The Phases of QCD

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Motivation

Different phases of QCD occur in the universe

Neutron Stars, Big Bang

Exploring the phase diagram is important to understanding the phase that we happen to live in

Structure of hadrons is determined by the structure of the vacuum

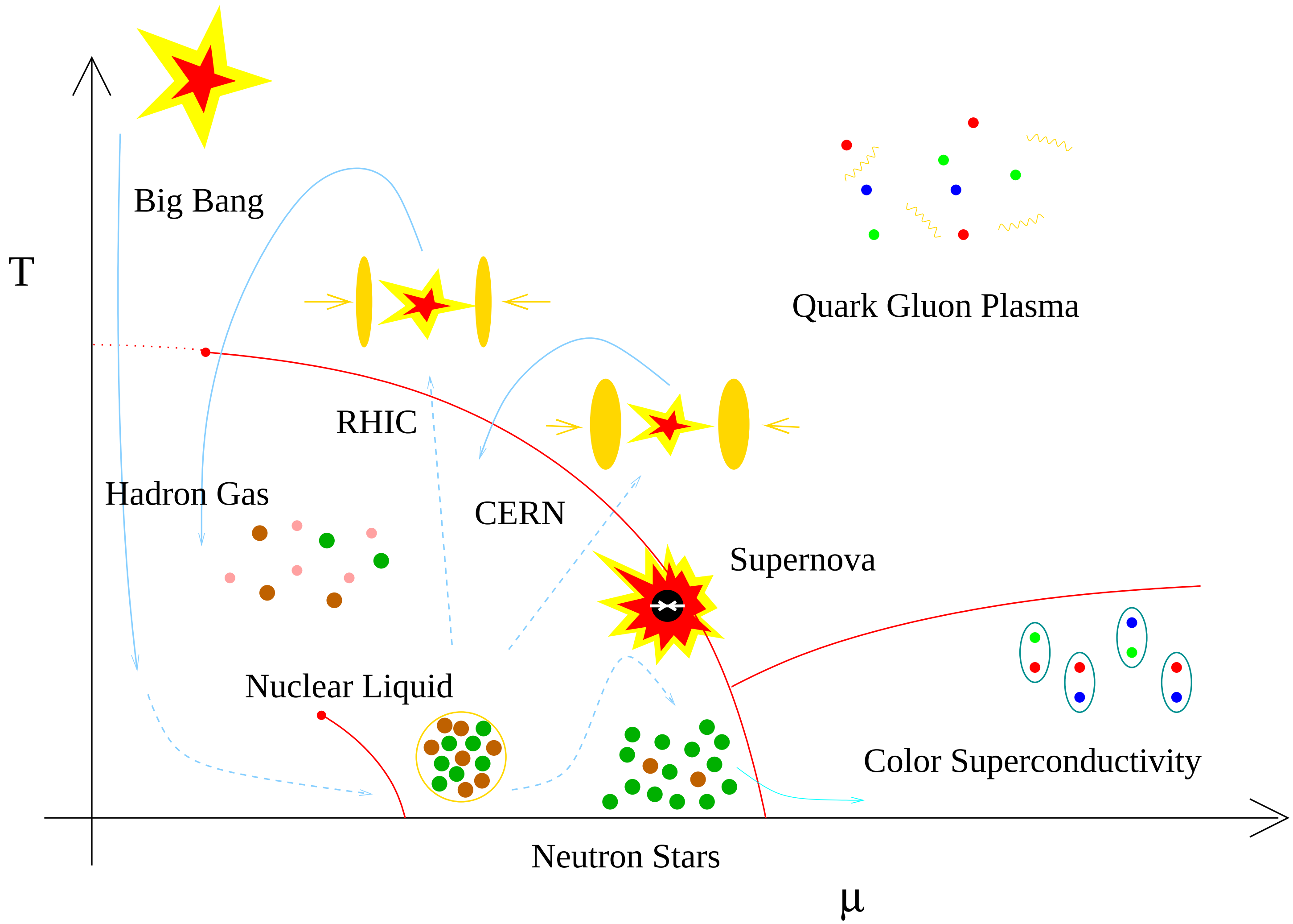
Need to understand how vacuum can be modified

QCD simplifies in extreme environments

Study QCD matter in a regime where quarks and gluons

are the correct degrees of freedom

QCD Phase Diagram



Quantumchromodynamics

Elementary fields: **Quarks** **Gluons**

$$(q_\alpha)^a_f \begin{cases} \text{color} & a = 1, \ldots, 3 \\ \text{spin} & \alpha = 1, 2 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$$

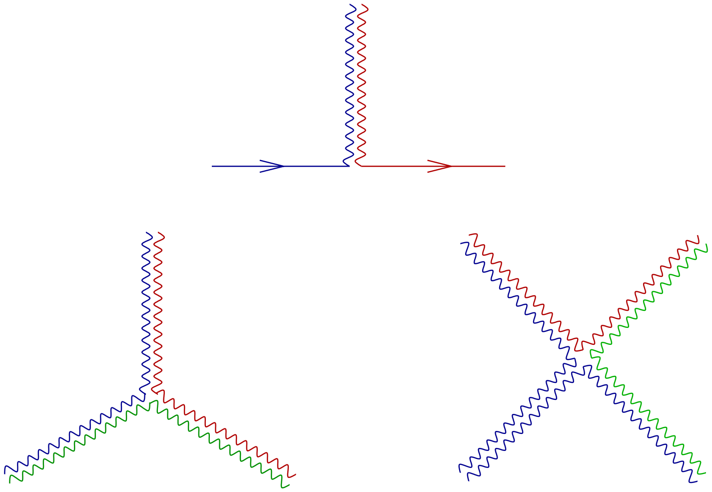
$$A^a_\mu \begin{cases} \text{color} & a = 1, \ldots, 8 \\ \text{spin} & \epsilon^\pm_\mu \end{cases}$$

Dynamics: non-abelian gauge theory

$$\mathcal{L} = \bar{q}_f (i \not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

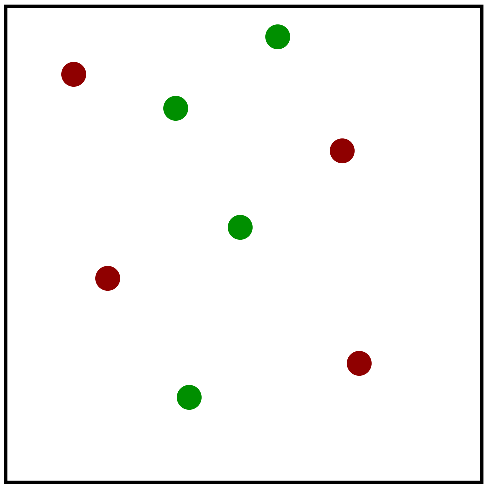
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$i \not{D} q = \gamma^\mu \left(i \partial_\mu + g A^a_\mu t^a \right) q$$

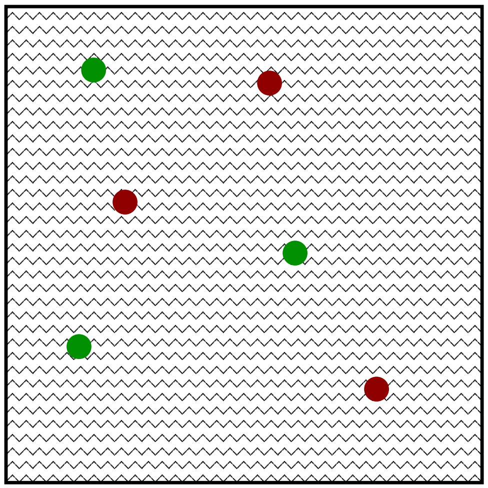


Phases of Gauge Theories

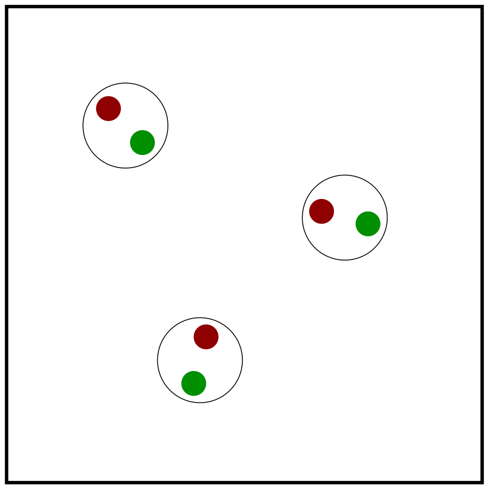
Coulomb



Higgs



Confinement



$$V(r) \sim \frac{e^2}{r}$$

$$V(r) \sim \frac{e^{-mr}}{r}$$

$$V(r) \sim kr$$

Standard Model: $U(1) \times SU(2) \times SU(3)$

Phases of Matter

phase	order param	broken symmetry	rigidity phenomenon	Goldstone boson
crystal	ρ_k	translations	rigid	phonon
magnet	$\vec{M}$	rotations	magnetization	magnon
superfluid	$\langle \Phi \rangle$	particle number	supercurrent	phonon
supercond.	$\langle \psi\psi \rangle$	gauge symmetry	supercurrent	none (Higgs)

Gauge Symmetry

Local gauge symmetry $U(x) \in SU(3)_c$

$$\psi \rightarrow U\psi \qquad\qquad D_\mu\psi \rightarrow UD_\mu\psi$$

$$A_\mu \rightarrow UA_\mu U^\dagger + iU\partial_\mu U^\dagger \qquad F_{\mu\nu} \rightarrow UF_{\mu\nu}U^\dagger$$

Gauge "symmetries" cannot be broken

Gauge "symmetries" can be realized in different modes

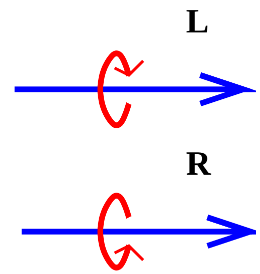
	Coulomb	Higgs	confined
d.o.f:	2 (massless)	3 (massive)	3 (massive)

Distinction between Higgs and confinement phase not always sharp

Chiral Symmetry

Define left and right handed fields

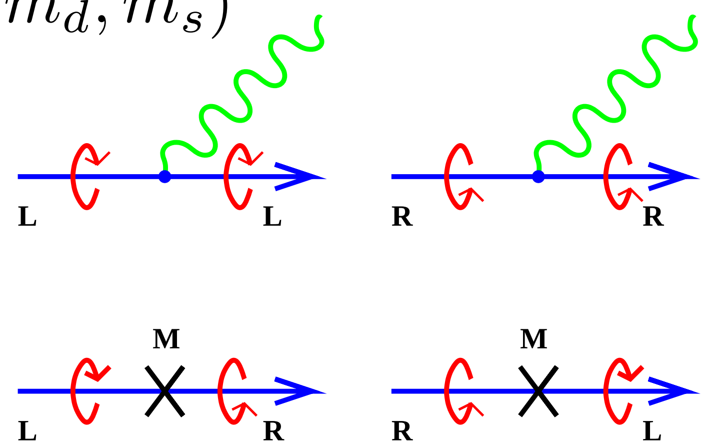
$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$



Fermionic lagrangian, $M = \mathrm{diag}(m_u, m_d, m_s)$

$$\mathcal{L} = \bar{\psi}_L (i\not{D})\psi_L + \bar{\psi}_R (i\not{D})\psi_R$$

$$+ \bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$



$M = 0$: Chiral symmetry $(L, R) \in SU(3)_L \times SU(3)_R$

$$\psi_L \to L\psi_L, \qquad \psi_R \to R\psi_R$$

Chiral Symmetry Breaking

Chiral symmetry implies massless, degenerate fermions

$$m_N^{(1/2)^+} = 935\,\mathrm{MeV} \qquad\qquad m_{N*}^{(1/2)^-} = 1535\,\mathrm{MeV}$$

Chiral symmetry is spontaneously broken

$$\langle \bar\psi_L^f \psi_R^g + \bar\psi_L^f \psi_R^g \rangle \simeq -(230\,\mathrm{MeV})^3\ \delta^{fg}$$

$$SU(3)_L \times SU(3)_R \to SU(3)_V$$

Consequences: dynamical mass generation $m_Q = 300\,\mathrm{MeV} \gg m_q$

$$m_N = 890\,\mathrm{MeV} + 45\,\mathrm{MeV} \qquad\qquad (\mathrm{QCD},\ 95\%) + (\mathrm{Higgs},\ 5\%)$$

Low Energy Effective Lagrangian

Low energy degrees of freedom: Goldstone modes

$$U(x) = \exp(i\pi^a\lambda^a/f_\pi)$$

Effective lagrangian

$$\mathcal{L} = \mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + (B\mathrm{Tr}[MU] + h.c.) + \ldots$$

controls

Goldstone boson scattering

Coupling to external currents

Quark mass dependence

Symmetries of the QCD Vacuum: Summary

Local $SU(3)$ gauge symmetry

confined: $V(r) \sim kr$

Chiral $SU(3)_L \times SU(3)_R$ symmetry

spontaneously broken to $SU(3)_V$

Axial $U(1)_A$ symmetry

$$\text{anomalous}: \qquad \partial_\mu A^0_\mu = \frac{N_f}{16\pi^2} G^a_{\mu\nu} \tilde{G}^a_{\mu\nu}$$

Vectorial $U(1)_B$ symmetry

unbroken: $B = \int d^3x \, \psi^\dagger \psi$ conserved

Notes

QCD with general N_f, N_c (with or without SUSY)

There are asymptotically free theories

without confinement and/or chiral symmetry breaking

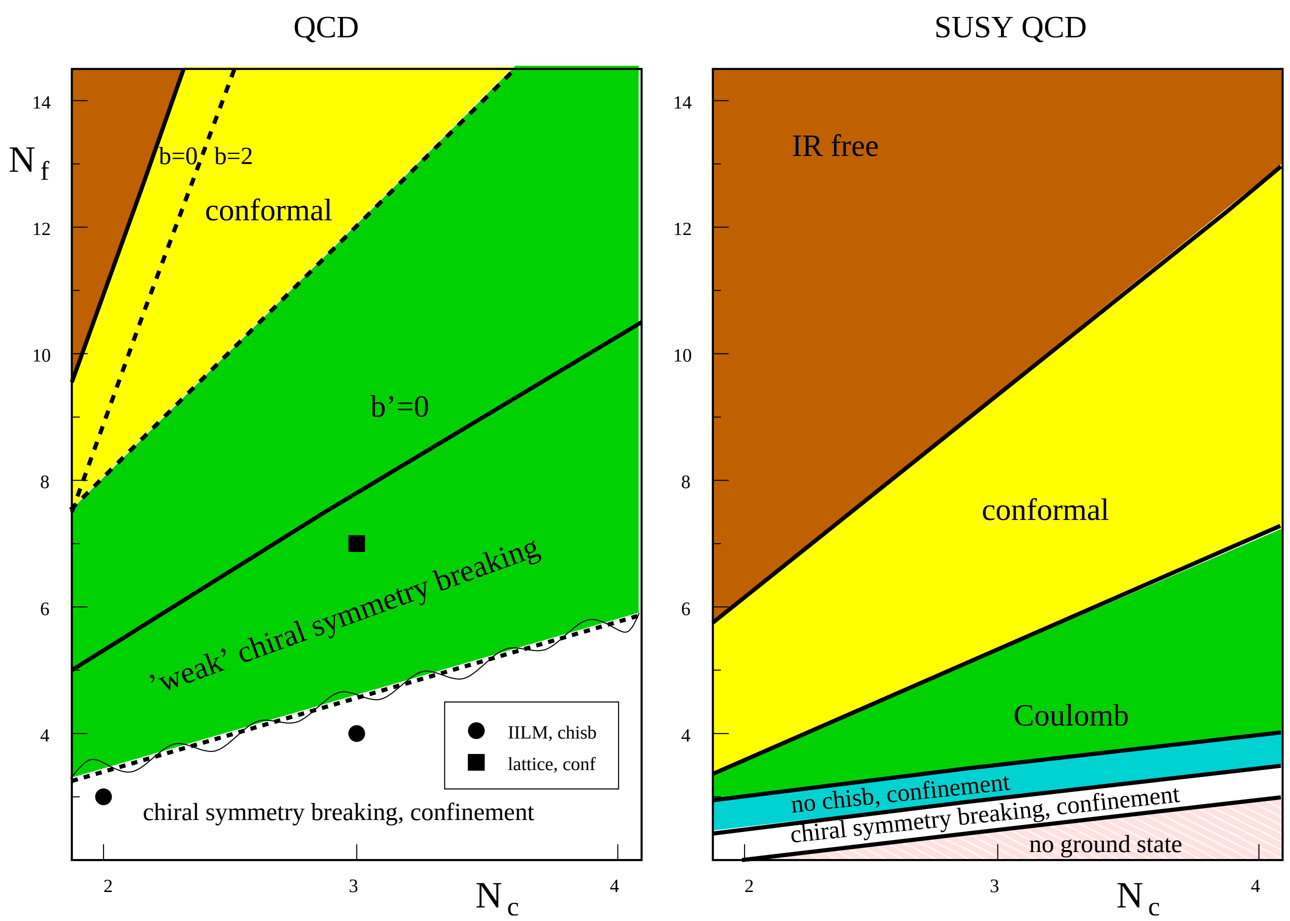
QCD with $N_f = 3$

confinement implies chiral symmetry breaking

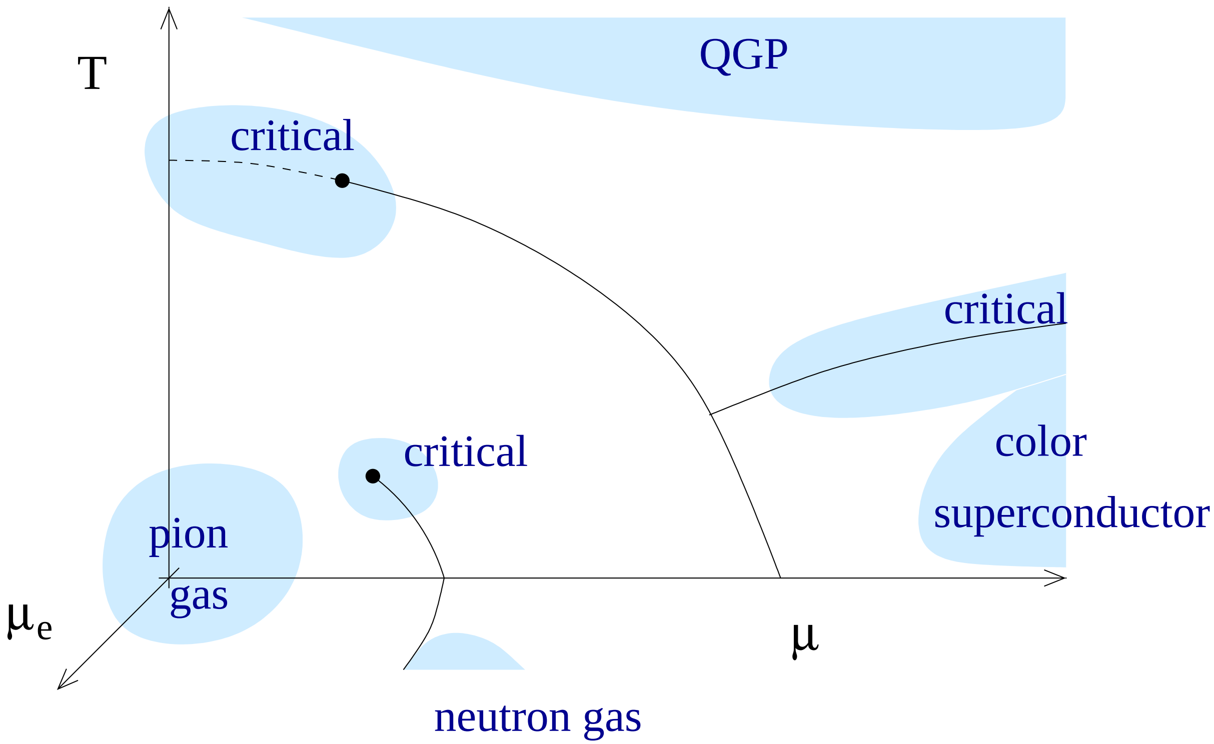
symmetry breaking pattern $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ unique*

order parameter $\langle \bar{\psi}\psi \rangle \neq 0$

QCD Phase Diagram: N_c and N_f

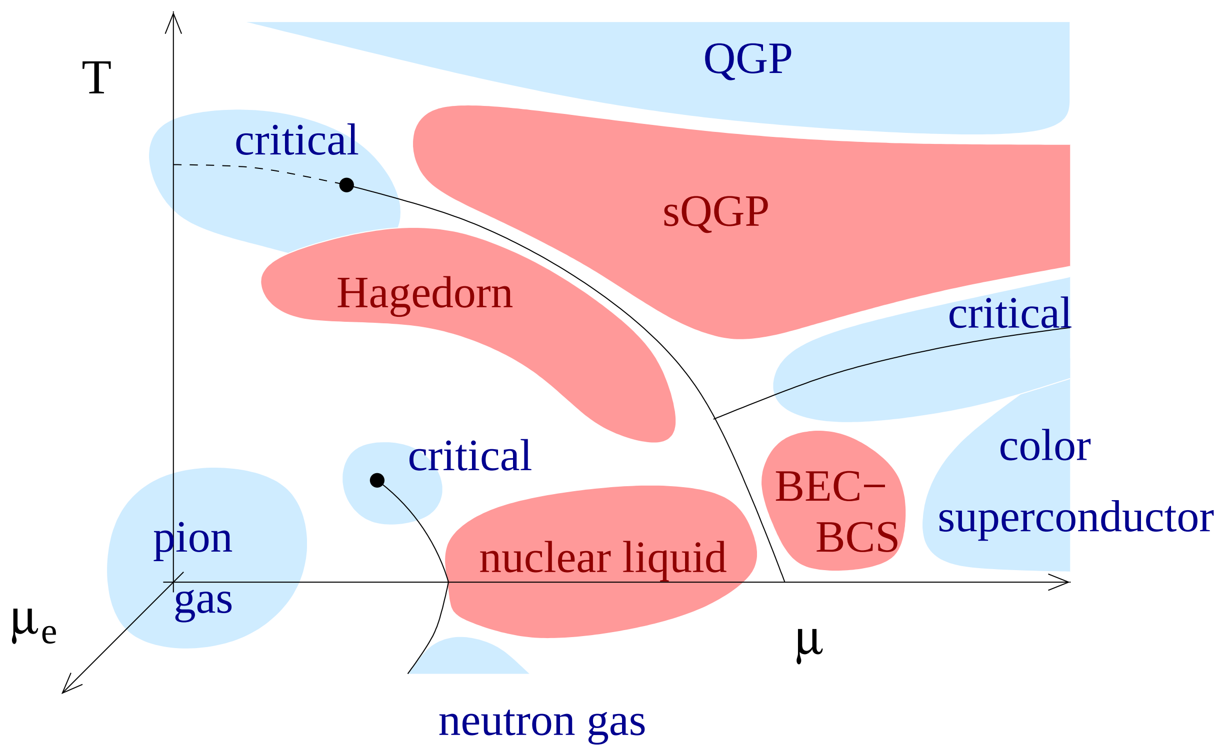


Approaching the Phase Diagram: Symmetries and Weak Coupling Arguments



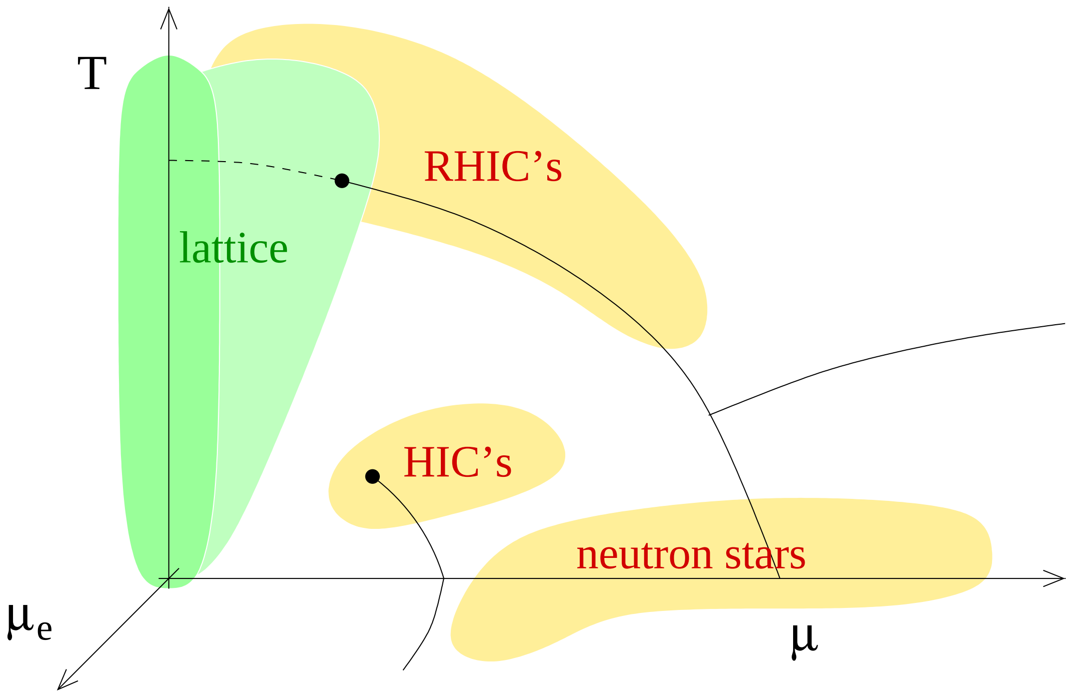
Approaching the Phase Diagram:

Strongly Correlated Phases



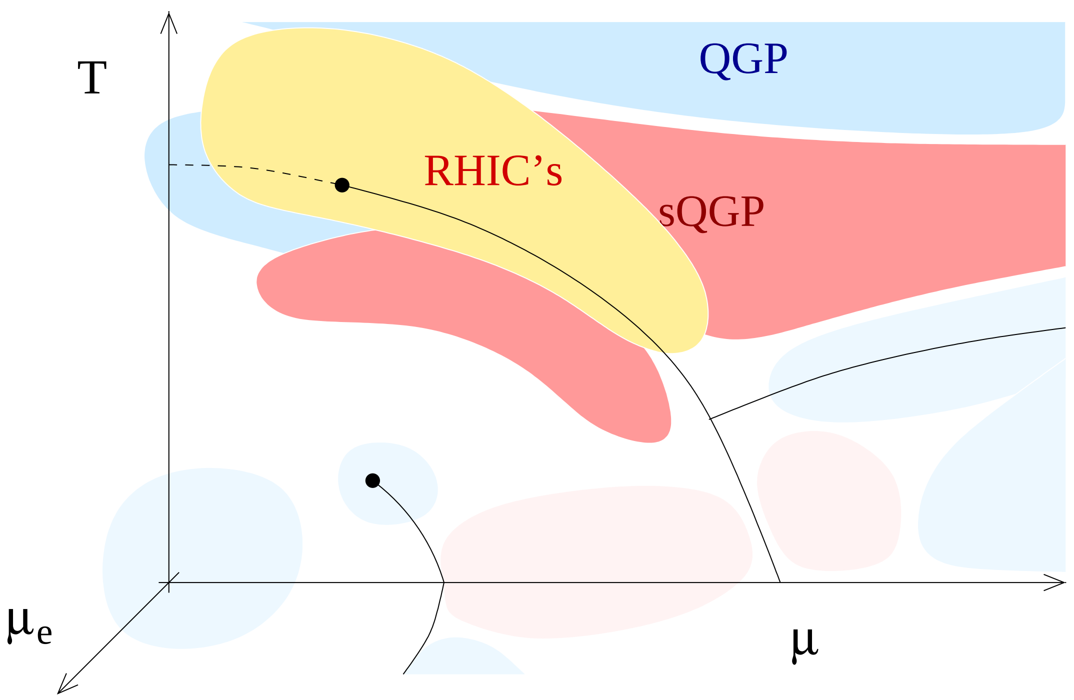
Approaching the Phase Diagram:

Experiments and Numerical Simulations



Part I: QCD at Finite Temperature

The Heavy Ion Program at RHIC



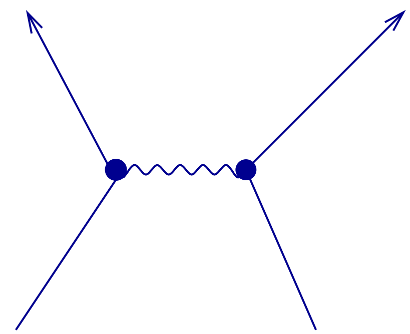
The High T Phase: Qualitative Argument

High T phase: Weakly interacting gas of quarks and gluons?

typical momenta $p \sim 3T$

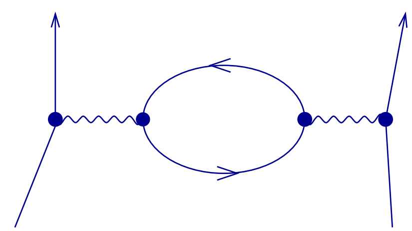
Large angle scattering involves large momentum transfer

effective coupling is small



Small angle scattering is screened (not anti-screened!)

coupling does not become large

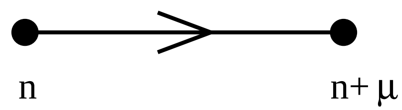


Quark Gluon Plasma

Lattice QCD

Euclidean partition function

$$Z = \int dA_\mu d\psi \exp(-S) = \int dA_\mu \det(i\not{D}\,) \exp(-S_G)$$

Lattice discretization: 

$$U_\mu(n) = \exp(igaA_\mu(n))$$

$$D_\mu \phi \;\rightarrow\; \frac{1}{a}[U_\mu(n)\phi(n+\mu) - \phi(n)]$$

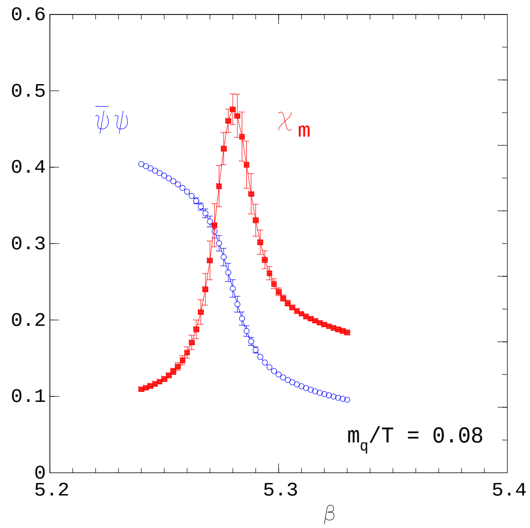
$$(G^a_{\mu\nu})^2 \;\rightarrow\; \frac{1}{a^4}\mathrm{Tr}[U_\mu(n)U_\nu(n+\mu)U_{-\mu}(n+\mu+\nu)U_{-\nu}(n+\nu) - 1]$$

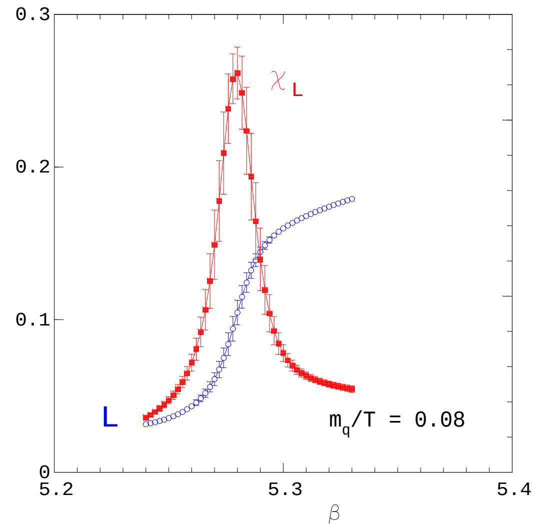
Monte Carlo:

$$\int dA_\mu\, e^{-S} \rightarrow \{U^{(1)}_\mu(n), U^{(2)}_\mu(n), \ldots\}$$

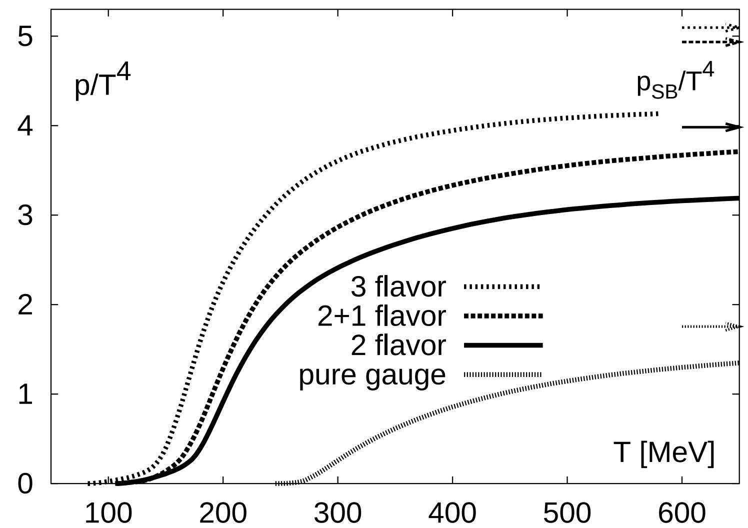
Lattice Results

order parameters

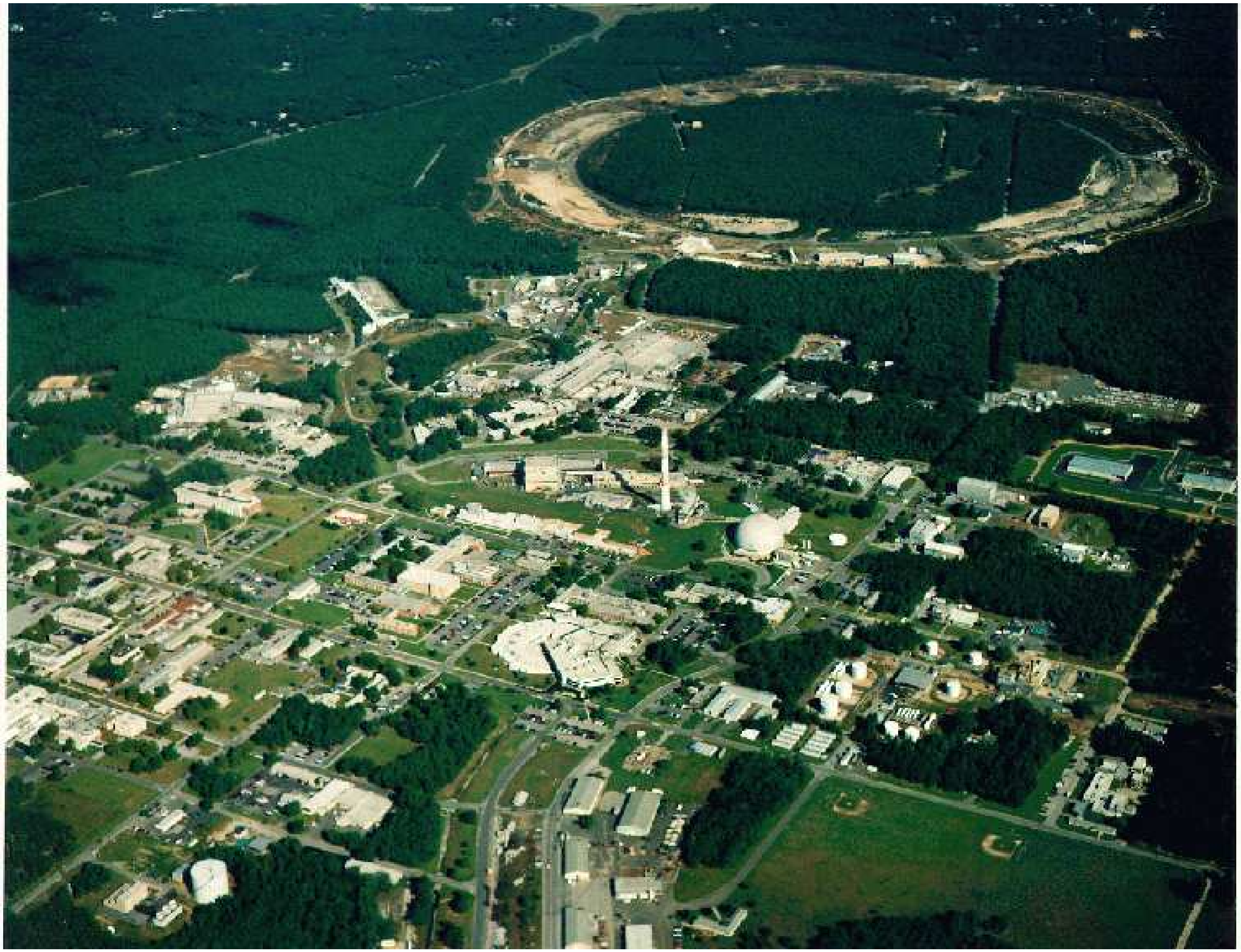




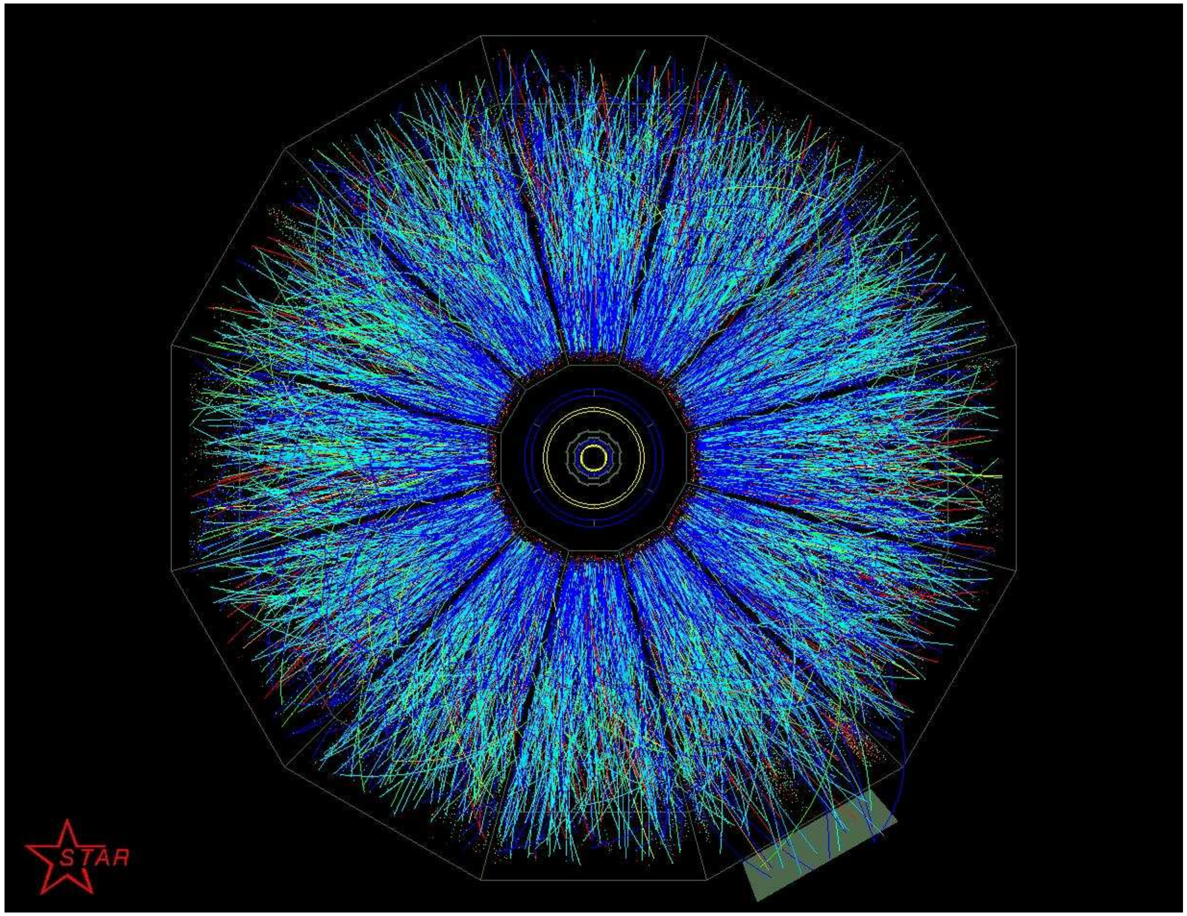
equation of state



BNL and RHIC

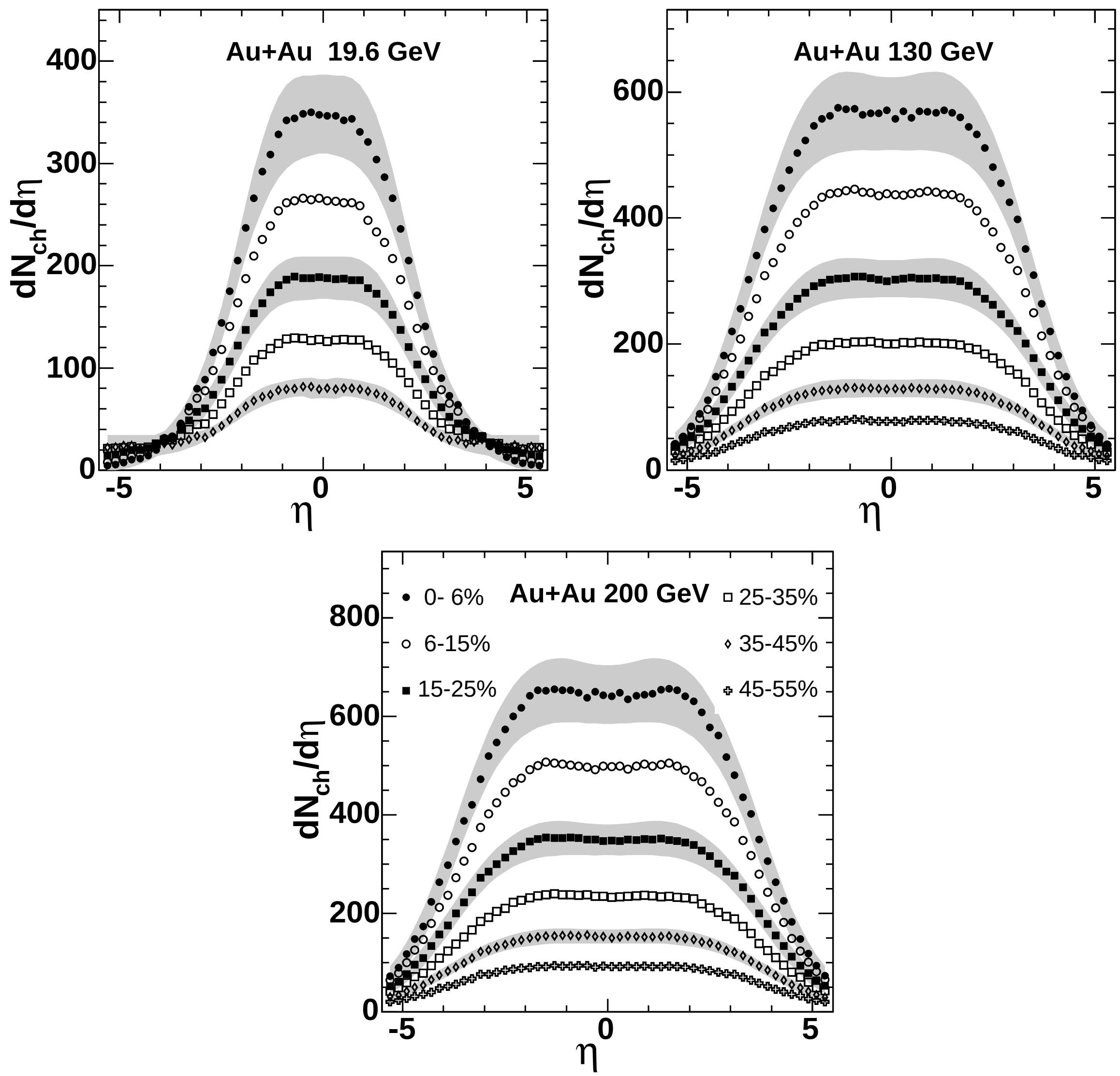


Multiplicities



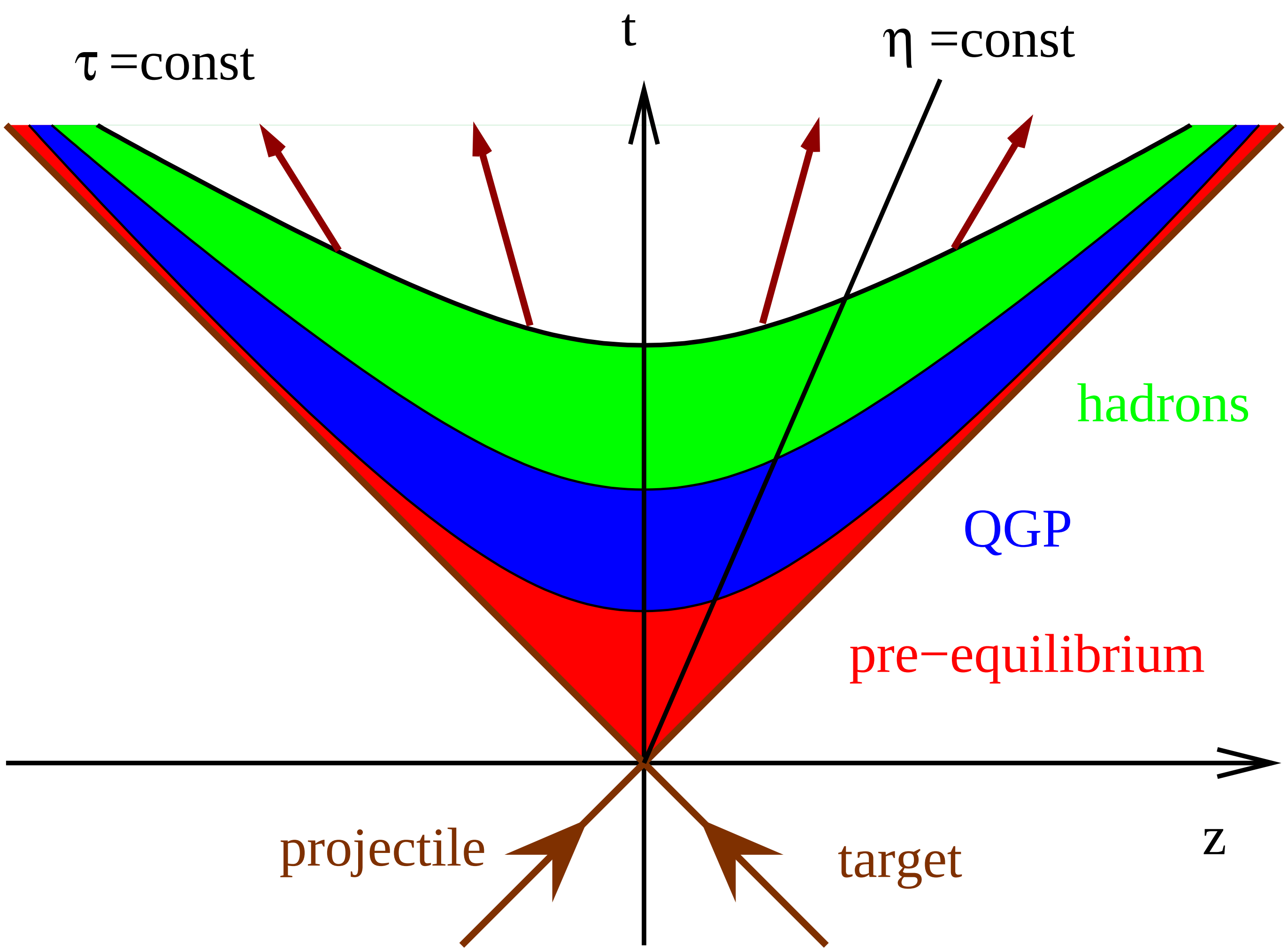
Star TPC

Multiplicities



Phobos White Paper (2005)

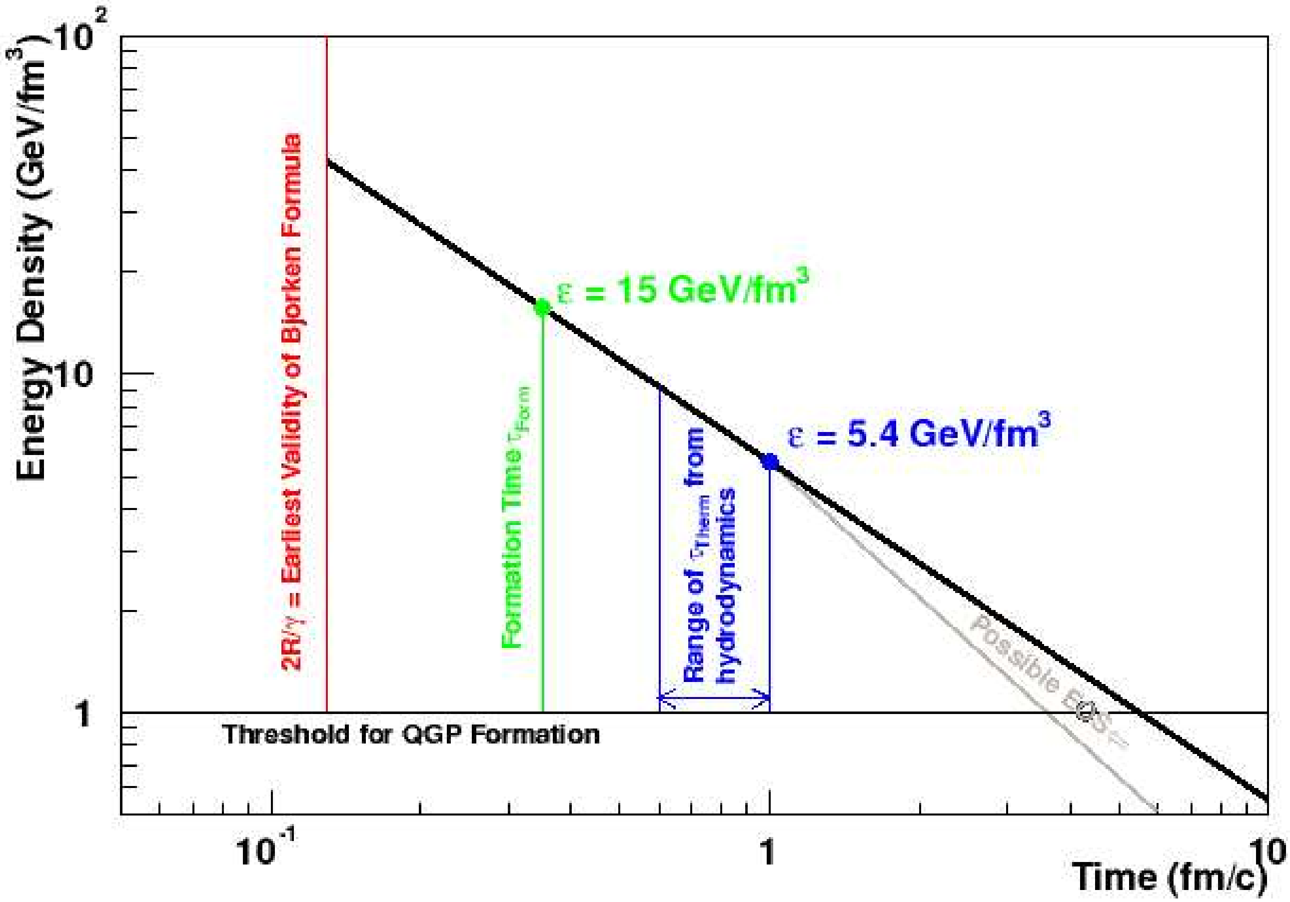
Heavy Ion Collisions



Scaling (Bjorken) expansion

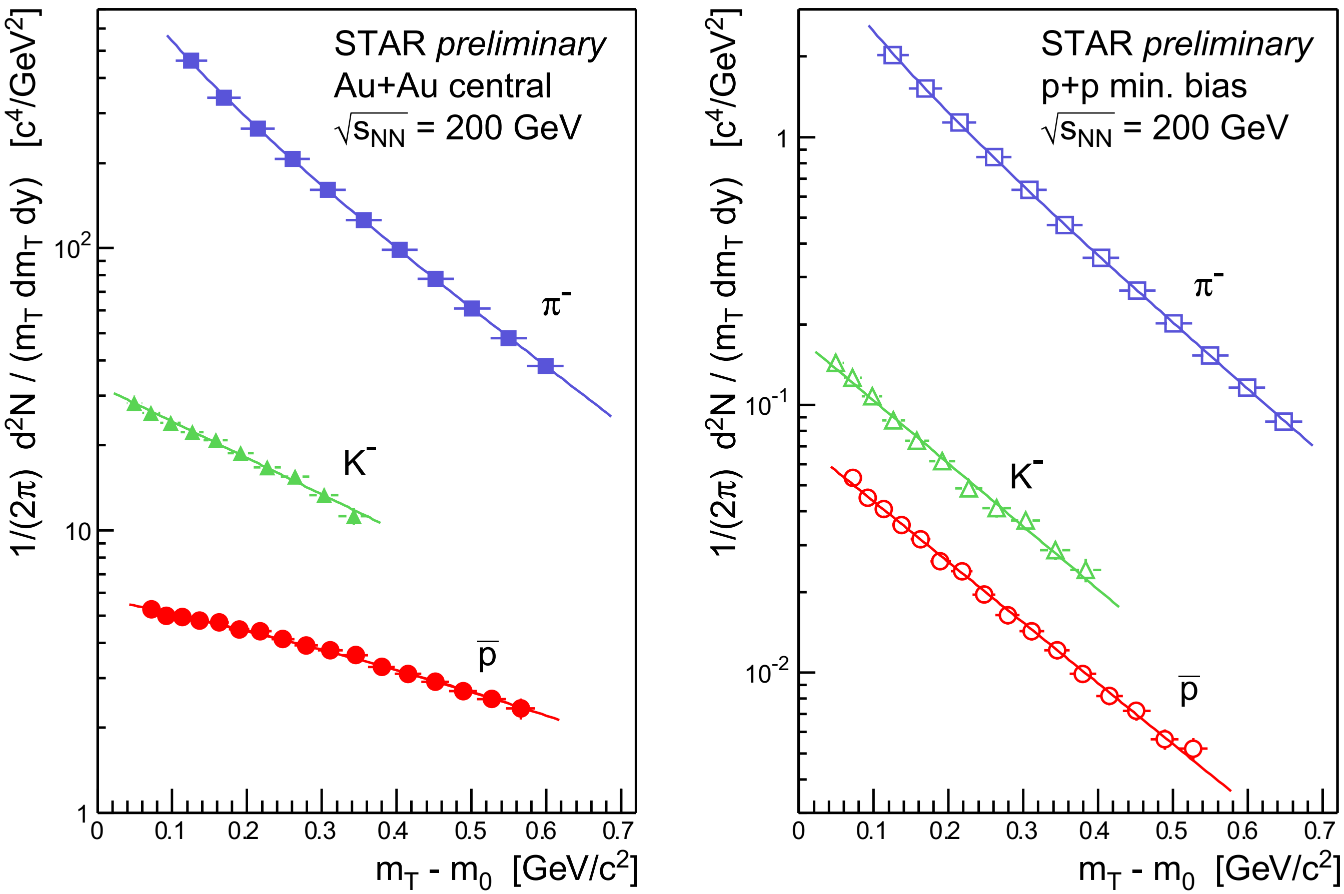
all comoving observers are equivalent

Bjorken Expansion



Radial Flow

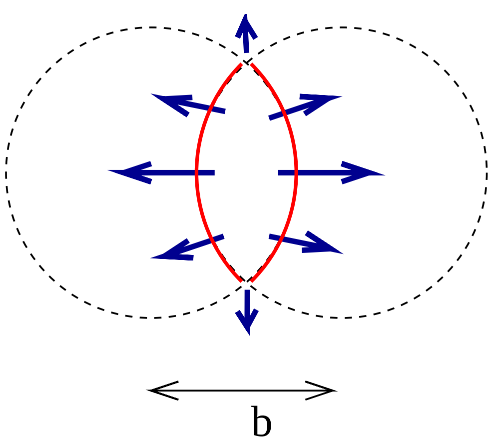
Radial expansion leads to blue-shifted spectra

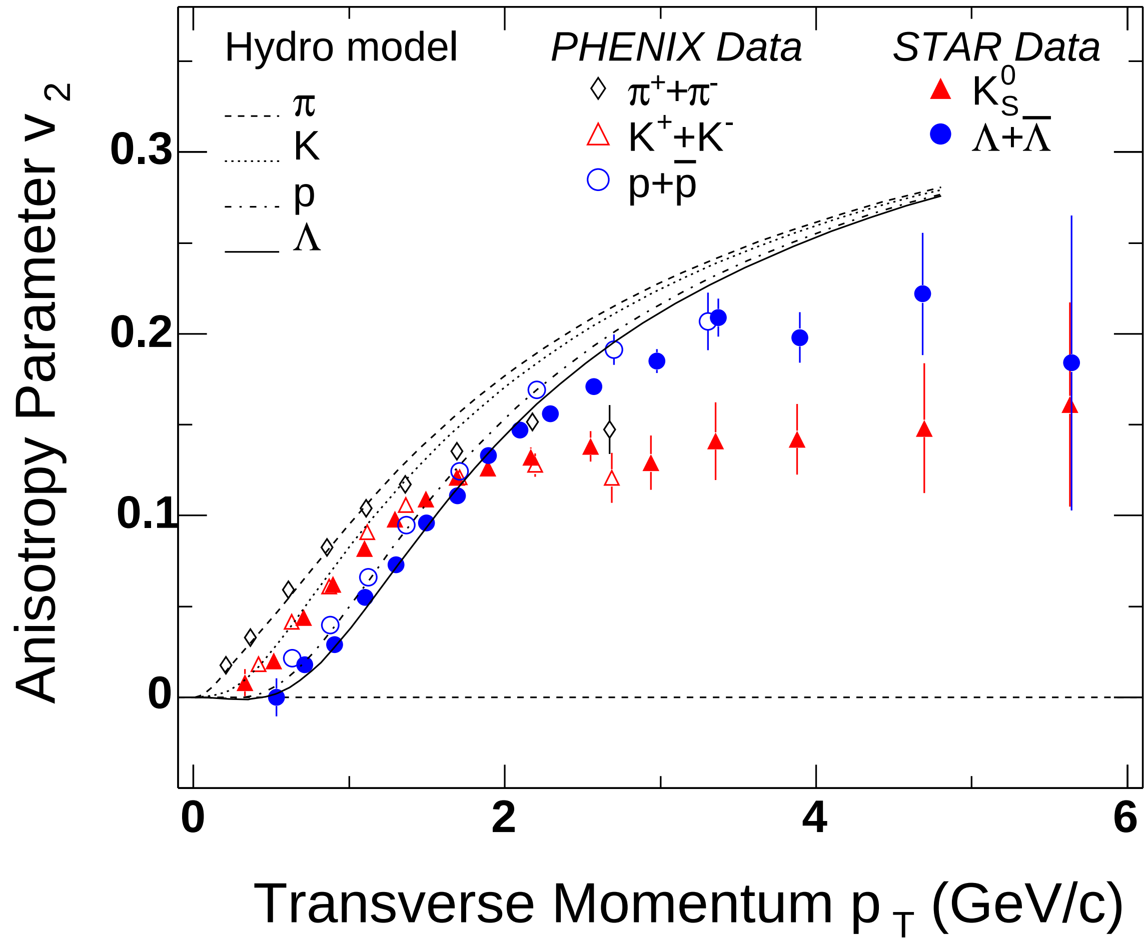


$v_T \sim 0.6c!$ $\qquad$ $m_T = \sqrt{p_T^2 + m^2}$

Elliptic Flow

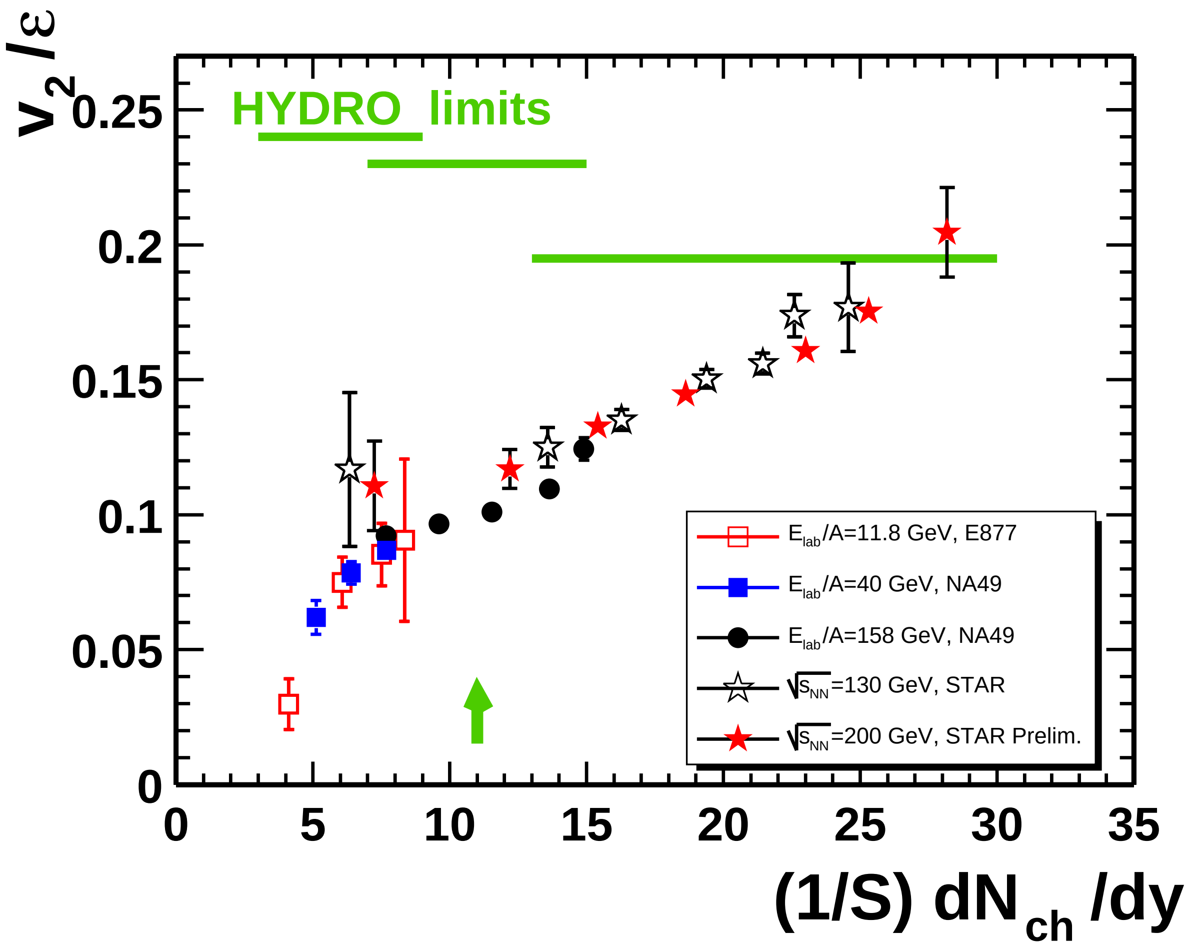
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy





source: U. Heinz (2005)

Elliptic Flow II



source: U. Heinz (2005)

Viscosity

Energy momentum tensor

$$T_{\mu\nu} \to T_{\mu\nu} + \eta(\nabla_\mu u_\nu + \nabla_\nu u_\mu - trace)$$

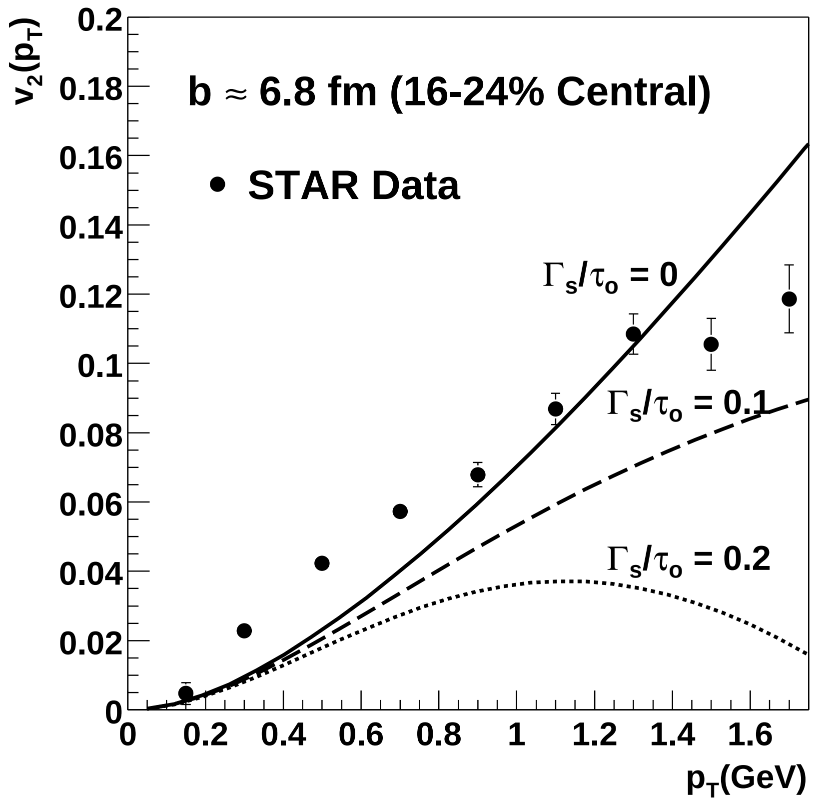
perturbative QCD

$$\eta = 107T^3/(g^4 \log(g^{-1}))$$

universal bound (D. Son)?

$$\eta/s \geq 1/(4\pi)$$

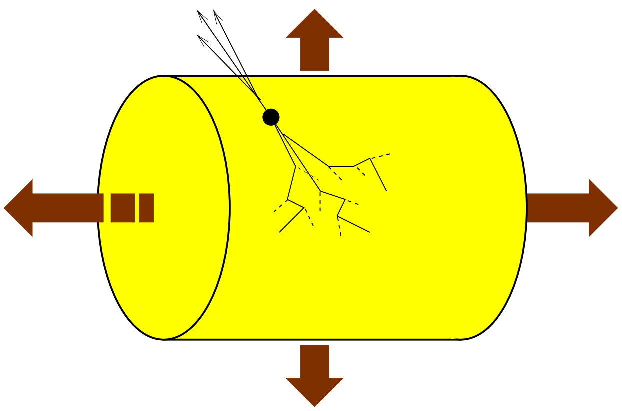
bound saturated in strong coupling SUSY theories with gravitational dual

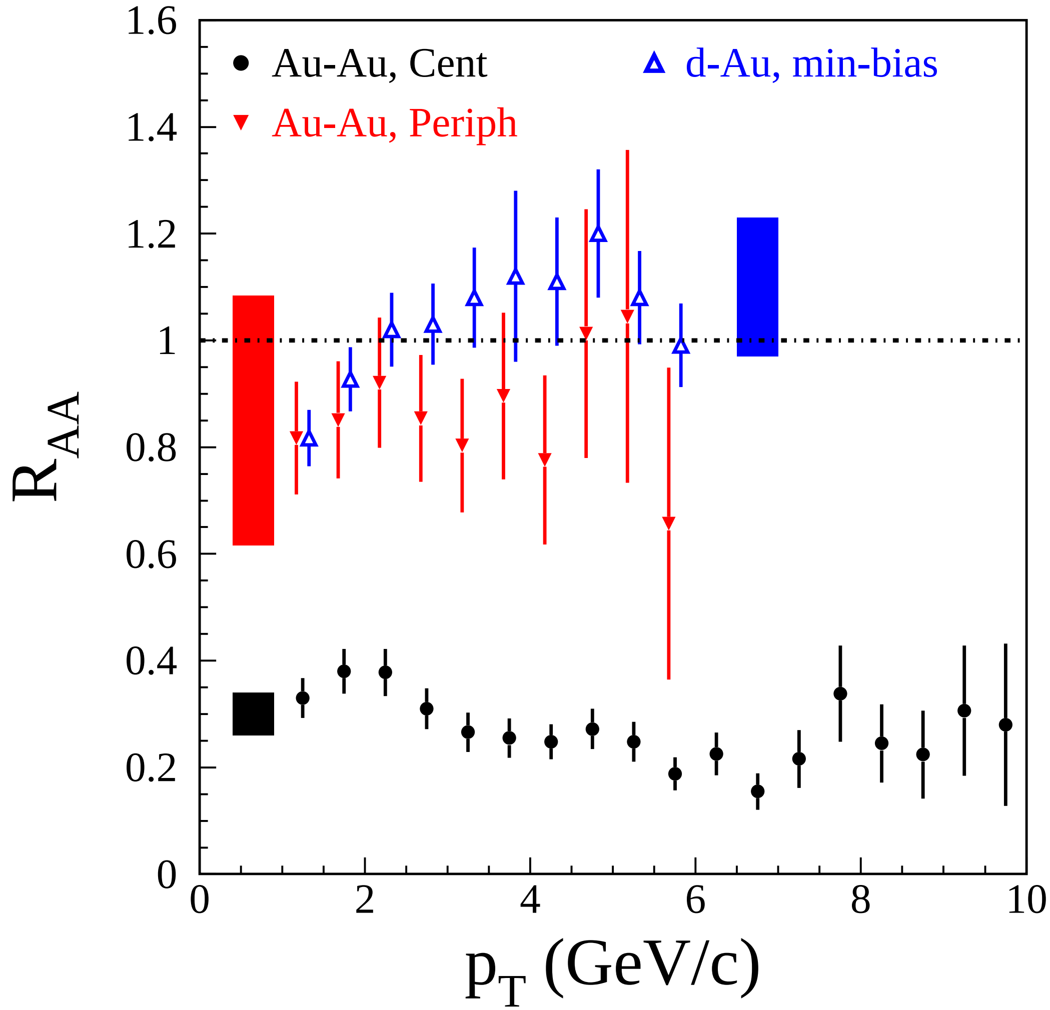


source: D. Teaney (2003)

Jet Quenching

$$R_{AA} = \frac{n_{AA}}{N_{coll} n_{pp}}$$

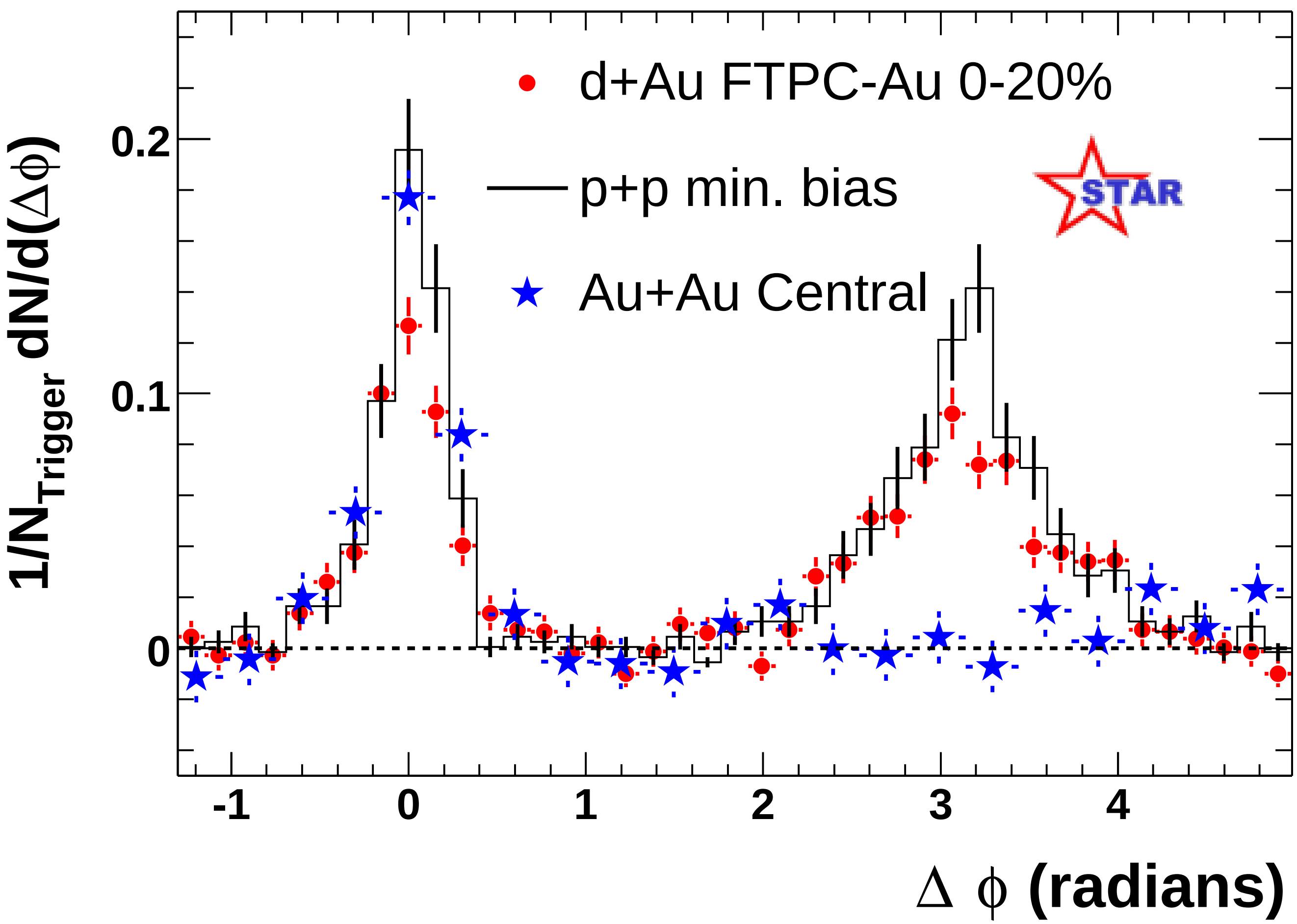




source: Phenix White Paper (2005)

Jet Quenching II

Disappearance of away-side jet

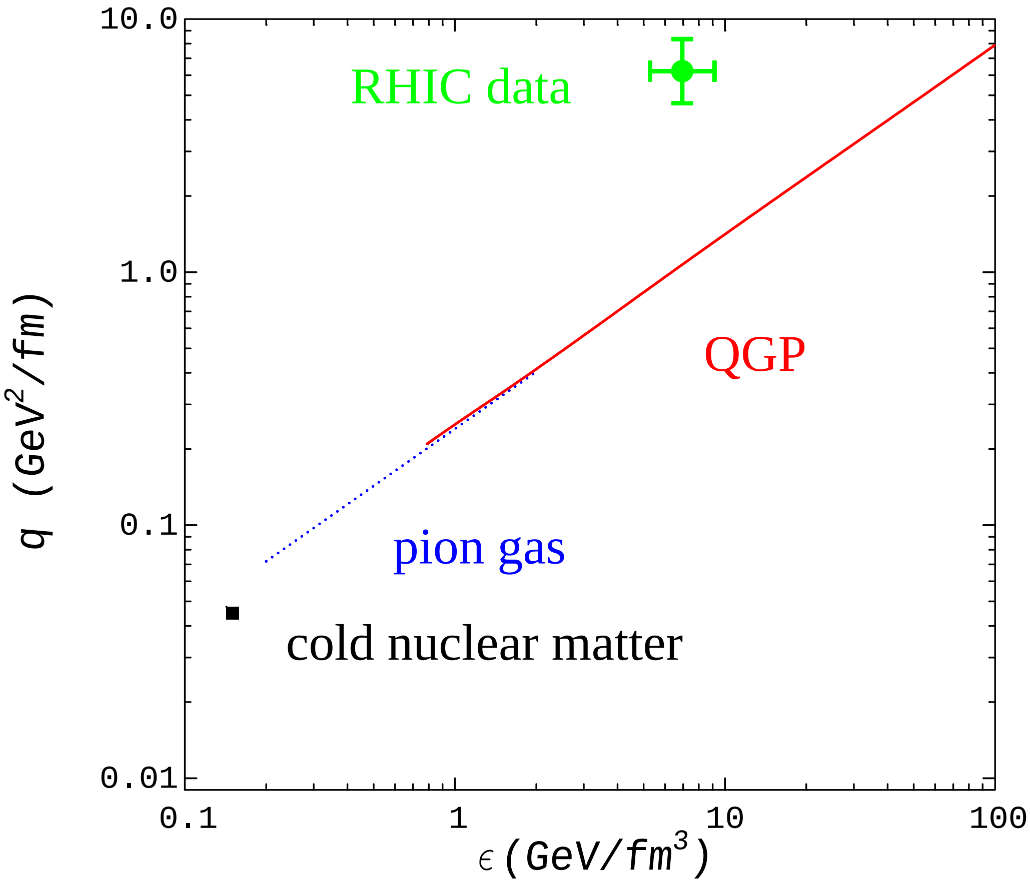


source: Star White Paper (2005)

Jet Quenching: Theory

energy loss governed by

$$\hat{q} = \rho \int q_\perp^2 \, dq_\perp^2 \; \frac{d\sigma}{dq_\perp^2}$$



larger than pQCD predicts?

source: R. Baier (2004)

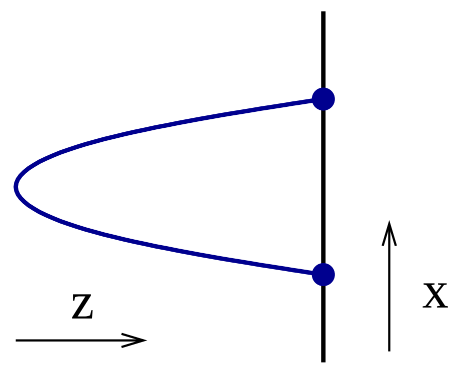
Gauge Theory at Strong Coupling: Holographic Duals

The AdS/CFT duality relates

large N_c (Conformal) gauge theory in 4 dimensions	$\Leftrightarrow$	string theory on 5 dimensional Anti-de Sitter space $\times S_5$
correlation fcts of gauge invariant operators	$\Leftrightarrow$	boundary correlation fcts of AdS fields

$$\langle \exp \int dx\ \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



The correspondence is simplest at strong coupling $g^2 N_c$

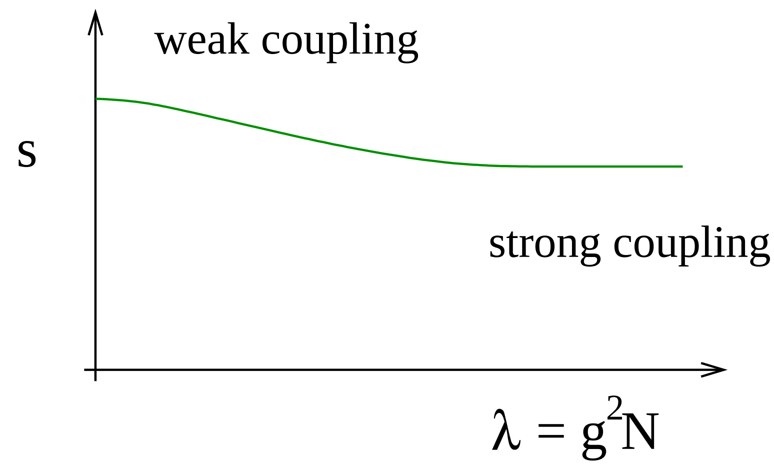
strongly coupled gauge theory $\Leftrightarrow$ classical string theory

Gauge Theory at Strong Coupling: Finite Temperature

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $=$area of event horizon

$$s = \frac{\pi^2}{2} N_c^2 T^3 = \frac{3}{4} s_0$$



Extended to transport properties by Policastro, Son and Starinets

$$\eta = \frac{\pi}{8} N_c^2 T^3$$

Summary (Experiment)

Matter equilibrates quickly and behaves collectively

Little Bang, not little fizzle

Initial energy density in excess of 10 GeV$/\mathrm{fm}^3$

Conditions for Plasma achieved

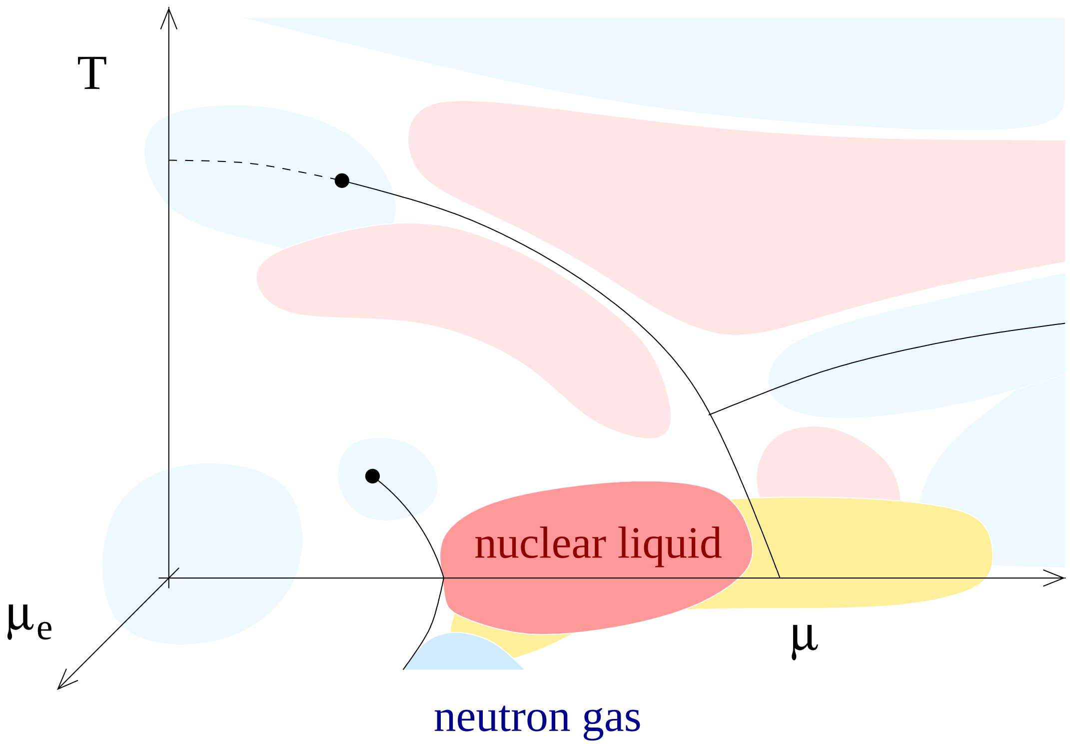
Evidence for strongly interacting Plasma ("sQGP")

Fast equilibration $\tau_0 \ll 1$ fm

Strong energy loss of leading partons

Part II: QCD at Finite Density

Nuclear Systems



QCD at Finite Density

Partition function

$$Z = \mathrm{Tr}\left[e^{-\beta(H-\mu N)}\right] \qquad \beta = 1/T \qquad N = \int d^3x\, \psi^\dagger \psi$$

Path integral representation (euclidean)

$$Z = \int dA_\mu \det(i\not{D} + i\mu\gamma_4)e^{-S} = \int dA_\mu e^{i\phi}|\det(i\not{D} + i\mu\gamma_4)|e^{-S}$$

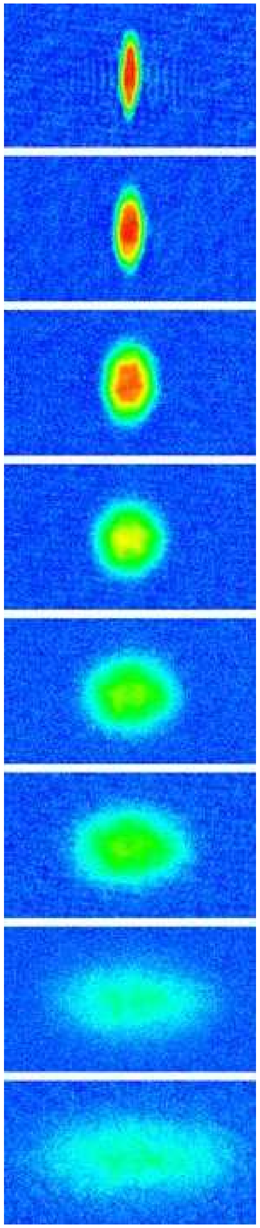
Sign problem: importance sampling does not work

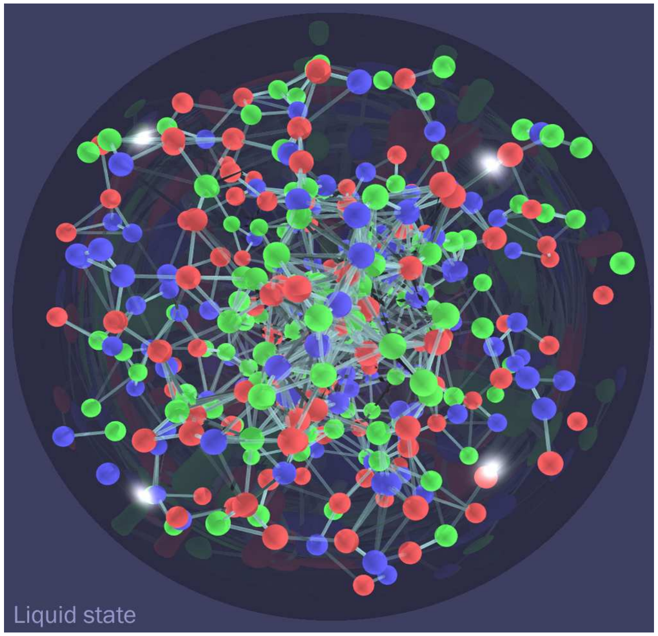
Also: No general theorems (a la Vafa-Witten)

Phase structure much richer

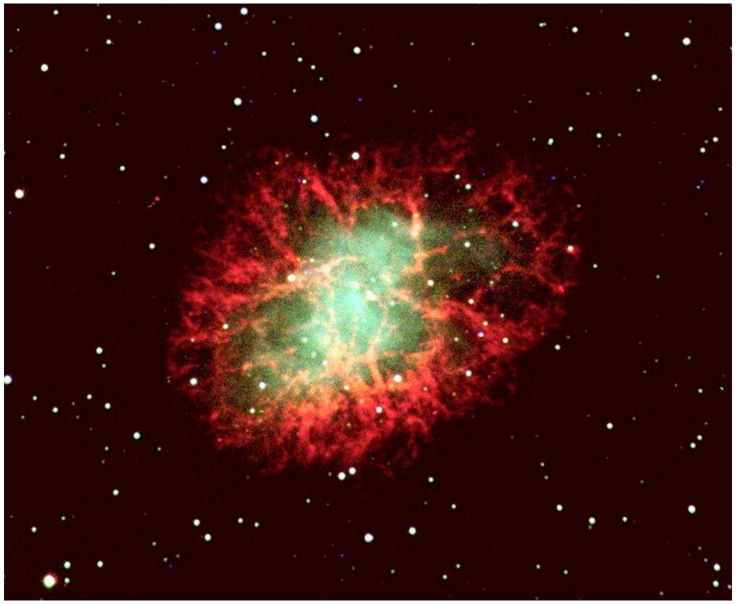
(breaking of translational, rotational, parity, isospin, ... symmetry)

Perfect Liquids





sQGP



Neutron Star (Crab)

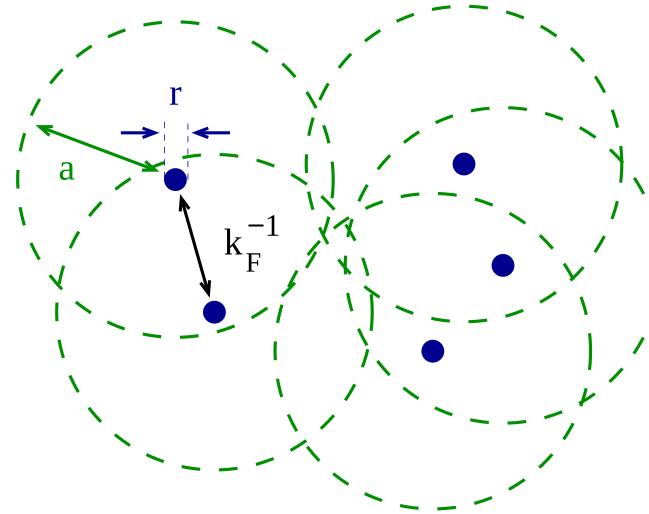
Trapped Fermi Gas

Universality

What do these systems have in common?

dilute: $r\rho^{1/3} \ll 1$

strongly correlated: $a\rho^{1/3} \gg 1$



Neutron Matter

$a_{nn} = -18$ fm, $r_{nn} = 2.7$ fm

$$0.001\rho_0 \leq \rho \leq 0.3\rho_0$$

Feshbach Resonance in ^{6}Li

$a_\infty = 45$ a.u., $k_F^{-1} \sim 10^3$ a.u.

$$a = a_\infty\left(1 + \frac{\Delta}{B - B_0}\right)$$

Universality

Consider limiting case ("Bertsch" problem)

$$(k_F a) \to \infty \qquad\qquad (k_F r) \to 0$$

Universal equation of state

$$\frac{E}{A} = \xi \left(\frac{E}{A}\right)_0 = \xi \frac{3}{5}\left(\frac{k_F^2}{2M}\right)$$

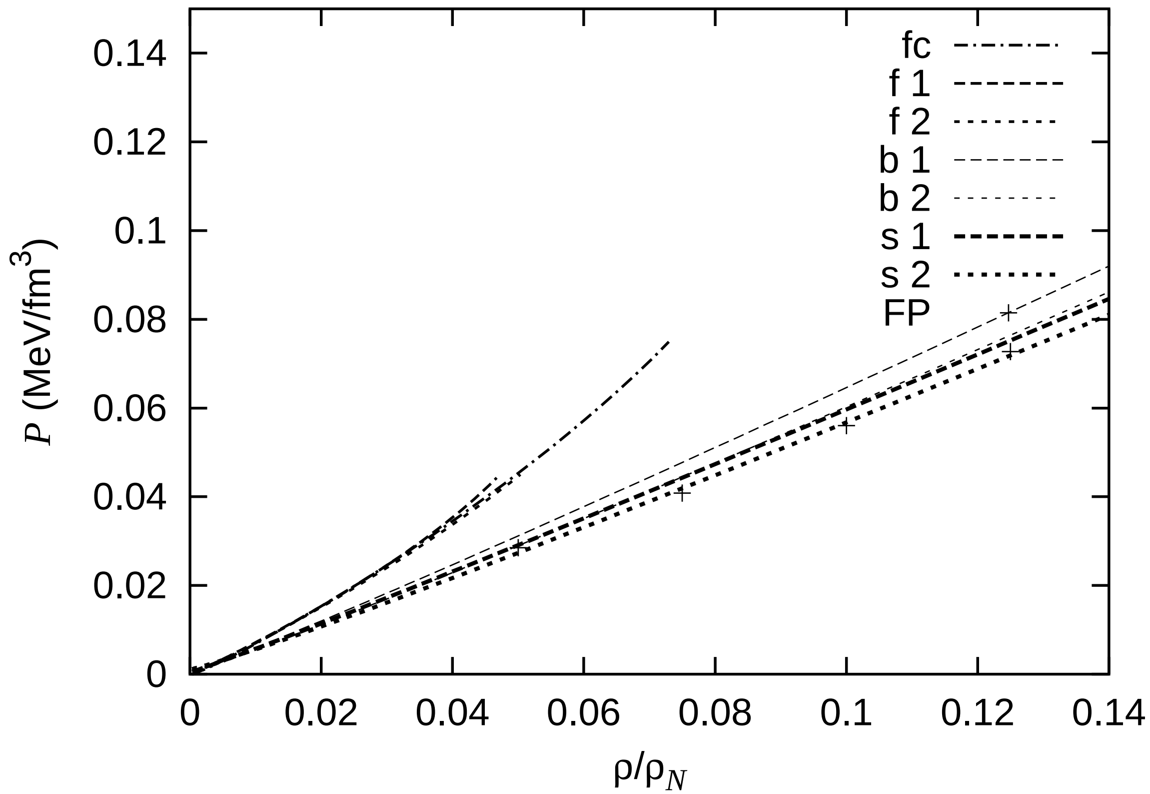
How to find ξ?

Numerical Simulations

Experiments with trapped fermions

Analytic Approaches

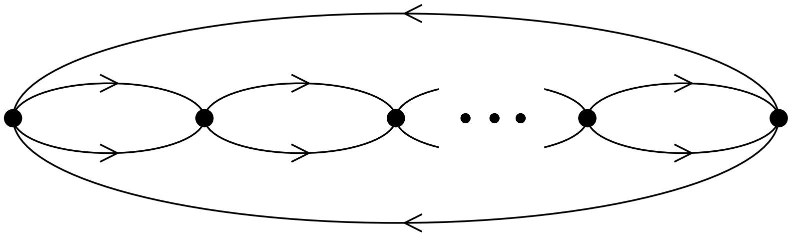
Neutron Matter on the Lattice

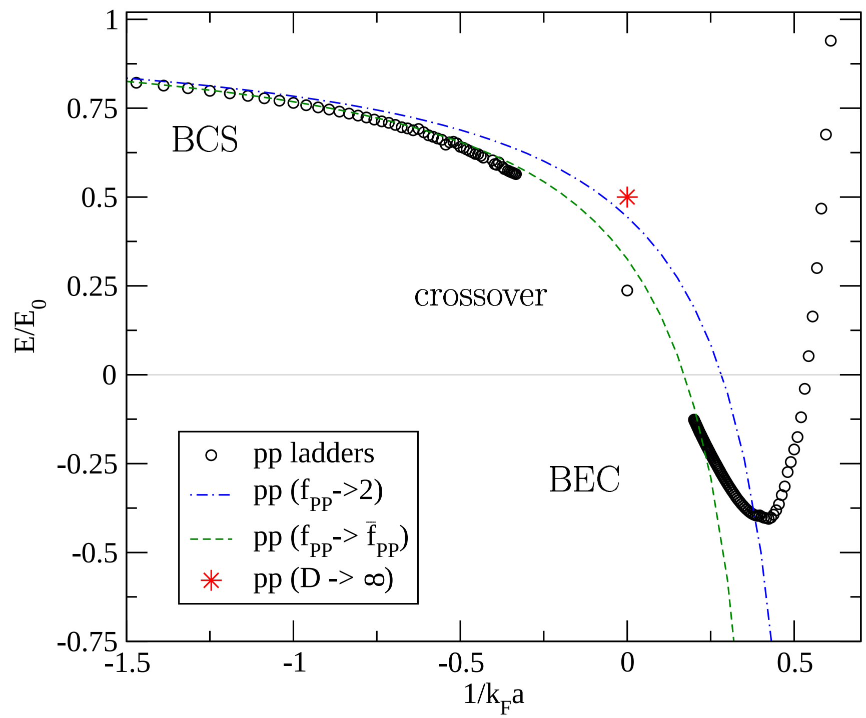


$$P \sim 0.5 P_0$$

Analytic Approaches

Two body correlations: Sum particle-particle ladders





$$\frac{E}{A} = \frac{k_F^2}{2M} \times$$

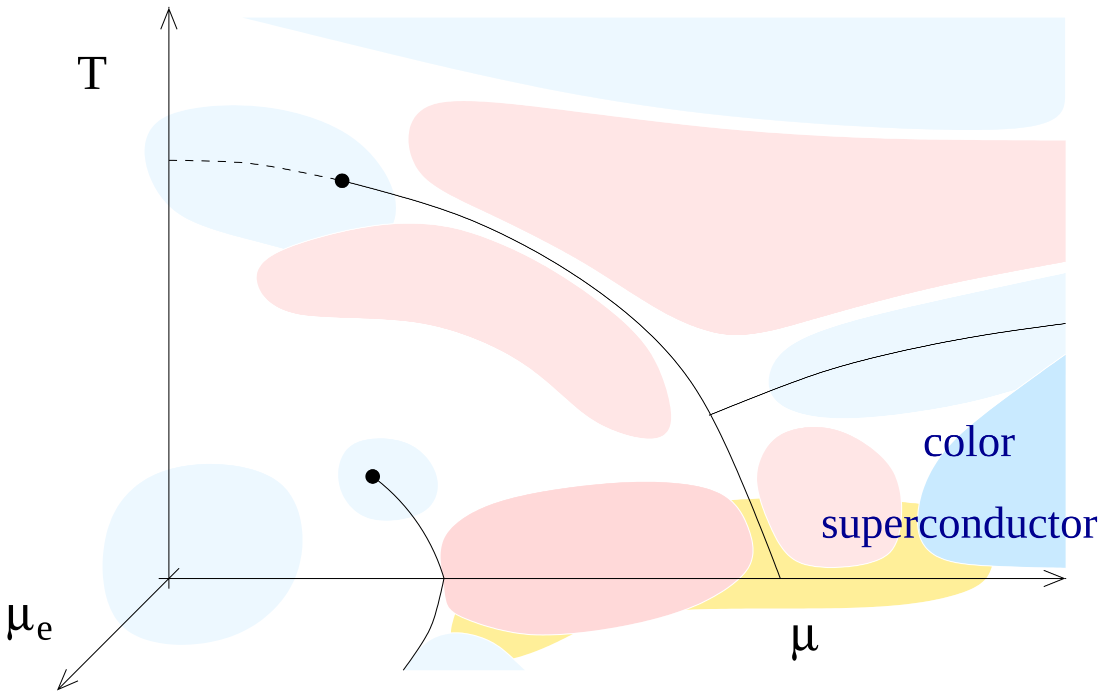
$$\frac{2(k_F a)/(3\pi)}{1 - \frac{6}{35\pi}(11 - 2\log(2))(k_F a)}$$

Systematic Approach: Large d Expansion

$$\xi = 0.5 + O(1/d)$$

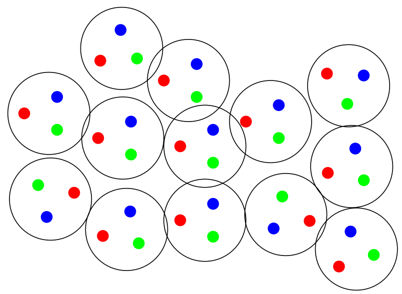
Part III: QCD at Finite Density

Color Superconductivity



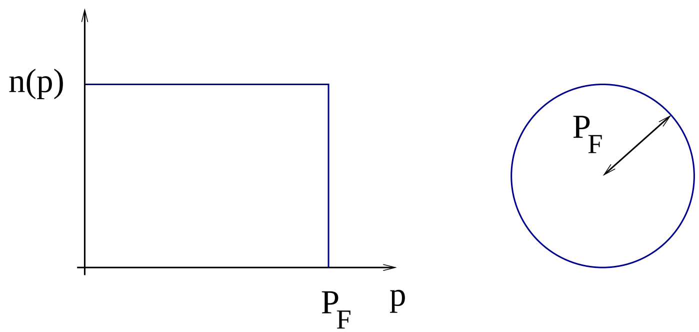
Very Dense Matter

Consider baryon density $n_B \gg 1\,\mathrm{fm}^{-3}$



quarks expected to move freely

Ground state: cold quark matter (quark fermi liquid)



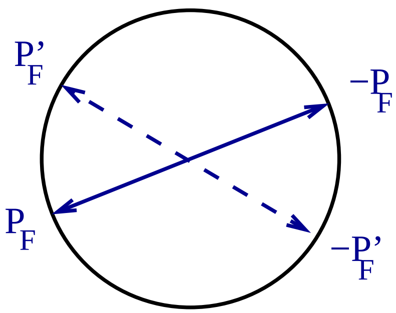
only quarks with $p \sim p_F$ scatter

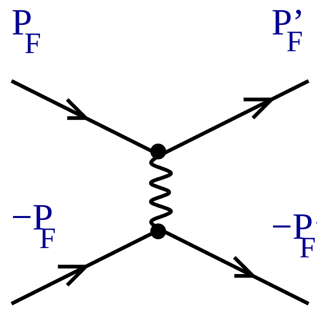
$p_F \gg \Lambda_{QCD} \rightarrow$ coupling is weak

No chiral symmetry breaking, confinement, or dynamically generated masses

Color Superconductivity

Is the quark liquid stable?





Dominant interaction:

Uses Fermi surface

coherently

Attractive interaction leads to instability

$\langle qq \rangle$ condensate, superfluidity/superconductivity, gap in fermion spectrum, transport without dissipation

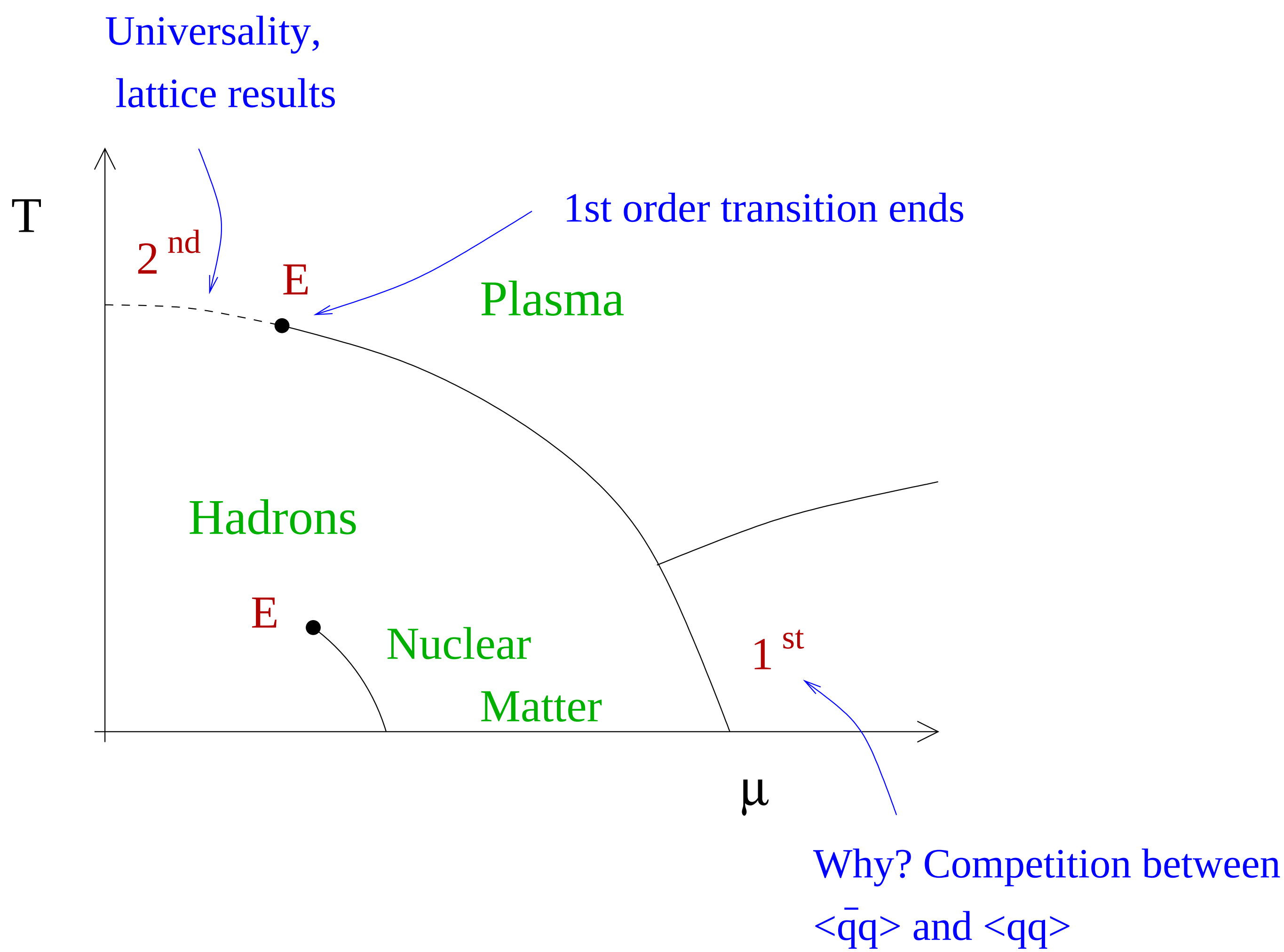
QCD: gluon exchange attractive in $\bar{3}$ channel

$$3 \times 3 = 6_S + 3_A \qquad \text{flux reduced} \Rightarrow \text{attractive}$$

Spin-flavor-color wave function

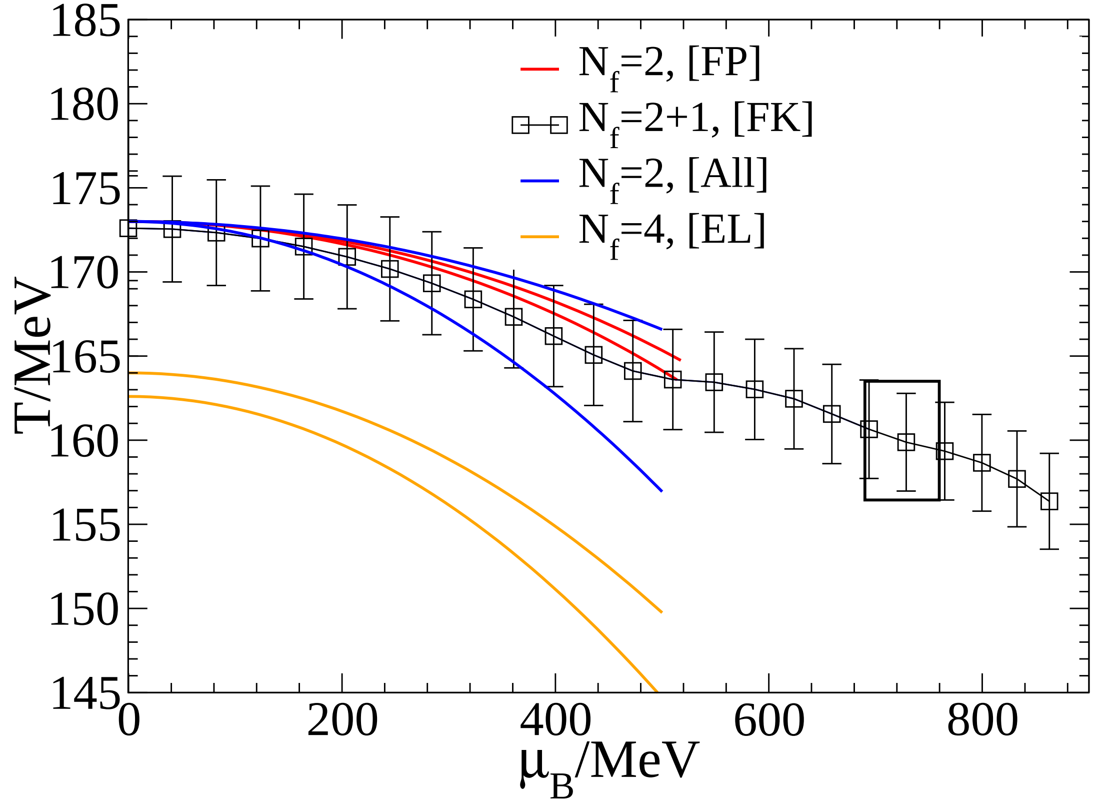
$$(\uparrow\downarrow - \downarrow\uparrow) \times (ud - du) \times (rb - br) \qquad s = 0,\ I = 0,\ c = \bar{3}$$

Phase Diagram: First Version



critical endpoint (E) persists even if $m \neq 0$

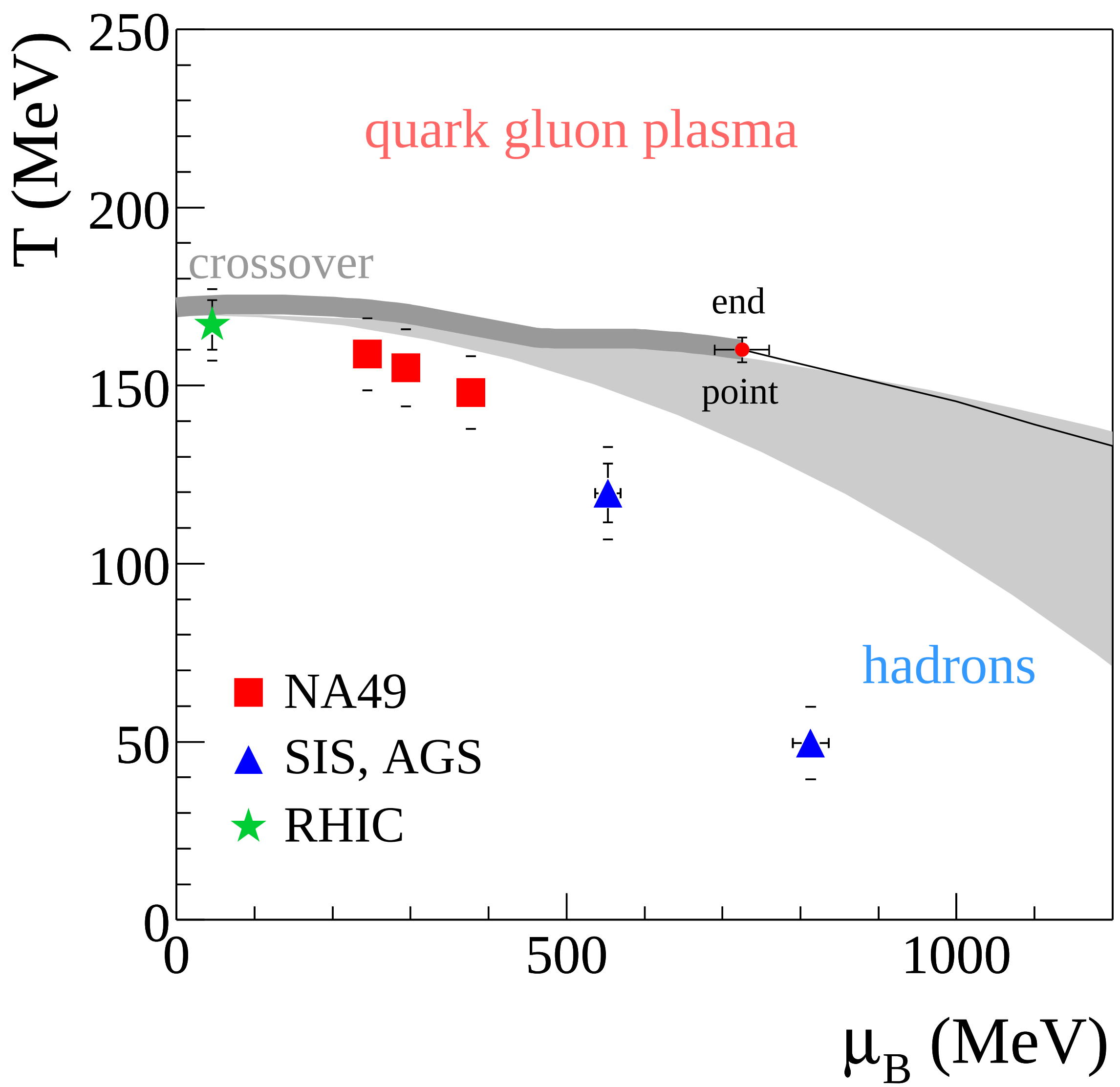
Lattice Results



[FK] Improved re-weighting, [FP] imaginary chemical potential

[All] Taylor expansions

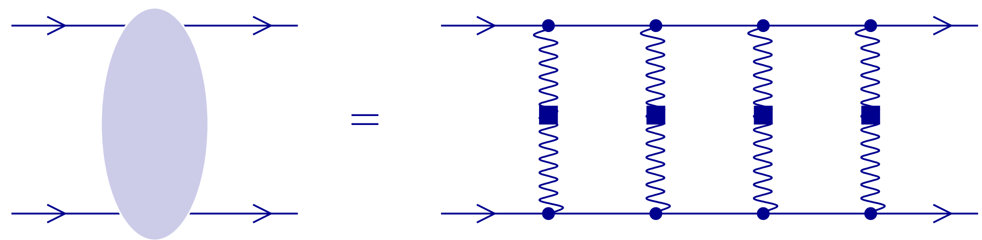
Phase Diagram: Freezeout



Superconductivity

quark-quark scattering

$(\mu \gg \Lambda_{QCD})$



Gap equation: double logarithmic behavior

$$\Delta(p_0) = \frac{g^2}{18\pi^2} \int dq_0 \left\{ \log\left(\frac{b_M}{|p_0 - q_0|} \right) + \ldots \right\} \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta(q_0)^2}}$$

collinear log BCS log

$$\Rightarrow \qquad \Delta_0 = 512\pi^4 \mu g^{-5} \exp\left(-\frac{\pi^2 + 4}{8} \right) \exp\left(-\frac{3\pi^2}{\sqrt{2}g} \right)$$

$\mu \to \infty$: CFL Phase

Consider $N_f = 3$ $(m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi\, \epsilon^{abI} \epsilon_{ijI}$$

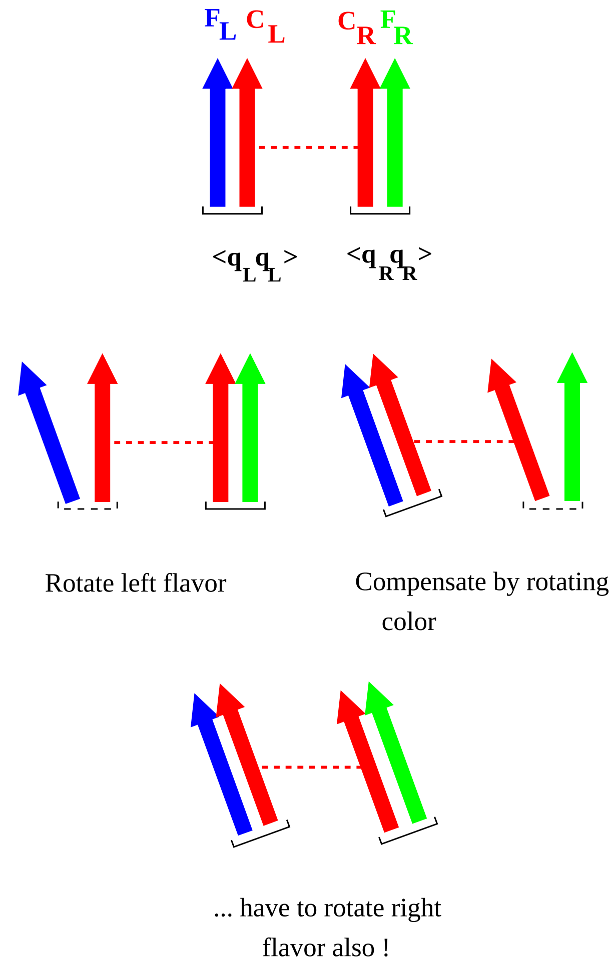
$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$

Symmetry breaking pattern:

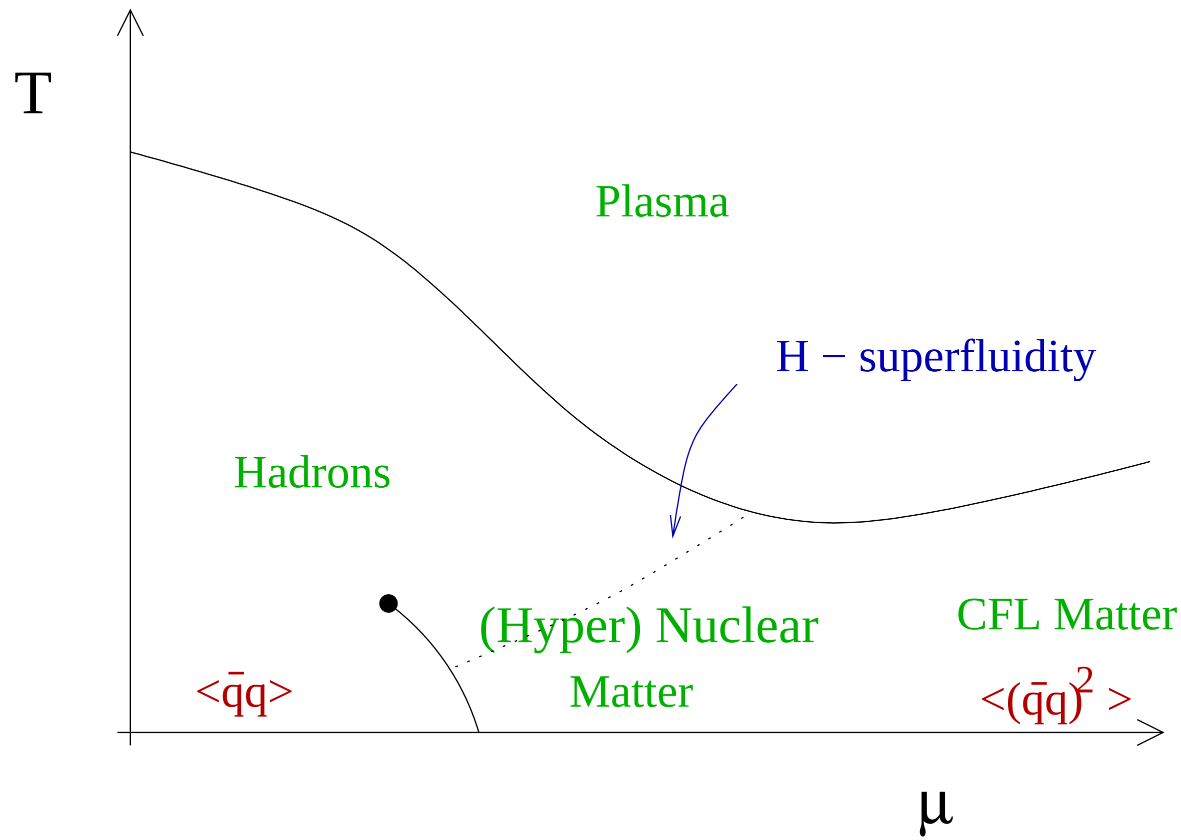
$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \to SU(3)_{C+F}$$

All quarks and gluons acquire a gap



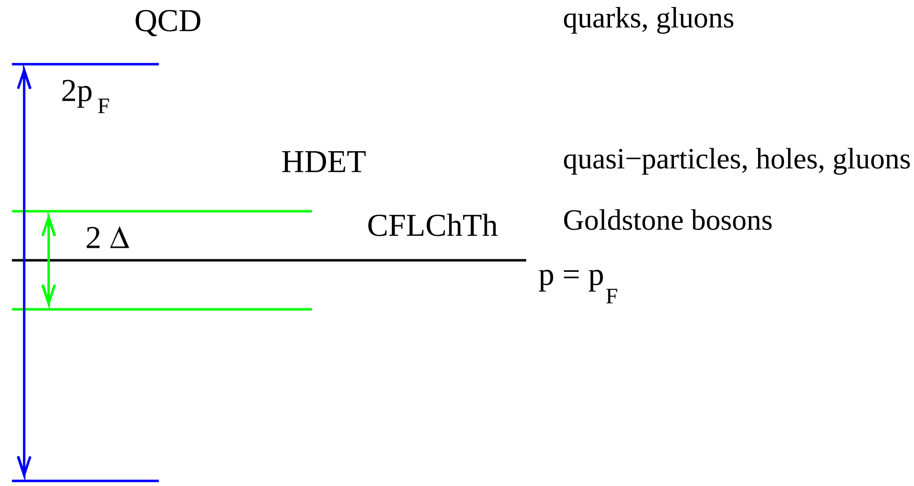
$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

Phase Diagram: $N_f = 3$ QCD



Quark Hadron Continuity

Effective Field Theories



Quantumchromodynamics

$$\mathcal{L} = \bar{\psi}(i\not{D} + \mu\gamma_0)\psi - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

High density effective theory

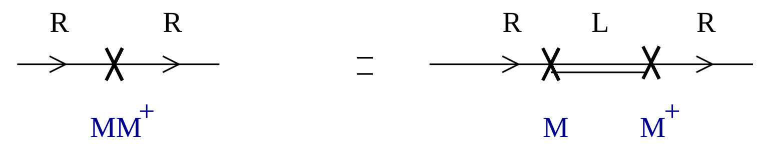
$$\mathcal{L} = \psi_v^\dagger(iv\cdot D)\psi_v - \frac{\Delta}{2}\psi_{-v}^T C\psi_v - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots$$

Chiral effective theory

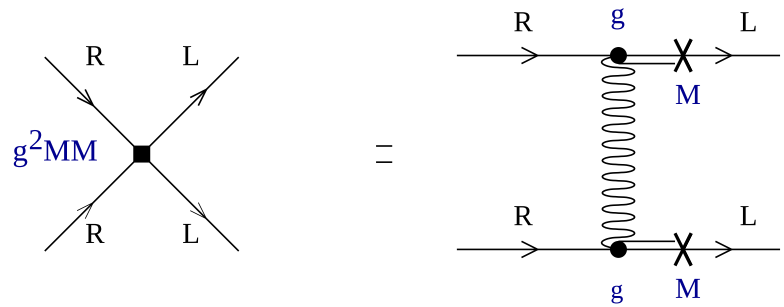
$$\mathcal{L} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[\nabla_0\Sigma\nabla_0\Sigma^\dagger - v^2\vec{\nabla}\Sigma\vec{\nabla}\Sigma^\dagger\right] + \mathrm{Tr}\left(N^\dagger iv^\mu D_\mu N\right) + \ldots$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu}\psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu}\psi_L$$



$$+ \frac{C}{\mu^2}(\psi_R^\dagger M \lambda^a \psi_L)(\psi_R^\dagger M \lambda^a \psi_L)$$



mass corrections to FL parameters $\hat{\mu}, v_F$ and V_0^{++--}

Mass Terms: Match HDET to CFLχTh

Kinetic term: $\psi_L^\dagger X_L \psi_L + \psi_R^\dagger X_R \psi_R$

$$\begin{aligned} D_0 N &= \partial_0 N + i[\Gamma_0, N], \qquad \Gamma_0 = \mathcal{V}_0 + \frac{1}{2}\left(\xi X_R \xi^\dagger + \xi^\dagger X_L \xi\right) \\ \nabla_0 \Sigma &= \partial_0 \Sigma + i X_L \Sigma - i \Sigma X_R \end{aligned}$$

vector (axial) potentials

Contact term: $(\psi_R^\dagger M \psi_L)(\psi_R^\dagger M \psi_L)$

$$\mathcal{L} = \frac{3\Delta^2}{4\pi^2}\left\{[\mathrm{Tr}(M\Sigma)]^2 - \mathrm{Tr}(M\Sigma M\Sigma)\right\}$$

meson mass terms

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2} \mathrm{Tr}\left(X_L \Sigma X_R \Sigma^\dagger\right) - A\,\mathrm{Tr}(M\Sigma^\dagger) - B_1 \left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

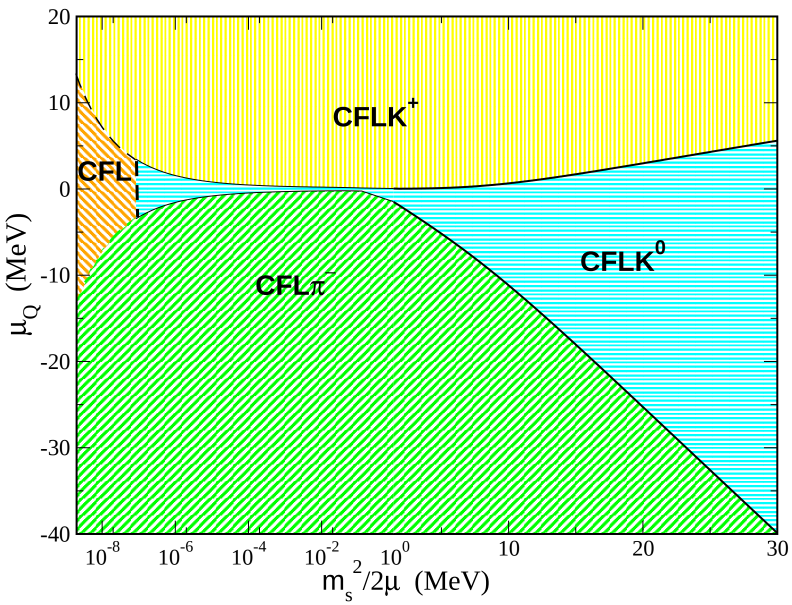
$$V(\Sigma_0) \equiv min$$

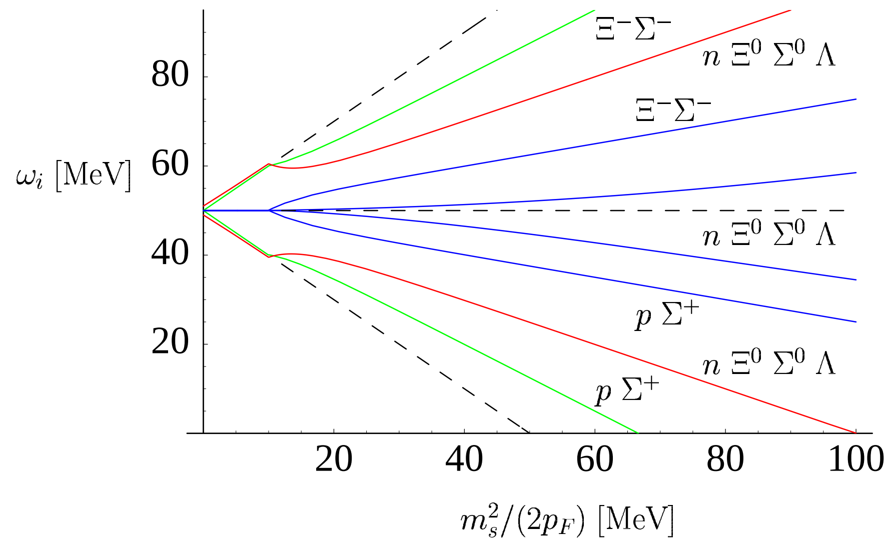
Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger \gamma_5 \rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$

$$\rho_{V,A} = \frac{1}{2}\left\{\xi \frac{M^\dagger M}{2p_F}\xi^\dagger \pm \xi^\dagger \frac{MM^\dagger}{2p_F}\xi\right\} \qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure and Spectrum





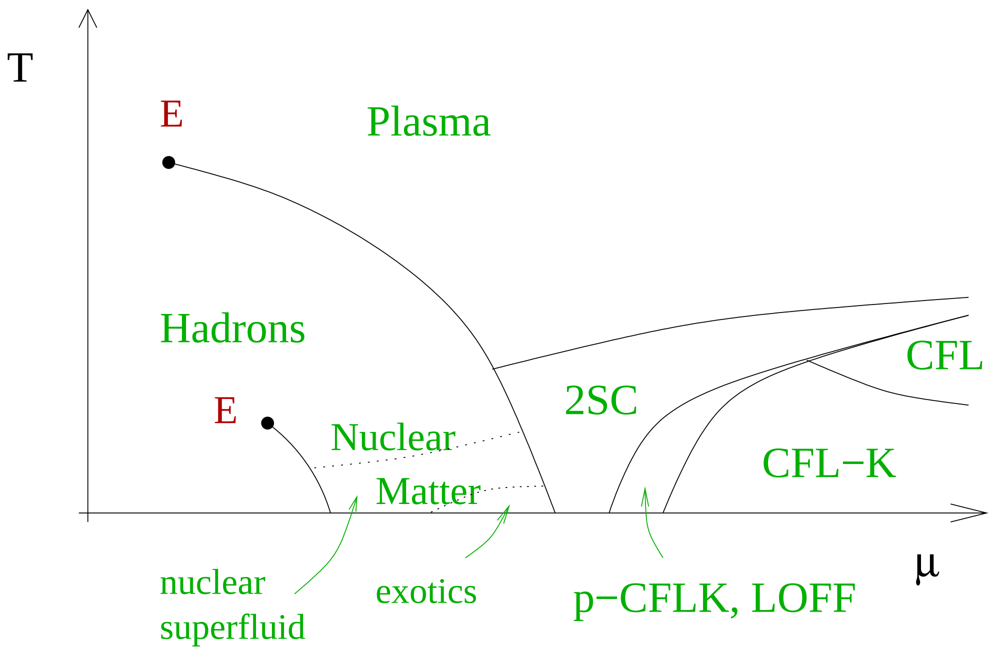
meson condensation: CFLK

$$m_s(crit) \sim m_u^{1/3}\Delta^{2/3}$$

gapless modes? (gCFLK)

$$\mu_s(crit) \sim \frac{4\Delta}{3}$$

Phase Diagram: $m_s \neq 0$



Phase structure at moderate μ (and $m_s, \mu_e \neq 0$) complicated and poorly understood. Systematic calculations

$$m_s^2 \ll \mu^2, m_s \Delta \ll \mu^2, g \ll 1$$

Use neutron stars to rule out certain phases

What are the most useful observables?

Conclusion: The Many Phases of QCD