Strongly interacting quantum fluids: From quarks to atoms

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What do these terms mean (roughly)?

strongly interacting: $\langle V_{pot} \rangle \simeq \langle T \rangle$

quantum: $l_{pp} \leq \lambda_{deB}$

fluid: $T_{ij} = T_{ij}(\rho, \vec{v}, \mathcal{E})$

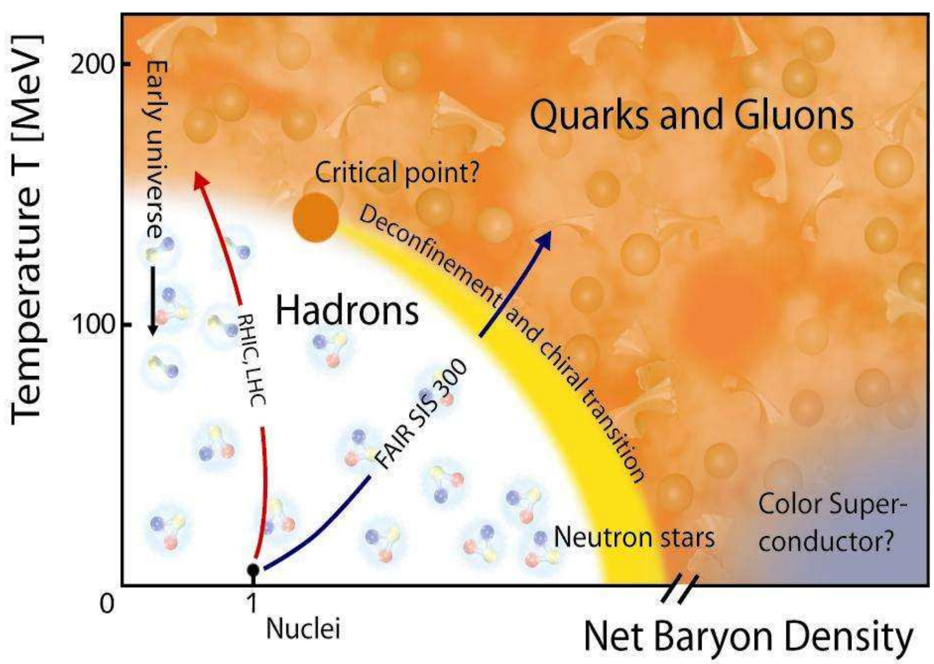
Plan of the lectures

1. Equilibrium properties
2. Transport: Hydro, kinetics, holography
3. Exploring nearly perfect fluids

QCD and the Quark Gluon Plasma

$$\boxed{\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu}}$$

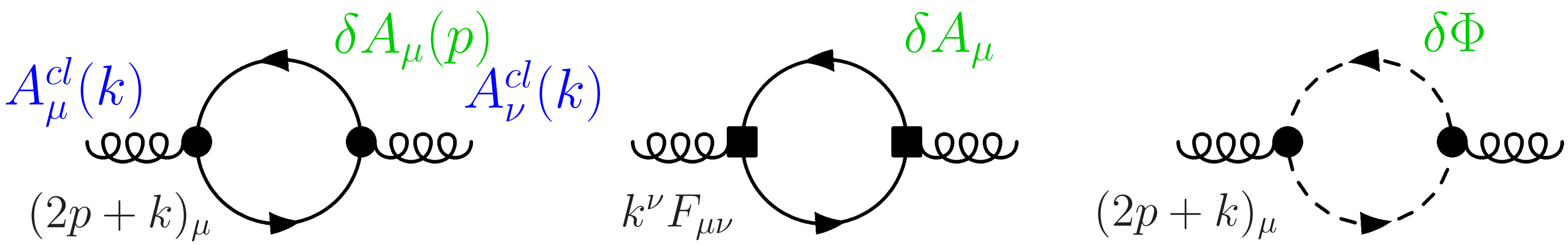
$$i\not{D}q = \gamma^\mu \left(i\partial_\mu + A^a_\mu t^a\right) q \qquad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$$



Asymptotic freedom

Classical field A_μ^{cl}. Modification due to quantum fluctuations:

$$A_\mu = A_\mu^{cl} + \delta A_\mu \qquad \frac{1}{g^2} F_{cl}^2 \to \left(\frac{1}{g^2} + c \log\left(\frac{k^2}{\mu^2}\right)\right) F_{cl}^2$$

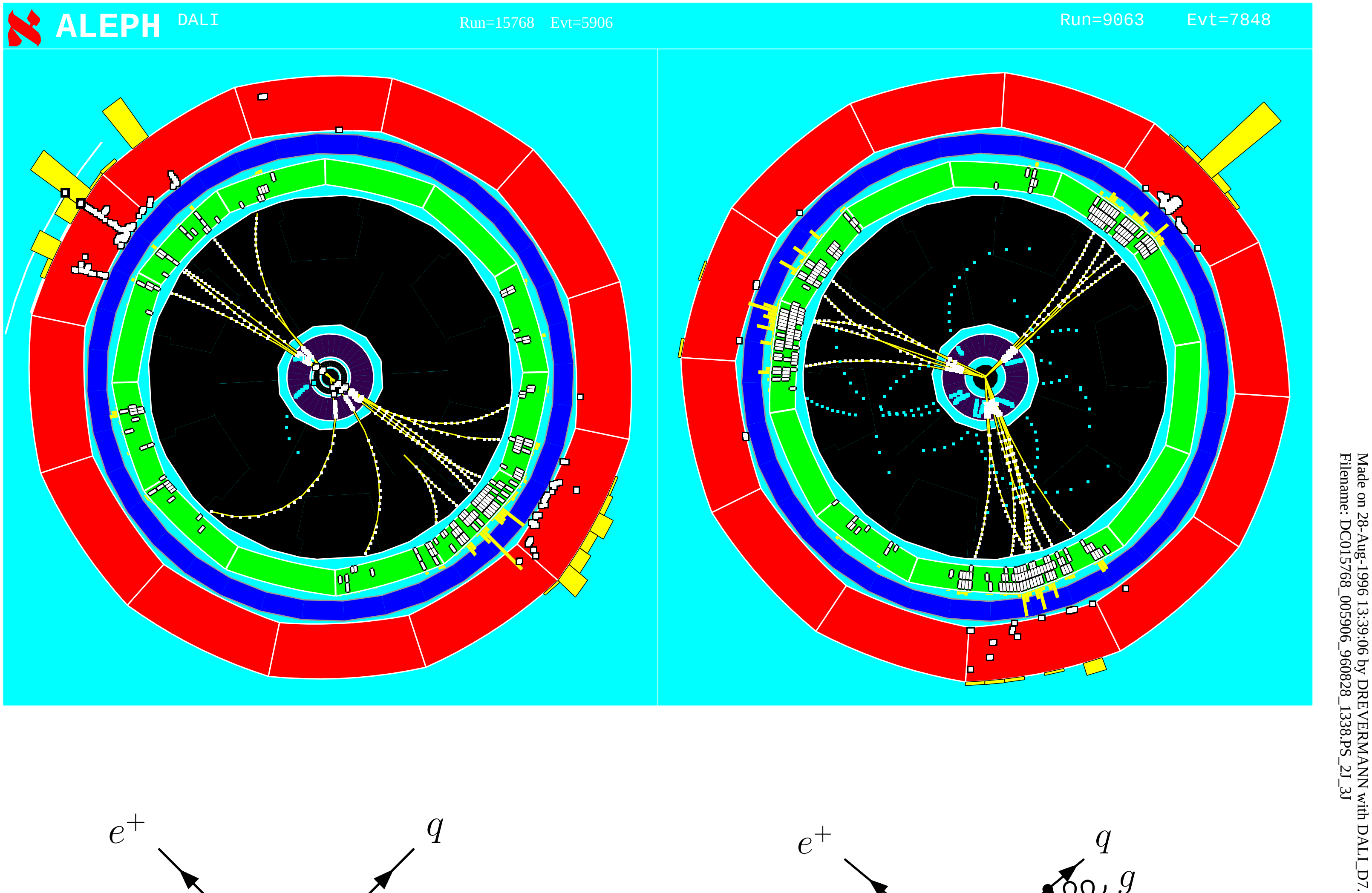


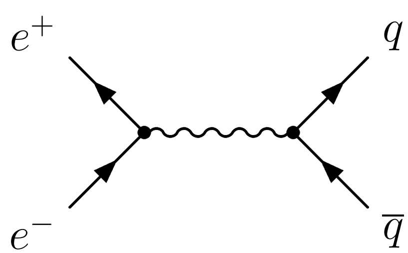
dielectric $\epsilon > 1$ paramagnetic $\mu > 1$ dielectric $\epsilon > 1$

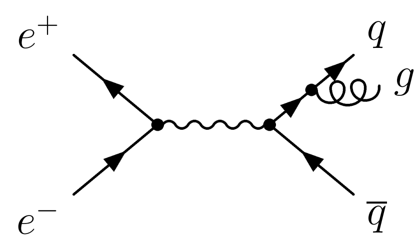
$$\mu\epsilon = 1 \;\Rightarrow\; \epsilon < 1$$

$$\beta(g) = \frac{\partial g}{\partial \log(\mu)} = \frac{g^3}{(4\pi)^2} \left\{ \left[\frac{1}{3} - 4\right] N_c + \frac{2}{3} N_f \right\} < 0$$

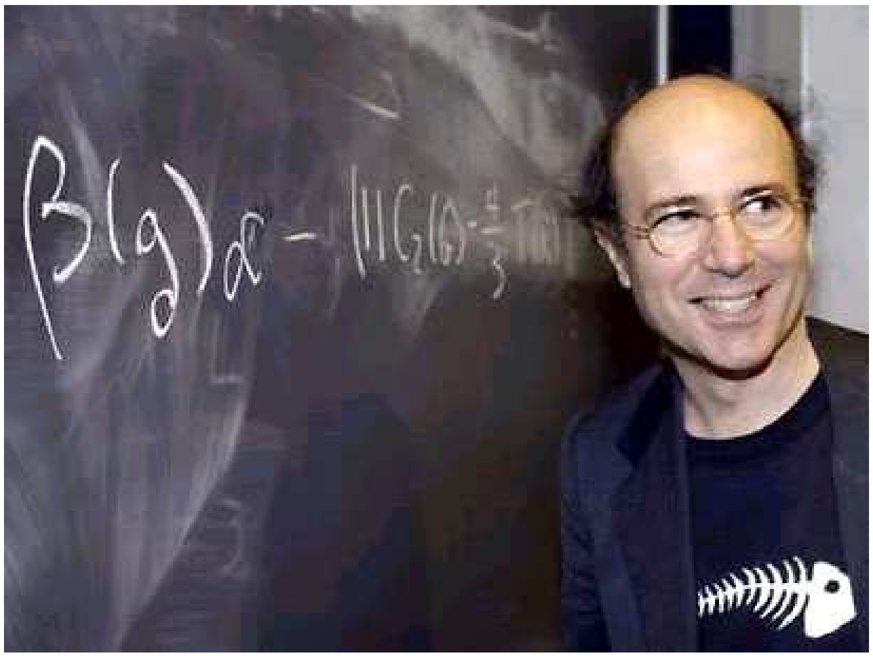
“Seeing” quarks and gluons

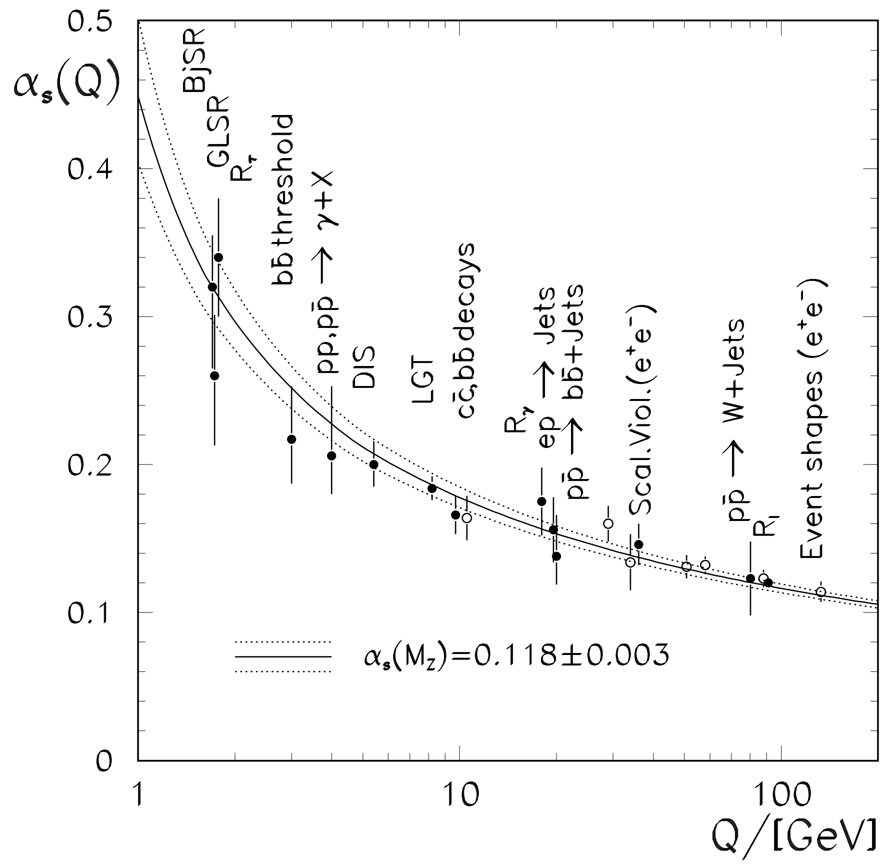






Running coupling constant





About units

Consider QCD Lite*

The lagrangian has a coupling constant, g, but no scale.

After renormalization g becomes scale dependent

g is traded for a scale parameter Λ

Λ is the only scale, the QCD "standard kilogram"

QCD Lite is a parameter free theory

Standard units: $\Lambda_{QCD} \simeq 200\,\mathrm{MeV} \simeq 1\,\mathrm{fm}^{-1}$

*QCD Lite is QCD in the limit $m_q \to 0$, $m_Q \to \infty$

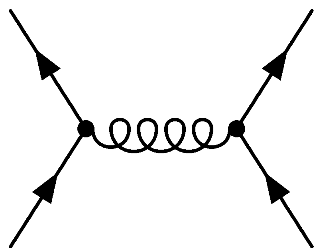
The high T phase: Qualitative argument

High T phase: Weakly interacting gas of quarks and gluons?

typical momenta $p \sim 3T$

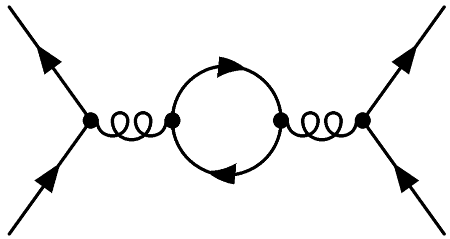
Large angle scattering involves large momentum transfer

effective coupling is small



Small angle scattering is screened (not anti-screened!)

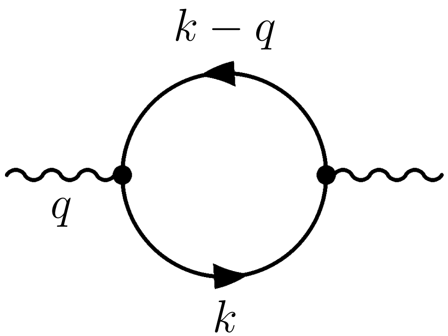
coupling does not become large



Quark Gluon Plasma

Gluon propagator

Warmup: Photon polarization function $\Pi_{\mu\nu}$



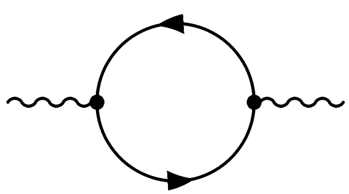
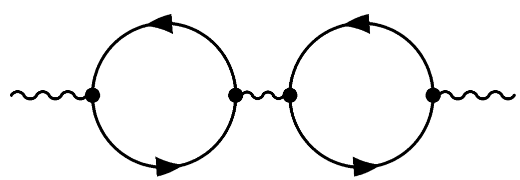
$$= e^2 T \sum_n \int \frac{d^3k}{(2\pi)^3} \mathrm{tr}[\gamma_\mu \not{k} \gamma_\nu (\not{k} - \not{q})] \Delta(k) \Delta(k-q)$$

Hard Thermal Loop (HTL) limit ($q \ll k \sim T$)

$$\Pi_{\mu\nu} = 2m^2 \int \frac{d\Omega}{4\pi} \Big(\frac{i\omega \hat{K}_\mu \hat{K}_\nu}{q \cdot \hat{K}} + \delta_{\mu 4} \delta_{\nu 4} \Big) \qquad \hat{K} = (-i, \hat{k})$$

$$2m^2 = \tfrac{1}{3} e^2 T^2 \text{ Debye mass}$$

Photon propagator: resum $\Pi_{\mu\nu}$ insertions

$D_{\mu\nu} =$  $+$  $+ \cdots = \dfrac{1}{(D^0_{\mu\nu})^{-1} + \Pi_{\mu\nu}}$

$D_{00}(\omega = 0, \vec{q})$ determines static potential

$$V(r) = e\int \frac{d^3q}{(2\pi)^3}\frac{e^{iqr}}{\vec{q}^{\,2}+\Pi_{00}} \simeq -\frac{e}{r}\exp(-m_D r)$$

screened Coulomb potential

D_{ij} determines magnetic interaction

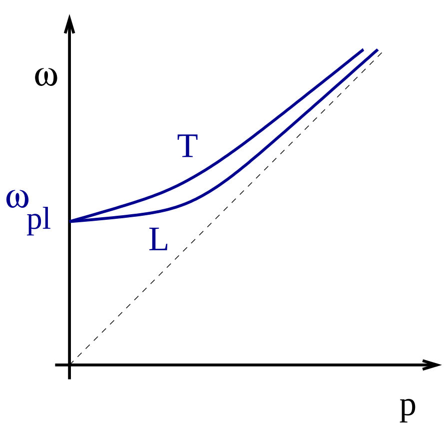
$$\Pi_{ii}(\omega \to 0, 0) = 0$$

no magnetic screening

$$Im\,\Pi_{ii}(\omega, q) \sim \frac{\omega}{q} m_D^2 \Theta(q-\omega)$$

Landau damping

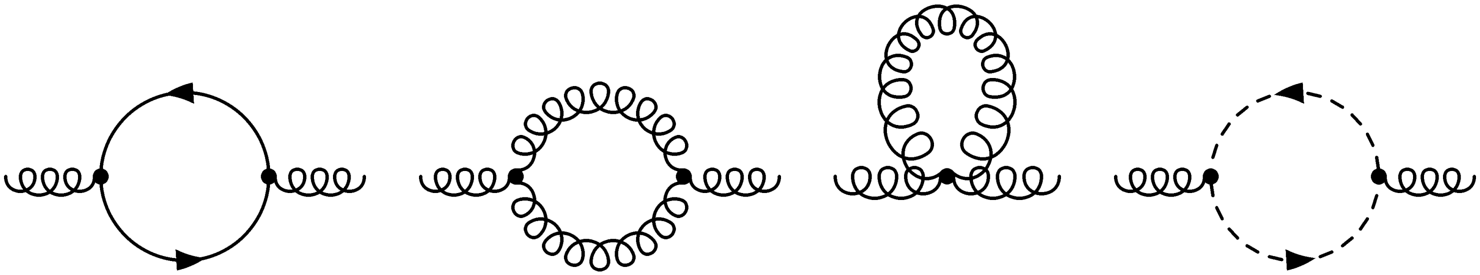
Poles of propagator: Plasmon dispersion relation



$$\text{pole}: \; D^{-1}_{L,T}(\omega, q) = 0$$

$$q \to 0: \quad \omega_L^2 = \omega_T^2 = \frac{1}{3} m_D^2$$

QCD looks more complicated



same result as QED with $m_D^2 = g^2T^2(1 + N_f/6)$

non-perturbative magnetic mass $m_M^2 \sim g^4T^2$

Conclusion: Perturbative Quark Gluon Plasma

quasi-quarks and quasi-gluons

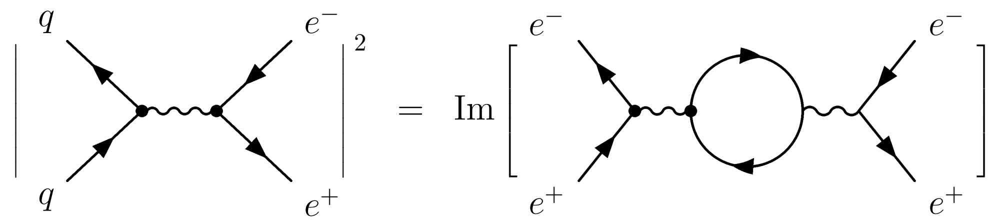
typical energies, momenta $\omega, p \sim T$

effective masses $m \sim gT$, width $\gamma \sim g^2T$

Note that $\gamma \ll \omega$ (long lived quasi-particles)

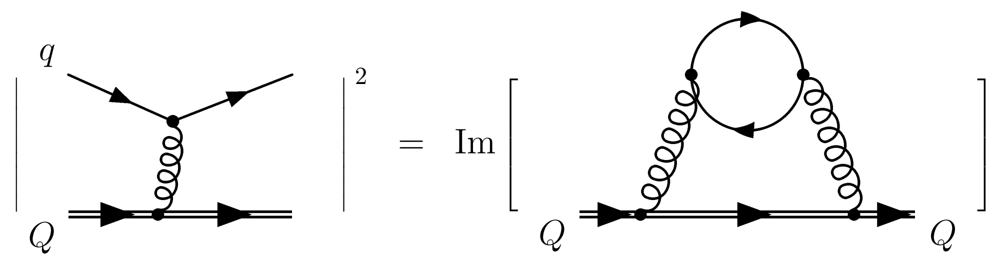
Physical applications

Dilepton production



$$\frac{dR}{d^4q} = \frac{\alpha^2}{48\pi^2}\Big(12\sum_q e_q^2\Big)e^{-E/T}$$

Collisional energy loss



$$\frac{dE}{dx} = \frac{8\pi}{3}\alpha_s^2 T^2\Big(1+\frac{N_f}{6}\Big)\log\Big(c\frac{\sqrt{ET}}{m_D}\Big) \qquad E \gg M^2/T$$

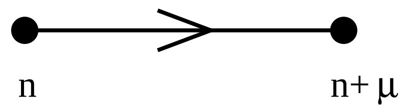
$E = 20$ GeV: $dE/dx \simeq 0.3\ \mathrm{GeV/fm}$ for c, b quarks

Note: for light quarks radiative energy loss dominates

Lattice QCD

Euclidean partition function

$$Z = \int dA_\mu d\psi \exp(-S) = \int dA_\mu \det(i\not{D}\,) \exp(-S_G)$$

Lattice discretization: 

$$U_\mu(n) = \exp(igaA_\mu(n))$$

$$D_\mu \phi \;\to\; \frac{1}{a}[U_\mu(n)\phi(n+\mu) - \phi(n)]$$

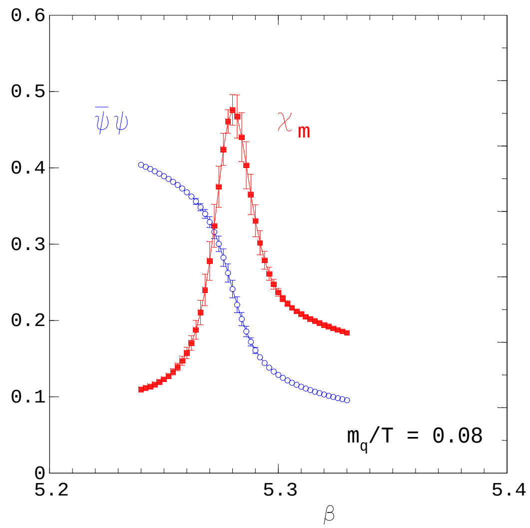
$$(G^a_{\mu\nu})^2 \;\to\; \frac{1}{a^4}\mathrm{Tr}[U_\mu(n)U_\nu(n+\mu)U_{-\mu}(n+\mu+\nu)U_{-\nu}(n+\nu) - 1]$$

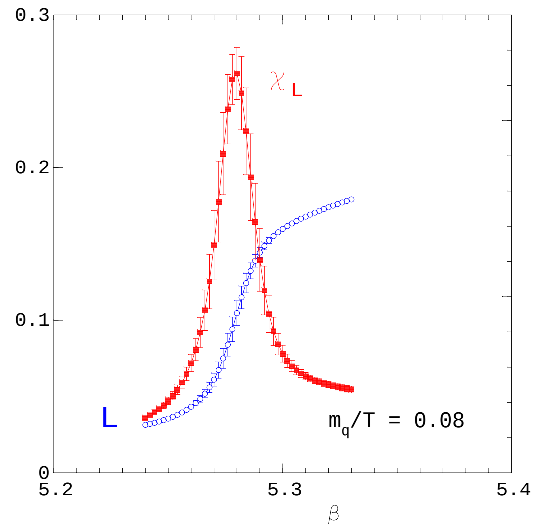
Monte Carlo:

$$\int dA_\mu \, e^{-S} \to \{U_\mu^{(1)}(n), U_\mu^{(2)}(n), \ldots\}$$

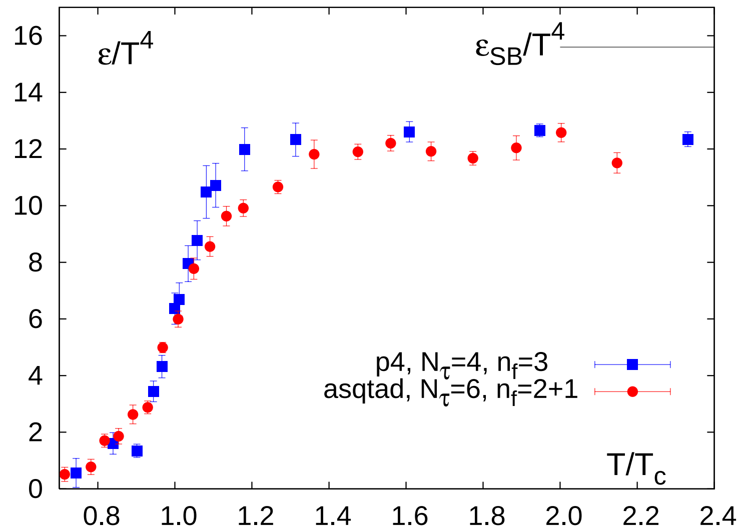
Lattice results

order parameters

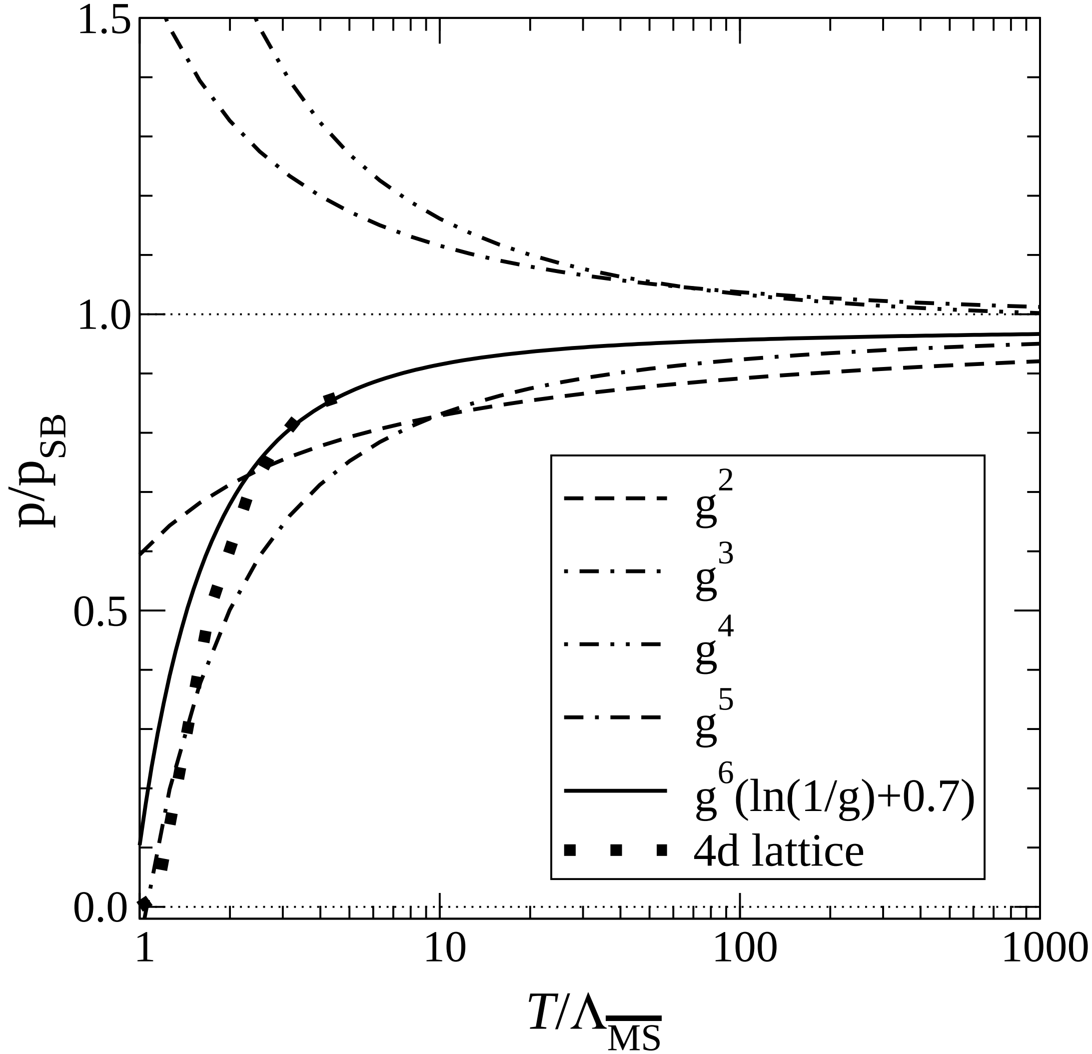




equation of state



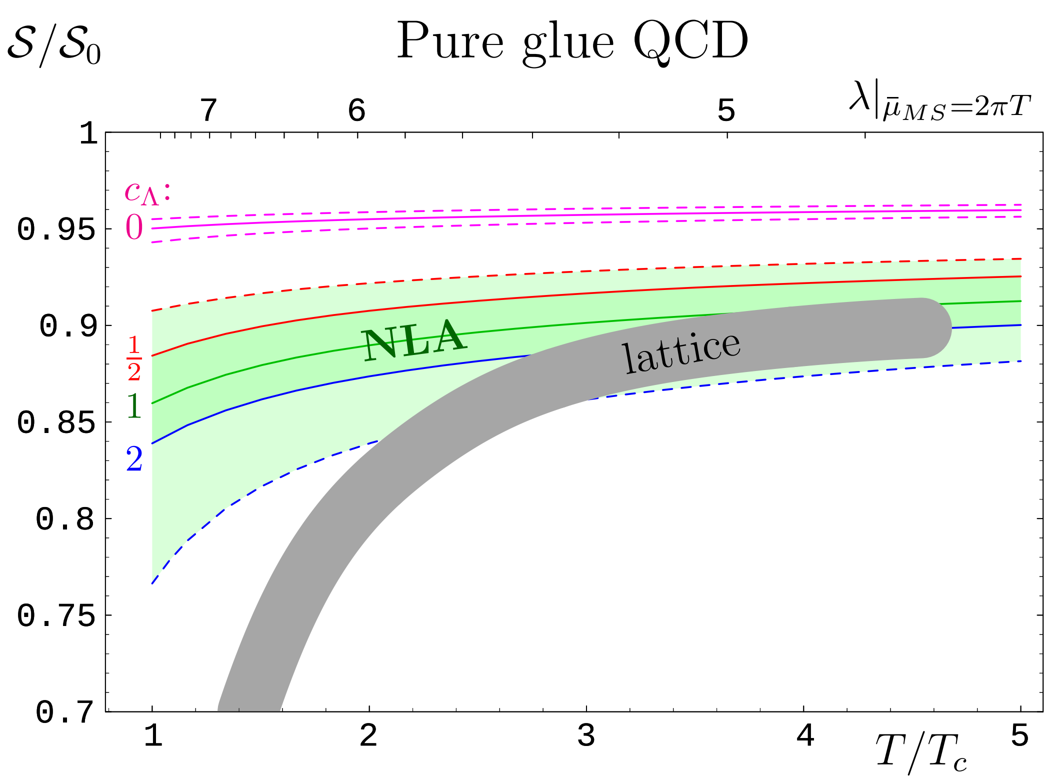
Lattice vs weak coupling thermodynamics



Kajantie et al. (2003)

convergence poor – related to non-analytic terms $(g^3, g^5, \ldots)$

HTL (resummed) perturbation theory



Blaizot et al. (2006)

convergence improved – agrees with lattice down to $\sim 2T_c$

$\mathcal{N} = 4$ Supersymmetric Yang-Mills Theory

Fields: Gluons, Gluinos, Higgses; all in the adjoint of $SU(N_c)$

$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \bar{\lambda}^a_A \sigma^\mu (D_\mu \lambda^A)^a + (D_\mu \Phi_{AB})^a (D_\mu \Phi^{AB})^a + \ldots$$

$$A^a_\mu \qquad \lambda^a_A\ (\bar{4}_R) \qquad \Phi^a_{AB}\ (6_R)$$

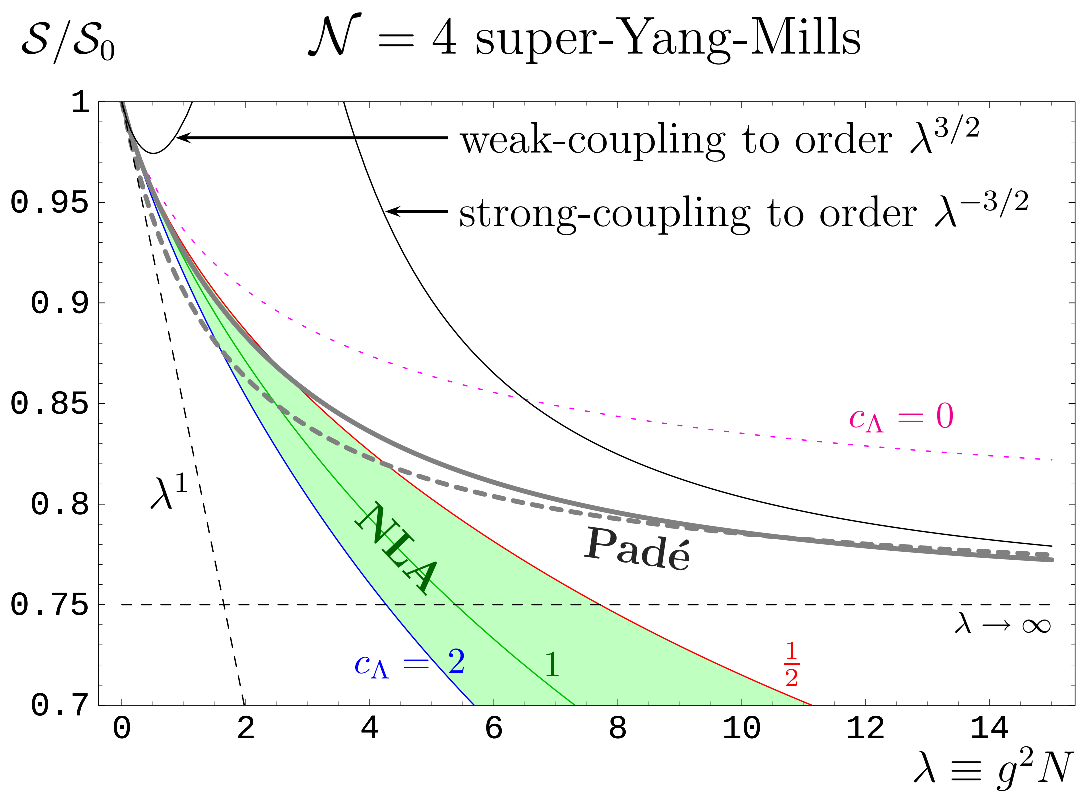
Global symmetries: Conformal and $SU(4)_R$

$$SO(4,2) \times SU(4)_R$$

Properties: Conformal $\beta(g) = 0$, extra scalars, no fundamental fermions, no chiral symmetry breaking, no confinement

strongly coupled SUSY-(Q)GP exactly solvable via AdS/CFT

SUSY QGP: weak vs strong coupling



smooth crossover near $g^2 N_c \sim 4$

The QGP plasma in equilibrium: where are we?

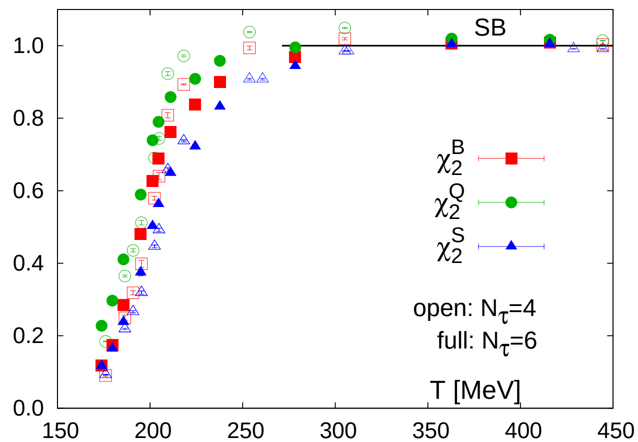
Strict perturbation theory does not work.

$$Need \quad g < 1$$

Resummed (quasi-particle) perturbation theory works down to $2T_c$.

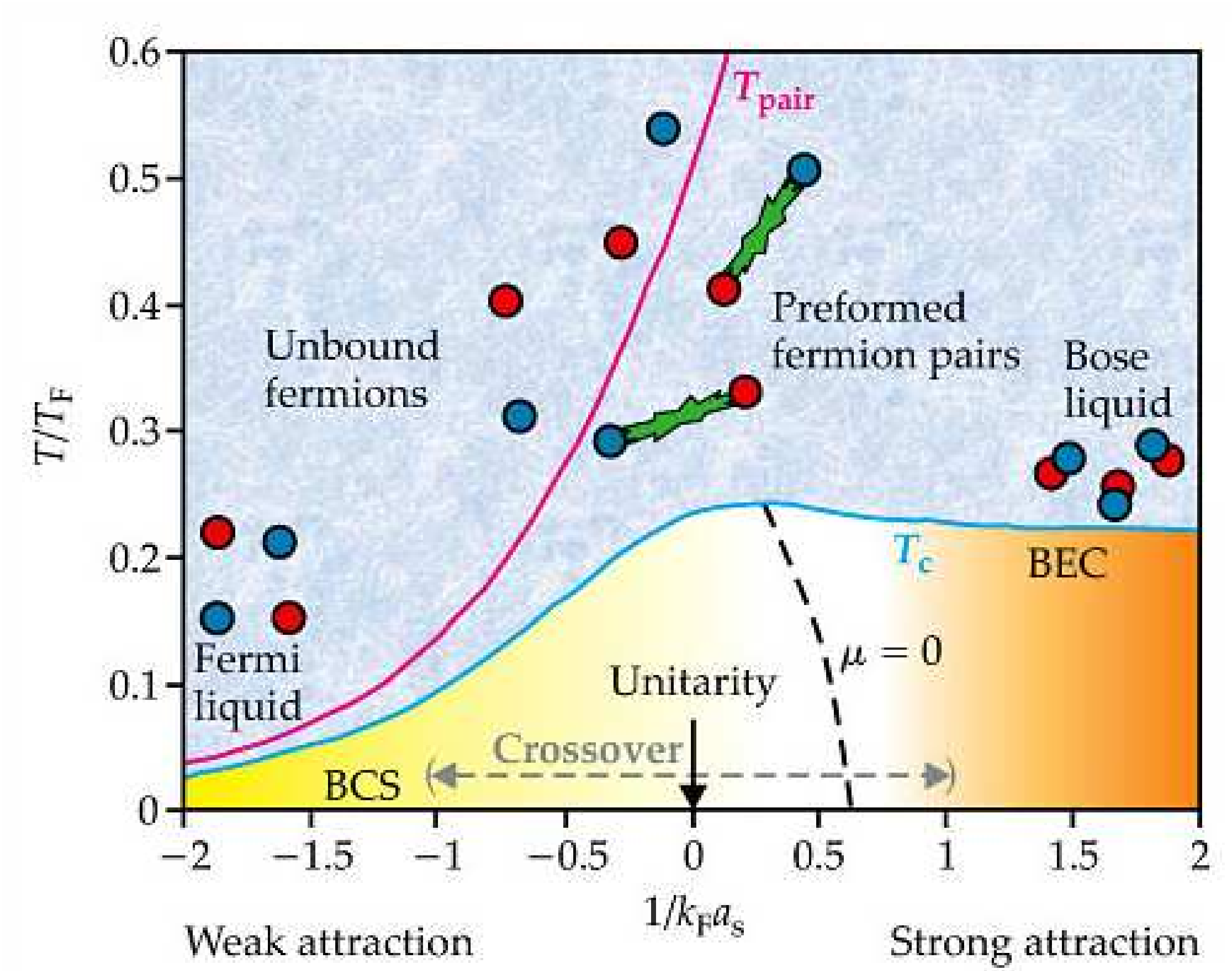
$$g^2 N_c \sim (4-8)$$

Other evidence in favor quasi-particles: quark flavor susceptibilities



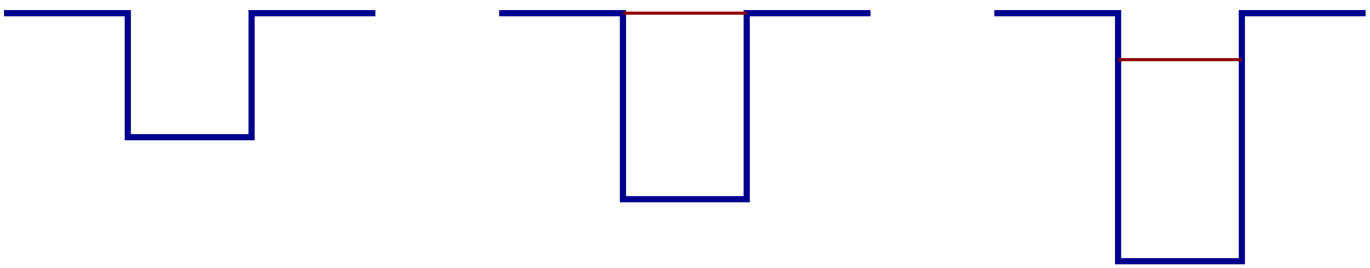
$$\chi_{qq} = \frac{\partial^2 \log Z}{\partial \mu_q^2} = \langle Q^2 \rangle - \langle Q \rangle^2$$

Dilute Fermi gas: BCS-BEC crossover



Unitarity limit

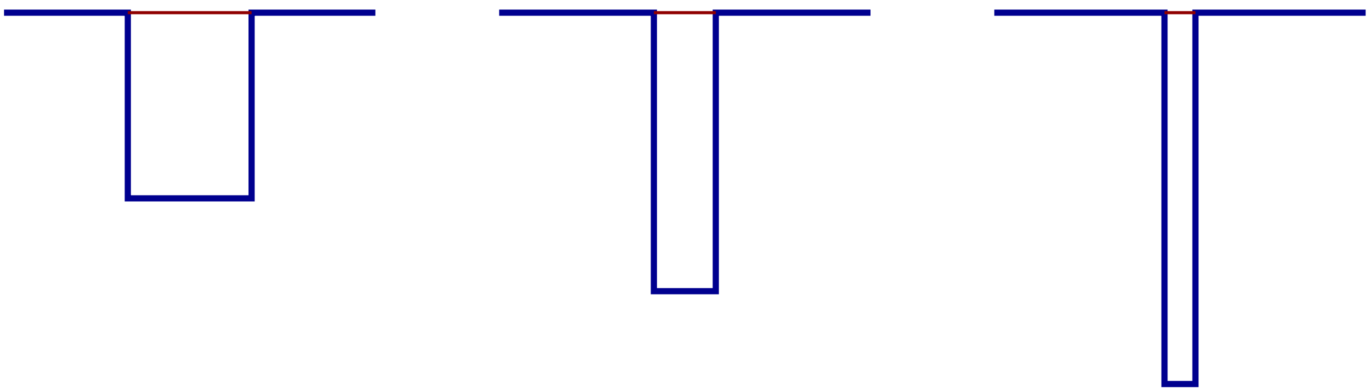
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$

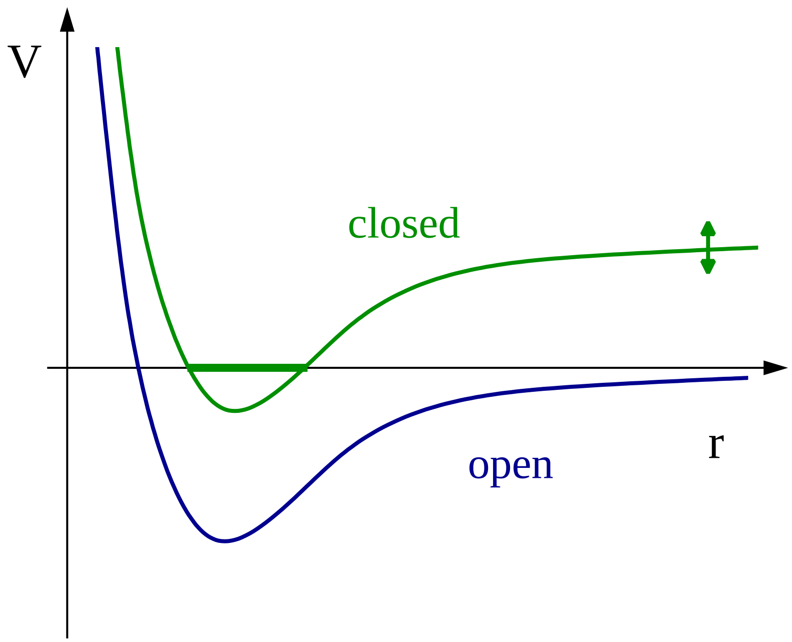


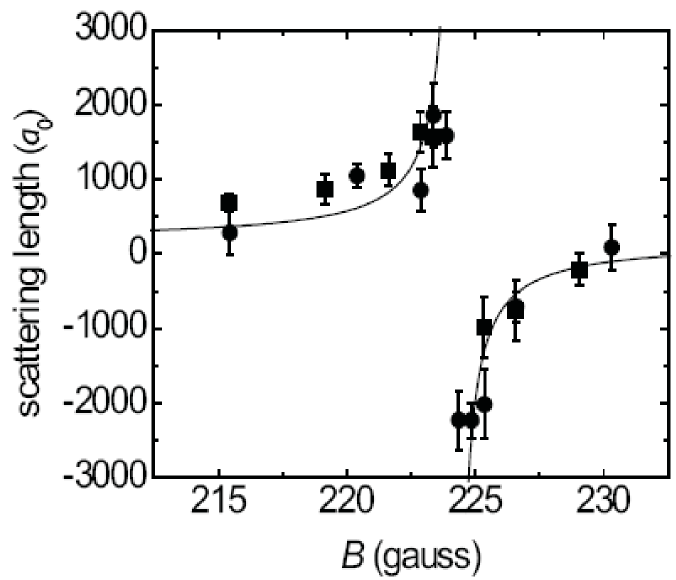
Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{a}r} \exp(-r/a)$$

Feshbach resonances

Atomic gas with two spin states: "↑" and "↓"



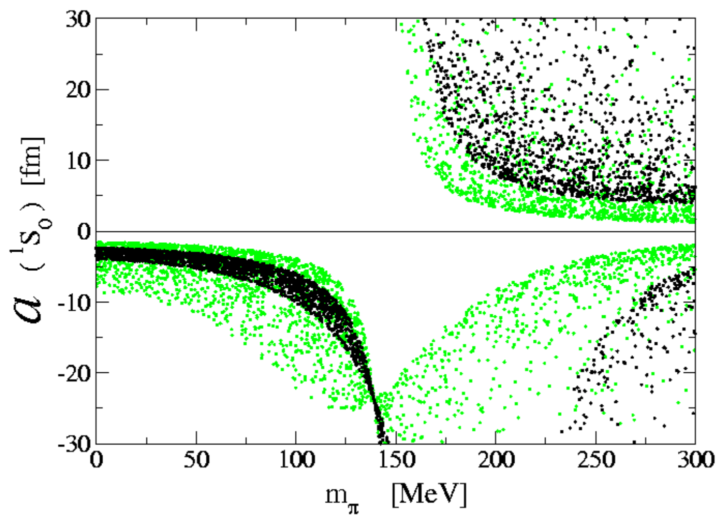


Feshbach resonance $$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

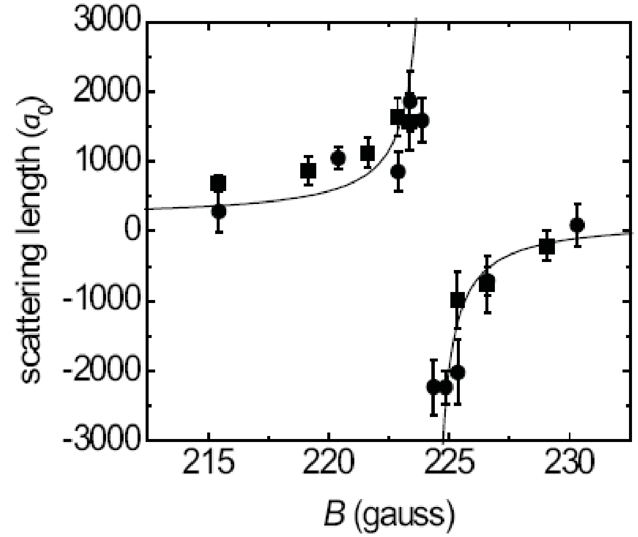
"Unitarity" limit $a \to \infty$ $$\sigma = \frac{4\pi}{k^2}$$

Universality

Neutron Matter



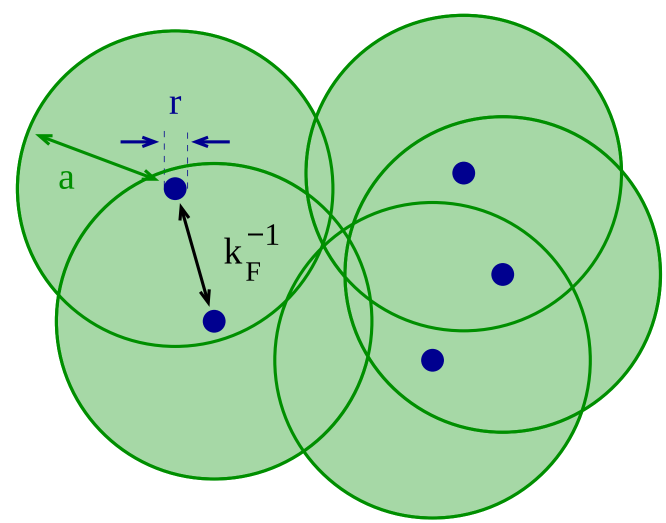
Feshbach Resonance in ^{6}Li



What do these systems have in common?

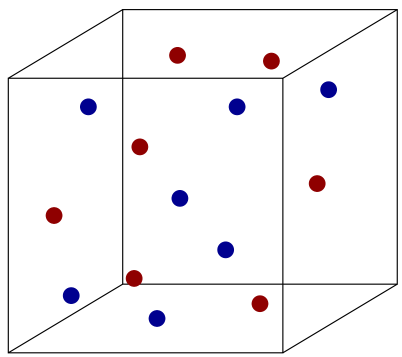
dilute: $r\rho^{1/3} \ll 1$

strongly correlated: $a\rho^{1/3} \gg 1$



Universality: Many body physics

Free fermi gas at zero temperature



$$\frac{E}{N} = \frac{3}{5}\frac{k_F^2}{2m} \qquad \frac{N}{V} = \frac{k_F^3}{3\pi^2}$$

Consider unitarity limit ($a \to \infty$, $r \to 0$)

$$\frac{E}{N} = \xi\,\frac{3}{5}\frac{k_F^2}{2m} \qquad k_F \equiv (3\pi^2 N/V)^{1/3}$$

Prize problem (Bertsch, 1998): Determine ξ

Similar problems: $\Delta = \alpha\epsilon_F$, $\qquad k_B T_c = \beta\epsilon_F$

Dilute Fermi gas: field theory

Non-relativistic fermions at low momentum

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

Unitary limit: $a \to \infty$, $\sigma \to 4\pi/k^2$ $(C_0 \to \infty)$

This limit is smooth: HS-trafo, $\Psi = (\psi_\uparrow, \psi_\downarrow^\dagger)$

$$\mathcal{L} = \Psi^\dagger \left[i\partial_0 + \sigma_3 \frac{\vec{\nabla}^2}{2m} \right] \Psi + \left(\Psi^\dagger \sigma_+ \Psi \phi + h.c. \right) - \frac{1}{C_0} \phi^* \phi \ ,$$

Low T $(T < T_c \sim \mu)$: Pairing and superfluidity

Upper and lower critical dimension

Zero energy bound state for arbitrary d

$$\psi''(r) + \frac{d-1}{r}\psi'(r) = 0 \qquad (r > r_0)$$

d=2: Arbitrarily weak attractive potential has a bound state

free fermions: $\mu = E_F$

d=4: Bound state wave function $\psi \sim 1/r^{d-2}$. Pairs do not overlap

free bosons: $\mu = 0$

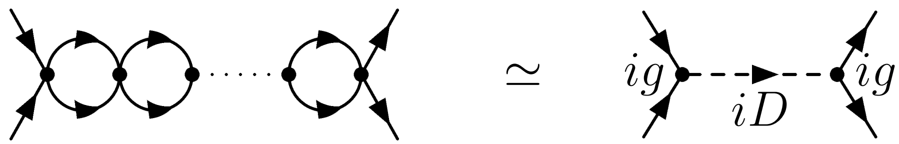
Conclude $\xi = \mu/E_F \sim 1/2$?

Try expansion around $d = 4$ or $d = 2$?

Nussinov & Nussinov (2004)

Epsilon expansion

EFT version: Compute scattering amplitude ($d = 4 - \epsilon$)

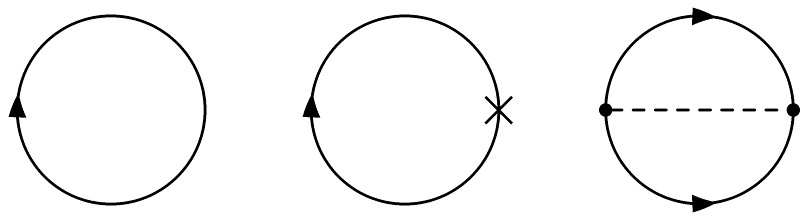


$$T = \frac{1}{\Gamma\left(1 - \frac{d}{2}\right)} \left(\frac{m}{4\pi}\right)^{-d/2} \left(-p_0 + \frac{\epsilon_p}{2}\right)^{1-d/2} \simeq \frac{8\pi^2\epsilon}{m^2} \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

$$g^2 \equiv \frac{8\pi^2\epsilon}{m^2} \qquad\qquad D(p_0, p) = \frac{i}{p_0 + \frac{\epsilon_p}{2} + i\delta}$$

Weakly interacting bosons and fermions

Results

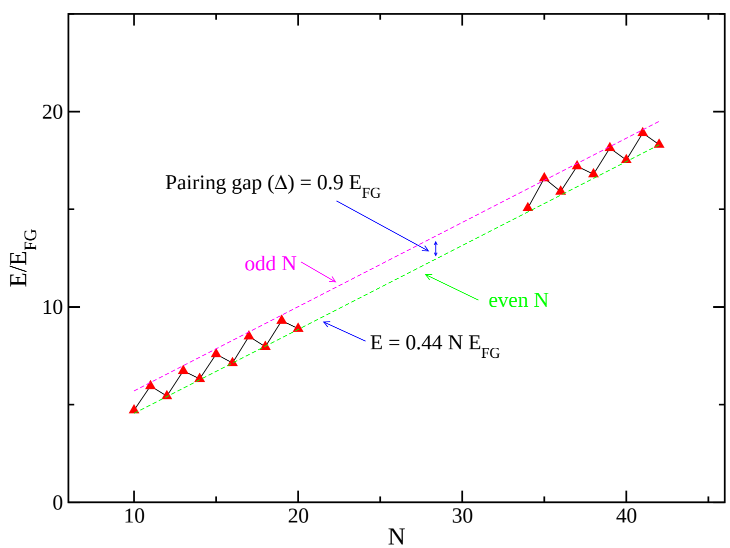


$O(1)$ $O(1)$ $O(\epsilon)$

$$\xi = \frac{1}{2}\epsilon^{3/2} + \frac{1}{16}\epsilon^{5/2}\ln\epsilon - 0.0246\,\epsilon^{5/2} + \ldots$$

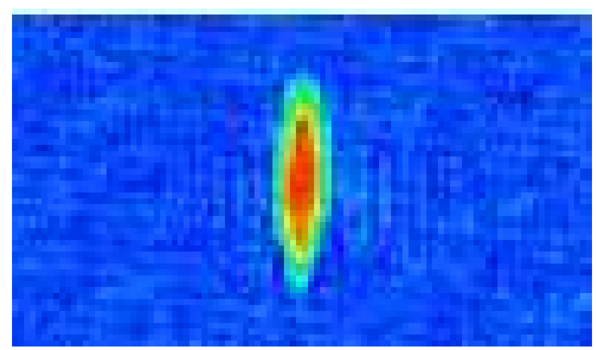
$$\xi(\epsilon=1) = 0.475$$

Green function MC



$\xi = 0.40$-0.44 (Carlson et al.)

Experiment



$\xi = 0.38(2)$ (Luo, Thomas)

Equation of state

Universality implies that

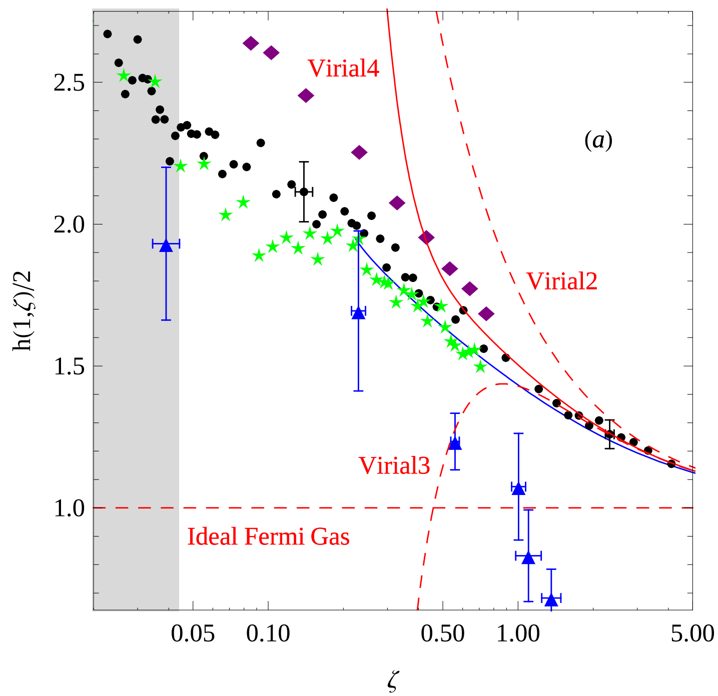
$$P(\mu, T) = P_0(\mu, T) h\left(\frac{\mu}{T}\right) \qquad \mathcal{E} = \frac{3}{2}P$$

At $T = 0$ have $P(\mu) = \xi^{-3/2} P_0(\mu)$ with $\xi \simeq 0.4$

Harmonic trap: $h(z)$ determined by column density ($dP = n d\mu$)

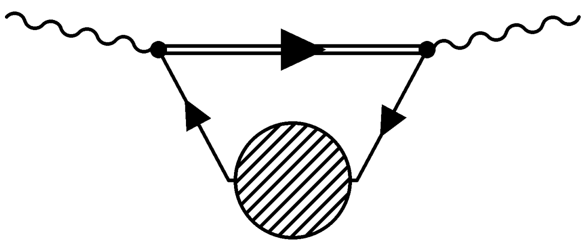
$$P(\mu(x), T) = \frac{m\omega_\perp^2}{\pi}\tilde{n}(x)$$

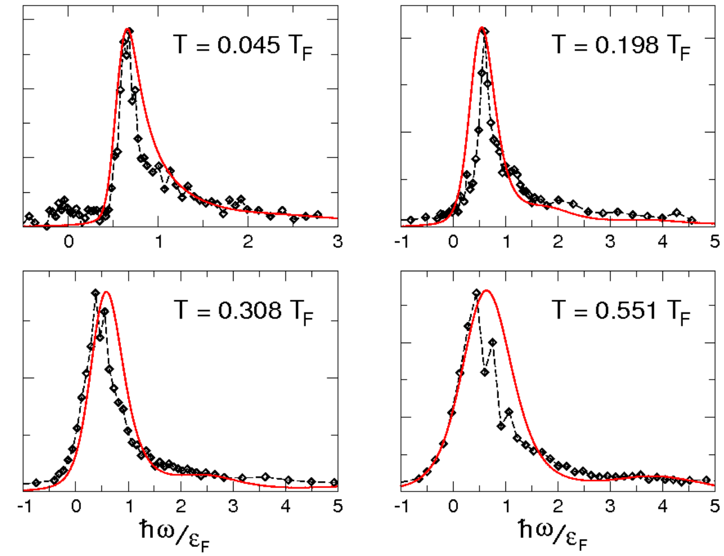
Nascimbene et al, Science (2010).



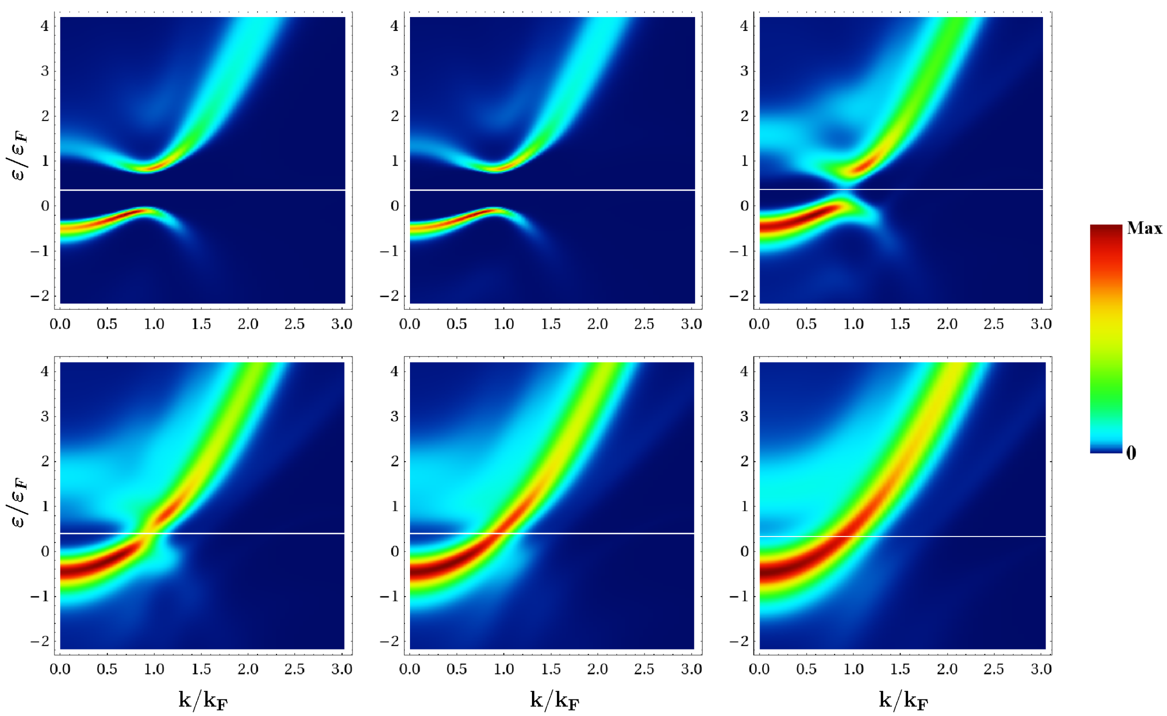
Evidence for quasi-particles: RF spectroscopy

$$\Gamma(\omega) \sim Im \int dt\, d^3x\, e^{i\omega t} \langle \psi_1^\dagger \psi_3(x,t) \psi_3^\dagger \psi_1 \rangle$$





Schirotzek et al.(2008)



T/T_F

0.01,0.06,0.14

0.16,0.18,0.30

Zwerger et al. (2010)

The Fermi gas in equilibrium: where are we?

Thermodynamics well under control (numerically and experimentally)

Theoretical approaches (BCS/BEC crossover, T-matrix, ERG, ...) “work”

Evidence for quasi-particles at large q and T