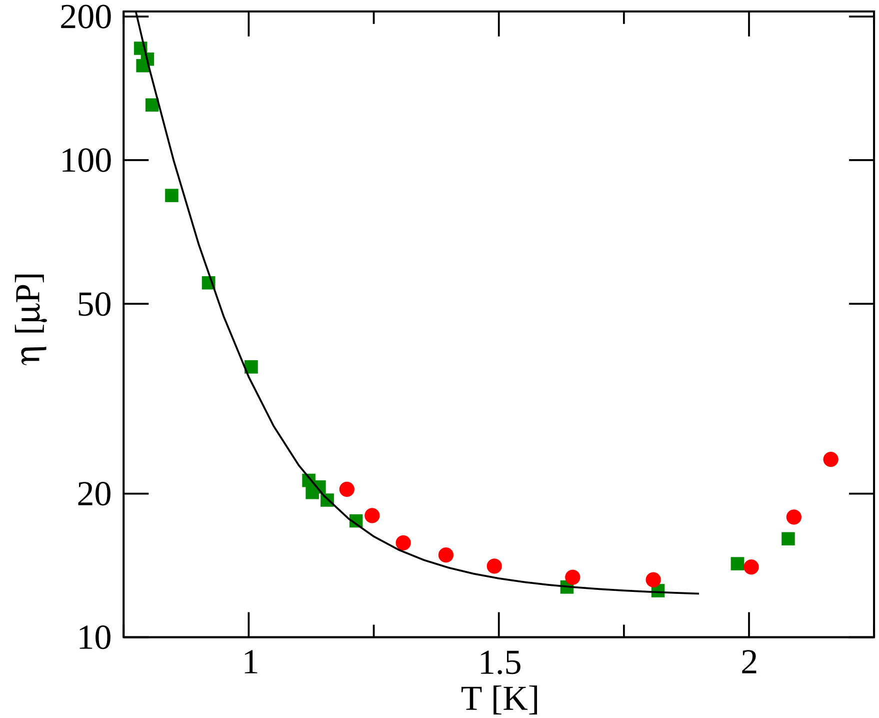


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.



Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

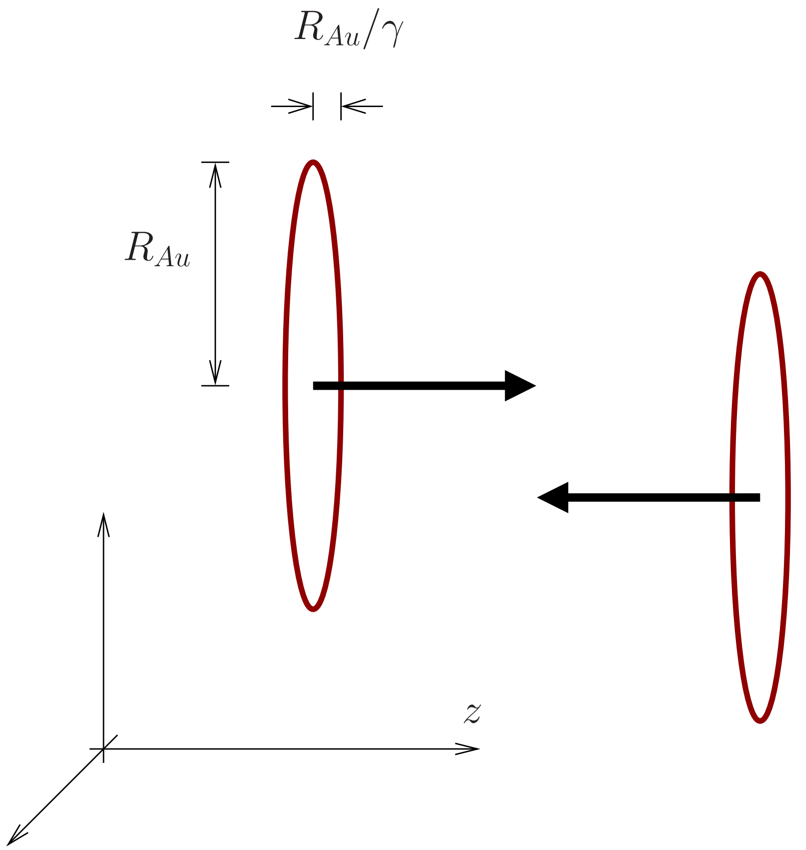
Hollis-Hallett (1955)

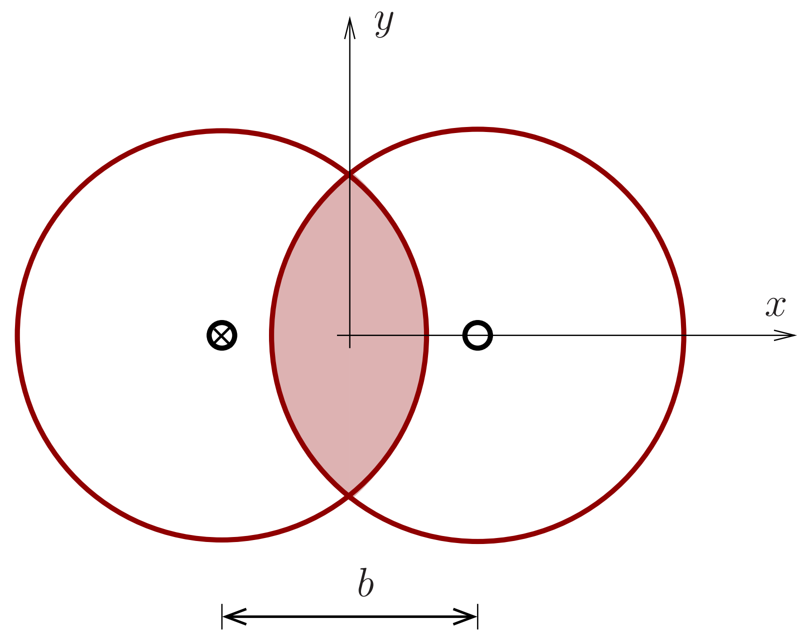
roton minimum, phonon rise

rotation viscometer

$$\eta/s \simeq 0.8\,\hbar/k_B$$

II. Heavy ion collision: Geometry





rapidity : $y = \dfrac{1}{2}\log\left(\dfrac{E+p_z}{E-p_z}\right)$

transverse momentum : $p_T^2 = p_x^2 + p_y^2$

Bjorken expansion

Experimental observation: At high energy ($\Delta y \to \infty$) rapidity distributions of produced particles (in both pp and AA) are "flat"

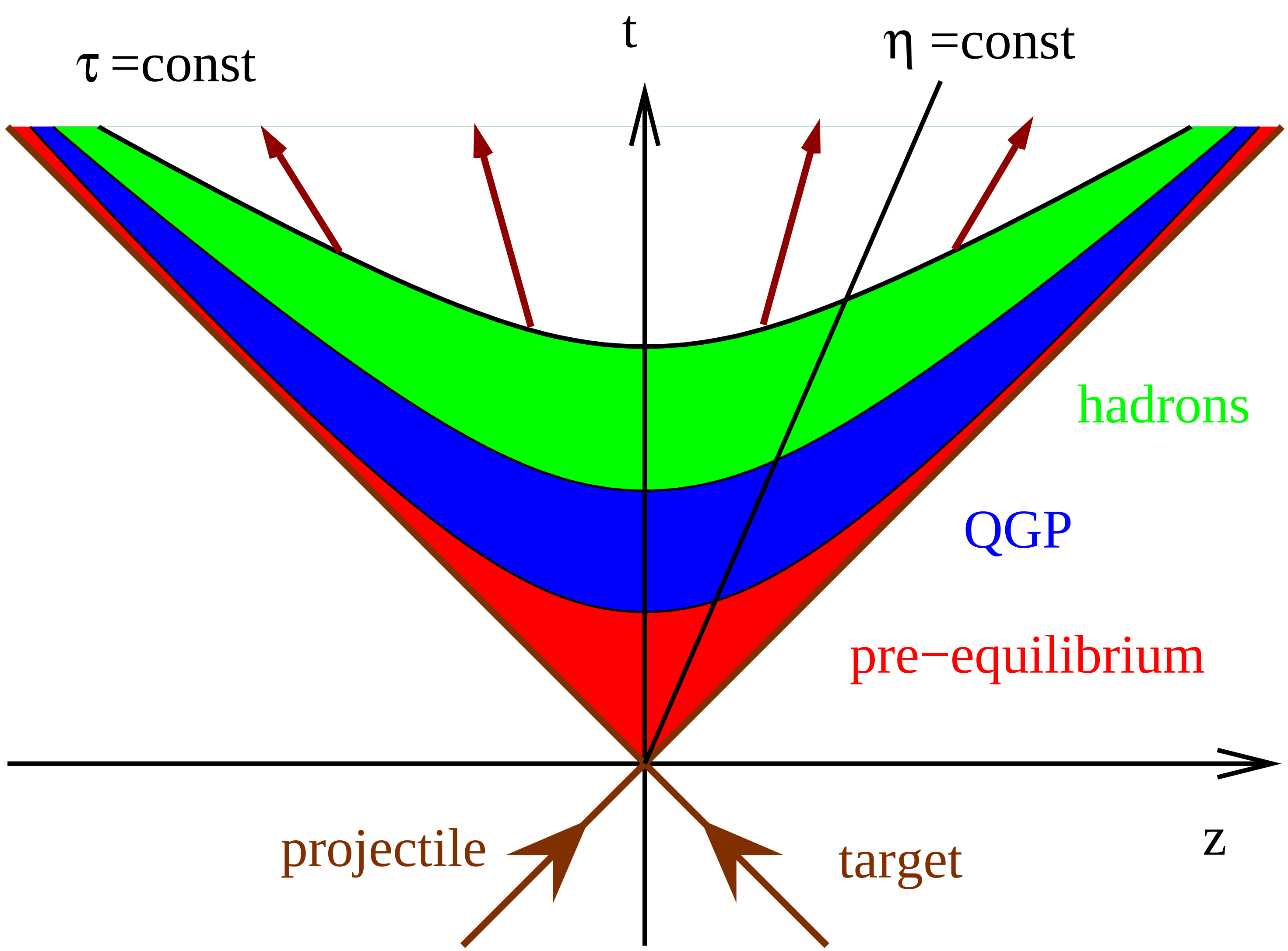
$$\frac{dN}{dy} \simeq const$$

Physics depends on proper time $\tau = \sqrt{t^2 - z^2}$, not on y

All comoving ($v = z/t$) observers are equivalent

Analogous to Hubble expansion

Bjorken expansion



Bjorken expansion: Hydrodynamics

Boost invariant expansion

$$u^\mu = \gamma(1, 0, 0, v_z) = (t/\tau, 0, 0, z/\tau)$$

solves Euler equation (no longitudinal acceleration)

$$\partial^\mu (s u_\mu) = 0 \qquad \Rightarrow \qquad \frac{d}{d\tau}\left[\tau s(\tau)\right] = 0$$

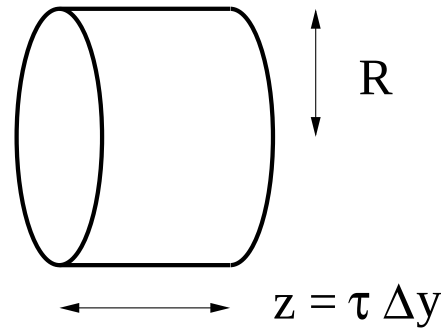
Solution for ideal Bj hydrodynamics

$$\boxed{s(\tau) = \frac{s_0 \tau_0}{\tau}} \qquad\qquad T = \frac{const}{\tau^{1/3}}$$

Exact boost invariance, no transverse expansion, no dissipation, . . .

Numerical estimates

Total entropy in rapidity interval $[y, y + \Delta y]$



$$S = s\pi R^2 z = s\pi R^2 \tau \Delta y = (s_0 \tau_0)\pi R^2 \Delta y$$

$$s_0 \tau_0 = \frac{1}{\pi R^2} \frac{S}{\Delta y}$$

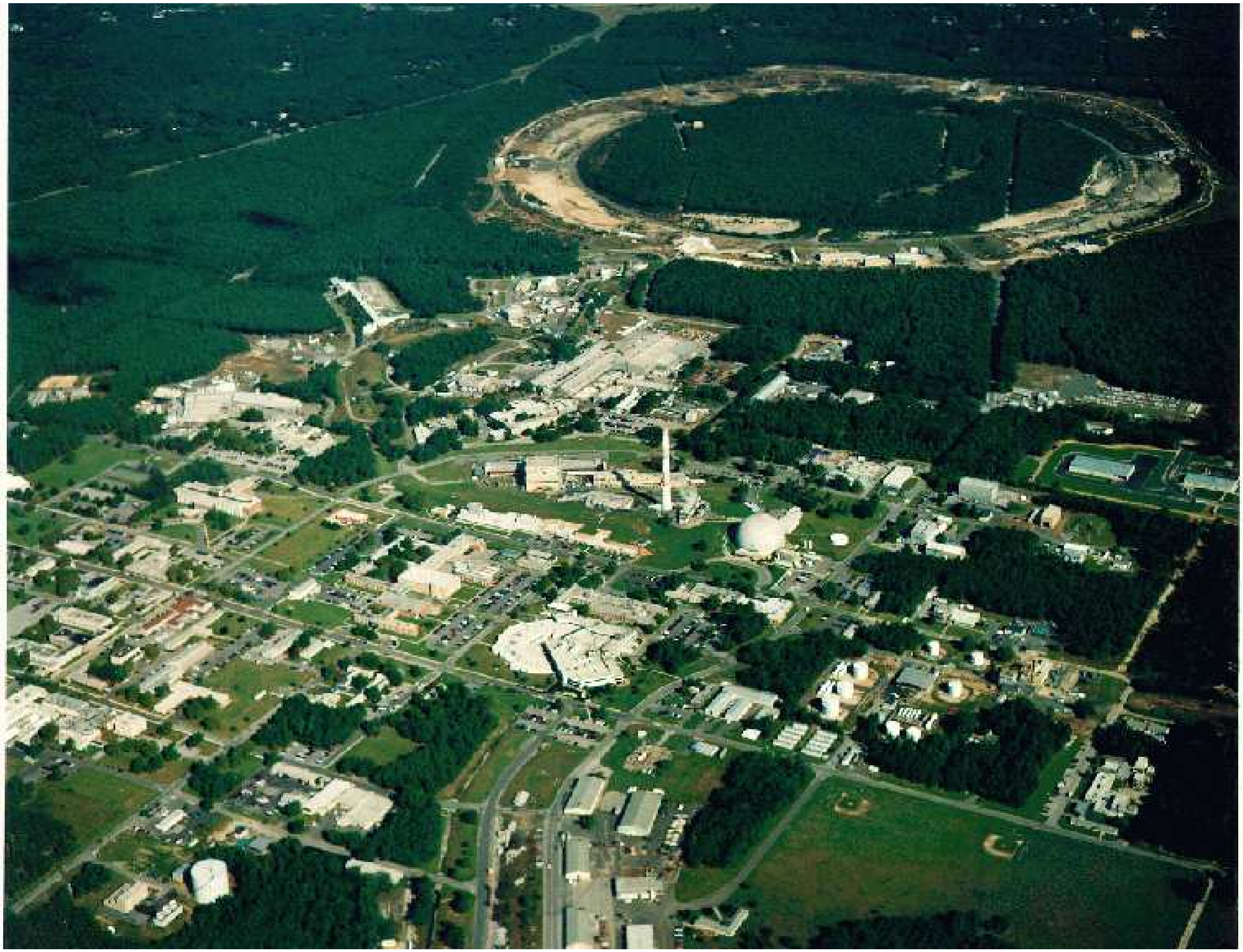
Use $S/N \simeq 3.6$

$$s_0 = \frac{3.6}{\pi R^2 \tau_0} \left(\frac{dN}{dy} \right) \qquad \text{Bj estimate}$$

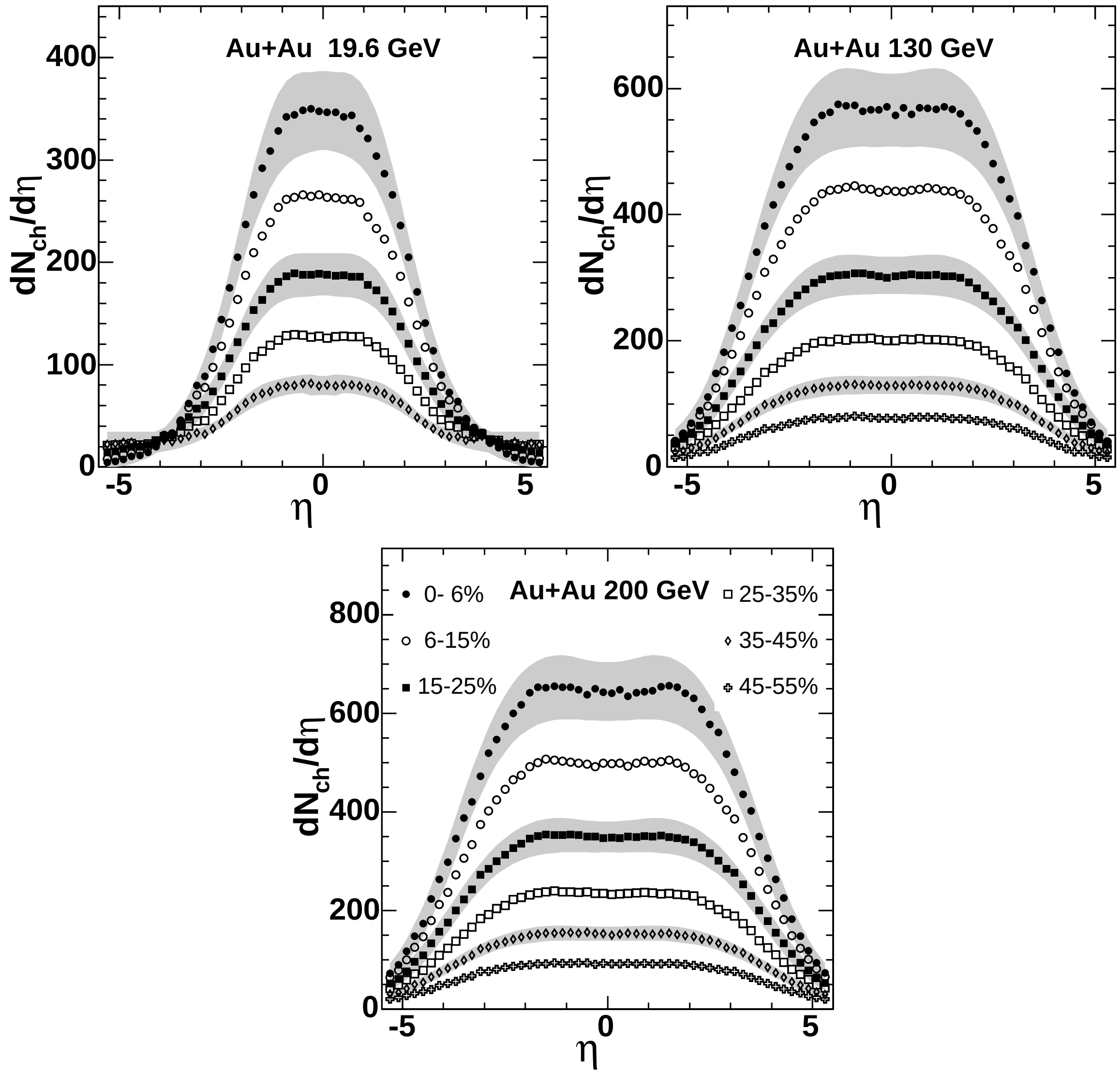
$$\epsilon_0 = \frac{1}{\pi R^2 \tau_0} \left(\frac{dE_T}{dy} \right)$$

Depends on initial time τ_0

BNL and RHIC

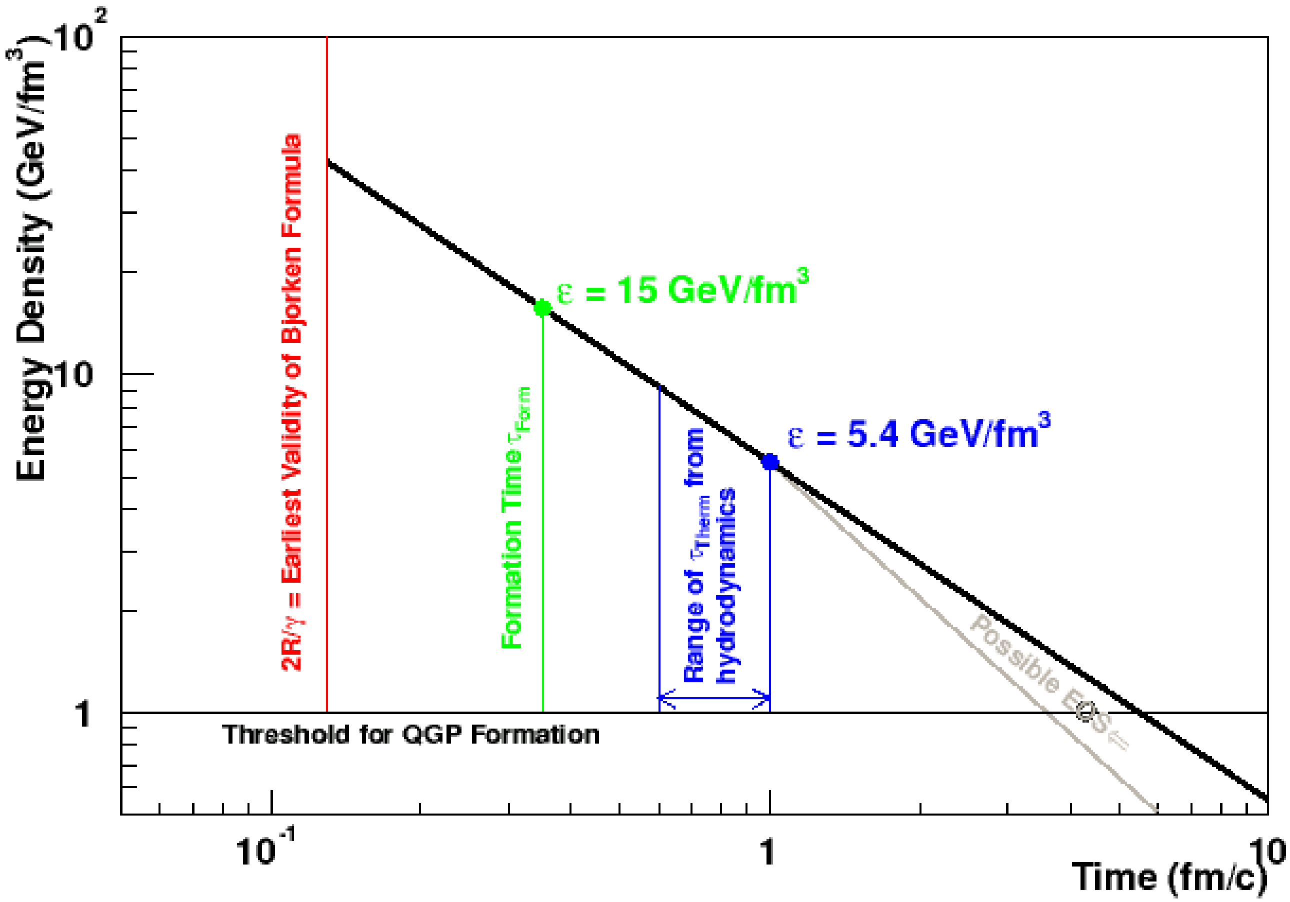


Multiplicities

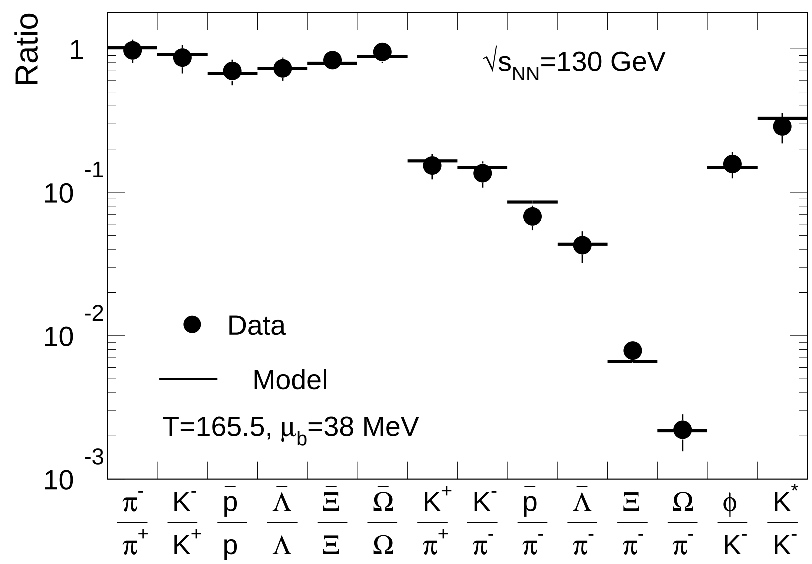


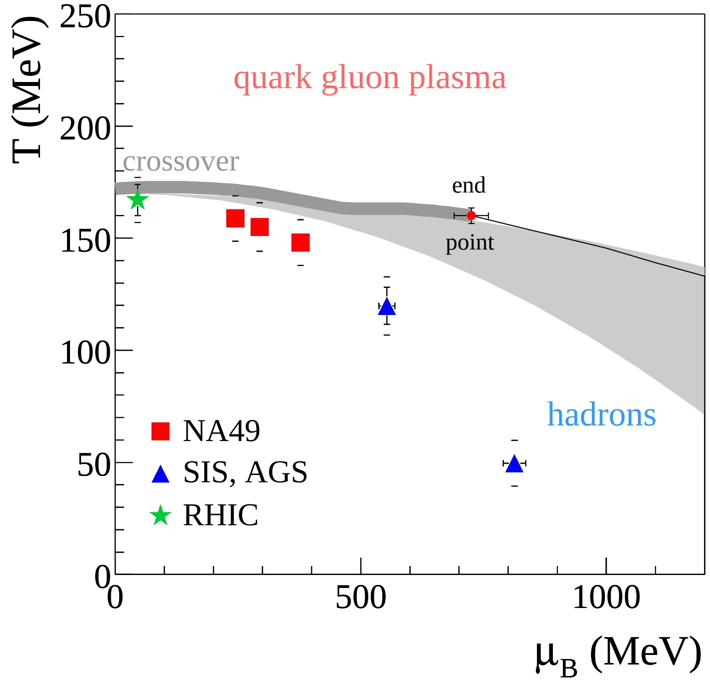
Phobos White Paper (2005)

Bjorken expansion



Chemical equilibrium at freezeout

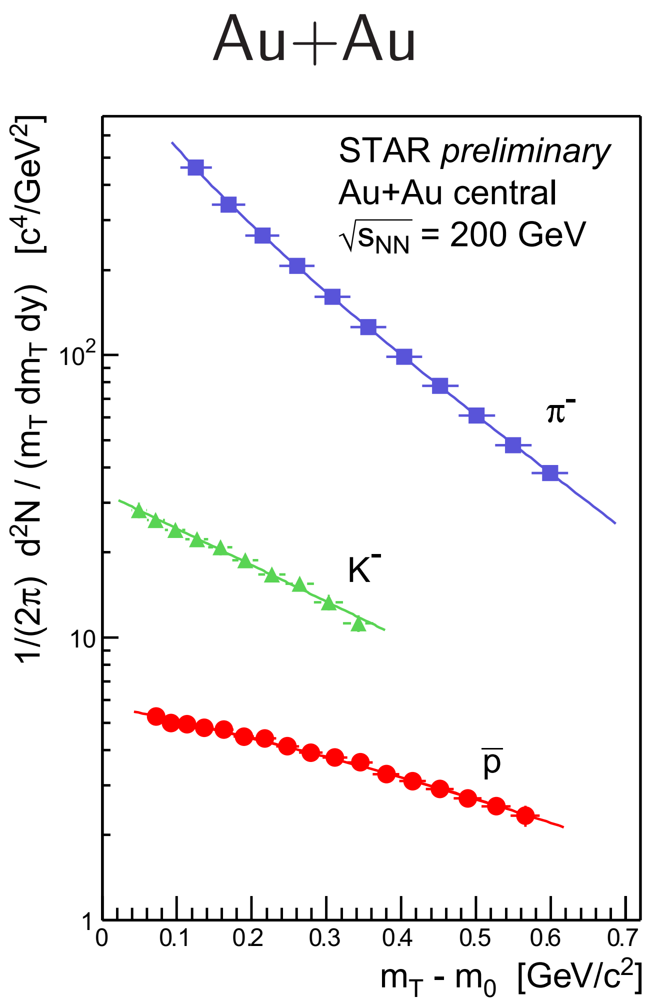


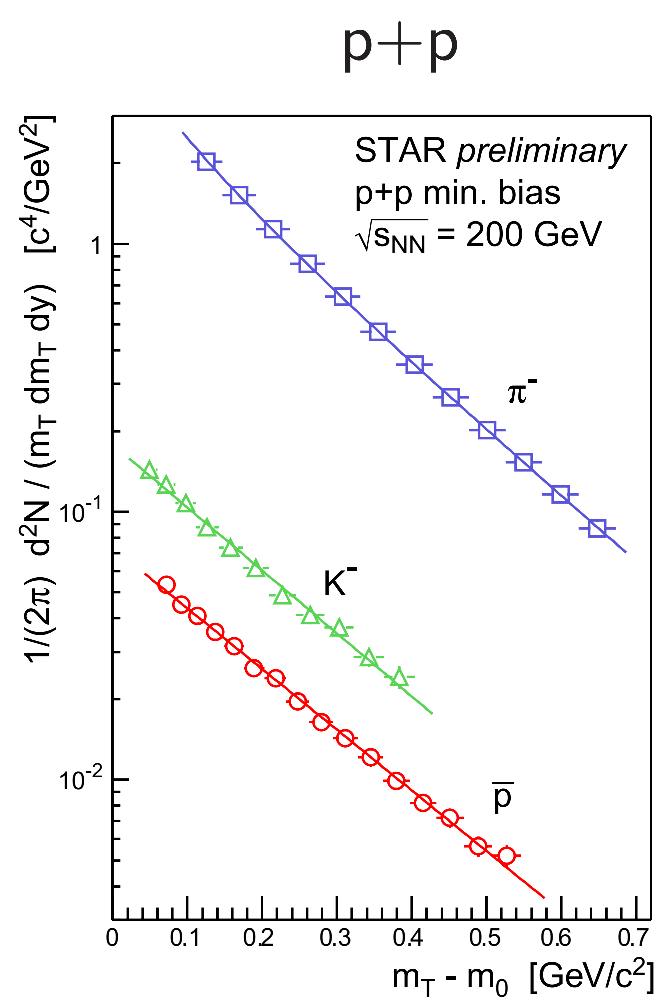


Andronic et al. (2006)

Collective behavior: Radial flow

Radial expansion leads to blue-shifted spectra in Au+Au





$v_T \sim 0.6c!$

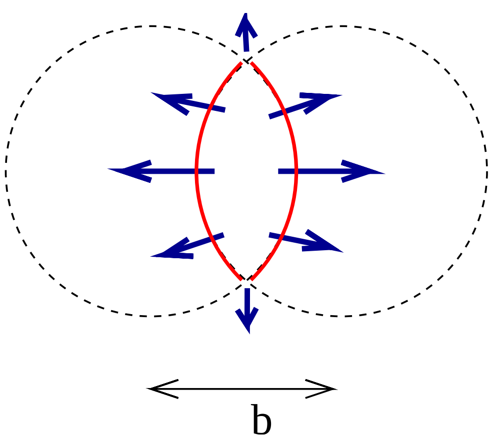
$$m_T = \sqrt{p_T^2 + m^2}$$

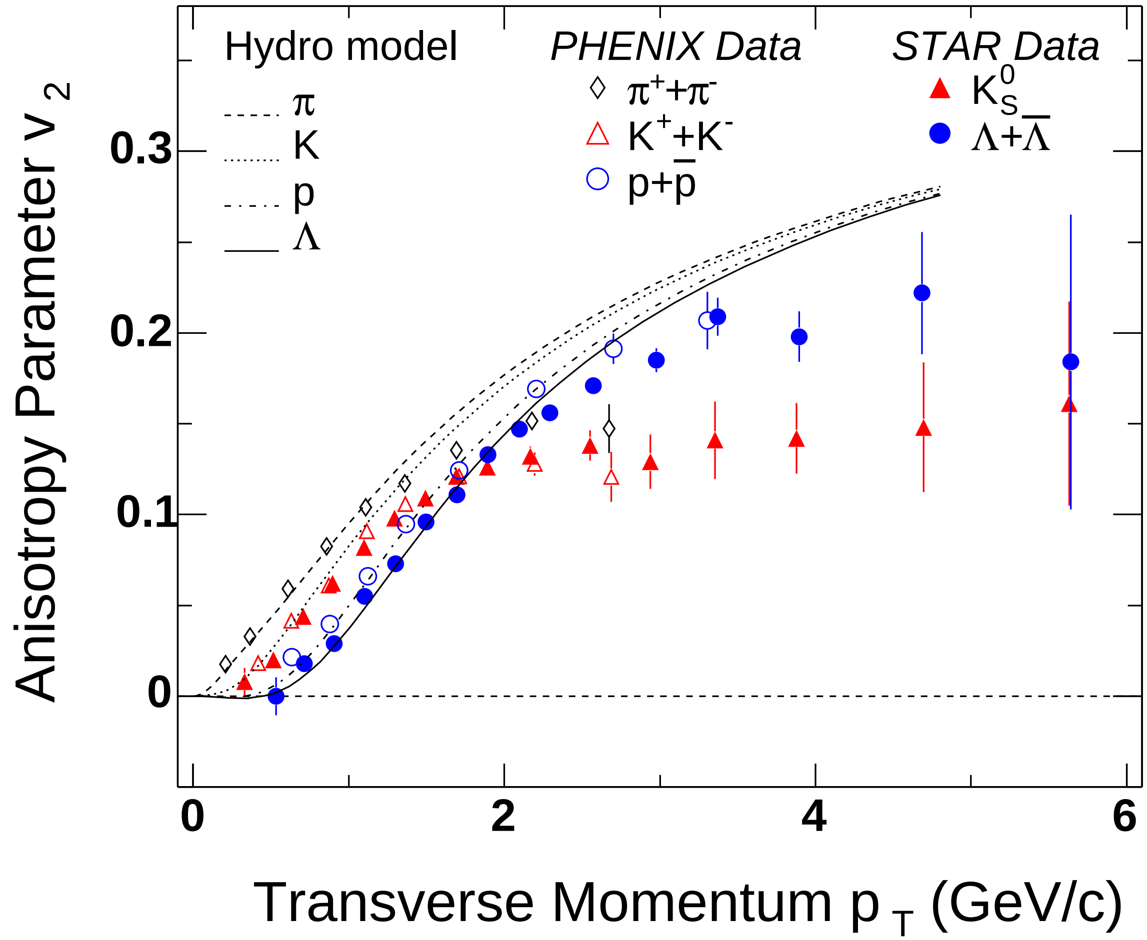
Collective behavior: Elliptic flow

Hydrodynamic expansion converts

coordinate space anisotropy

to momentum space anisotropy

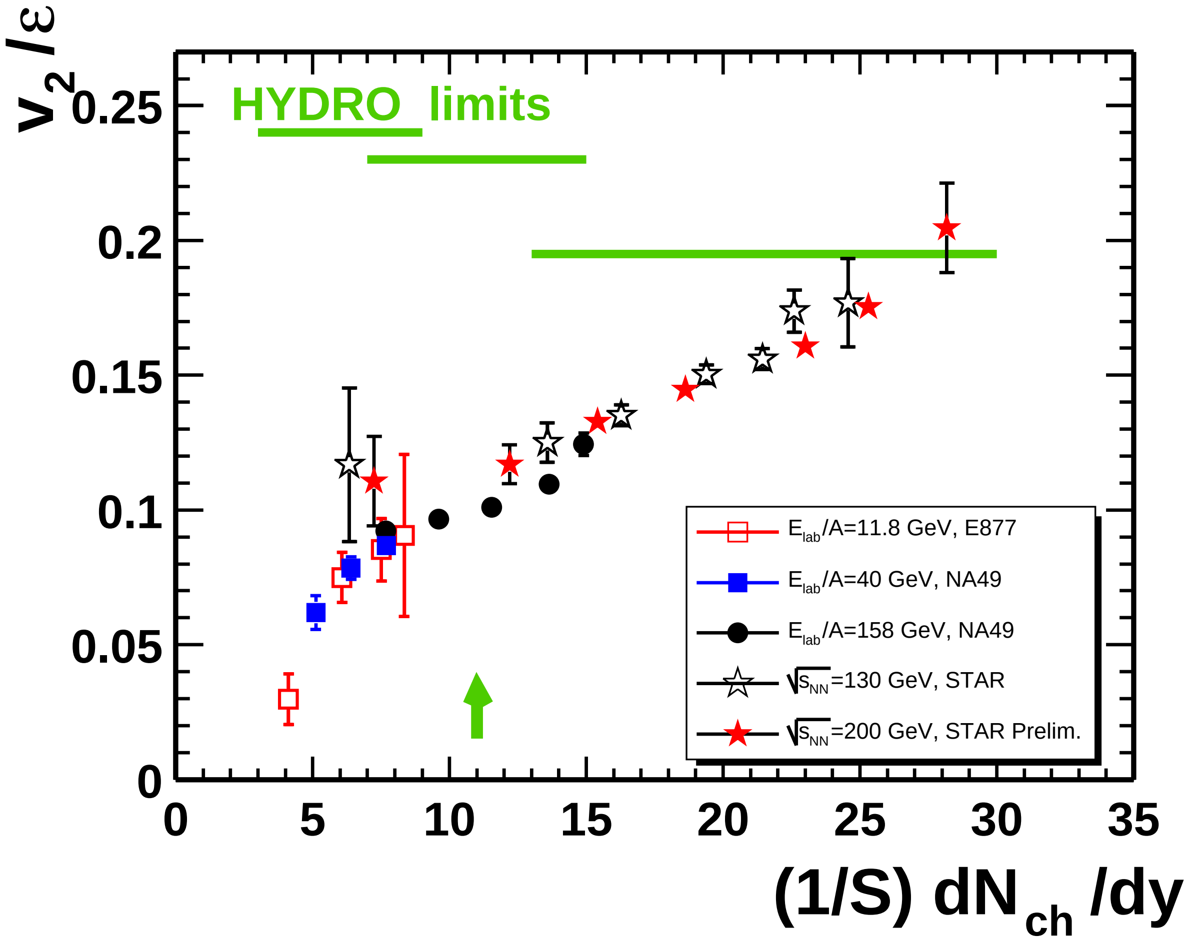




source: U. Heinz (2005)

$$p_0 \left. \frac{dN}{d^3p} \right|_{p_z=0} = v_0(p_\perp)\,(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots)$$

Elliptic flow II: Multiplicity scaling



source: U. Heinz (2005)

Viscous Corrections

Longitudinal expansion: Bj expansion solves Navier-Stokes equation

entropy equation $$\frac{1}{s}\frac{ds}{d\tau} = -\frac{1}{\tau}\left(1 - \frac{\frac{4}{3}\eta + \zeta}{sT\tau}\right)$$

Viscous corrections small if $\frac{4}{3}\frac{\eta}{s} + \frac{\zeta}{s} \ll (T\tau)$

early	$T\tau \sim \tau^{2/3}$	$\eta/s \sim const$	$\eta/s < \tau_0 T_0$
late	$T\tau \sim const$	$\eta \sim T/\sigma$	$\tau^2/\sigma < 1$

Hydro valid for $\tau \in [\tau_0, \tau_{fr}]$

Viscous corrections to T_{ij} (radial expansion)

$$T_{zz} = P - \frac{4}{3}\frac{\eta}{\tau} \qquad T_{xx} = T_{yy} = P + \frac{2}{3}\frac{\eta}{\tau}$$

increases radial flow (central collision)

decreases elliptic flow (peripheral collision)

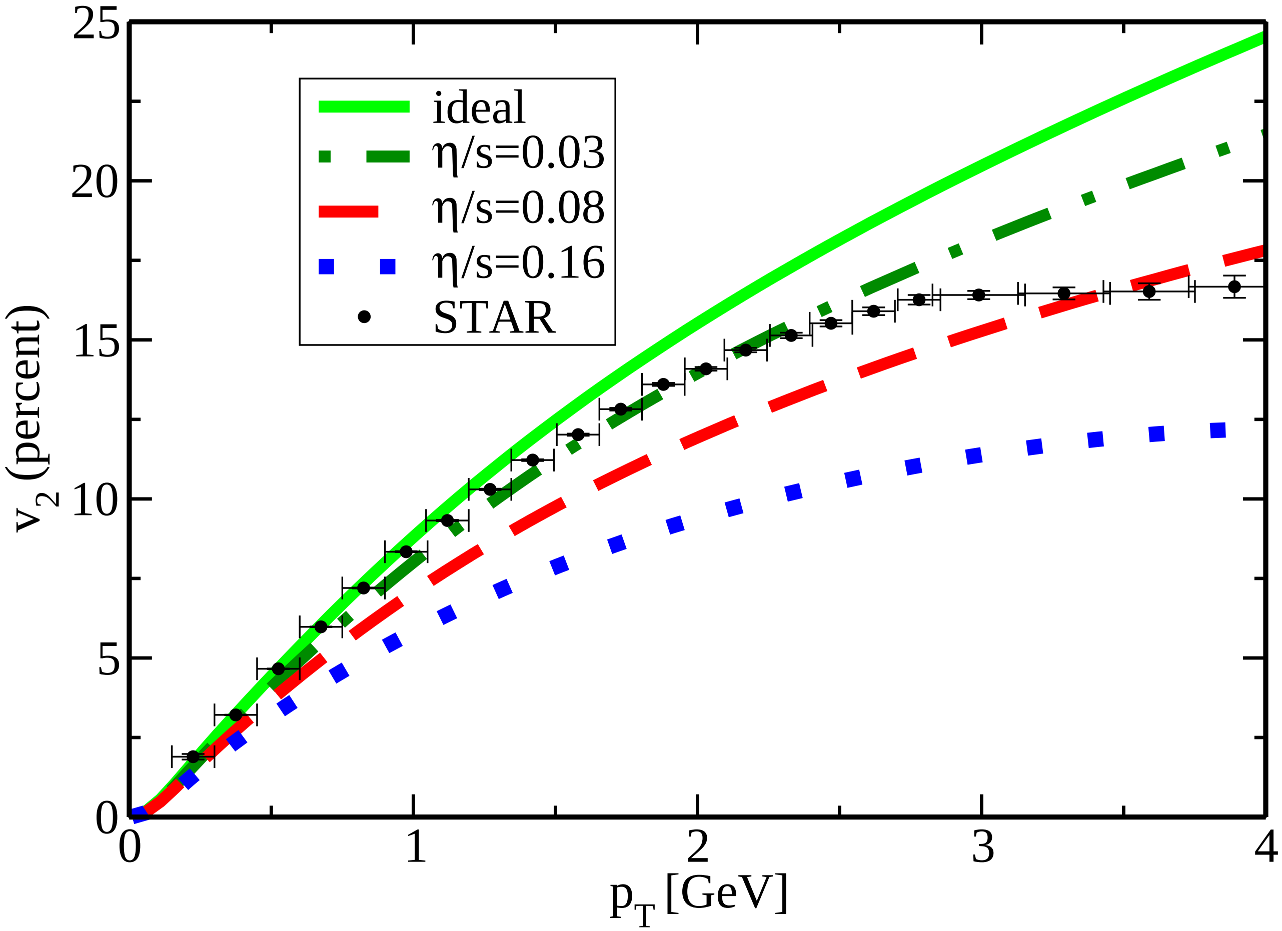
Modification of distribution function

$$\delta f = \frac{3}{8}\frac{\Gamma_s}{T^2} f_0(1+f_0) p_\alpha p_\beta \nabla^{\langle\alpha} u^{\beta\rangle}$$

Correction to spectrum grows with $p_\perp^2$

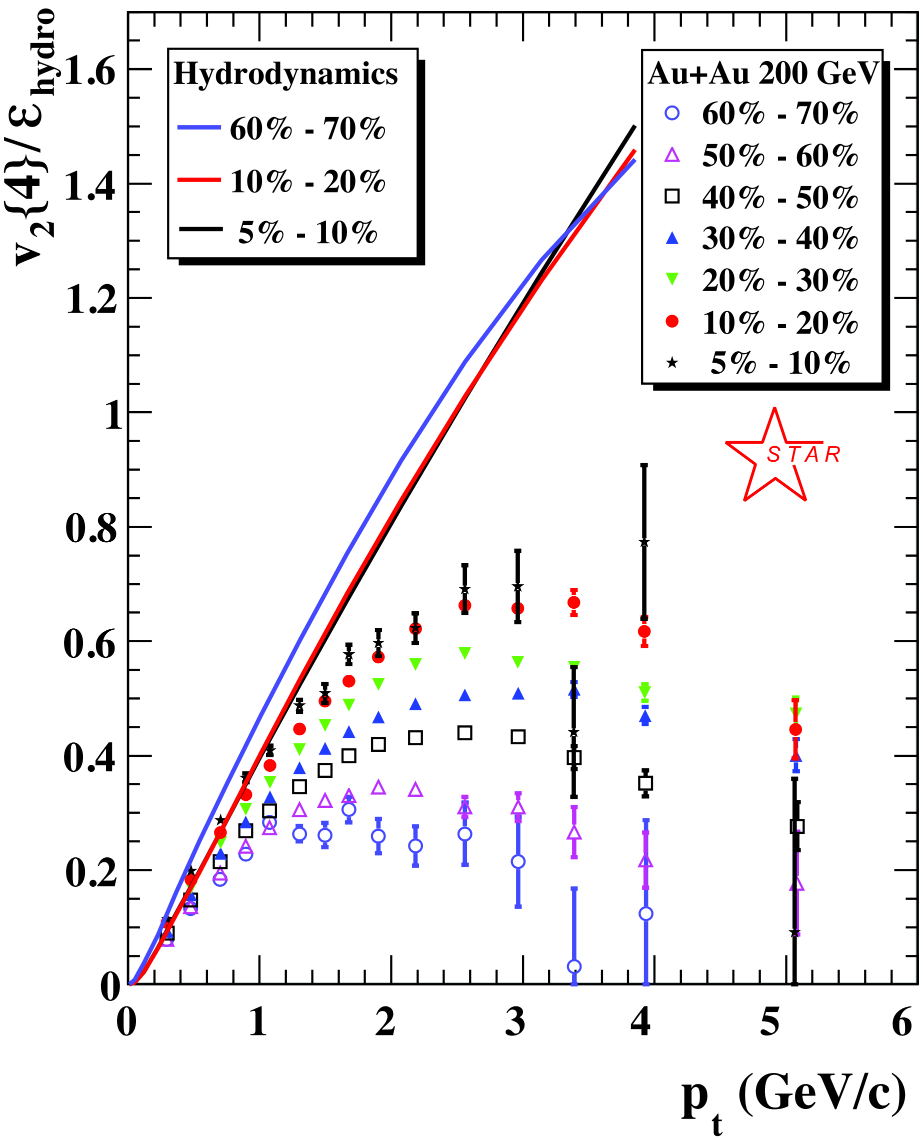
$$\frac{\delta(dN)}{dN_0} = \frac{\Gamma_s}{4\tau_f}\left(\frac{p_\perp}{T}\right)^2$$

Elliptic flow III: Viscous effects



Romatschke (2007), Teaney (2003)

Elliptic flow IV: Systematic trends



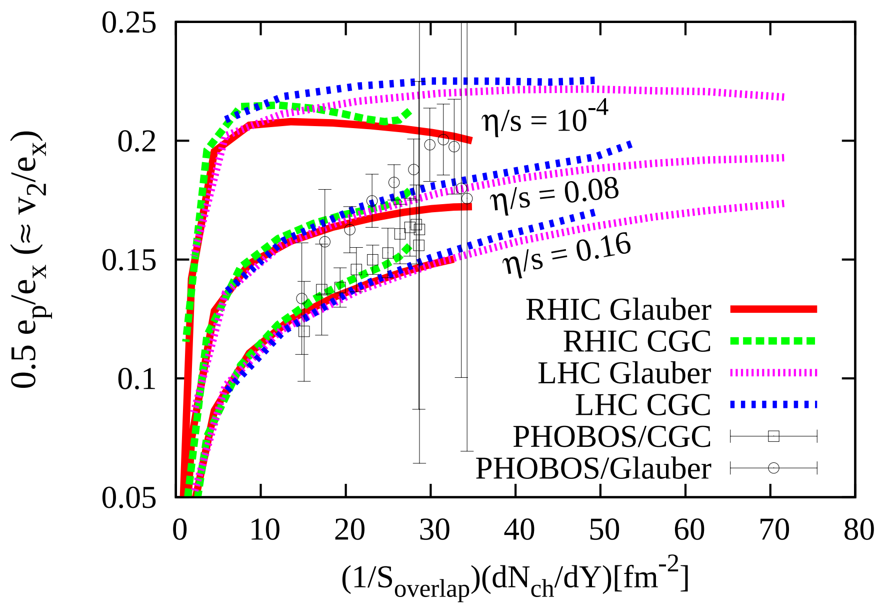
source: R. Snellings (STAR)

Deviation from ideal hydro

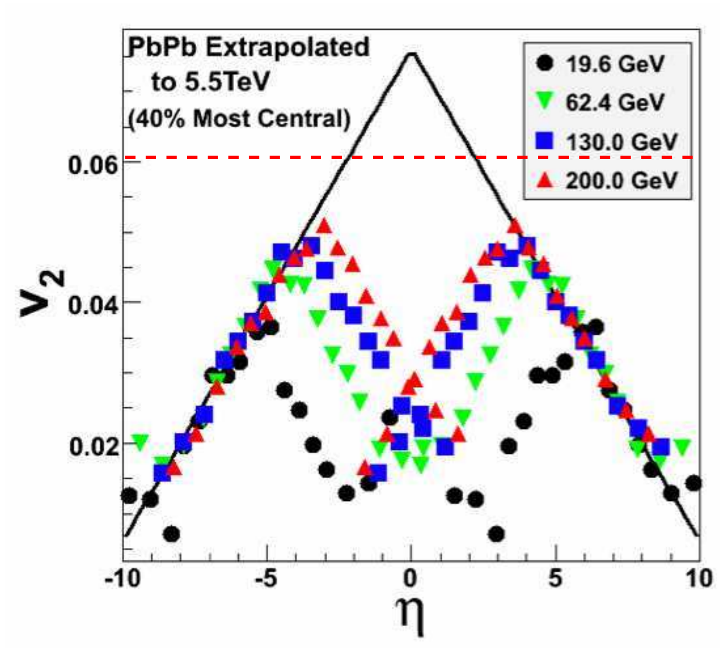
increases for more peripheral events

increases with $p_\perp$

Elliptic flow V: Predictions for LHC



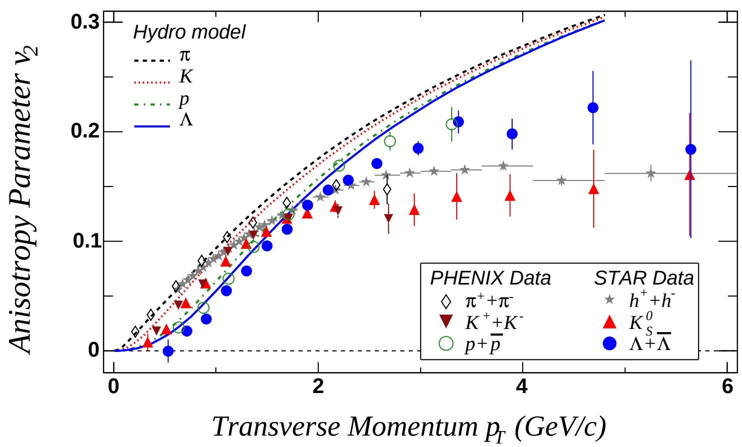
Romatschke, Luzum (2009)

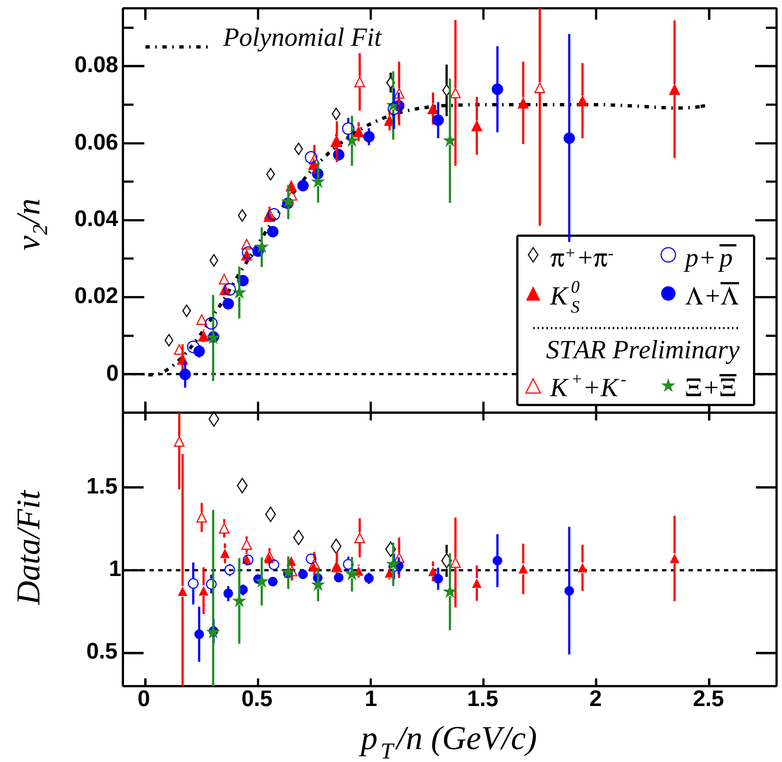


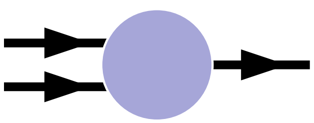
Busza (QM 2009)

Elliptic flow VI: Recombination

"quark number" scaling of elliptic flow

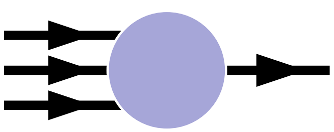






(qq̄) (mes)

$$p_\perp^{mes} = 2p_\perp^{qu}$$

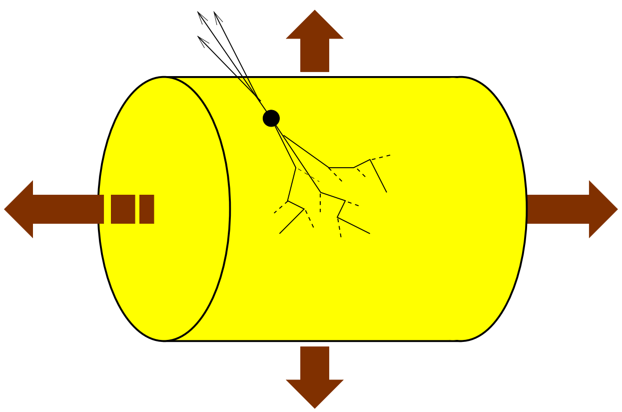


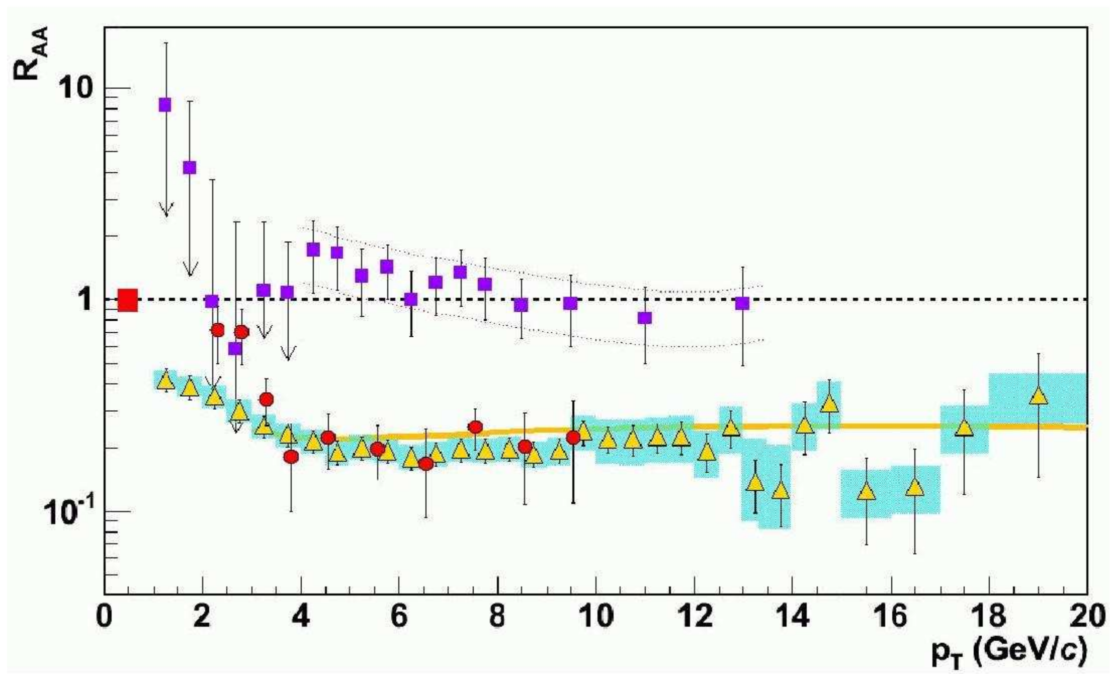
(qqq) (bar)

$$p_\perp^{bar} = 3p_\perp^{qu}$$

Jet quenching

$$R_{AA} = \frac{n_{AA}}{N_{coll} n_{pp}}$$

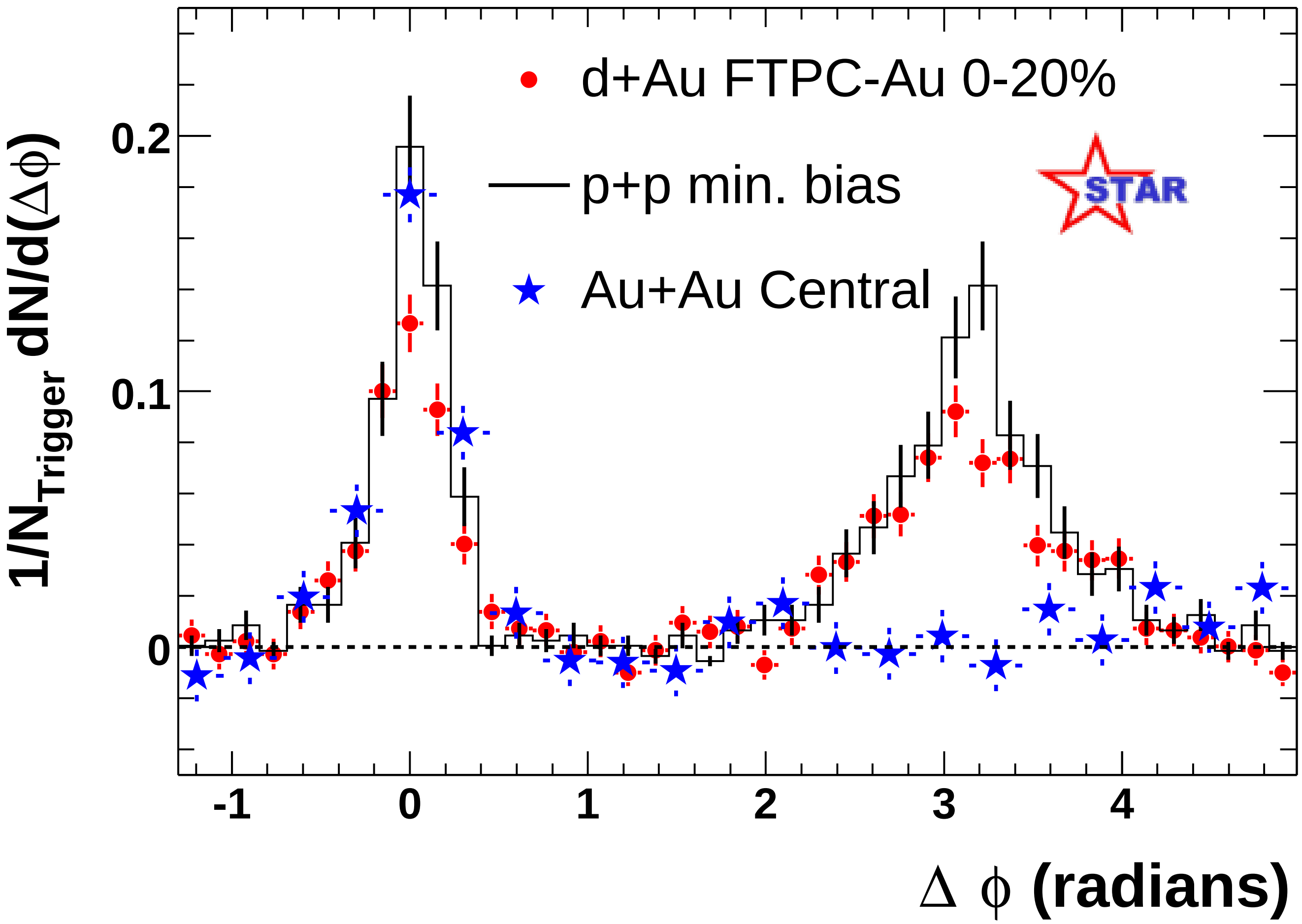




source: Akiba [Phenix] (2006)

Jet quenching II

Disappearance of away-side jet

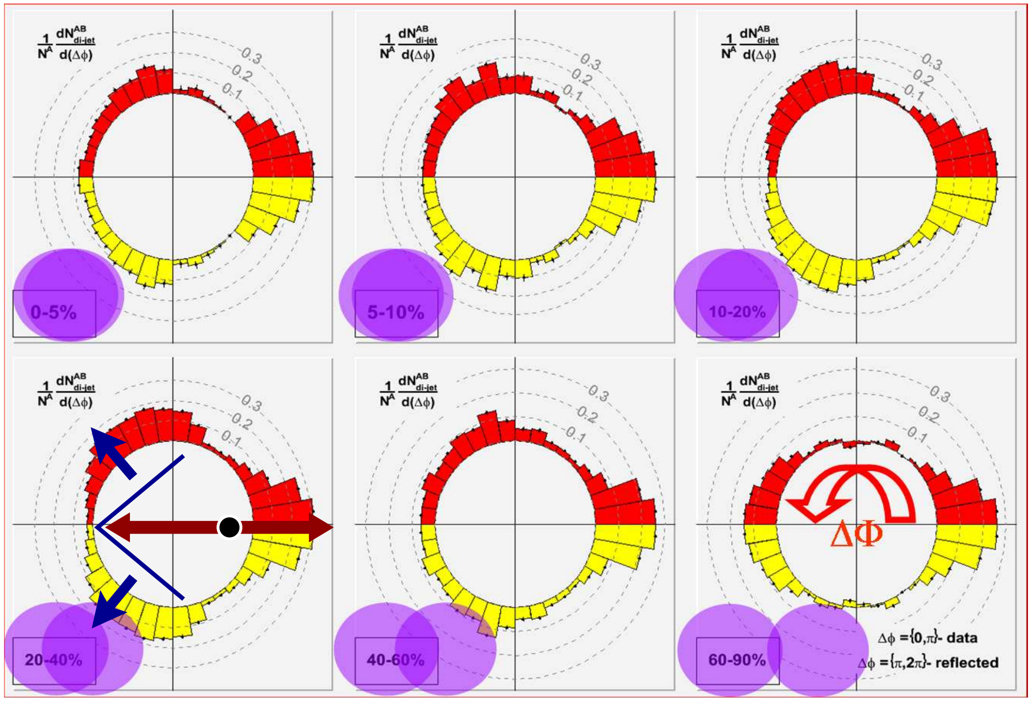


source: Star White Paper (2005)

Jet quenching III: The Mach cone

azimuthal multiplicity $dN/d\phi$

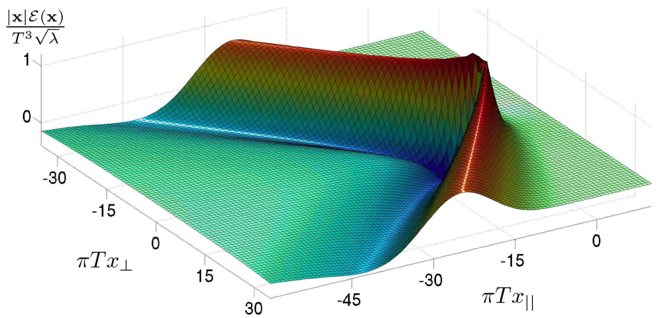
(high energy trigger particle at $\phi = 0$)



source: Phenix (PRL, 2006), W. Zajc (2007)

wake of a fast quark

in $\mathcal{N} = 4$ plasma

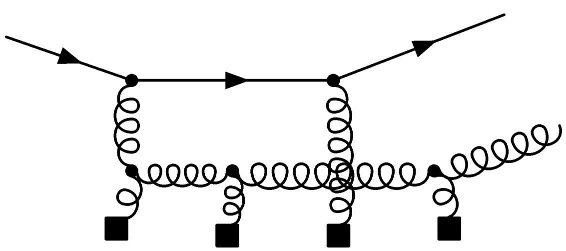


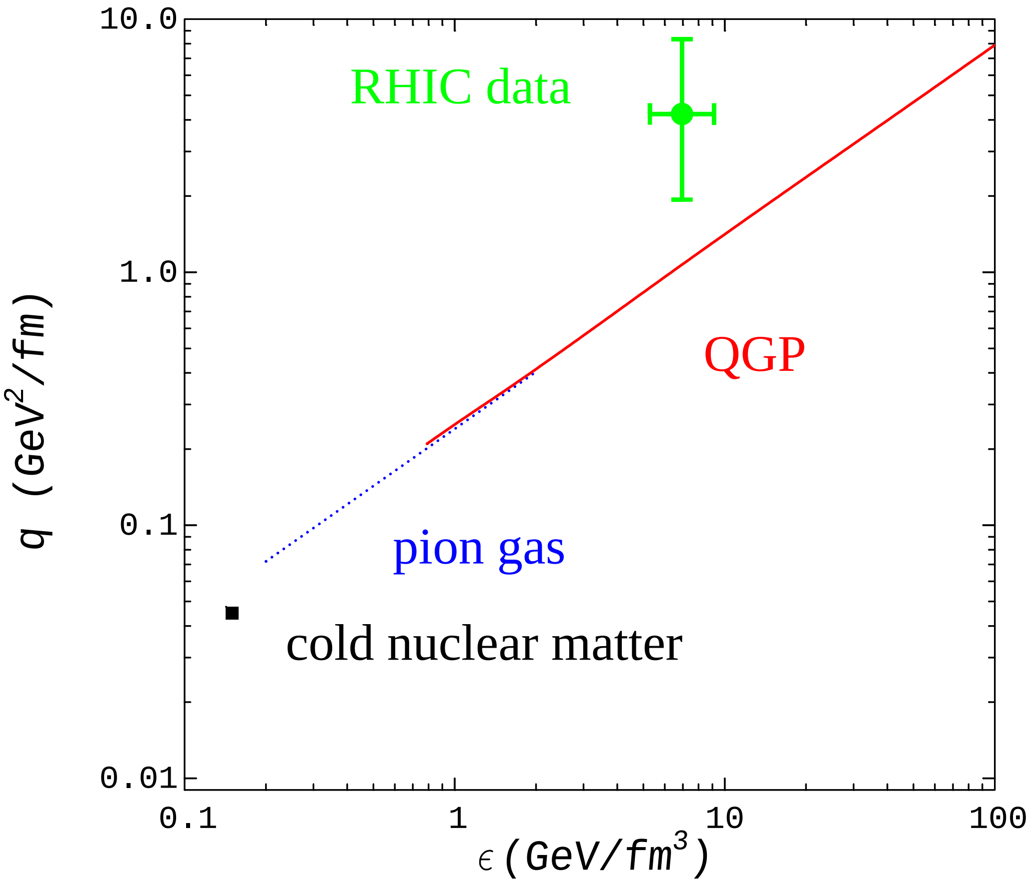
Chesler and Yaffe (2007)

Jet quenching: Theory

energy loss governed by

$$\hat{q} = \rho \int q_\perp^2 \, dq_\perp^2 \; \frac{d\sigma}{dq_\perp^2}$$





larger than pQCD predicts? relation to η? ($\hat{q} \sim 1/\eta$?)

also: large energy loss of heavy quarks

source: R. Baier (2004)

Where are we?

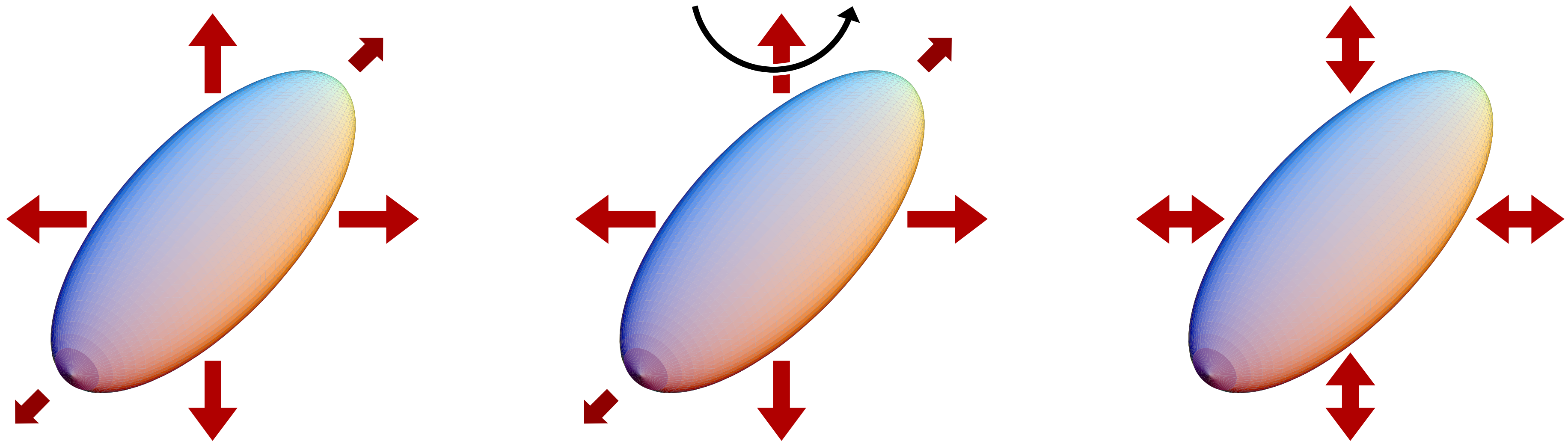
observe almost ideal fluid behavior, initial conditions well above critical energy density.

systematics require $0 < \eta/s < 0.4$; more studies needed, LHC elliptic flow will be very interesting.

jet quenching large; very detailed studies under way. LHC will provide unprecedented range.

heavy flavors: large energy loss seen, flavor studies (c/b) under way.

III. Experiment: Cold gases



transverse expansion   expansion (rotating trap)   collective modes

$$\begin{aligned}
\frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) &= 0 \\
mn\frac{\partial \vec{v}}{\partial t} + mn\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} &= -\vec{\nabla}P - n\vec{\nabla}V
\end{aligned}$$

Scaling flows

Universal equation of state $$P = \frac{n^{5/3}}{m} f\left(\frac{mT}{n^{2/3}}\right)$$

Equilibrium density profile (local density approximation)

$$n_0(x) = n(\mu(x), T) \qquad \mu(x) = \mu_0\left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right)$$

Scaling Flow: Stretch and rotate profile

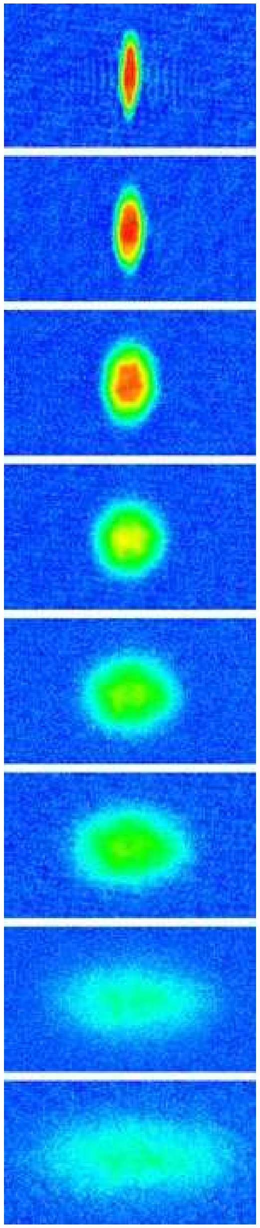
$$\mu_0 \to \mu_0(t), \qquad T \to T_0(\mu_0(t)/\mu_0), \qquad R_x \to R_x(t), \quad \ldots$$

Linear velocity profile

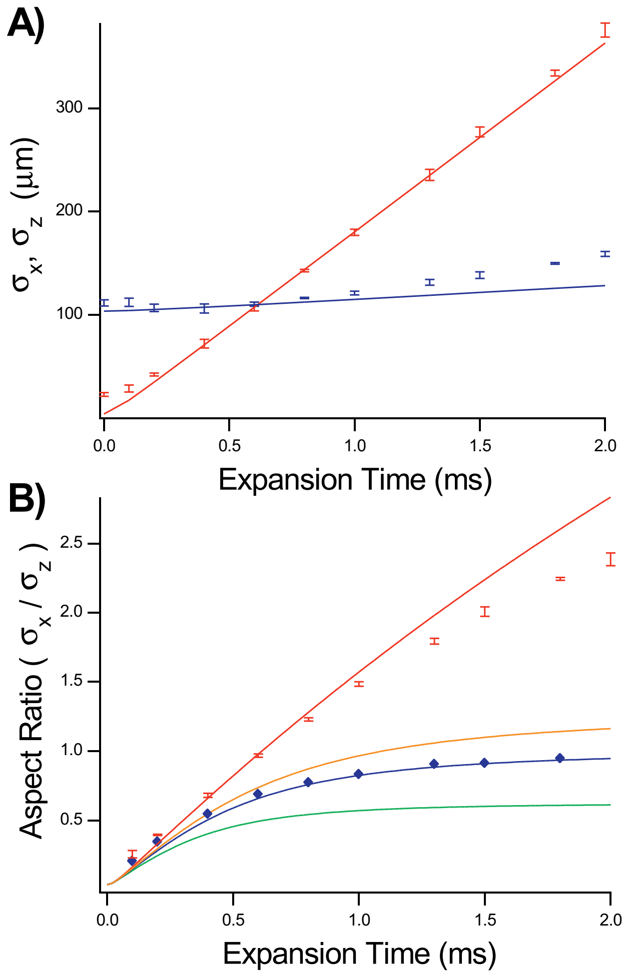
$$\vec{v}(x,t) = \frac{1}{2}\vec{\nabla}\left(\alpha_x x^2 + \alpha_y y^2 + \alpha_z z^2 + \alpha x y\right) + \omega \hat{z} \times \vec{x}.$$

"Hubble flow"

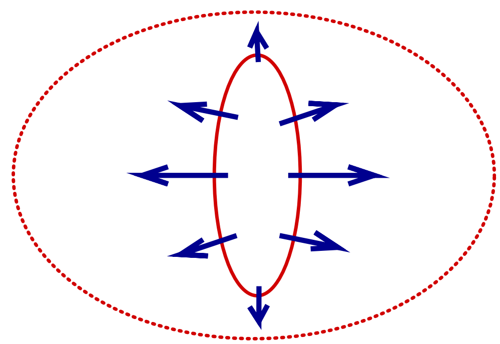
Almost ideal fluid dynamics



O'Hara et al. (2002)

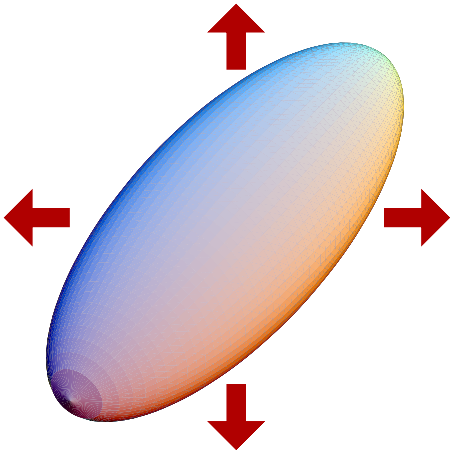


Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Almost ideal fluid dynamics

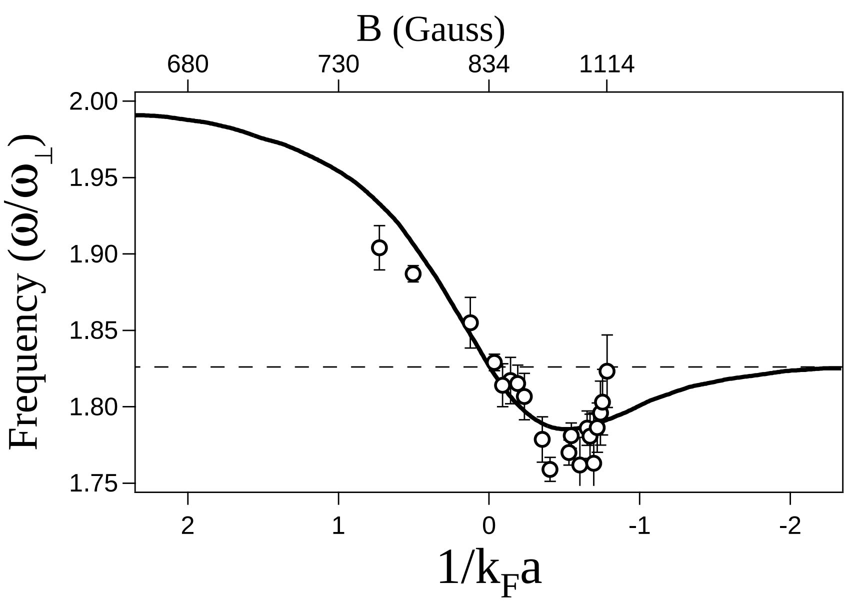
Radial breathing mode



Ideal fluid hydrodynamics ($P \sim n^{5/3}$)

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) \;=\; 0$$

$$\frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla}\right) \vec{v} \;=\; -\frac{\vec{\nabla} P}{mn} - \frac{\vec{\nabla} V}{m}$$



Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}}\,\omega_\perp$$

experiment: Kinast et al. (2005)

Dissipation (scaling flows)

Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

$$\dot{E} = -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2$$

Have $\zeta = 0$ and $T(x) = const$. Universality implies

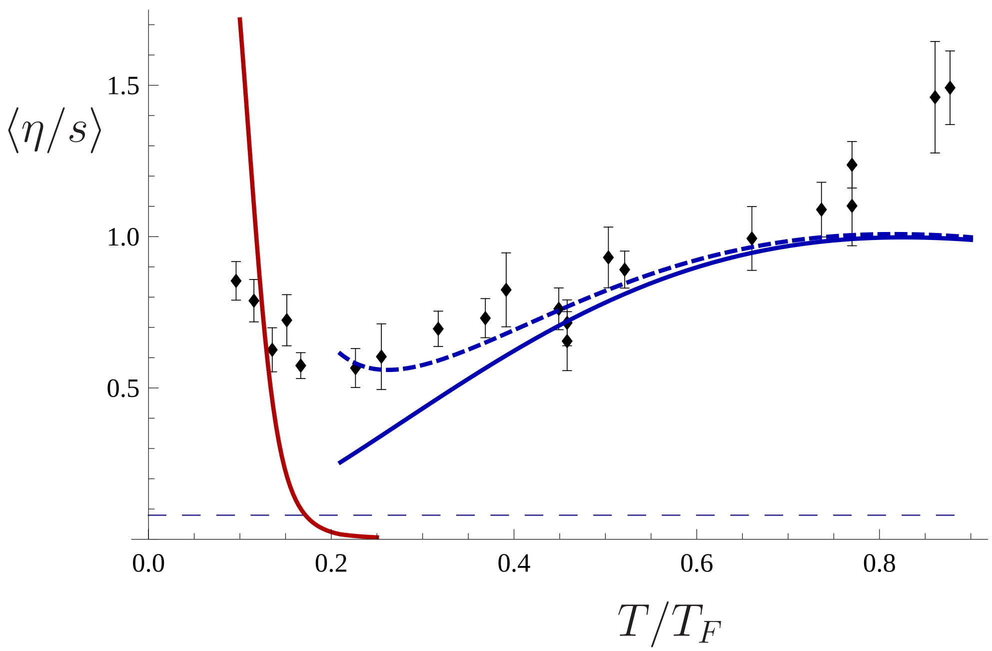
$$\eta(x) = s(x)\,\alpha_s\left(\frac{T}{\mu(x)}\right)$$

$$\int d^3x\,\eta(x) = S\langle\alpha_s\rangle$$

Collective modes: Small viscous correction exponentiates

$$a(t) = a_0 \cos(\omega t) \exp(-\Gamma\, t)$$

$$\langle \eta/s \rangle = (3N\lambda)^{1/3} \left(\frac{\Gamma}{\omega_\perp}\right) \left(\frac{E_0}{E_F}\right) \left(\frac{N}{S}\right)$$



Kinast et al. (2006), Schaefer (2007)

Navier-Stokes equation

Option 1: Moment method

$$\int d^3x\, x_k \,(\rho \dot{v}_i + \ldots) = \int d^3x\, x_k \,(-\nabla_i P - \nabla_j \delta\Pi_{ij})$$

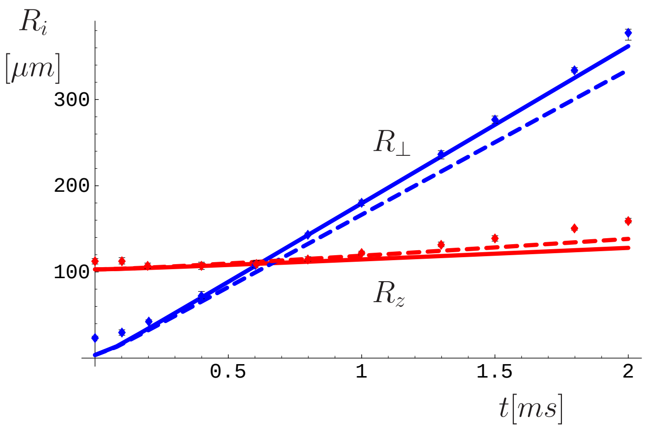
Only involves $\langle\eta\rangle/E_0$.

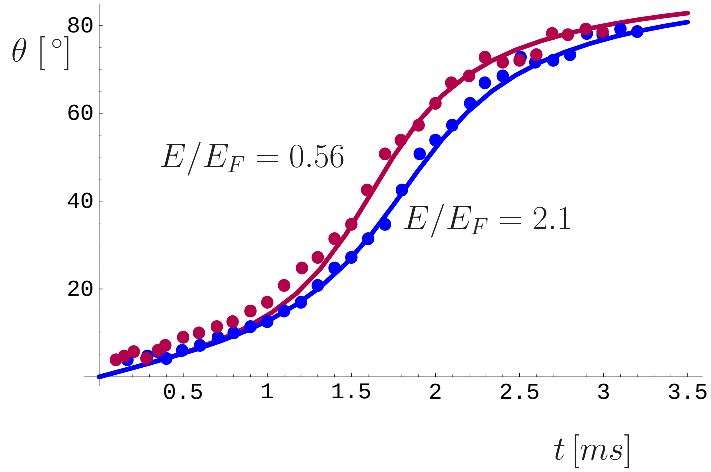
Option 2: Scaling ansatz for $\eta(\mu, T)$

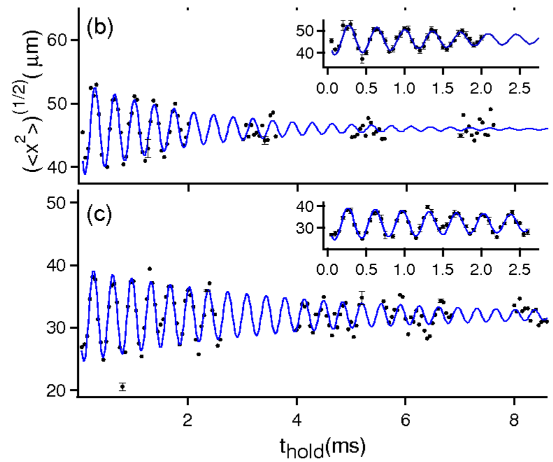
$$\eta(n,T) = \eta_0 (mT)^{3/2} + \eta_1 \frac{P(n,T)}{T}$$

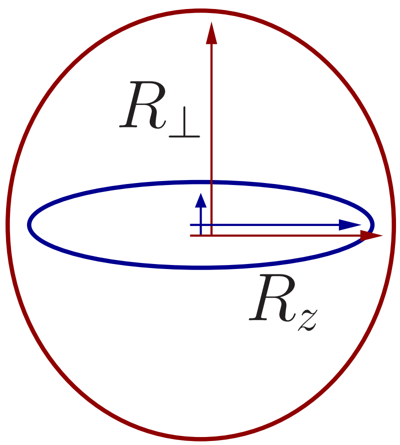
Option 3: Numerical solutions.

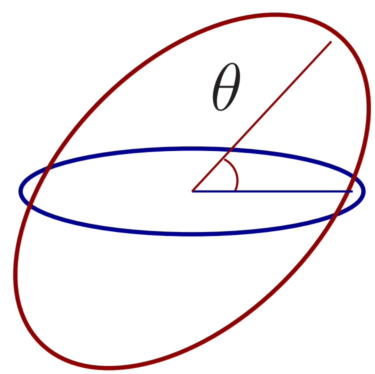
Dissipation

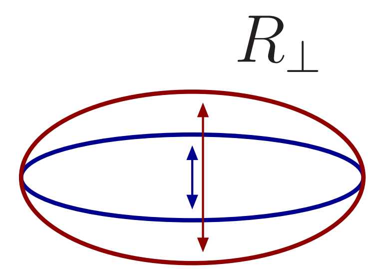






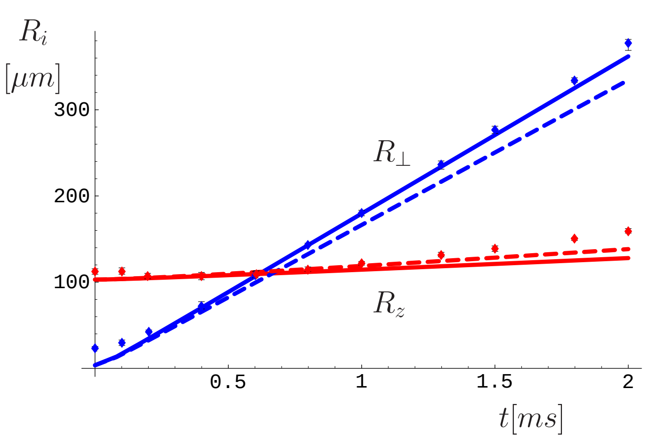


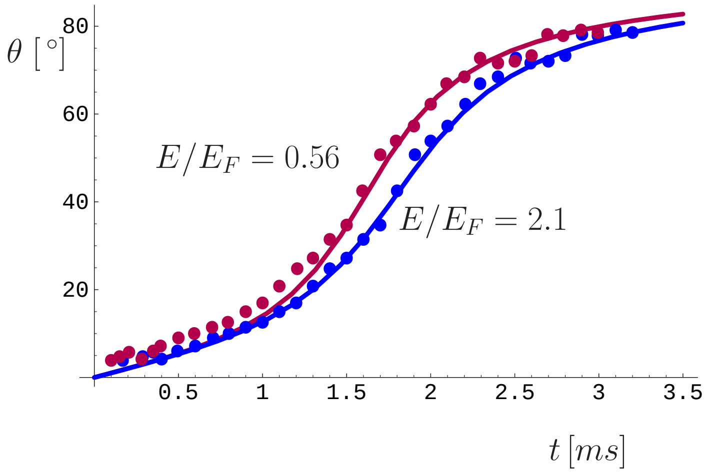


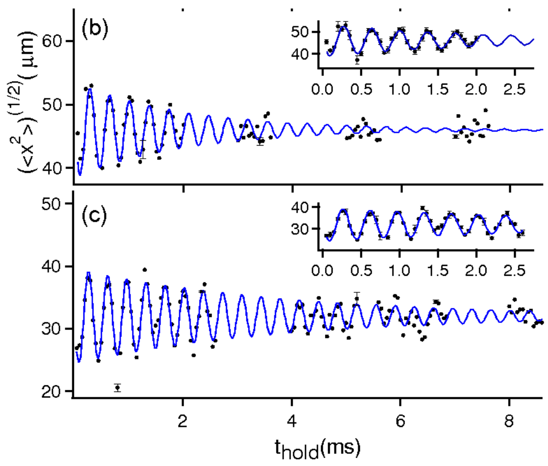


O'Hara et al (2002), Kinast et al (2005), Clancy et al (2007)

Dissipation





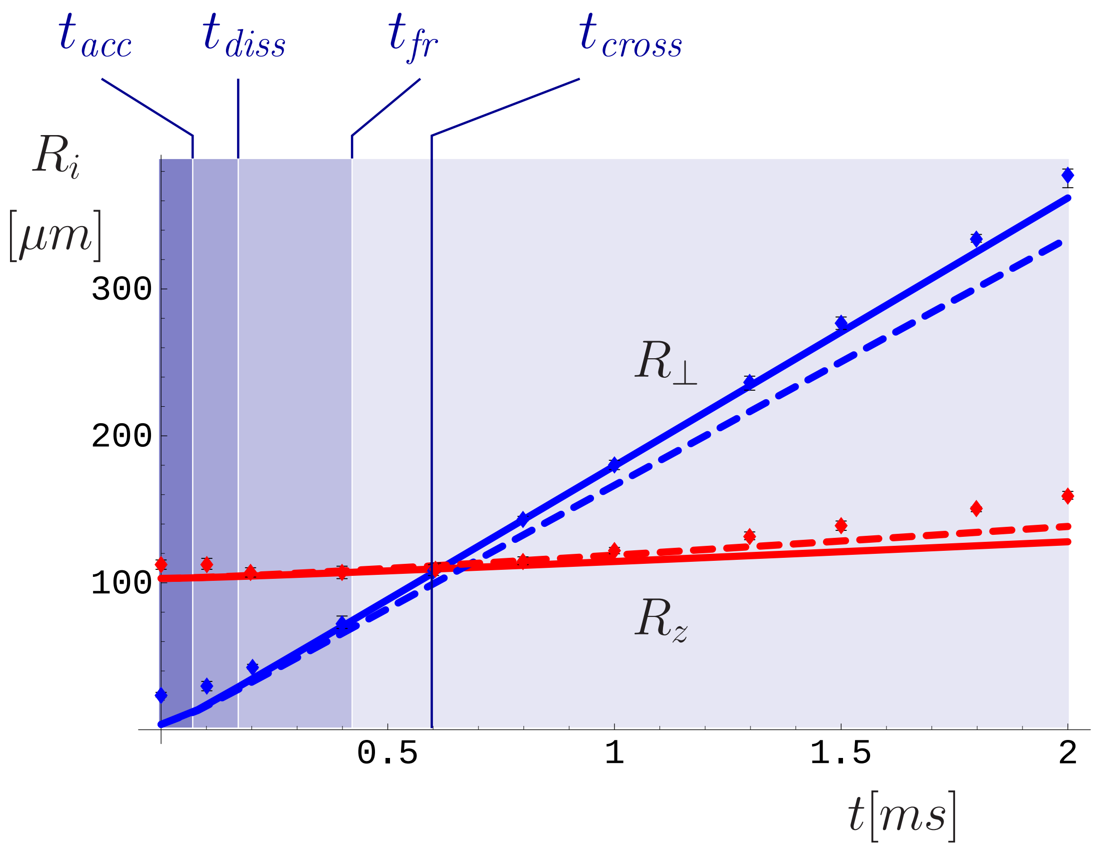


$$\left.\begin{array}{c}(\delta t_0)/t_0\\(\delta a)/a\end{array}\right\} = \left\{\begin{array}{c}0.008\\0.024\end{array}\right\}\left(\frac{\langle\alpha_s\rangle}{1/(4\pi)}\right)\left(\frac{2\cdot 10^5}{N}\right)^{1/3}\left(\frac{S/N}{2.3}\right)\left(\frac{0.85}{E_0/E_F}\right)$$

t_0: "Crossing time" $(b_\perp = b_z,\ \theta = 45^\circ)$

a: amplitude

Time Scales



Where are we?

high temperature ($T > 2.5T_c$) dominated by corona

low temperature ($T \sim T_c$): evidence for low viscosity ($\eta/s < 0.4$) core

also seen in "irrotational flow" data

full (2nd order hydro or hydro+kin) analysis needed

The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.