

Nearly Perfect Fluidity: From Cold Atoms to Hot Quarks

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RHIC serves the perfect fluid



Experiments at RHIC and the LHC are consistent with the idea that a thermalized plasma is produced, and that the equation of state is that of a weakly coupled gas of quarks and gluons.

But: Transport properties of the system (primarily viscosity and energy loss) are in dramatic disagreement with expectations for a weakly coupled QGP. The plasma must be very strongly coupled.

In this talk I will try to explain this statement, review the current evidence, and put the results in a broader perspective (by comparing with another strongly coupled fluid, the dilute atomic Fermi gas at “unitarity”).

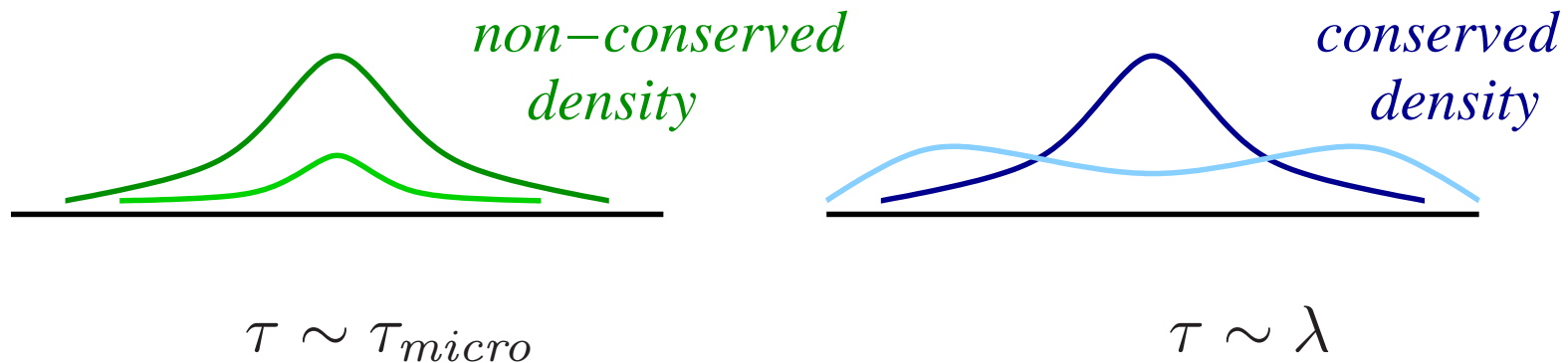
Hydrodynamics

Hydrodynamics (undergraduate version): Newton's law for continuous, deformable media.



Fluids: Gases, liquids, plasmas, ...

Hydrodynamics (postmodern): Effective theory of non-equilibrium long-wavelength, low-frequency dynamics of any many-body system.



$\tau \gg \tau_{micro}$: Dynamics of conserved charges.

Water: $(\rho, \epsilon, \vec{\pi})$

Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} = -\vec{\nabla} \cdot (\rho \vec{v}) \qquad \frac{\partial \epsilon}{\partial t} = -\vec{\nabla}_j \vec{j}^\epsilon$$

$$\frac{\partial}{\partial t} (\rho v_i) = -\nabla_j \Pi_{ij}$$

mass \times acceleration = force

Constitutive relations: Stress tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3} \delta_{ij} \nabla_k v_k \right) + O(\nabla^2)$$

reactive

dissipative

2nd order

$$\text{Expansion } \Pi_{ij}^0 \gg \delta \Pi_{ij}^1 \gg \delta \Pi_{ij}^2$$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$\frac{1}{Re} = \frac{\eta}{\hbar n} \times \frac{\hbar}{mvL}$$

fluid property flow property



-1

Bath tub : $mvL \gg \hbar$ hydro reliable

Heavy ions : $mvL \sim \hbar$ need $\eta < \hbar n$

Note: Bacteria swim in the regime $Re^{-1} \gg 1$ but $Ma^2 \cdot Re^{-1} \ll 1$.

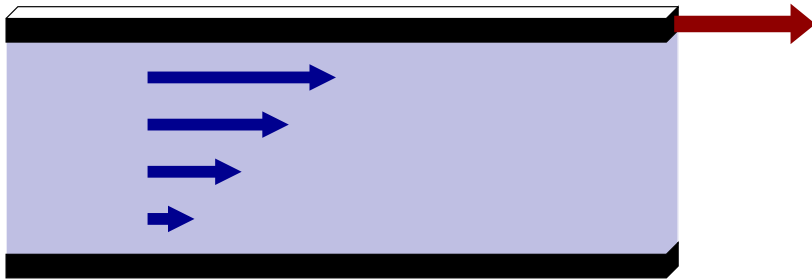
Shear viscosity and friction

Momentum conservation at $O(\nabla v)$

$$\rho \left(\frac{\partial}{\partial t} \vec{v} + (\vec{v} \cdot \vec{\nabla}) \vec{v} \right) = -\vec{\nabla} P + \eta \nabla^2 \vec{v}$$

Navier-Stokes equation

Viscosity determines shear stress (“friction”) in fluid flow



$$F = A \eta \frac{\partial v_x}{\partial y}$$

Kinetic theory

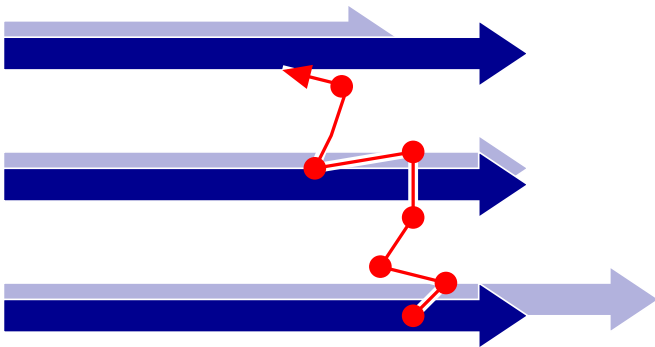
Kinetic theory: conserved quantities carried by quasi-particles.
Quasi-particles described by distribution functions $f(x, p, t)$.

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = -C[f_p]$$

$$C[f_p] = \begin{array}{c} p \\ \swarrow \quad \searrow \\ \bullet \\ \nearrow \quad \nwarrow \\ \end{array} - \begin{array}{c} \swarrow \quad \searrow \\ \bullet \\ \nearrow \quad \nwarrow \\ \end{array} \begin{array}{c} p \\ \swarrow \quad \searrow \\ \bullet \\ \nearrow \quad \nwarrow \\ \end{array}$$



Shear viscosity corresponds to momentum diffusion



$$\eta \sim \frac{1}{3} n \bar{p} l_{mfp}$$

Shear viscosity: Additional properties

Weakly interacting gas, $l_{mfp} \sim \frac{1}{n\sigma}$:

$$\eta \sim \frac{1}{3} \bar{p} \sigma$$

shear viscosity independent of density

Non-interacting gas ($\sigma \rightarrow 0$):

$$\eta \rightarrow \infty$$

non-interacting and hydro limit ($T \rightarrow \infty$) limit do not commute

strongly interacting gas:

$$\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$$

but: kinetic theory not reliable!

Historical digression: Mott's minimal conductivity

(Sir) Nevill Mott predicted that the metal-insulator transition cannot be continuous; there is a minimal conductivity.

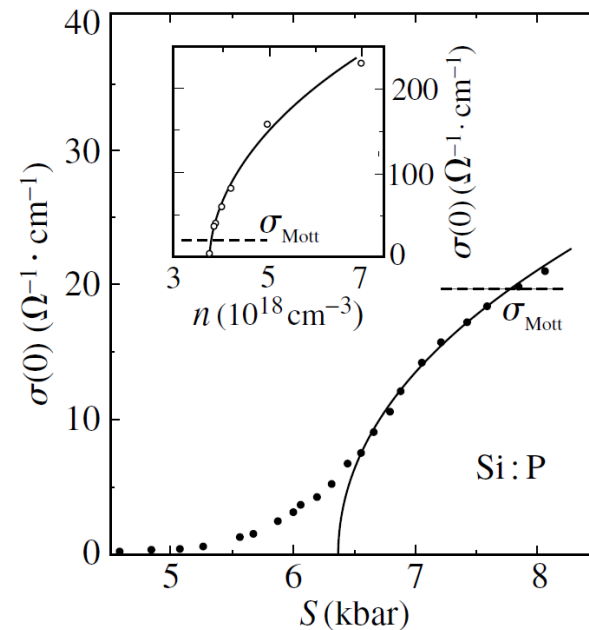
Conduction in Non-crystalline Systems
IX. The Minimum Metallic Conductivity

By N. F. MOTT
Cavendish Laboratory, Cambridge

[Received 27 July 1972]

$$\frac{\sigma}{n^{1/3}} \geq \frac{1}{(3\pi^2)^{2/3}} \frac{e^2}{\hbar}$$

This idea is not correct,
the metal-insulator transition can
be continuous.



Historical digression: Minimal shear viscosity

Danielewicz & Gyulassy argue that the shear viscosity cannot be zero.

PHYSICAL REVIEW D

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1 JANUARY 1985

Dissipative phenomena in quark-gluon plasmas

P. Danielewicz* and M. Gyulassy

Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

(Received 12 April 1984; revised manuscript received 24 September 1984)

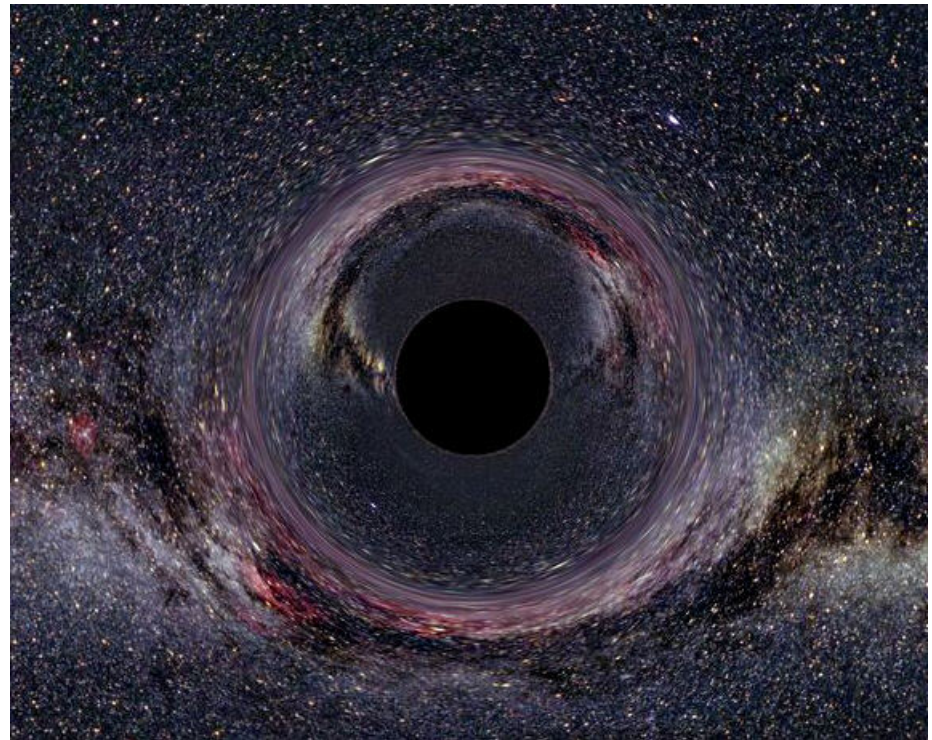
than $\langle p \rangle^{-1}$. Requiring $\lambda_i \gtrsim \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \gtrsim \frac{1}{3}n, \quad (3.3)$$

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of

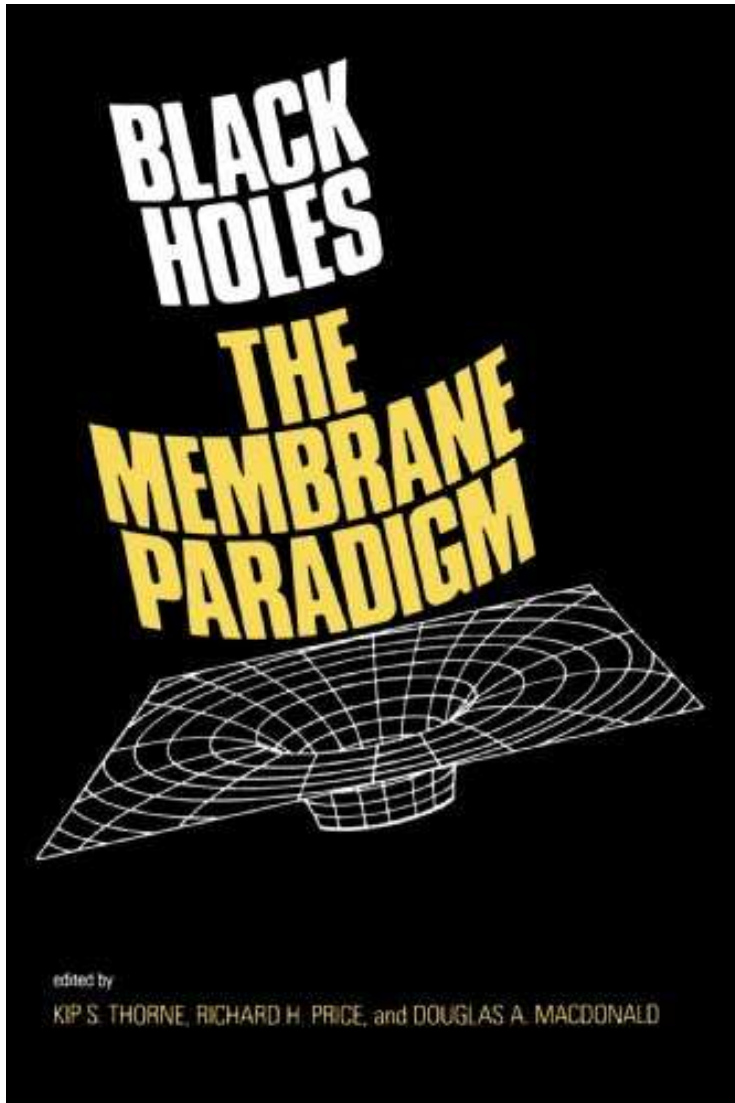
Is this idea correct?

And now for something completely different ...



This is an irreversible process, $\Delta S > 0$.

And now for something completely different ...



Ringdown can be described in terms of stretched horizon that behaves as a sheared fluid

$$\eta = \frac{s}{4\pi}$$

Note: Unusual thermodynamics, e.g. $\zeta, C < 0$.

Idea can be made precise using the “AdS/CFT correspondence”

Strongly coupled thermal
field theory on R^4



Weakly coupled string theory
on AdS_5 black hole

CFT temperature

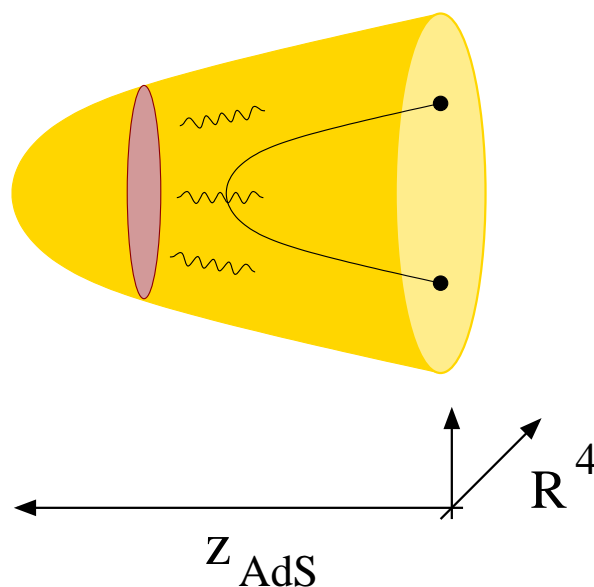


Hawking temperature of
black hole

CFT entropy



Hawking-Bekenstein entropy
 \sim area of event horizon



Holographic duals: Transport properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy \Leftrightarrow

Hawking-Bekenstein entropy

\sim area of event horizon

shear viscosity \Leftrightarrow

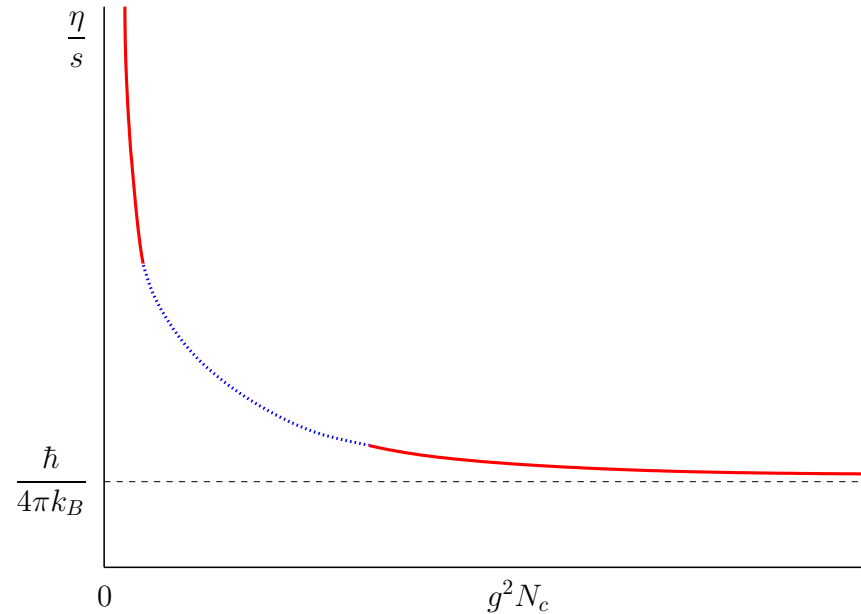
Graviton absorption cross section

\sim area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

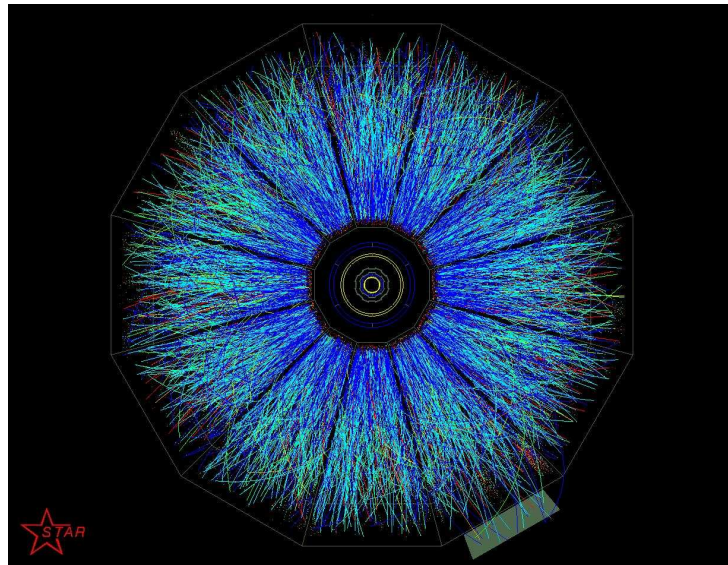
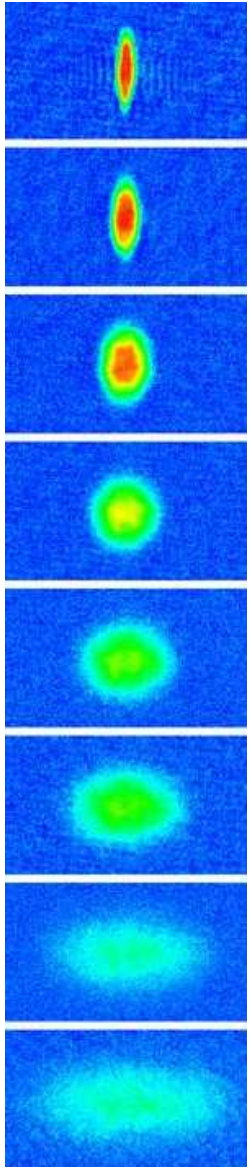
Son and Starinets (2001)



Strong coupling limit universal? Provides lower bound for all theories?

Answer appears to be no; e.g. theories with higher derivative gravity duals.

Perfect Fluids: The contenders



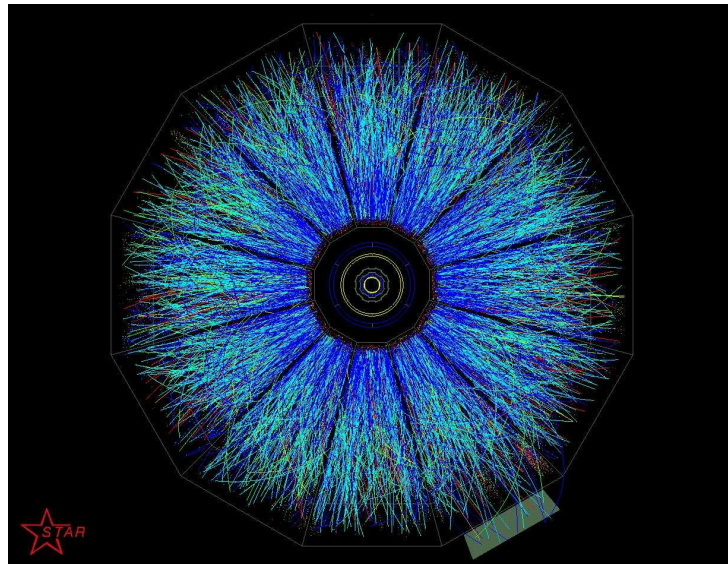
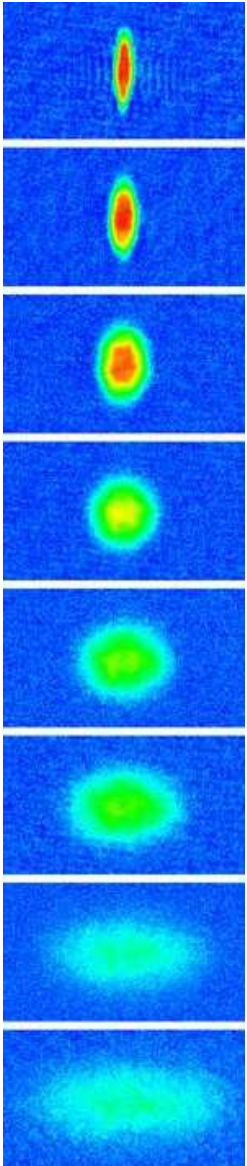
QGP ($T=180$ MeV)

Trapped Atoms
($T=0.1$ neV)



Liquid Helium
($T=0.1$ meV)

Perfect Fluids: The contenders



QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η/s

Perfect Fluids: Not a contender



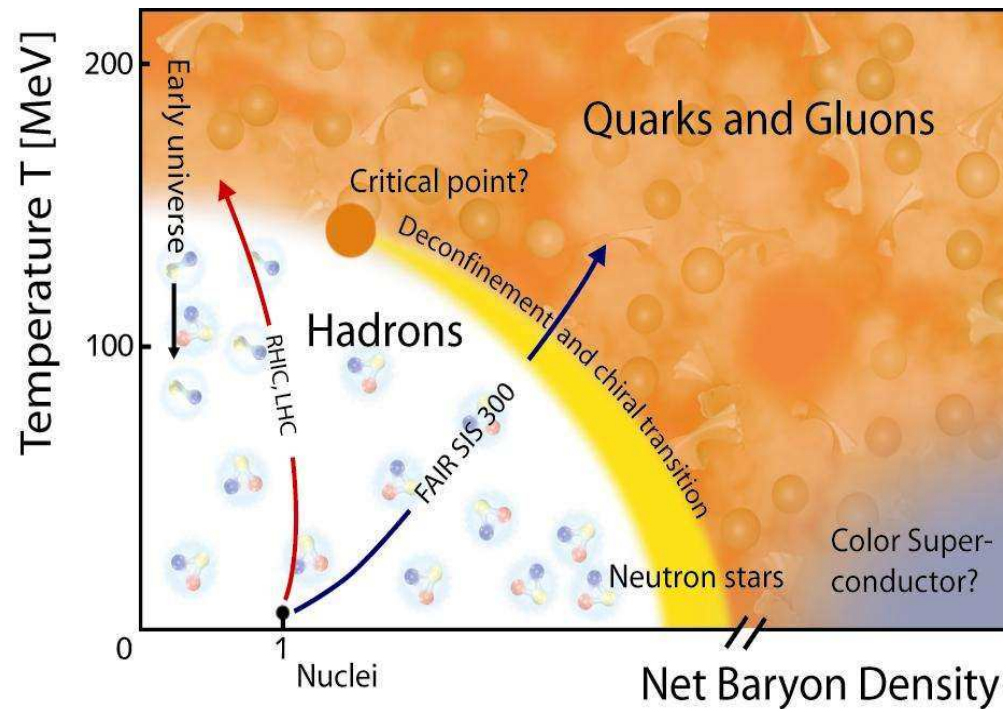
Queensland pitch-drop
experiment

1927-2011 (8 drops)

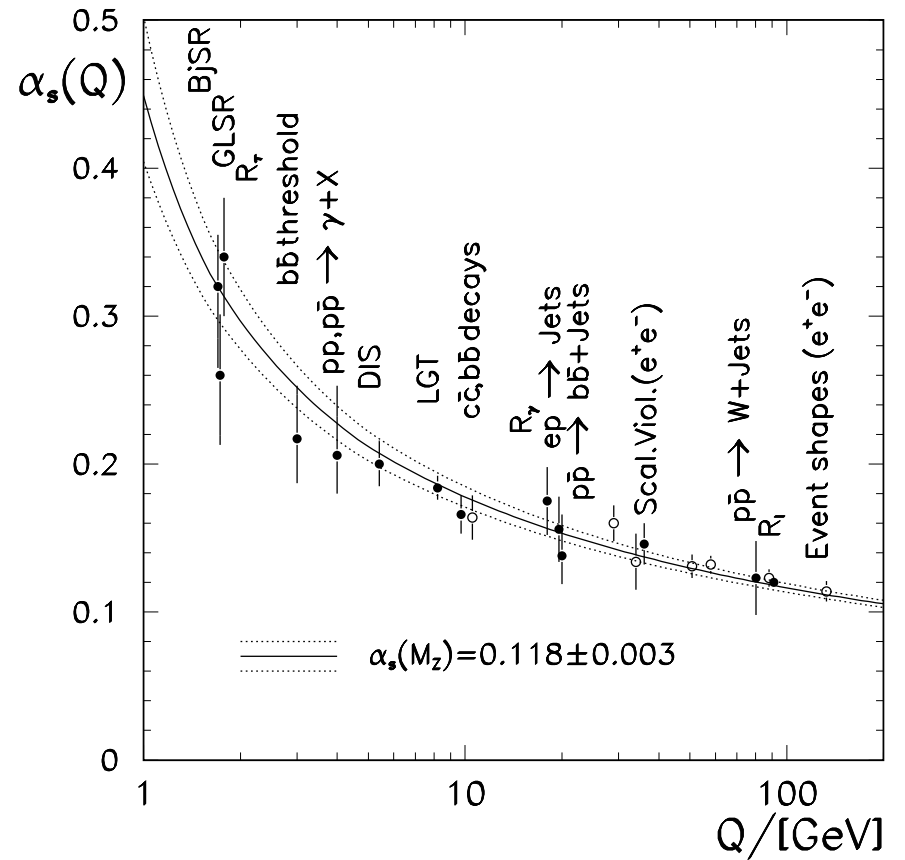
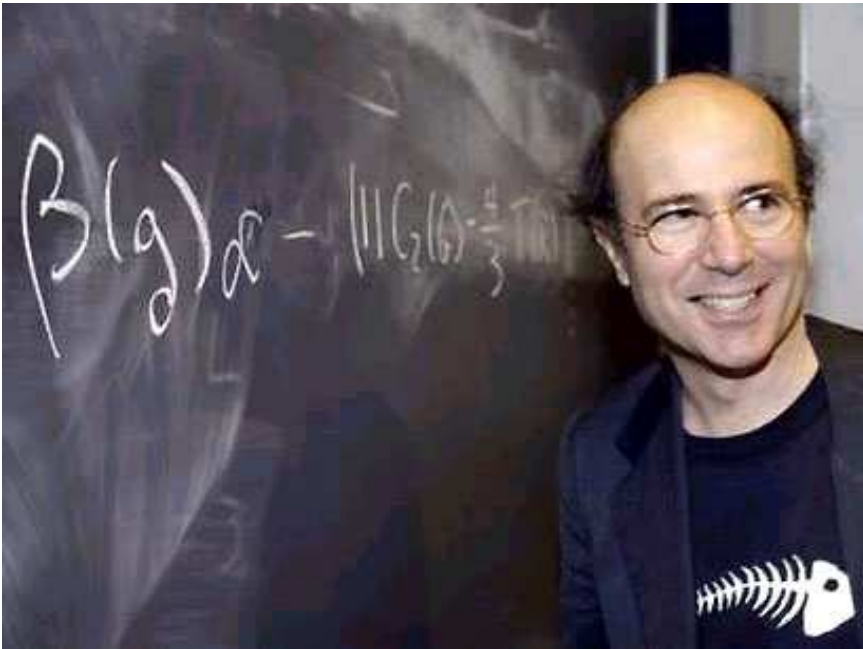
$$\eta = (2.3 \pm 0.5) \cdot 10^8 \text{ Pa s}$$

I. QCD and the Quark Gluon Plasma

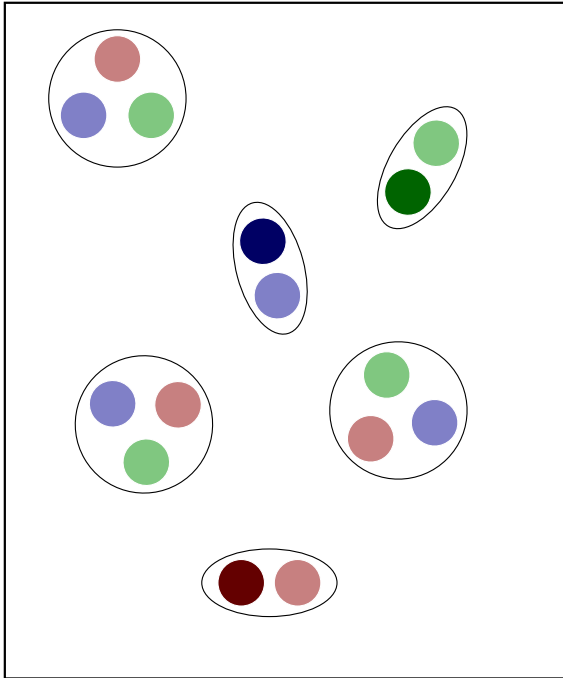
$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a$$



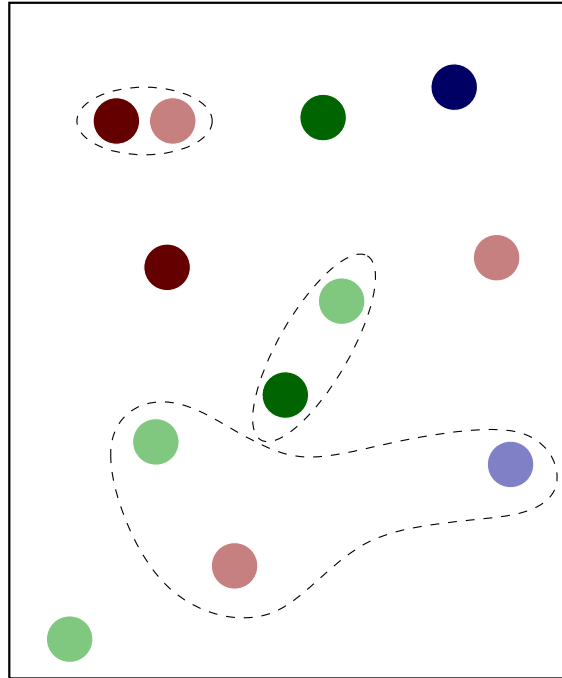
Running coupling constant



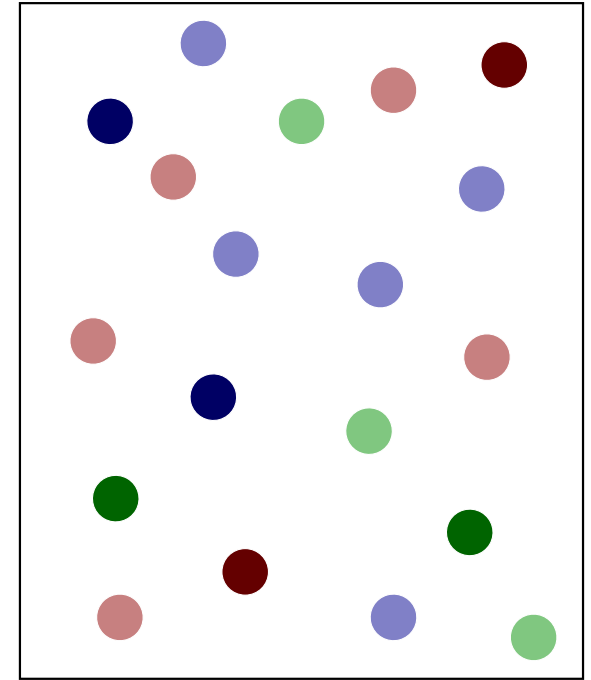
From hadrons to quarks



weakly coupled
hadron gas



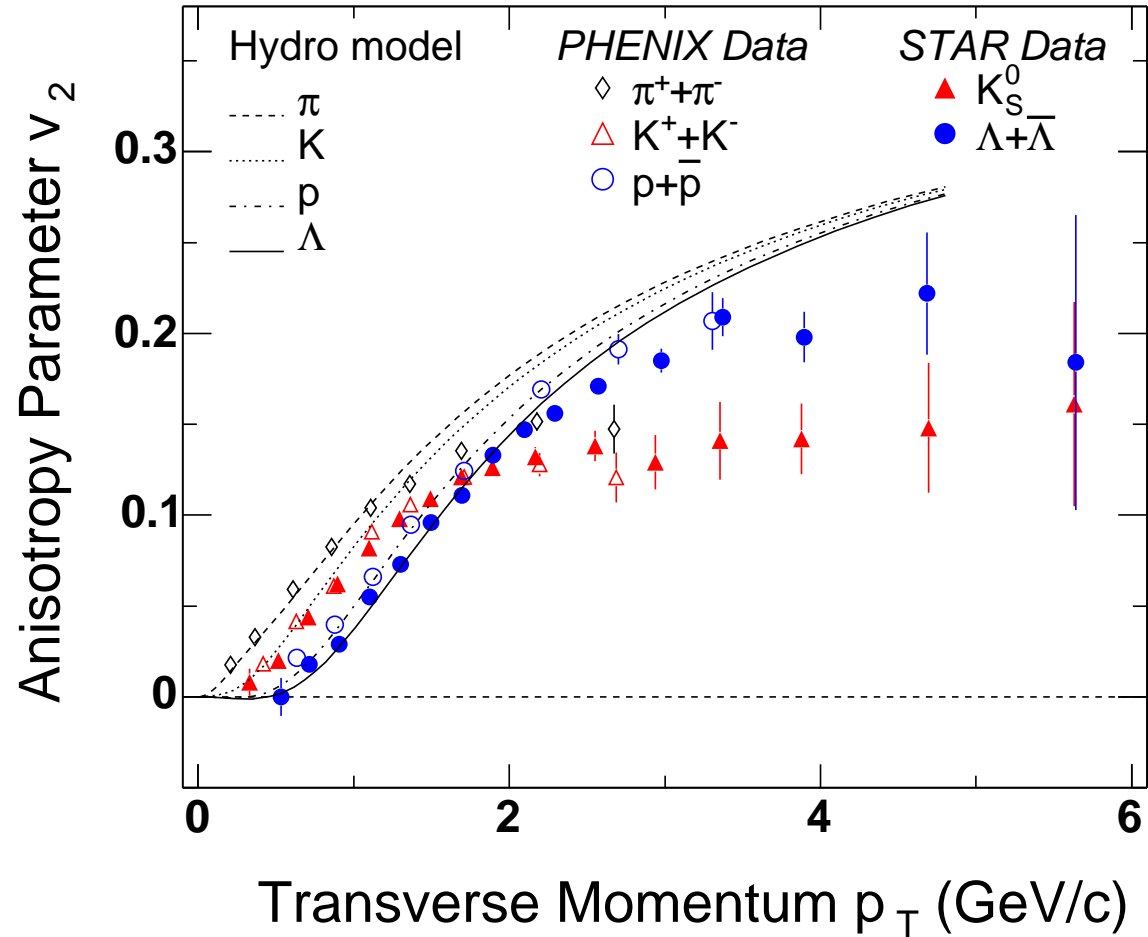
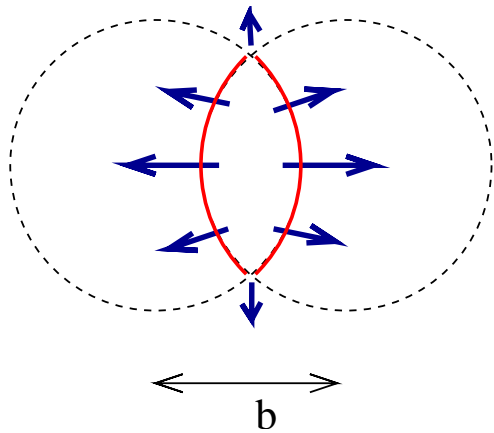
strongly correlated
fluid



weakly coupled
quark gluon plasma

Elliptic Flow (QGP)

Hydrodynamic expansion converts
 coordinate space
 anisotropy
 to momentum space
 anisotropy



source: U. Heinz (2005)

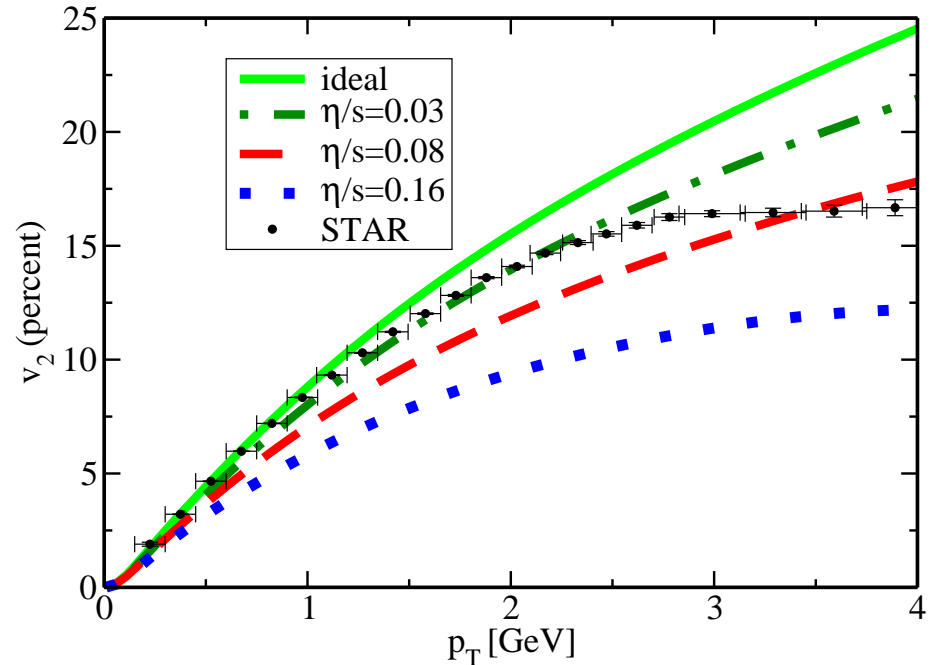
$$p_0 \left. \frac{dN}{d^3p} \right|_{p_z=0} = v_0(p_\perp) (1 + 2v_2(p_\perp) \cos(2\phi) + \dots)$$

Viscosity and Elliptic Flow

Viscous correction to v_2 (blast wave model)

$$\frac{\delta v_2}{v_2} = -\frac{1}{3} \frac{1}{\tau_f T_f} \left(\frac{\eta}{s}\right) \left(\frac{p_\perp}{T_f}\right)^2$$

Grows with p_\perp , decreases with system size



Romatschke (2007), Teaney (2003)

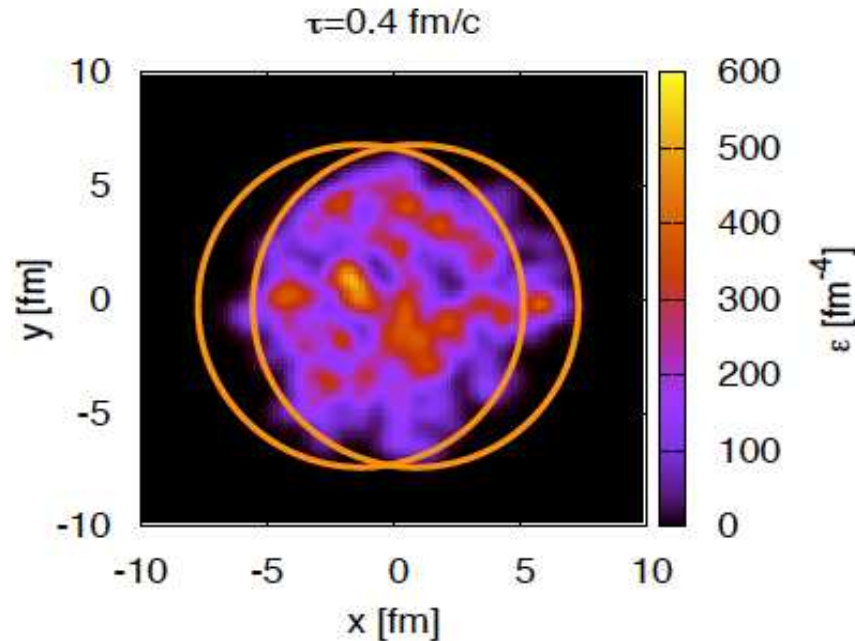
Many details: Dependence on initial conditions, freeze out, etc.

conservative bound

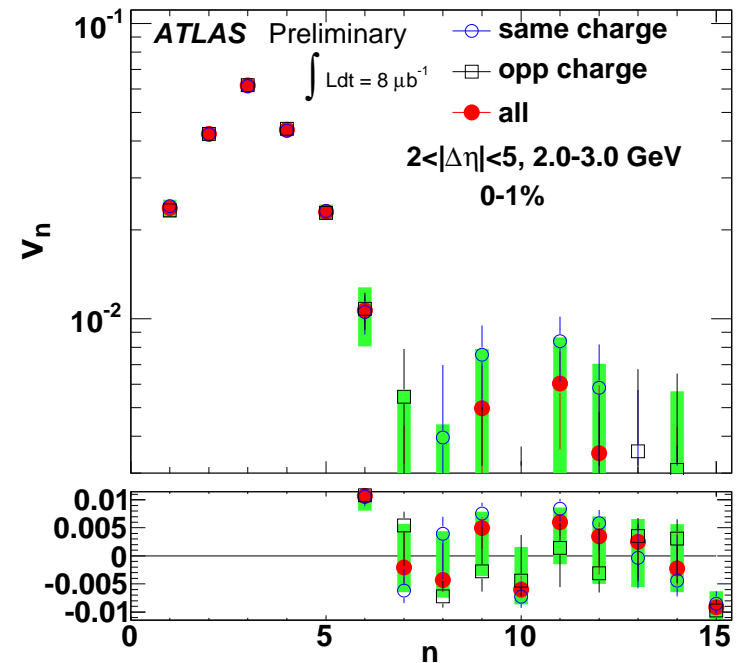
$$\frac{\eta}{s} < 0.25$$

Frontier I: Higher moments of flow

Hydro converts moments of initial deformation to moments of flow



B. Schenke



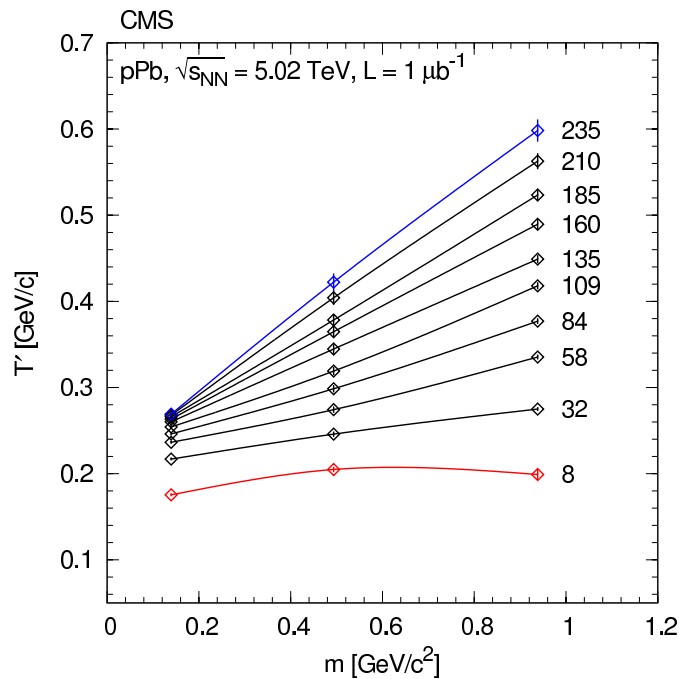
Atlas (J. Jia, QM 2011)

Glauber predicts flat initial spectrum ($n \geq 3$). Observed flow spectrum consistent with sound attenuation

$$\delta T^{\mu\nu}(t) = \exp\left(-\frac{2}{3} \frac{\eta}{s} \frac{k^2 t}{T}\right) \delta T^{\mu\nu}(0)$$

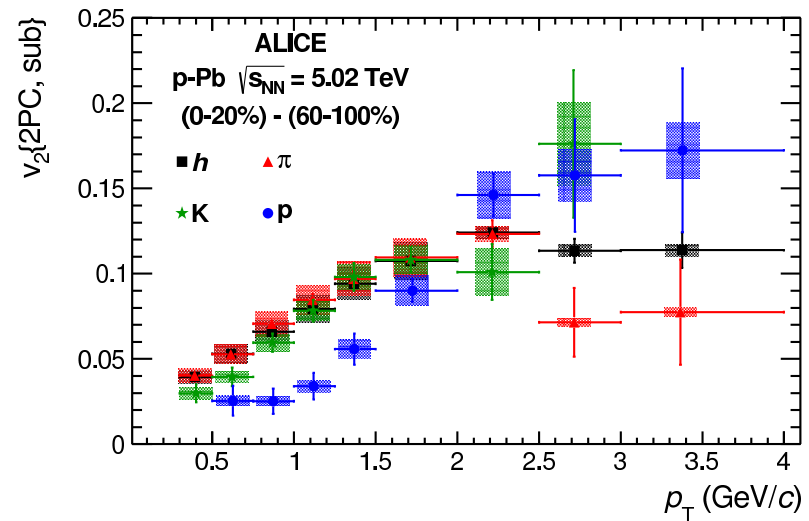
Frontier II: Everything flows (even p+Pb)

Signatures of collective expansion (radial and elliptic flow) in high multiplicity p+Pb collisions.



Mass ordering of mean p_T

CMS (2013)

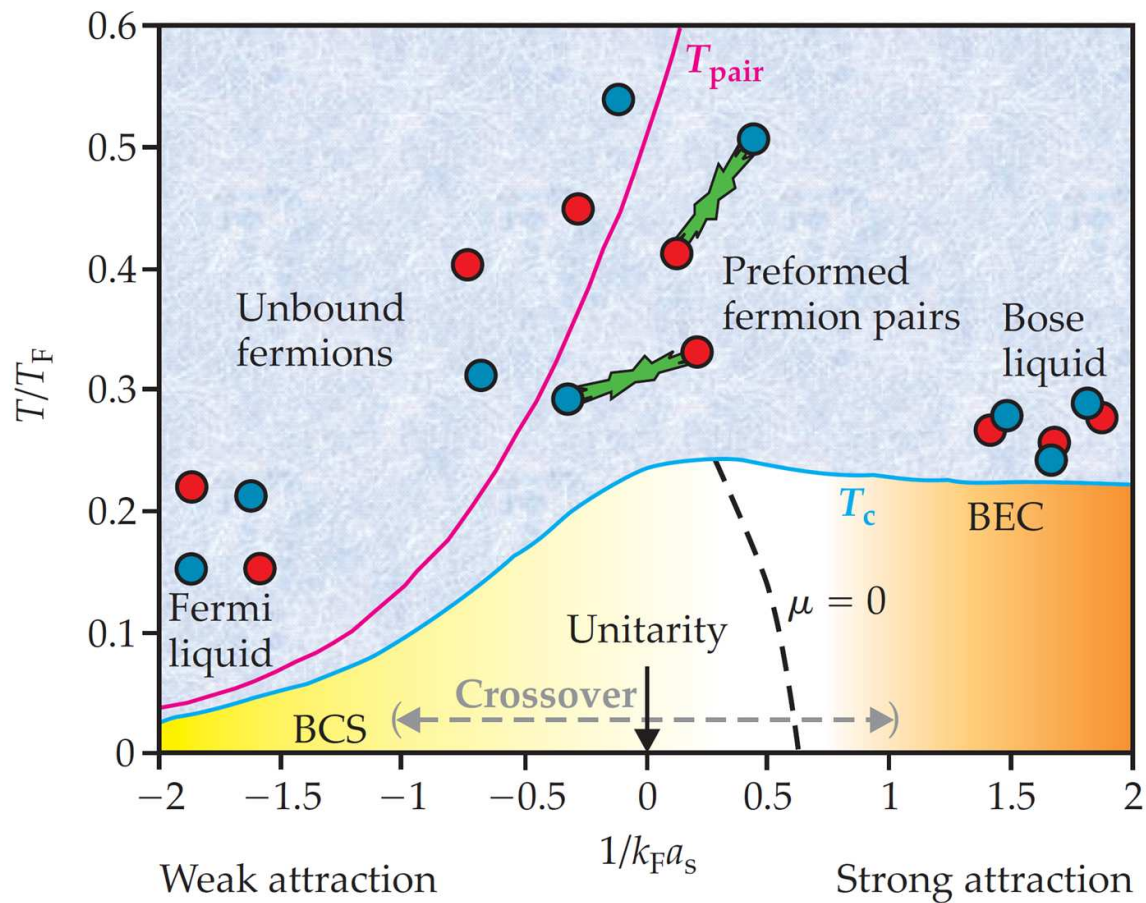


Mass ordering of $v_2(p_T)$

Alice (2013)

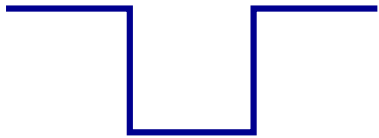
II. Dilute Fermi gas: BCS-BEC crossover

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

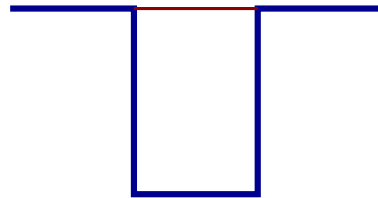


Unitarity limit

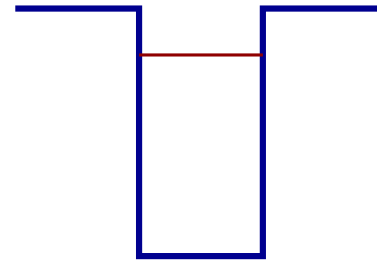
Consider simple square well potential



$$a < 0$$



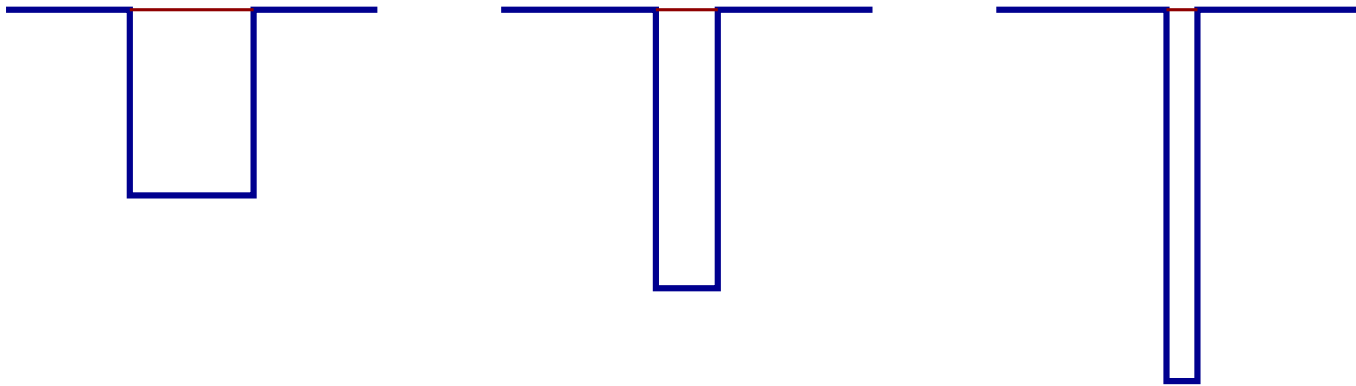
$$a = \infty, \epsilon_B = 0$$



$$a > 0, \epsilon_B > 0$$

Unitarity limit

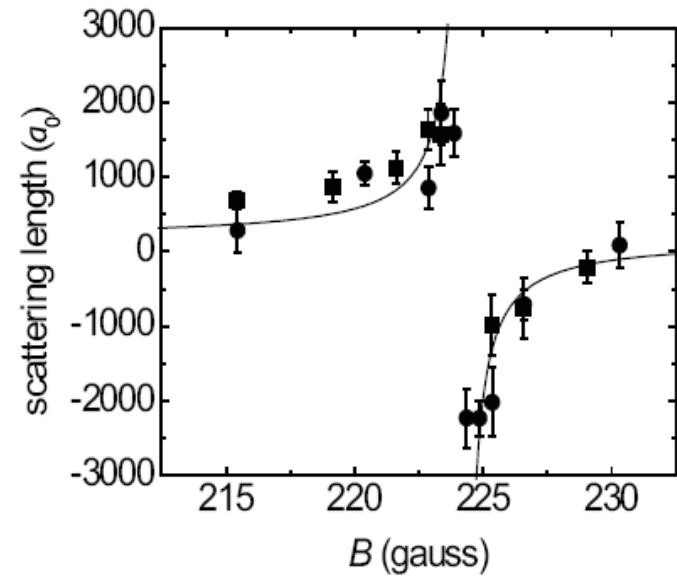
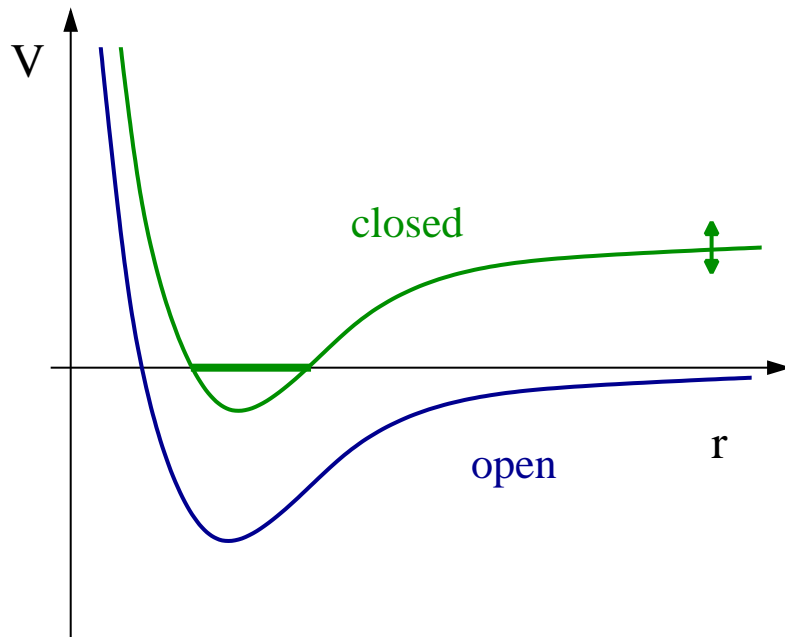
Now take the range to zero, keeping $\epsilon_B \simeq 0$



Universal scattering amplitude $\mathcal{T} = \frac{1}{ik}$

Feshbach resonances

Atomic gas with two spin states: “↑” and “↓”

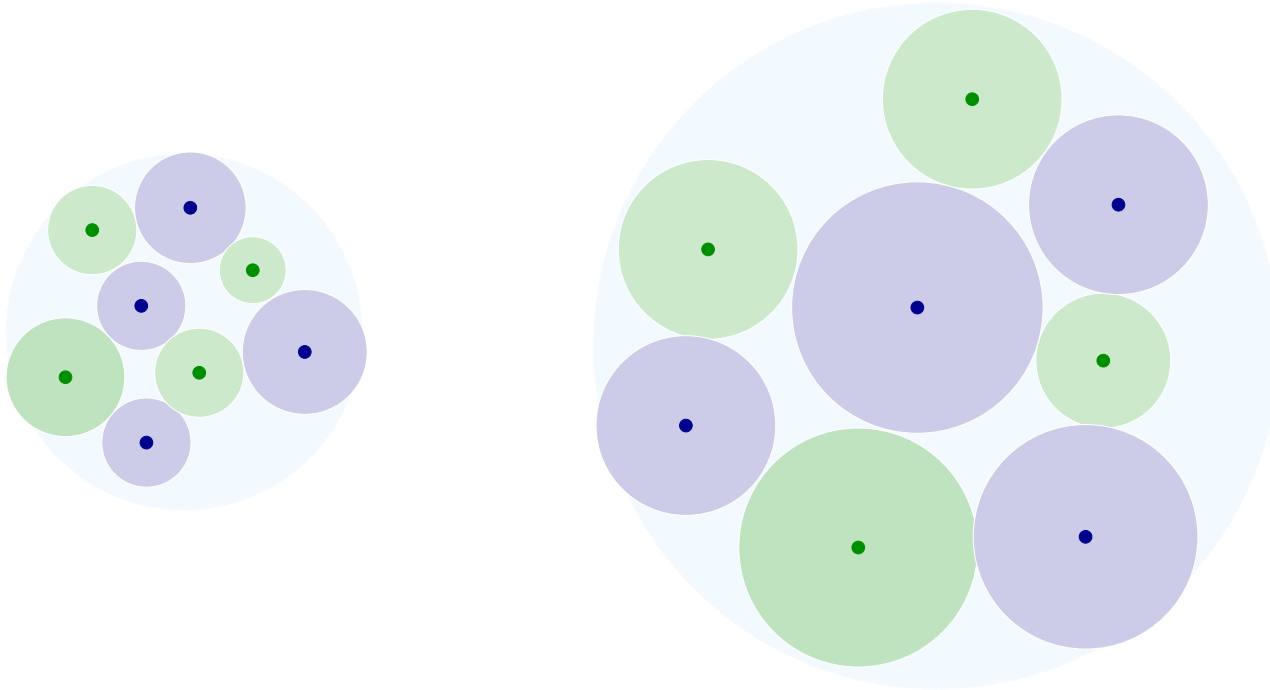


Feshbach resonance

$$a(B) = a_0 \left(1 + \frac{\Delta}{B - B_0} \right)$$

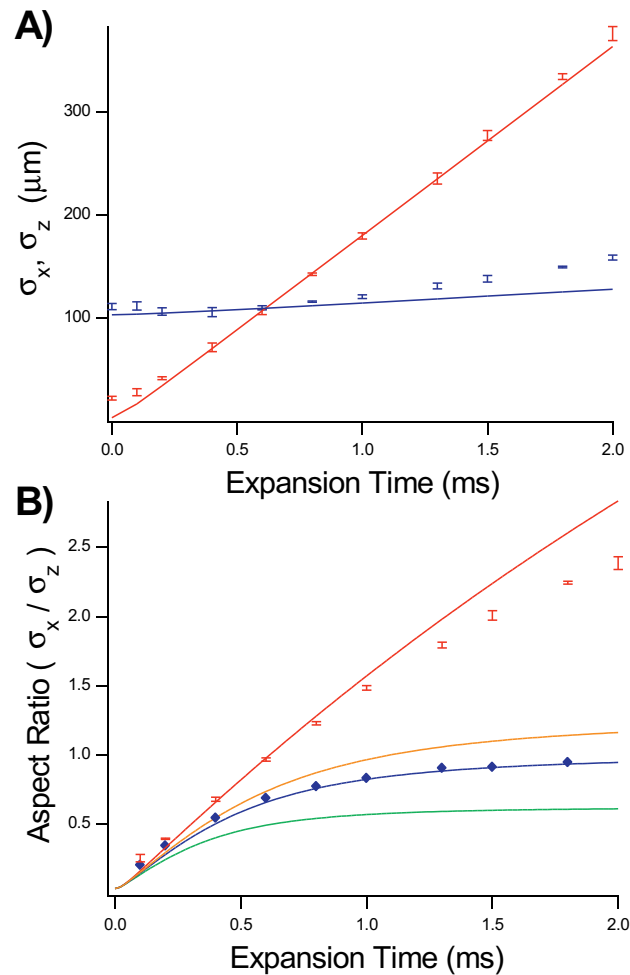
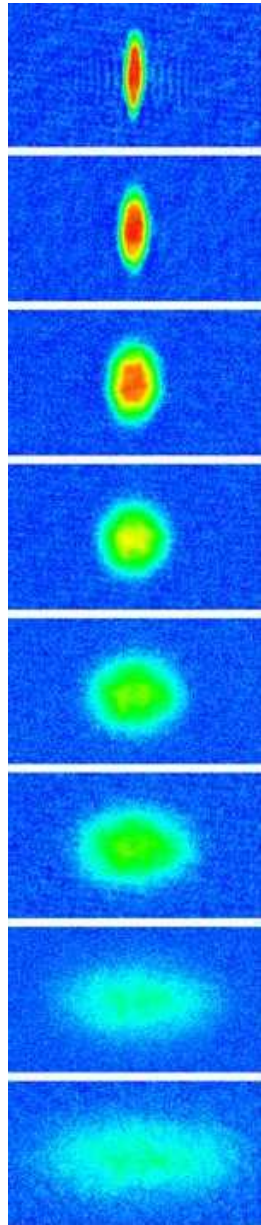
Universal fluid dynamics

Many body system: Effective cross section $\sigma_{tr} \sim n^{-2/3}$ (or $\sigma_{tr} \sim \lambda^2$)

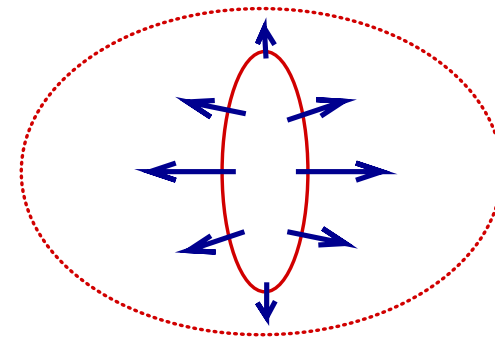


Systems remains hydrodynamic despite expansion

Almost ideal fluid dynamics

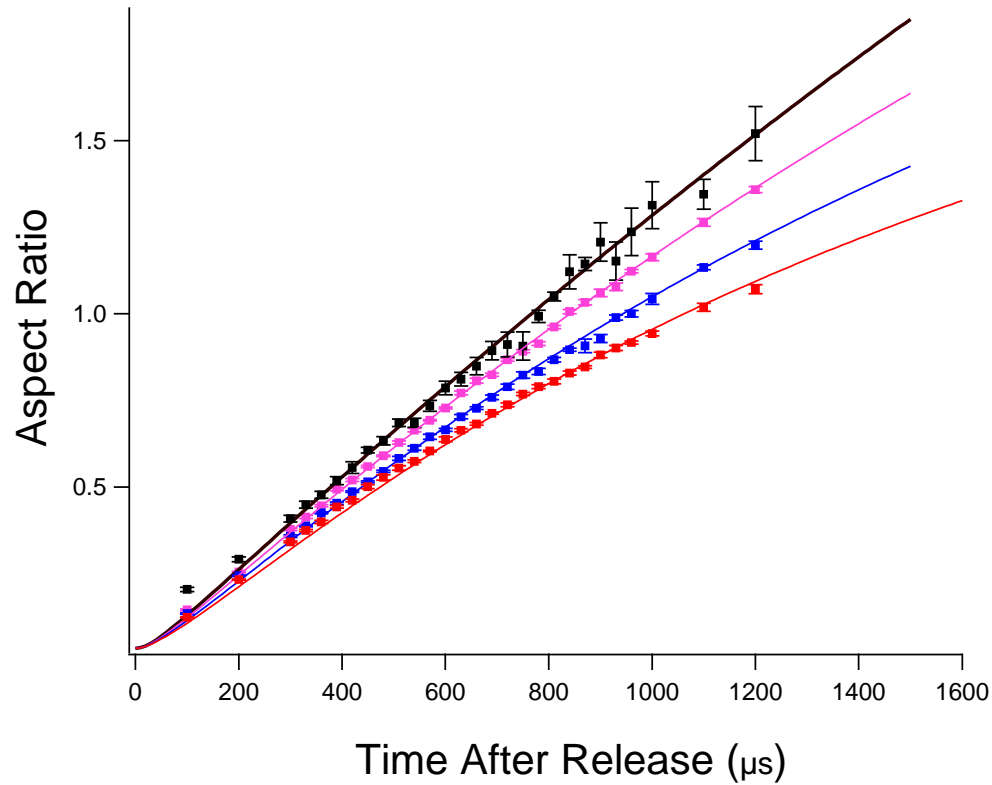
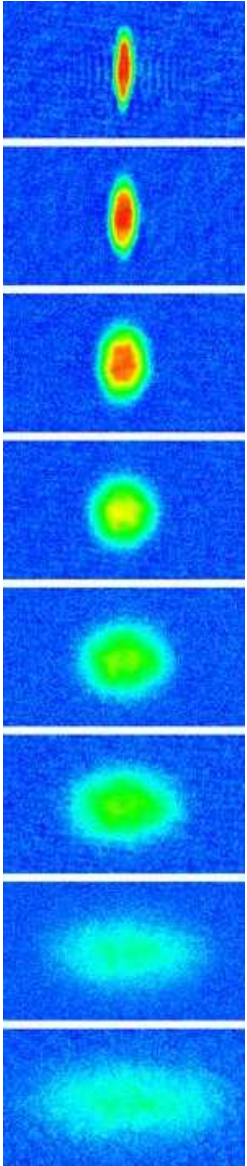


Hydrodynamic
expansion converts
coordinate space
anisotropy
to momentum space
anisotropy



Elliptic flow: High T limit

$$\text{Quantum viscosity } \eta = \eta_0 \frac{(mT)^{3/2}}{\hbar^2}$$



$$\eta = \eta_0 (mT)^{3/2}$$

$$\tau_R = \eta / P$$

Cao, T.S. et al., Science (2010)

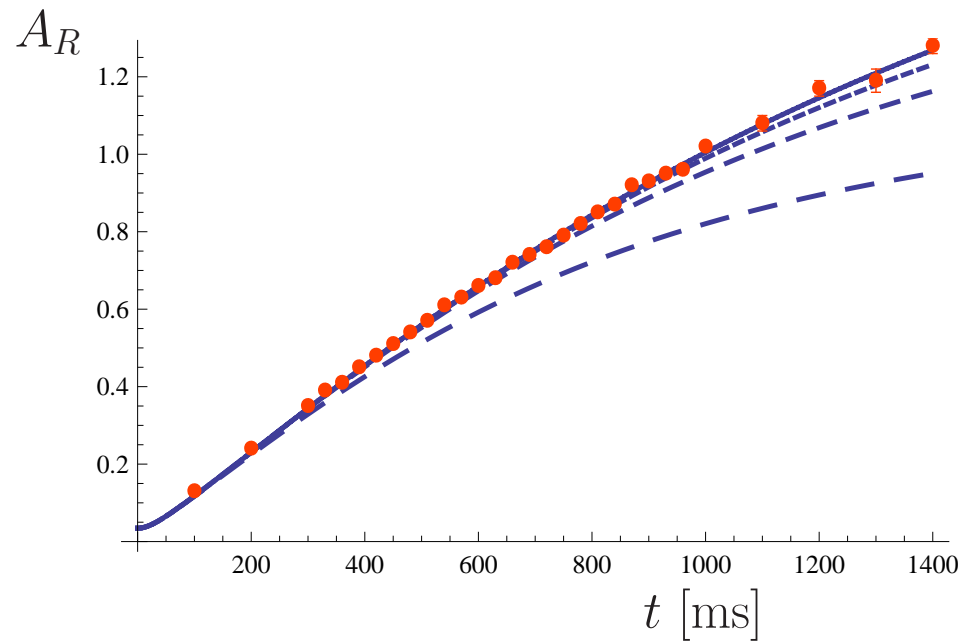
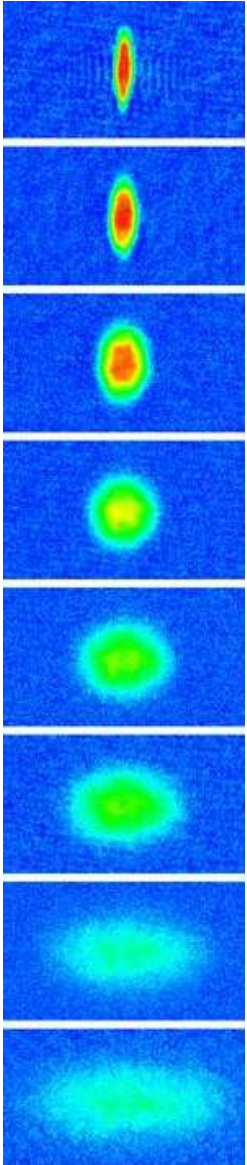
$$\text{fit: } \eta_0 = 0.33 \pm 0.04$$

$$\text{theory: } \eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$$

Elliptic flow: Freezeout?

switch from hydro to (weakly collisional) kinetics

at scale factor $b_{\perp}^{fr} = 1, 5, 10, 20$

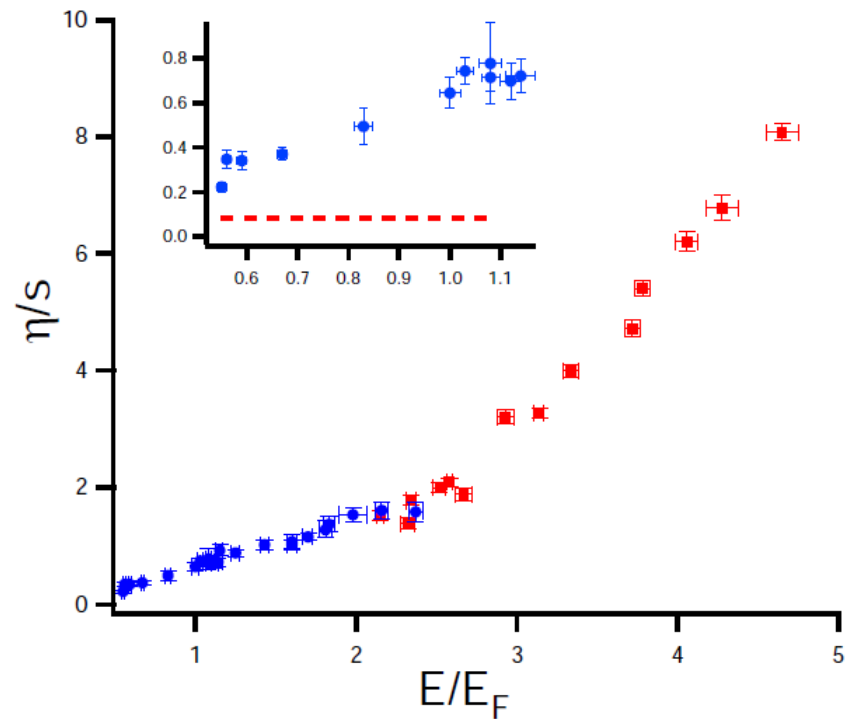
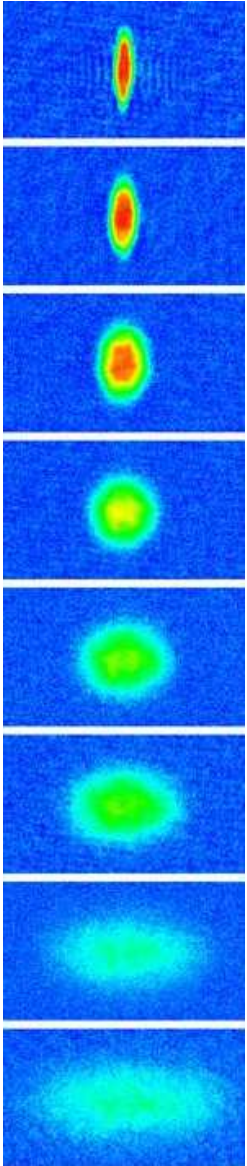


Dusling, Schaefer (2010)

no freezeout seen in the data

Viscosity to entropy density ratio

consider both collective modes (low T)
and elliptic flow (high T)



Cao, T.S. et al., Science (2010)

$$\eta/s \leq 0.4$$

The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of black holes in 5 (and more) dimensions.

We still do not know whether there is a fundamental lower bound on η .

Outlook

Improved determinations of η/s for both the QGP and cold atomic gases. Need to unfold T , ρ dependence.

Work in progress.

Other transport properties: Bulk viscosity, diffusion constants, relaxation times, etc.

ζ (QGP), T.S., K. Dusling (2012), ζ (CAG) in progress.

Transport dominated by quasi-particles? How can we tell?

Possible path: Spectral fcts, see T.S. (2010), Drut et al.