From Trapped Atoms

To Liberated Quarks

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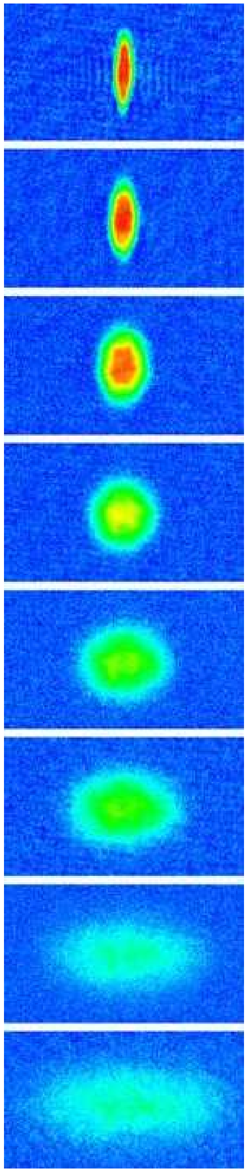
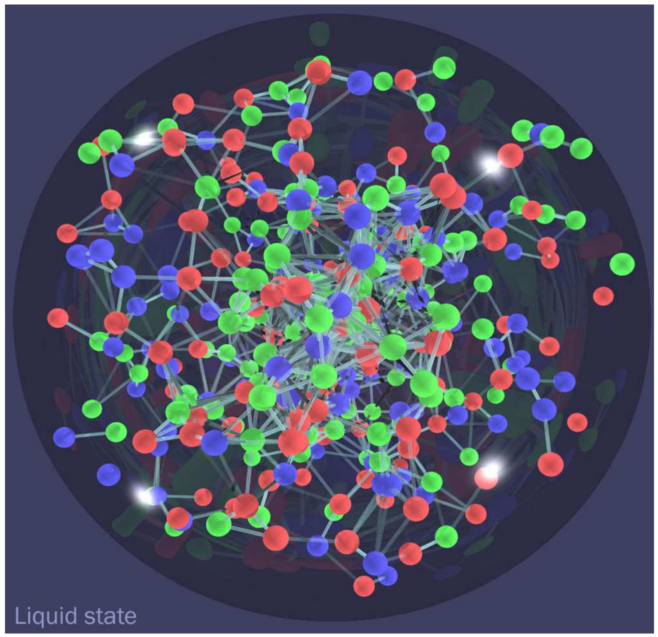
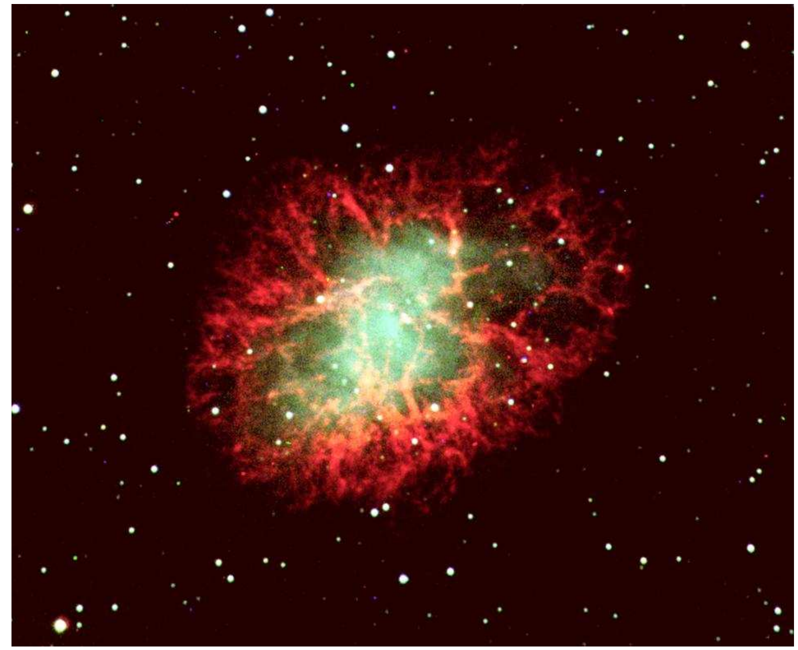
Perfect Liquids





sQGP (T=180 MeV)



Neutron Matter (T=1 MeV)

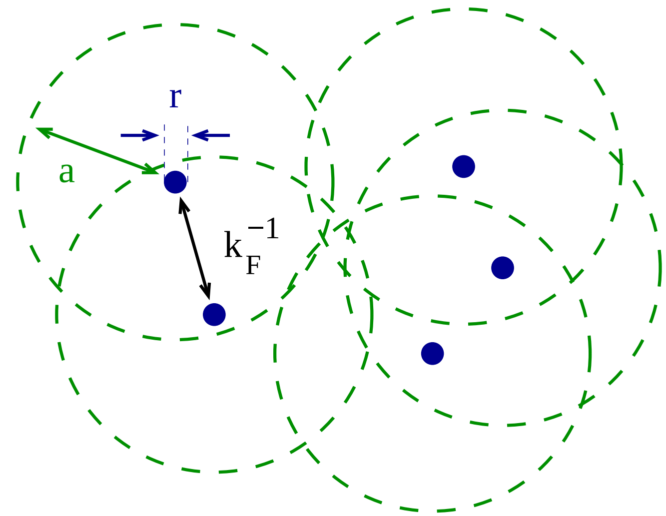
Trapped Atoms (T=5 peV)

Universality

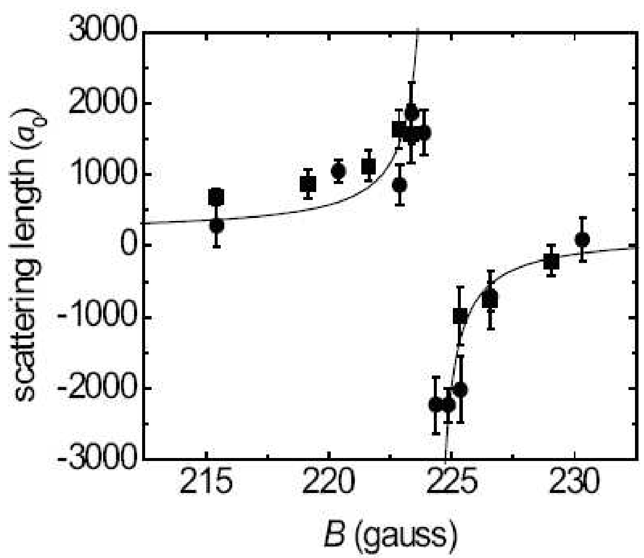
What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

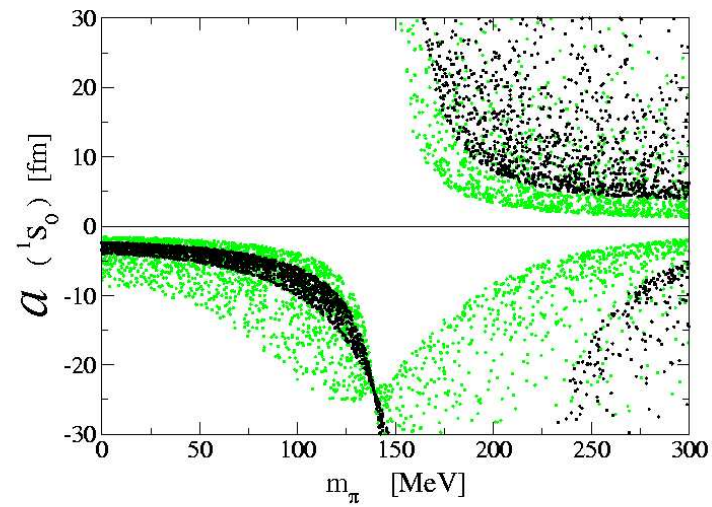
strongly correlated: $a\rho^{1/3} \gg 1$



Feshbach Resonance in ^{6}Li



Neutron Matter



I. Equation of State

Universal Equation of State

Consider limiting case ("Bertsch" problem)

$$(k_F a) \to \infty \qquad\qquad (k_F r) \to 0$$

Universal equation of state

$$\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0 = \xi \frac{3}{5}\left(\frac{k_F^2}{2M}\right)$$

How to find ξ?

Numerical Simulations

Experiments with trapped fermions

Analytic Approaches

Effective Field Theory

Effective field theory for pointlike, non-relativistic neutrons

$$\mathcal{L}_{\rm eff} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + h.c. \right] + \ldots$$

$$(a, r, \ldots) \Rightarrow (C_0, C_2, \ldots)$$

Partition Function (Hubbard-Stratonovich field s, Fermion matrix Q)

$$Z = \int Ds \exp\left[-S' \right], \qquad S' = -\log(\det(Q)) + V(s)$$

$C_0 < 0$ (attractive): $\det(Q) \geq 0$

Continuum Limit

Fix coupling constant at finite lattice spacing

$$\frac{M}{4\pi a} = \frac{1}{C_0} + \frac{1}{2}\sum_{\vec{p}} \frac{1}{E_{\vec{p}}}$$

Take lattice spacing b, b_τ to zero

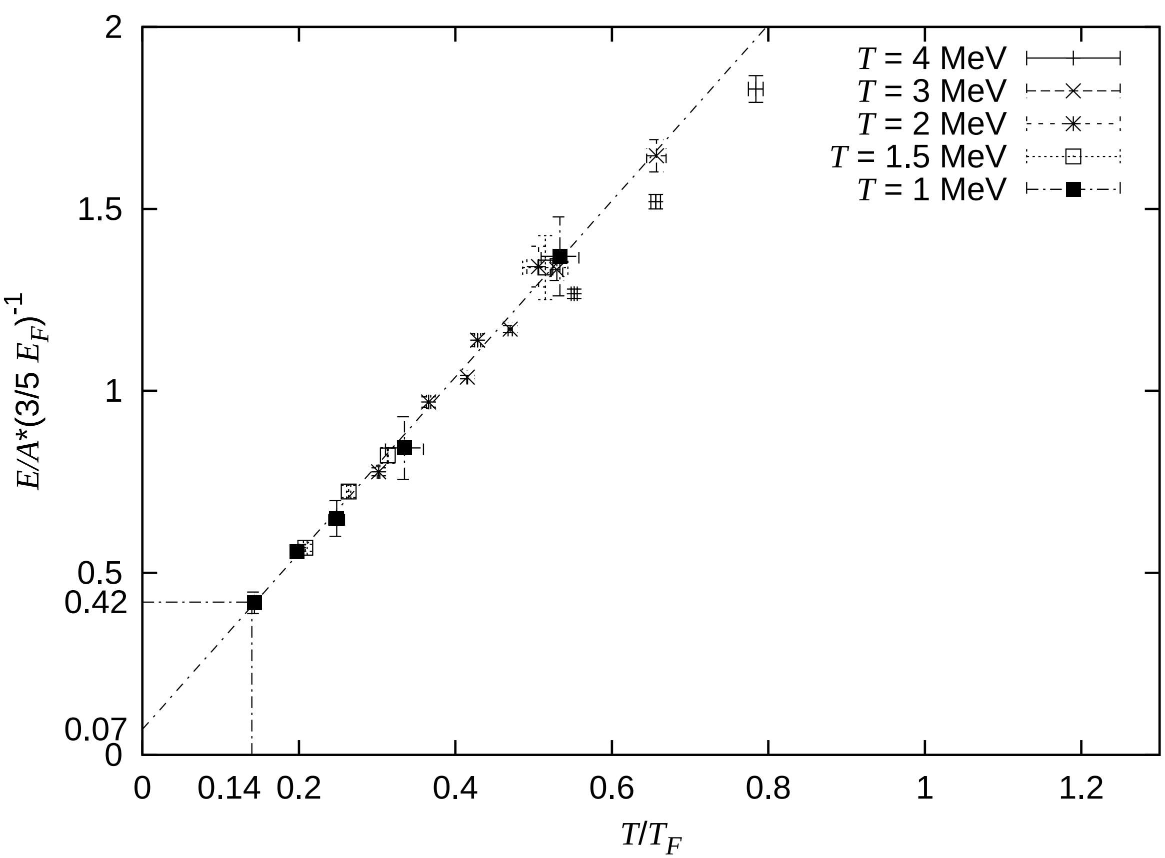
$$\mu b_\tau \to 0 \qquad n^{1/3} b \to 0 \qquad n^{1/3} a = const$$

Physical density fixed, lattice filling $\to 0$

Consider universal (unitary) limit

$$n^{1/3} a \to \infty$$

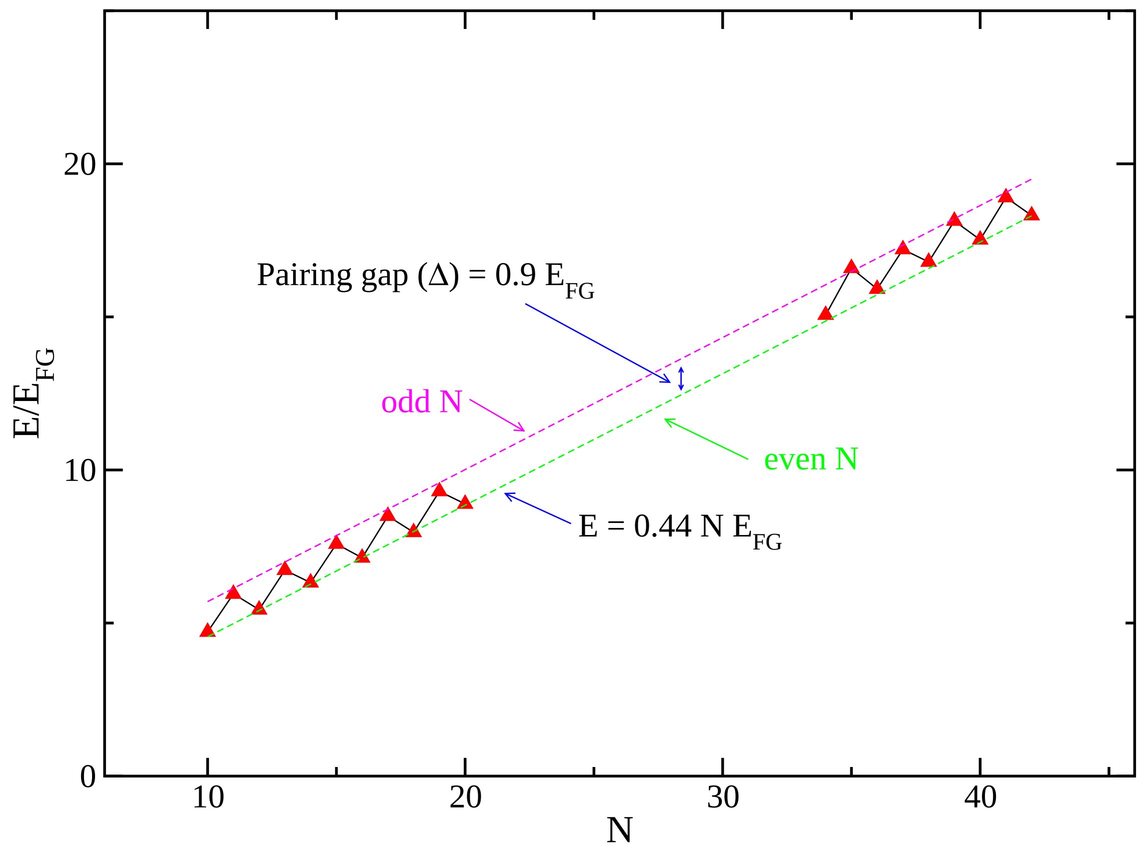
Lattice Results



Canonical $T = 0$ calculation: $\xi = 0.25(3)$ (D. Lee)

Not extrapolated to zero lattice spacing

Green Function Monte Carlo



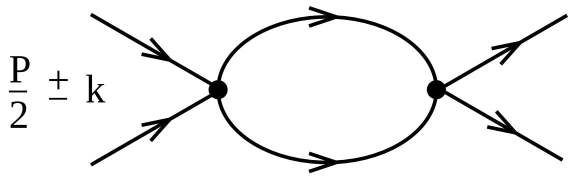
Other lattice results: $\xi = 0.4$ (Burovski et al., Bulgac et al.)

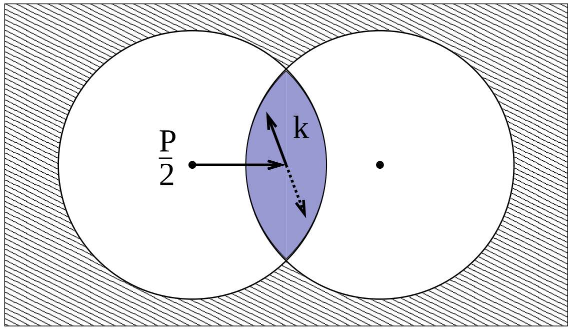
Experiment: $\xi = 0.27^{+0.12}_{-0.09}\,[1],\ 0.51 \pm 0.04\,[2],\ 0.74 \pm 0.07\,[3]$

[1] Bartenstein et al., [2] Kinast et al., [3] Gehm et al.

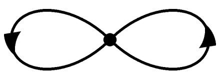
Large d Limit

In medium scattering strongly restricted by phase space

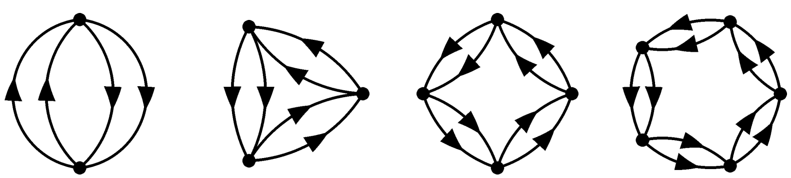




Find limit in which ladders are leading order



$$(C_0/d) \cdot 1/d$$



$$(C_0/d)^k \cdot 1/d$$

$$\lambda \equiv \left[\frac{\Omega_d C_0 k_F^{d-2} M}{d(2\pi)^d} \right]$$

$$\lambda = const \ \ (d \to \infty)$$

$$\xi = \frac{1}{2} + O(1/d)$$

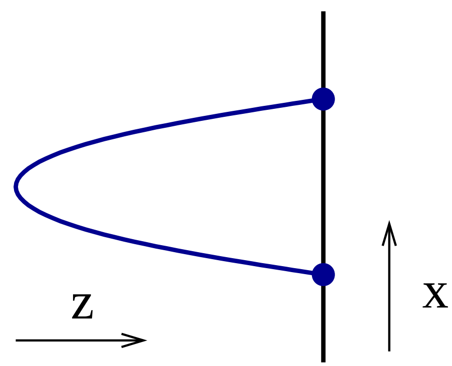
Gauge Theory at Strong Coupling: Holographic Duals

The AdS/CFT duality relates

large N_c (Conformal) gauge theory in 4 dimensions	$\Leftrightarrow$	string theory on 5 dimensional Anti-de Sitter space $\times S_5$
correlation fcts of gauge invariant operators	$\Leftrightarrow$	boundary correlation fcts of AdS fields

$$\langle \exp \int dx \; \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



The correspondence is simplest at strong coupling $g^2 N_c$

strongly coupled gauge theory $\Leftrightarrow$ classical string theory

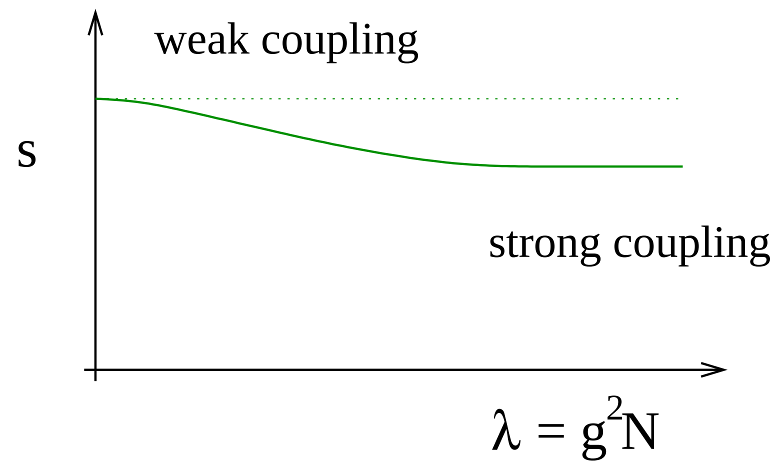
Gauge Theory at Strong Coupling: Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy =area of event horizon

$$s = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0$$

Gubser and Klebanov

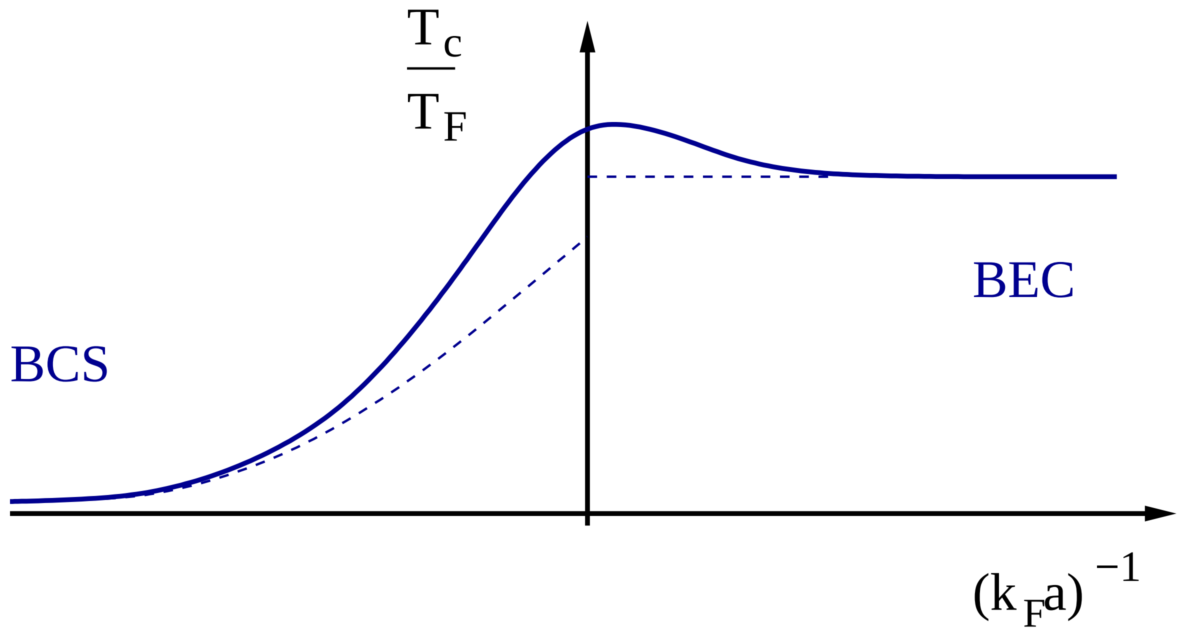


Extended to transport properties by Policastro, Son and Starinets

$$\eta = \frac{\pi}{8} N_c^2 T^3$$

II. How Large Can T_c Get?

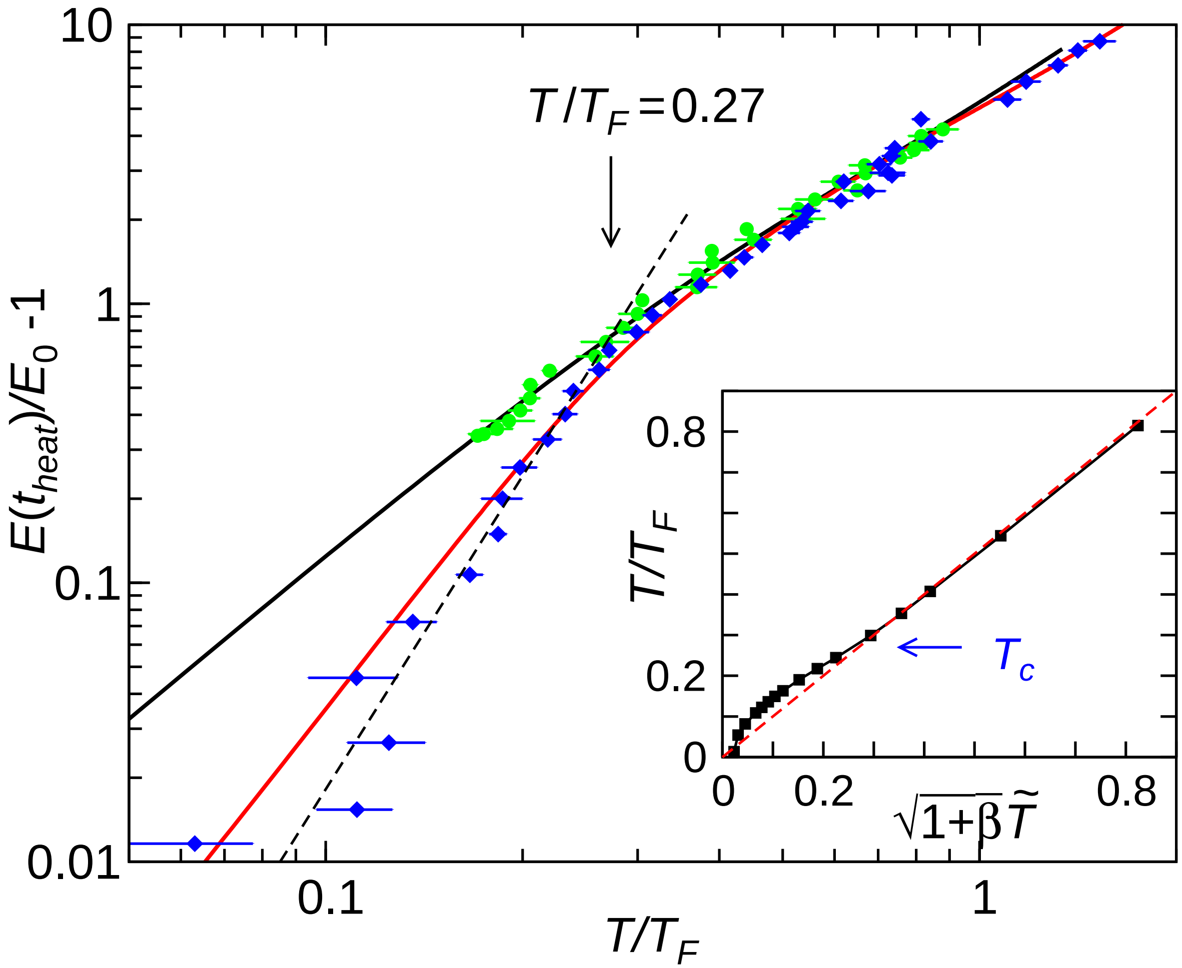
Critical Temperature: From BCS to BEC



$$T_c^{BCS} = \frac{4 \cdot 2^{1/3} e^{\gamma}}{e^{7/3}\pi} \epsilon_F \exp\left(-\frac{\pi}{|k_F a|}\right) \qquad T_c^{BEC} = 3.31 \left(\frac{n^{2/3}}{m}\right)$$

$$T_c(a \to \infty) = 0.28\epsilon_F \qquad T_c = 0.34\epsilon_F + O(a_B n^{1/3})$$

Experimental Results



Kinast et al. (2005)

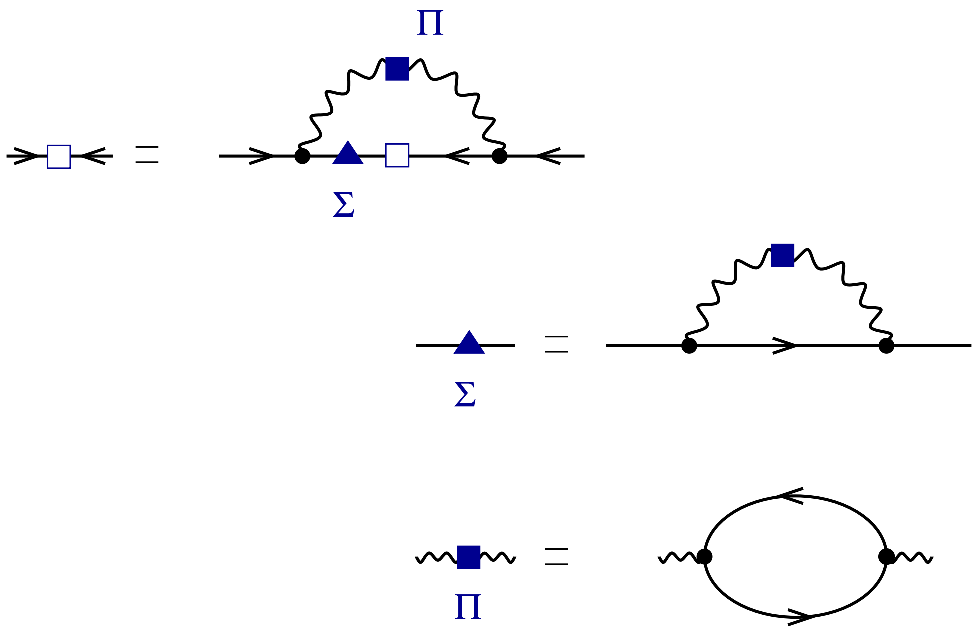
Lattice results: $T_c/T_F = 0.15$ (UMass)

Quark Matter: Color Superconductivity

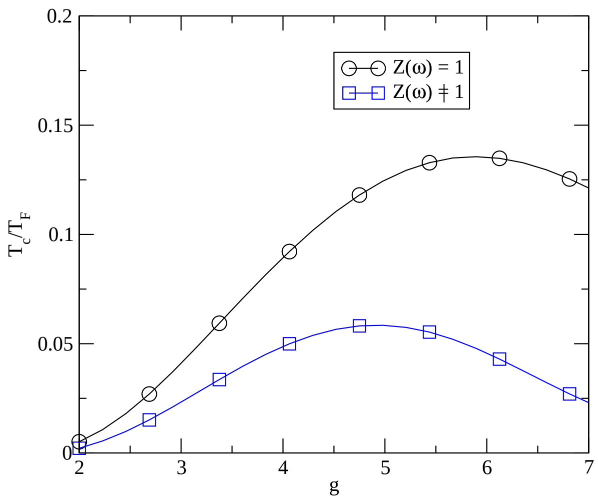
Weak coupling result

$$\frac{T_c}{T_F} = \frac{be^\gamma}{\pi}\exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$

$$b = \frac{512\pi^4}{g^5}\left(\frac{2}{N_f}\right)^{\frac{5}{2}} e^{-\frac{\pi^2+4}{8}}$$



Maximum $T_c/T_F = 0.025$. Strong coupling?



Note: Transition to χSB

Consider $N_c = 2$ QCD?

III. Stressed Pairing

Polarized Fermions: From BEC to BCS

BEC limit: Tightly bound bosons, no polarization for $\delta\mu < \Delta$

$\delta\mu > \Delta$: Mixture of Fermi and Bose liquid, no phase separation

Stable? Consider EFT for gapless fermions interacting with GB's

$$\mathcal{L} = \psi^\dagger \left(i\partial_0 - \epsilon(-i\vec{\partial}) - (\vec{\partial}\varphi)\cdot\frac{\overleftrightarrow{\partial}}{2m} \right)\psi + \frac{f_t^2}{2}\dot{\varphi}^2 - \frac{f^2}{2}(\vec{\partial}\varphi)^2$$

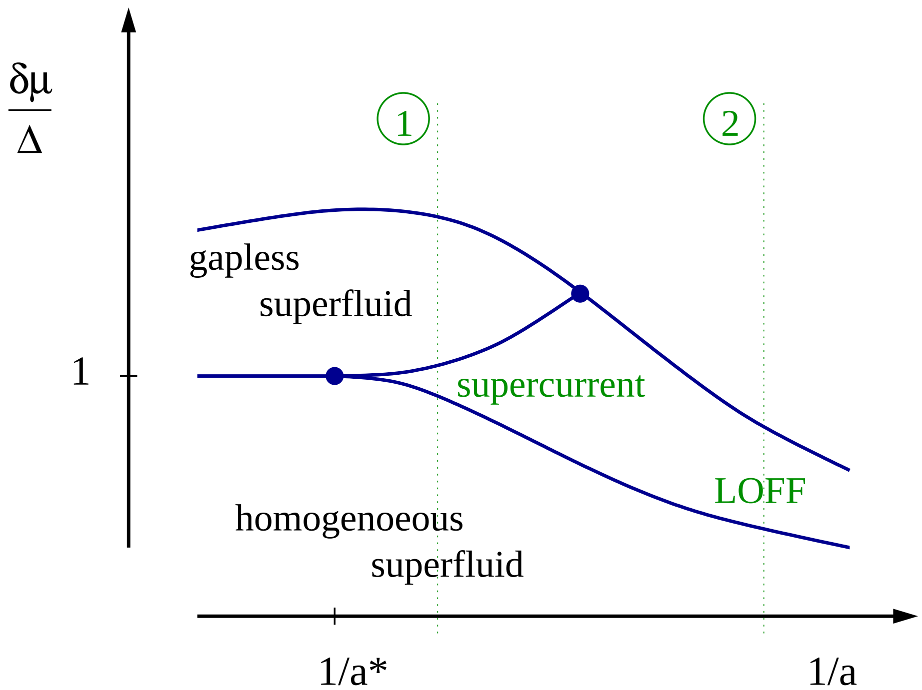
$$\epsilon_v(\vec{p}) = \epsilon(\vec{p}) + \vec{v}_s\cdot\vec{p} - \delta\mu$$

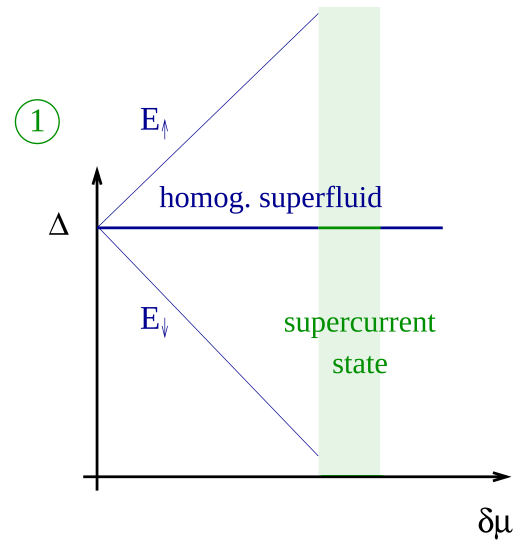
Free energy of state with non-zero current

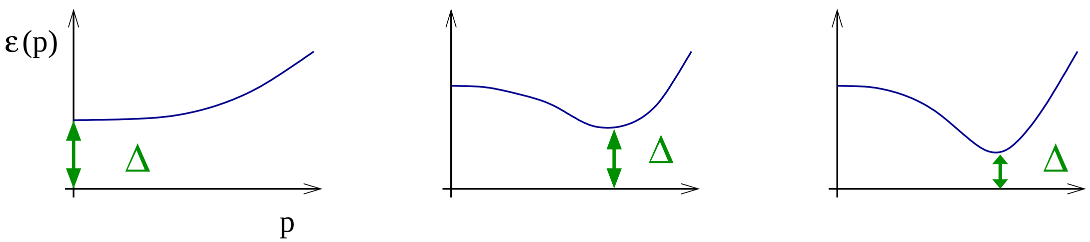
$$F(v_s) = \frac{1}{2}nmv_s^2 + \int \frac{d^3p}{(2\pi)^3}\,\epsilon_v(\vec{p})\Theta\left(-\epsilon_v(\vec{p})\right)$$

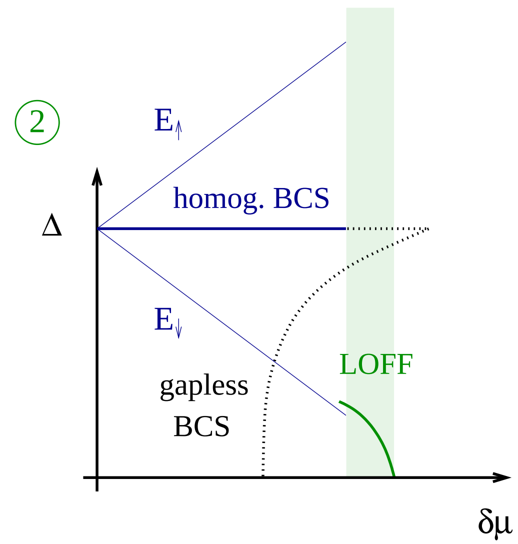
Unstable for BCS-type dispersion relation

Schematic Phase Diagram



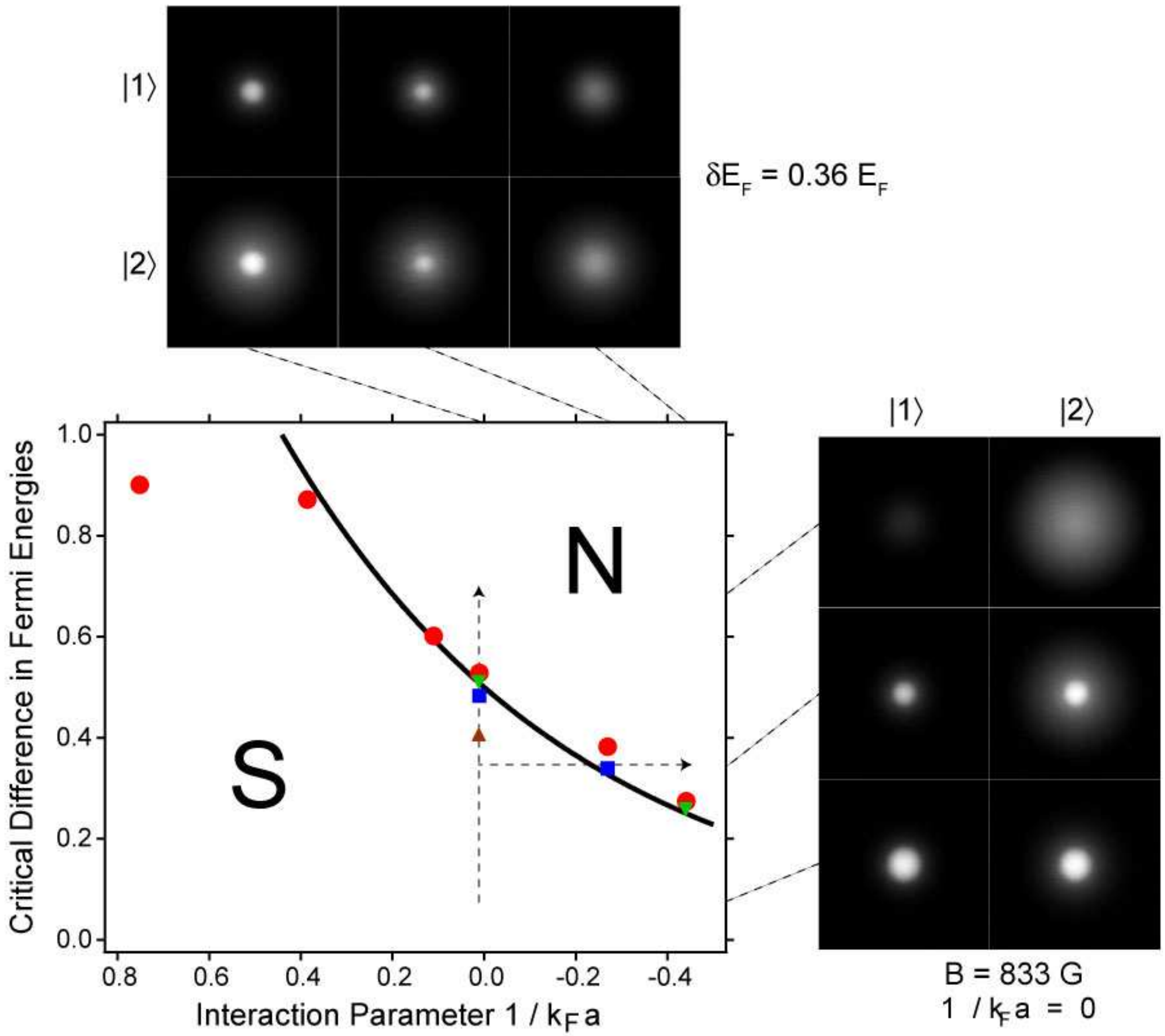






Son & Stephanov (2005)

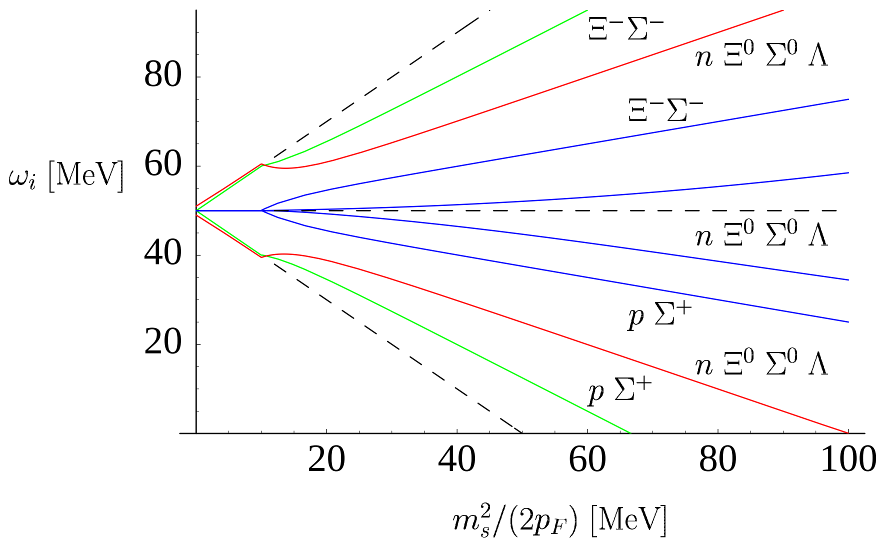
Experimental Situation

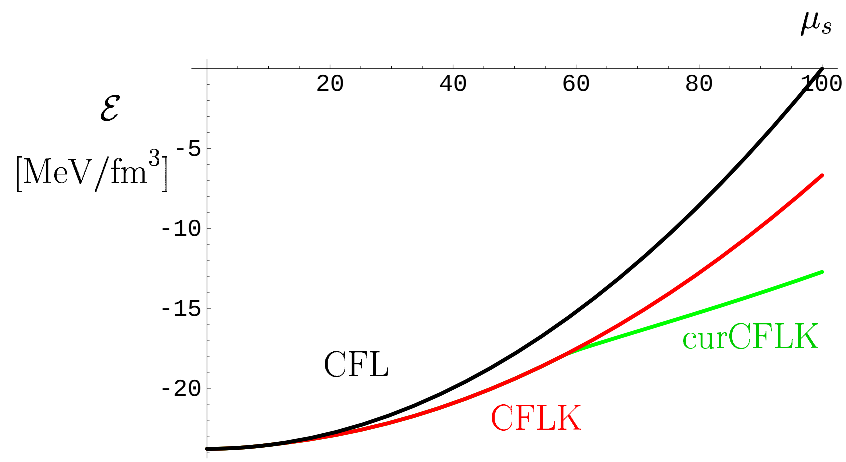


Zwierlein et al.(MIT group)

Phase Structure of CFL Quark Matter

How does CFL ($\langle ud\rangle = \langle ds\rangle = \langle su\rangle$) pairing responds to m_s?





Excitation energy of fermions

Gapless modes appear at

$$\mu_s(crit) \sim \frac{4\Delta}{3}$$

Energy density of superfluid phases

$$\mu_s(K-cond) \sim m_u^{2/3}\Delta^{4/3}/\mu$$

$$\mu_s(GB-cur) \sim 4\Delta/3$$

Figures: Kryjevski & Schäfer (2004)

Schäfer (2005)

Summary

Trapped atoms near a Feshbach resonance provide interesing model system for

- equation of state of strongly correlated systems (neutron matter, sQGP)
- viscosity of strongly correlated systems (sQGP?)
- superfluidity at strong coupling (T_c/T_F, response to pair breaking fields, precursor phenomena)