A Tale of Two Effective Field Theories

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CALCULATION OF SPIN-DEPENDENT PARAMETERS IN THE LANDAU-MIGDAL THEORY OF NUCLEI †

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Abstract: Contributions to the spin-dependent parameter G_0', the coefficient of $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\, \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ in the Fermi-liquid interaction, and to the tensor invariants, are related back to elementary-particle exchange. Once finite-range pion-nucleon interactions are used, almost all of G_0' comes from the ρ-exchange nucleon-nucleon potential. Using modern parameterizations of the strength in the ρ-channel, we find G_0' to be in the region of 1.5 to 2.4 which agrees well with an empirical determination.

1. Introduction

In the sixties a model for nuclei, based on Landau's theory of normal Fermi liquids [1]), was proposed by Migdal [2]). In this theory a set of Fermi-liquid parameters, describing the particle-hole interaction, is assigned to nuclei heavy enough to develop a central region of saturated matter. In so far as the central density of these nuclei is the same, one set of parameters would describe all nuclei. Assuming spin-isospin isotropy the particle-hole interaction in symmetric nuclear matter is given by [2])

$$\mathscr{F}(\boldsymbol{k}_1, \boldsymbol{k}_2) = F(\boldsymbol{k}_1, \boldsymbol{k}_2) + F'(\boldsymbol{k}_1, \boldsymbol{k}_2)\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + G(\boldsymbol{k}_1, \boldsymbol{k}_2)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + G'(\boldsymbol{k}_1, \boldsymbol{k}_2)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \tag{1.1}$$

Motivation

There is a successful effective theory of fermionic many body systems

Landau Fermi-Liquid Theory

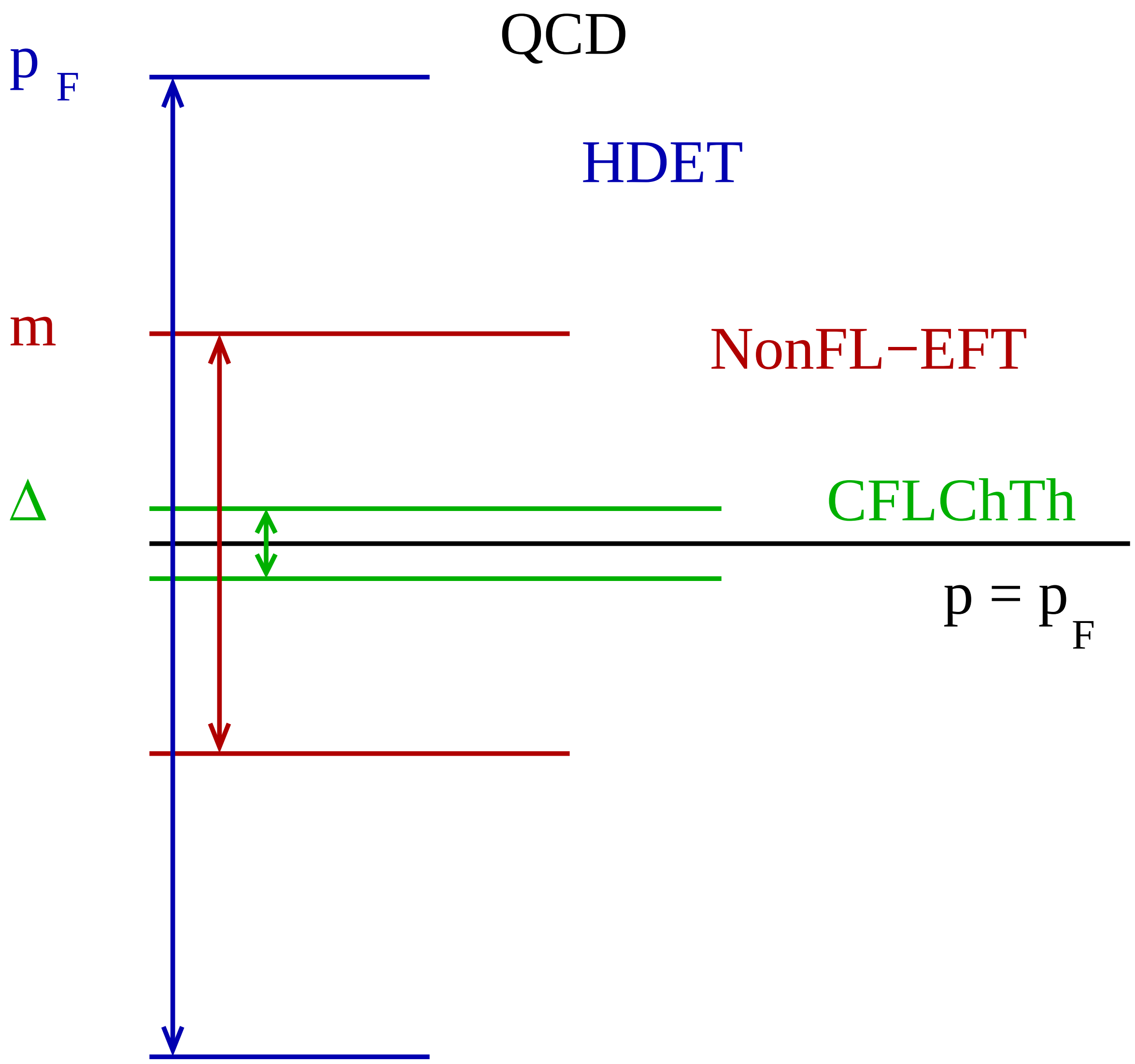
FLT theory: Quasi-particles near the Fermi surface. Interactions characterized by FL parameters. Does not rely on weak coupling.

Predicts collective modes, thermodynamics, transport, ...

Gauge Theories: Unscreened long range forces

Does a quasi-particle EFT exist?

Effective Field Theories

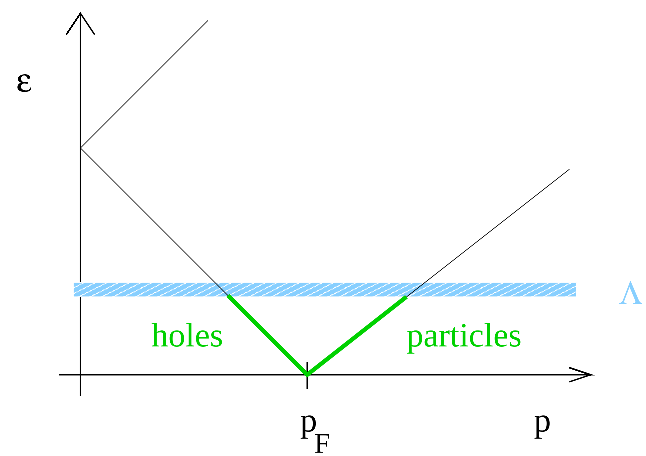


High Density Effective Theory

QCD lagrangian

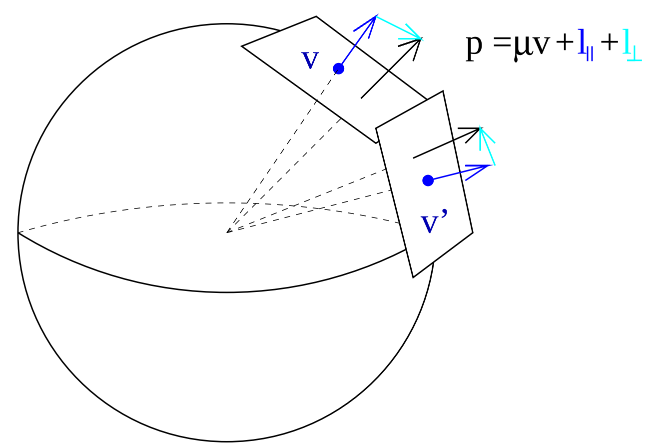
$$\mathcal{L} = \bar{\psi}\left(i\not{D} + \mu\gamma_0 - m\right)\psi - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

Quasi-particles (holes)



$$E_\pm = -\mu \pm \sqrt{\vec{p}^2 + m^2} \simeq -\mu \pm |\vec{p}|$$

Effective field theory on v-patches



$$\psi_{v\pm} = e^{-i\mu v\cdot x}\left(\frac{1 \pm \vec{\alpha}\cdot\vec{v}}{2}\right)\psi$$

High Density Effective Theory, cont

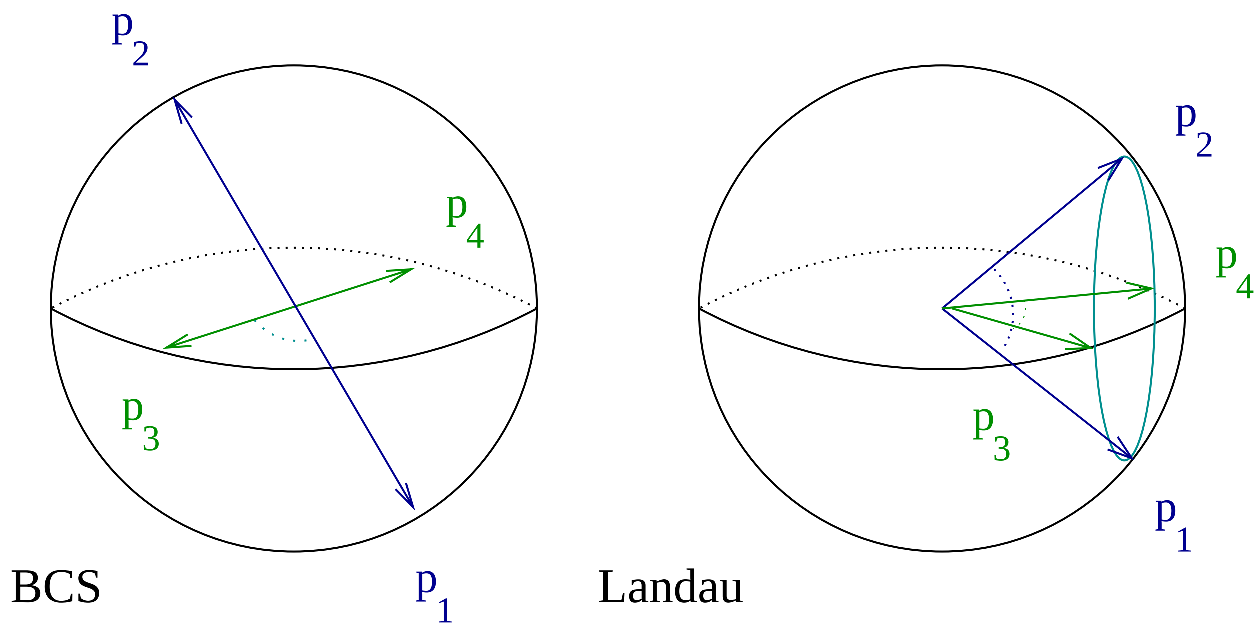
Effective lagrangian for ψ_{v+}

$$\mathcal{L} = \sum_v \psi_v^\dagger \left(iv \cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a + \ldots$$

Four Fermion Operators

quark-quark scattering

$$(v_1, v_2) \to (v_3, v_4)$$

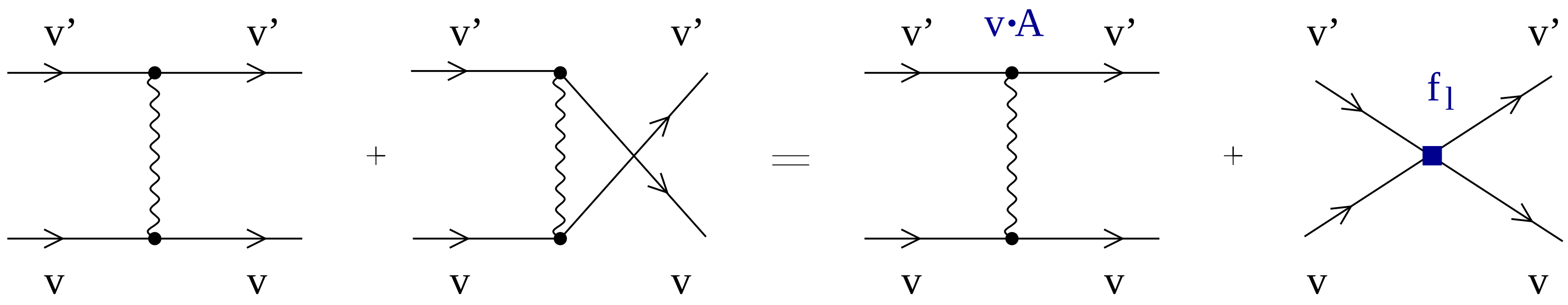


$$\mathcal{L}_{BCS} = \frac{1}{\mu^2} \sum V_l^{\Gamma\Gamma'} R_l^{\Gamma\Gamma'}(\vec{v} \cdot \vec{v}') \Big(\psi_v \Gamma \psi_{-v} \Big) \Big(\psi_{v'}^\dagger \Gamma' \psi_{-v'}^\dagger \Big),$$

$$\mathcal{L}_{FL} = \frac{1}{\mu^2} \sum F_l^{\Gamma\Gamma'}(\phi) R_l^{\Gamma\Gamma'}(\vec{v} \cdot \vec{v}') \Big(\psi_v \Gamma \psi_{v'} \Big) \Big(\psi_{\tilde{v}}^\dagger \Gamma' \psi_{\tilde{v}'}^\dagger \Big)$$

Four Fermion Operators: Matching

Match scattering amplitudes on Fermi surface: forward scattering



Color-flavor-spin symmetric terms

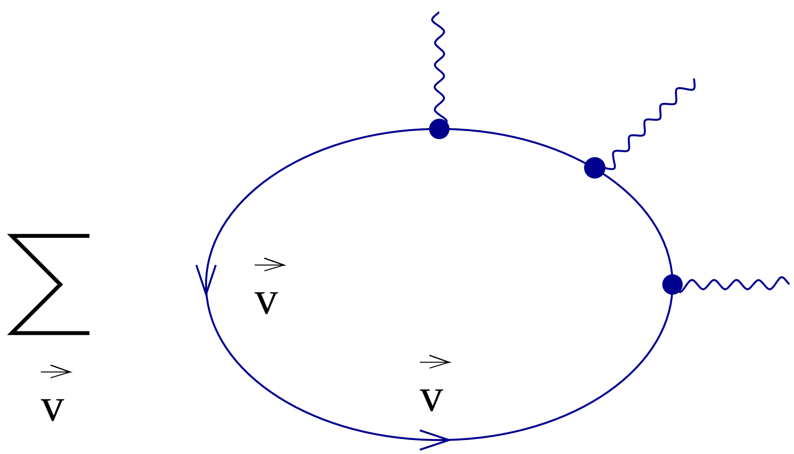
$$f_0^s = \frac{C_F}{4N_c N_f}\frac{g^2}{p_F^2}, \qquad f_i^s = 0 \;\; (i > 1)$$

Power Counting

Naive power counting

$$\mathcal{L} = \hat{\mathcal{L}}\left(\psi, \psi^\dagger, \frac{D_{||}}{\mu}, \frac{D_\perp}{\mu}, \frac{\bar{D}_{||}}{\mu}, \frac{m}{\mu}\right)$$

Problem: hard loops (large $N_{\vec{v}}$ graphs)



$$\frac{1}{2\pi}\sum_{\vec{v}}\int \frac{d^2 l_\perp}{(2\pi)^2} = \frac{\mu^2}{2\pi^2}\int \frac{d\Omega}{4\pi}.$$

Have to sum large $N_{\vec{v}}$ graphs

Effective Theory for $l < m$

$$\mathcal{L} = \psi_v^\dagger \left(iv \cdot D - \frac{D_\perp^2}{2\mu} \right) \psi_v + \mathcal{L}_{4f} - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + \mathcal{L}_{HDL}$$

$$\mathcal{L}_{HDL} = -\frac{m^2}{2} \sum_v G^a_{\mu\alpha} \frac{v^\alpha v^\beta}{(v \cdot D)^2} G^b_{\mu\beta}$$

Transverse gauge boson propagator

$$D_{ij}(k) = \frac{\delta_{ij} - \hat{k}_i \hat{k}_j}{k_0^2 - \vec{k}^2 + i\frac{\pi}{2} m^2 \frac{k_0}{|\vec{k}|}},$$

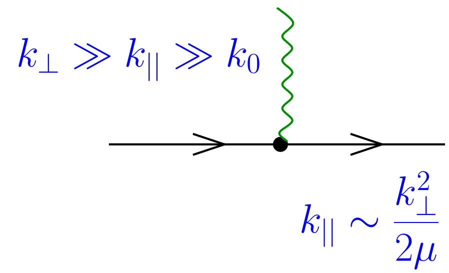
Scaling of gluon momenta

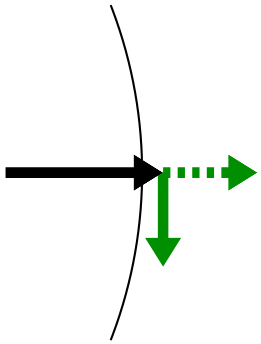
$$|\vec{k}| \sim k_0^{1/3} m^{2/3} \gg k_0 \quad \text{gluons are very spacelike}$$

Non-Fermi Liquid Effective Theory

Gluons very spacelike $|\vec{k}| \gg |k_0|$. Quark kinematics?

$$k_0 \simeq k_{||} + \frac{k_\perp^2}{2\mu}$$





Scaling relations

$$k_\perp \sim m^{2/3} k_0^{1/3}, \qquad k_{||} \sim m^{4/3} k_0^{2/3}/\mu$$

Propagators

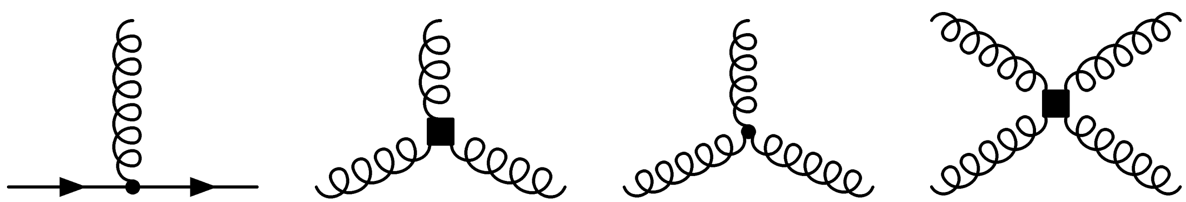
$$S_{\alpha\beta} = \frac{-i\delta_{\alpha\beta}}{p_{||} + \frac{p_\perp^2}{2\mu} - i\epsilon sgn(p_0)} \qquad D_{ij} = \frac{-i\delta_{ij}}{k_\perp^2 - i\frac{\pi}{2} m^2 \frac{k_0}{k_\perp}},$$

Non-Fermi Liquid Expansion

Scale momenta $(k_0, k_{||}, k_\perp) \to (sk_0, s^{2/3}k_{||}, s^{1/3}k_\perp)$

$$[\psi] = 5/6 \qquad [A_i] = 5/6 \qquad [S] = [D] = 0$$

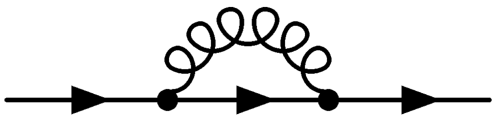
Scaling behavior of vertices



$s^{1/6}$ $\qquad$ $s^{1/2}$ $\qquad$ $s^{5/6}$ $\qquad$ s

Systematic expansion in $\epsilon^{1/3} \equiv (\omega/m)^{1/3}$

Loop Corrections: Quark Self Energy



$$= g^2 C_F \int \frac{dk_0}{2\pi} \int \frac{dk_\perp^2}{(2\pi)^2} \frac{k_\perp}{k_\perp^3 + i\eta k_0}$$

$$\times \int \frac{dk_{||}}{2\pi} \frac{\Theta(p_0 + k_0)}{k_{||} + p_{||} - \frac{(k_\perp + p_\perp)^2}{2\mu} + i\epsilon}$$

Transverse momentum integral logarithmic

$$\int \frac{dk_\perp^3}{k_\perp^3 + i\eta k_0} \sim \log\left(\frac{\Lambda}{k_0}\right)$$

Quark self energy

$$\Sigma(p) = \frac{g^2}{9\pi^2} p_0 \log\left(\frac{\Lambda}{|p_0|}\right)$$

Quark Self Energy, cont

Higher order corrections?

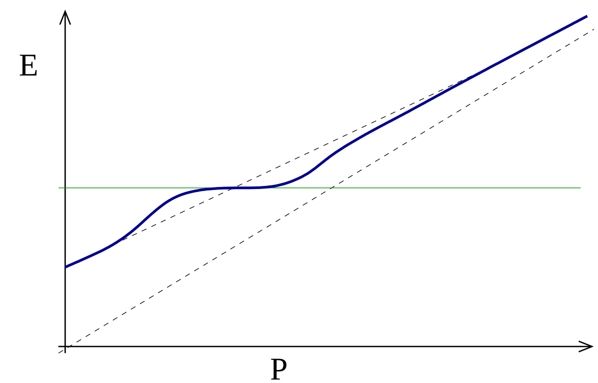
$$\Sigma(p) = \frac{g^2}{9\pi^2}\left(p_0 \log\left(\frac{2^{5/2}m}{\pi|p_0|}\right) + i\frac{\pi}{2}p_0\right) + O\left(\epsilon^{5/3}\right)$$

Scale determined by electric gluon exchange

No $p_0[\alpha_s \log(p_0)]^n$ terms

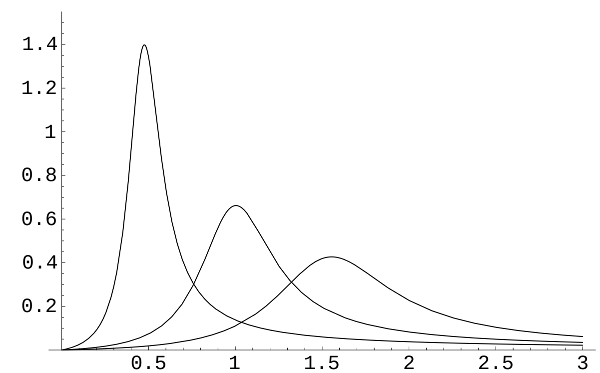
quasi-particle velocity vanishes as

$$v \sim \log(\Lambda/\omega)^{-1}$$



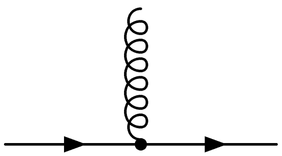
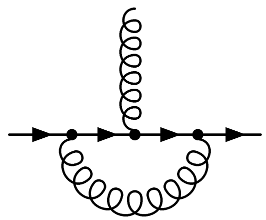
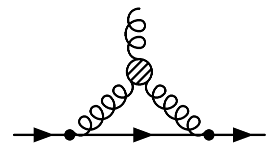
anomalous term in the specific heat

$$c_v \sim \gamma T \log(T)$$



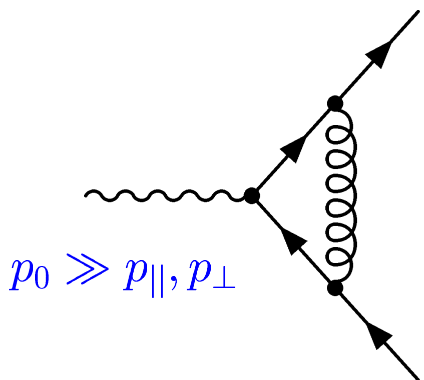
Vertex Corrections, Migdal's Theorem

Corrections to quark gluon vertex

 $+$  $+$  $\sim \; gv(1 + O(\epsilon^{1/3}))$

Analogous to electron-phonon coupling

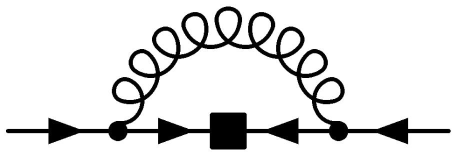
Can this fail? Yes, if external momenta fail to satisfy $p_\perp \gg p_0$



$$= \quad \frac{eg^2}{9\pi^2} v_\mu \log(\epsilon)$$

Superconductivity

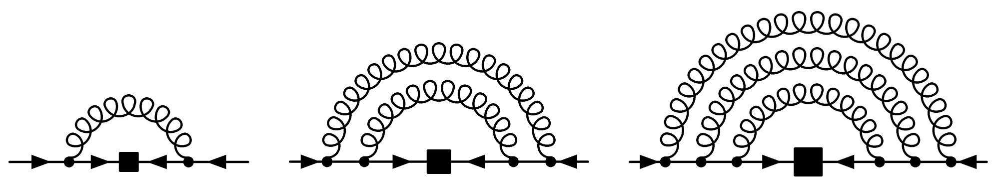
Same phenomenon occurs in anomalous self energy



$$= \frac{g^2}{18\pi^2} \int dq_0 \log\left(\frac{\Lambda_{BCS}}{|p_0 - q_0|}\right) \frac{\Delta(q_0)}{\sqrt{q_0^2 + \Delta(q_0)^2}}$$

$\Lambda_{BCS} = 256\pi^4 g^{-5}\mu$ determined by electric exchanges

Have to sum all planar diagrams, non-planar suppressed by $\epsilon^{1/3}$



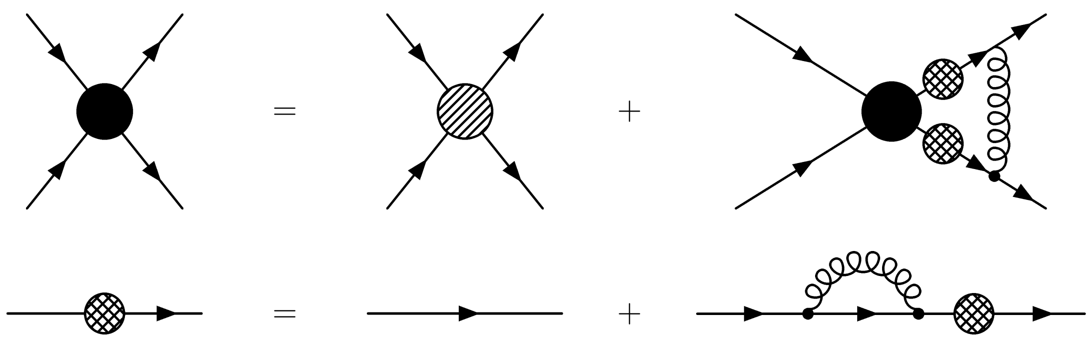
Solution at next-to-leading order (includes normal self energy)

$$\Delta_0 = 2\Lambda_{BCS} \exp\left(-\frac{\pi^2+4}{8}\right) \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right) \qquad \Delta_0 \sim 50\,\mathrm{MeV}$$

Summary

Systematic low energy expansion in $(\omega/m)^{1/3}$ and $\log(\omega/m)$

Standard FL channels (BCS, ZS, ZS'): Ladder diagrams have to be summed, kernel has perturbative expansion



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Pion condensation and density isomerism in nuclear matter

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We follow the treatment of the σ model[10] in the mean-field approximation and make the ansatz

$$\langle\sigma\rangle = f_\pi \cos\theta, \quad \langle\pi^\pm\rangle = f_\pi \sin\theta e^{\pm ikr} \quad \langle\pi^0\rangle = 0 \tag{1}$$

for the meson fields, the chiral invariant being $f_\pi^2 = \sigma^2 + \pi\cdot\pi$ with $f_\pi = 94.5$ MeV, the pion decay constant, k is the pion momentum. This ansatz results in a liquid condensate (the nuclear matter

phases see Dautry's article.[17] With the ansatz (1) we get the total Hamilton density

$$\mathcal{H} = \mathcal{E}_M + \mathcal{H}_{N+\pi N}, \tag{2a}$$

where the meson part is given by

$$\mathcal{E}_M = \tfrac{1}{2} f_\pi^2 k^2 \sin^2\theta + f_\pi^2 m_\pi^2 (1-\cos\theta), \tag{2b}$$

where $f_\pi^2 m_\pi^2$ is added in order to set $\mathcal{E}_M = 0$ for $\theta = 0$. The nucleon and interaction part $\mathcal{H}_{N+\pi N}$ is (in nonrelativistic approximation) given by

$$\mathcal{H}_{N+\pi N} = \varphi_N^* \left(\frac{(\vec{p} - \vec{k}\frac{1}{2}\tau_3\cos\theta)^2}{2M} - \vec{\sigma}\cdot\vec{k} g_A \tfrac{1}{2}\tau_2 \sin\theta \right) \varphi_N, \tag{3}$$

where $g_A = f_\pi g/M$ with $g^2/4\pi = 14$ and $M = m_{\mathrm{nucleon}} = 6.7\, m_\pi$. Diagonalization of $\mathcal{H}_{N+\pi N}$ in isospin space gives the quasiparticle energies

$$E_\pm(\vec{p}) = \frac{\vec{p}^2}{2M} + \frac{k^2\cos^2\theta}{8M} \pm (a^2+b^2)^{1/2}, \tag{4a}$$

where

$$a = \frac{-\vec{p}\cdot\vec{k}}{2M}\cos\theta, \quad b = \tfrac{1}{2} g_A k \sin\theta. \tag{4b}$$

The ground state energy density of the system is then obtained by minimization of

$$\mathcal{E} = \mathcal{E}_M + 2\sum_\pm \int \frac{d^3p}{(2\pi)^3} E_\pm(\vec{p})\Theta[\lambda - E_\pm(\vec{p})] \tag{5}$$

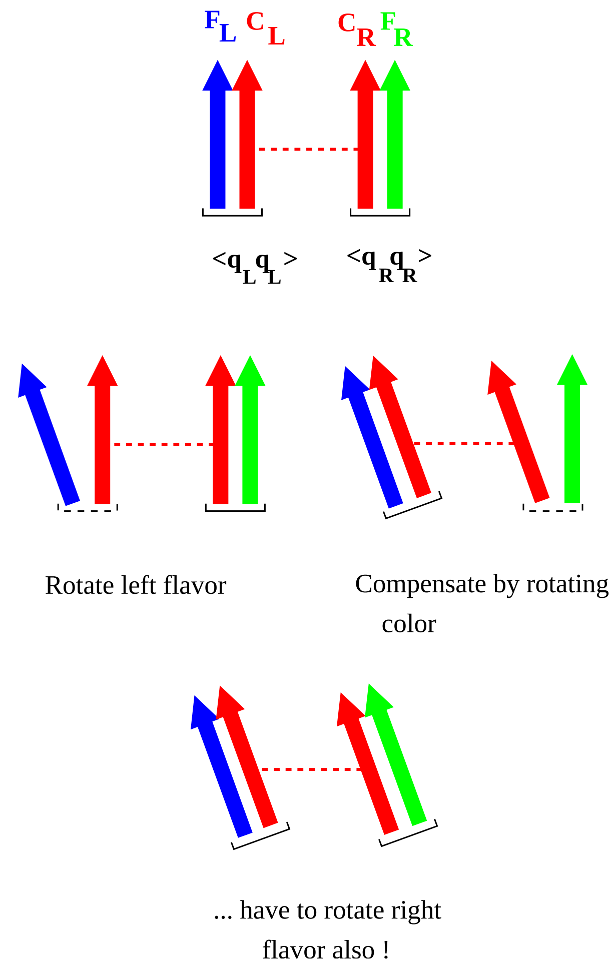
CFL Phase

Consider $N_f = 3 \ (m_i = 0)$

$$\langle q_i^a q_j^b \rangle = \phi \, \epsilon^{abI} \epsilon_{ijI}$$

$$\langle ud \rangle = \langle us \rangle = \langle ds \rangle$$

$$\langle rb \rangle = \langle rg \rangle = \langle bg \rangle$$



Symmetry breaking pattern:

$$SU(3)_L \times SU(3)_R \times [SU(3)]_C \times U(1) \to SU(3)_{C+F}$$

All quarks and gluons acquire a gap

$$\langle \psi_L \psi_L \rangle = -\langle \psi_R \psi_R \rangle$$

EFT in the CFL Phase

Consider HDET with a CFL gap term

$$\mathcal{L} = \mathrm{Tr}\left(\psi_L^\dagger (iv\cdot D)\psi_L\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(X^\dagger\psi_L X^\dagger\psi_L\right) - \kappa\left[\mathrm{Tr}\left(X^\dagger\psi_L\right)\right]^2\right\}$$
$$+ (L\leftrightarrow R, X\leftrightarrow Y)$$

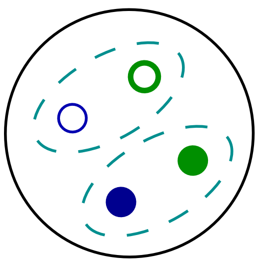
$$\psi_L \to L\psi_L C^T,\; X\to LXC^T, \qquad \langle X\rangle = \langle Y\rangle = \mathbb{1}$$

Quark loops generate a kinetic term for X, Y

Integrate out gluons, identify low energy fields ($\xi = \Sigma^{1/2}$)

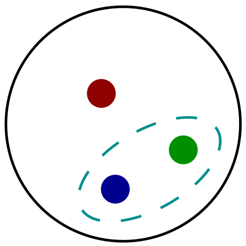
$$\Sigma = XY^\dagger$$

[8]+[1] GBs



$$N_L = \xi(\psi_L X^\dagger)\xi^\dagger$$

[8]+[1] Baryons



Effective theory: (CFL) baryon chiral perturbation theory

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{f_\pi^2}{4}\left\{\mathrm{Tr}\left(\nabla_0\Sigma\nabla_0\Sigma^\dagger\right) - v_\pi^2\,\mathrm{Tr}\left(\nabla_i\Sigma\nabla_i\Sigma^\dagger\right)\right\} \\
& + \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) - D\,\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left\{\mathcal{A}_\mu, N\right\}\right) \\
& - F\,\mathrm{Tr}\left(N^\dagger v^\mu \gamma_5 \left[\mathcal{A}_\mu, N\right]\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\}
\end{aligned}
$$

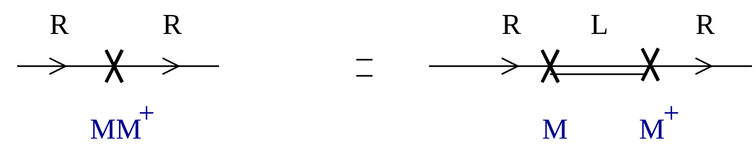
with $D_\mu N = \partial_\mu N + i[\mathcal{V}_\mu, N]$

$$
\begin{aligned}
\mathcal{V}_\mu &= -\frac{i}{2}\left(\xi\partial_\mu\xi^\dagger + \xi^\dagger\partial_\mu\xi\right) \\
\mathcal{A}_\mu &= -\frac{i}{2}\xi\left(\partial_\mu\Sigma^\dagger\right)\xi
\end{aligned}
$$

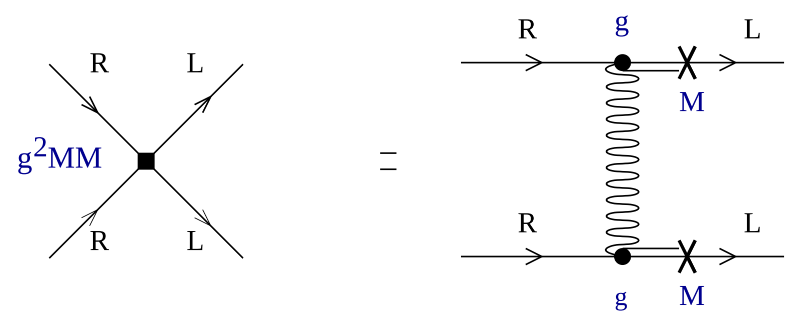
$$
f_\pi^2 = \frac{21 - 8\log 2}{18}\,\frac{\mu^2}{2\pi^2} \qquad v_\pi^2 = \frac{1}{3} \qquad D = F = \frac{1}{2}
$$

Mass Terms: Match HDET to QCD

$$\mathcal{L} = \psi_R^\dagger \frac{MM^\dagger}{2\mu}\psi_R + \psi_L^\dagger \frac{M^\dagger M}{2\mu}\psi_L$$



$$+ \frac{C}{\mu^2}(\psi_R^\dagger M \lambda^a \psi_L)(\psi_R^\dagger M \lambda^a \psi_L)$$



mass corrections to FL parameters $\hat{\mu}$ and $F^0(++ \to --)$

Phase Structure and Spectrum

Phase structure determined by effective potential

$$V(\Sigma) = \frac{f_\pi^2}{2}\mathrm{Tr}\left(X_L \Sigma X_R \Sigma^\dagger\right) - A\mathrm{Tr}(M\Sigma^\dagger) - B_1\left[\mathrm{Tr}(M\Sigma^\dagger)\right]^2 + \ldots$$

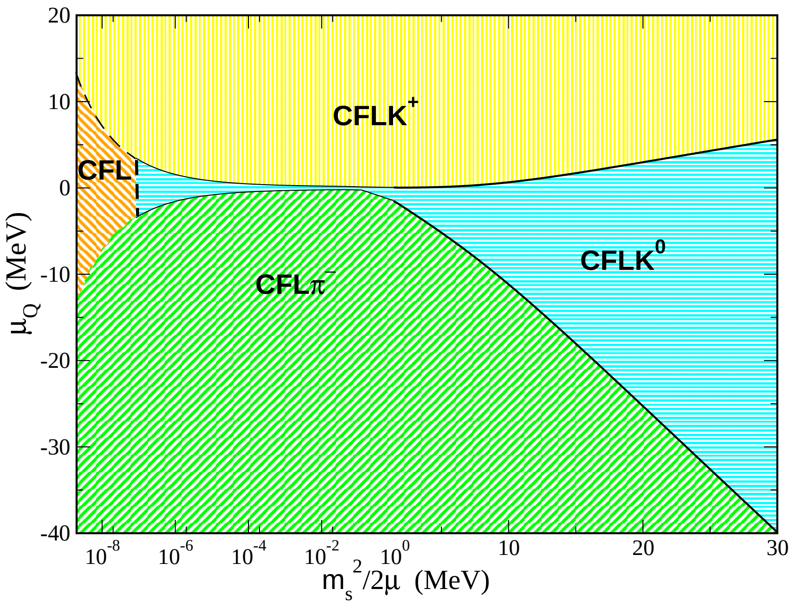
$$V(\Sigma_0) \equiv min$$

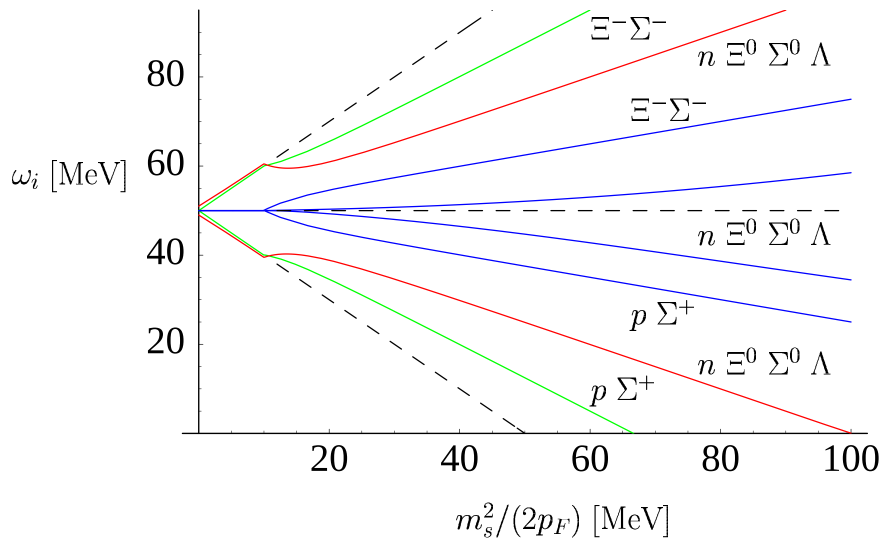
Fermion spectrum determined by

$$\mathcal{L} = \mathrm{Tr}\left(N^\dagger i v^\mu D_\mu N\right) + \mathrm{Tr}\left(N^\dagger \gamma_5 \rho_A N\right) + \frac{\Delta}{2}\left\{\mathrm{Tr}\left(NN\right) - \left[\mathrm{Tr}\left(N\right)\right]^2\right\},$$

$$\rho_{V,A} = \frac{1}{2}\left\{\xi\frac{M^\dagger M}{2p_F}\xi^\dagger \pm \xi^\dagger\frac{MM^\dagger}{2p_F}\xi\right\} \qquad \xi = \sqrt{\Sigma_0}$$

Phase Structure and Spectrum





meson condensation: CFLK

s-wave condensate

gapless modes? (gCFLK)

p-wave condensation

Instabilities

Consider meson current

$$\Sigma(x) = U_Y(x)\Sigma_K U_Y(x)^\dagger \qquad U_Y(x) = \exp(i\phi_K(x)\lambda_8)$$

$$\vec{\mathcal{V}}(x) = \frac{\vec{\nabla}\phi_K}{4}(-2\hat{I}_3 + 3\hat{Y}) \qquad \vec{\mathcal{A}}(x) = \vec{\nabla}\phi_K(e^{i\phi_K}\hat{u}^+ + e^{-i\phi_K}\hat{u}^-)$$

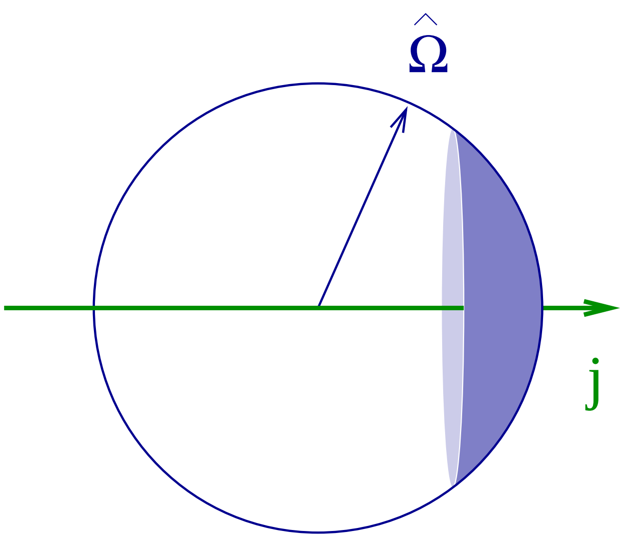
Gradient energy

$$\mathcal{E} = \frac{f_\pi^2}{2} v_\pi^2 \jmath_K^2 \qquad \vec{\jmath}_k = \vec{\nabla}\phi_K$$

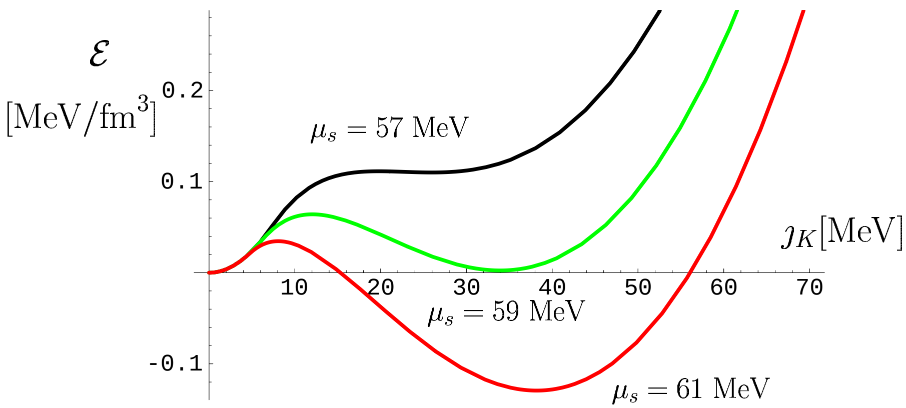
Fermion spectrum

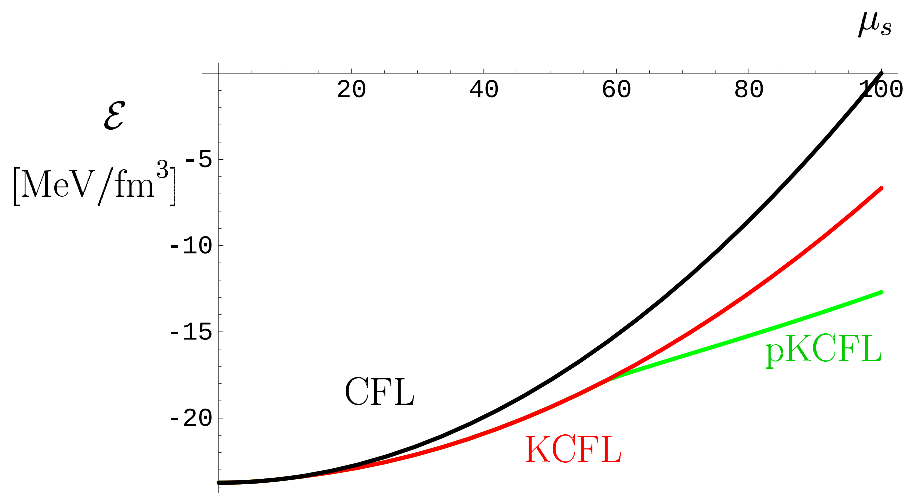
$$\omega_l = \Delta + \frac{l^2}{2\Delta} - \frac{4\mu_s}{3} - \frac{1}{4}\vec{v}\cdot\vec{\jmath}_K$$

$$\mathcal{E} = \frac{\mu^2}{2\pi^2}\int dl \int d\hat{\Omega}\; \omega_l \Theta(-\omega_l)$$



Energy Functional





$$\left.\frac{3\mu_s - 4\Delta}{\Delta}\right|_{crit} = a h_{crit} \qquad h_{crit} = -0.067 \qquad a = \frac{2}{15^2 c_\pi^2 v_\pi^4}$$

[Figures include baryon current $j_B = \alpha_B/\alpha_K j_K$]

Notes

No net current, meson current canceled by backflow of gapless modes

$$(\delta\mathcal{E})/(\delta\nabla\phi) = 0$$

Instability related to "chromomagnetic instability"

CFL phase: gluons carry $SU(3)_F$ quantum numbers

Meson current equivalent to a color gauge field

Happy Birthday

Wolfram, Peter & Gerry