Nearly Perfect Fluidity: From Cold Atoms to Hot Quarks

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RHIC serves the perfect fluid

Experiments at RHIC and the LHC are consistent with the idea that a thermalized plasma is produced, and that the equation of state is that of a weakly coupled gas of quarks and gluons.

But: Transport properties of the system (primarily viscosity and energy loss) are in dramatic disagreement with expectations for a weakly coupled QGP. The plasma must be very strongly coupled.

In this talk I will try to explain this statement, review the current evidence, and put the results in a broader perspective (by comparing with another strongly coupled fluid, the dilute atomic Fermi gas at "unitarity").

Hydroynamics

Hydrodynamics (undergraduate version): Newton's law for continuous, deformable media.

Fluids: Gases, liquids, plasmas, . . .

Hydrodynamics (postmodern): Effective theory of non-equilibrium long-wavelength, low-frequency dynamics of any many-body system.

$\tau \sim \tau_{micro}$ $\tau \sim \lambda$

$\tau \gg \tau_{micro}$: Dynamics of conserved charges.

Water: $(\rho, \epsilon, \vec{\pi})$

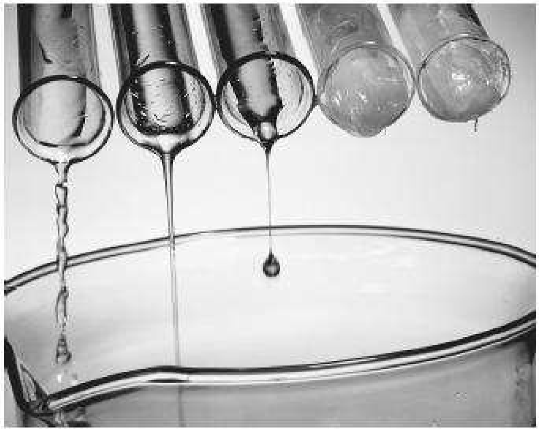
Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \nabla_j \Pi_{ij} = 0$$



Constitutive relations: Stress tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3}\delta_{ij} \nabla_k v_k \right) + O(\nabla^2)$$

reactive　　dissipative　　2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$\frac{1}{Re} = \frac{\eta}{\hbar n} \times \frac{\hbar}{mvL}$$

	$\frac{\eta}{\hbar n}$		$\frac{\hbar}{mvL}$
	fluid property		flow property

Bath tub :	$mvL \gg \hbar$	hydro reliable
Heavy ions :	$mvL \sim \hbar$	need $\eta < \hbar n$

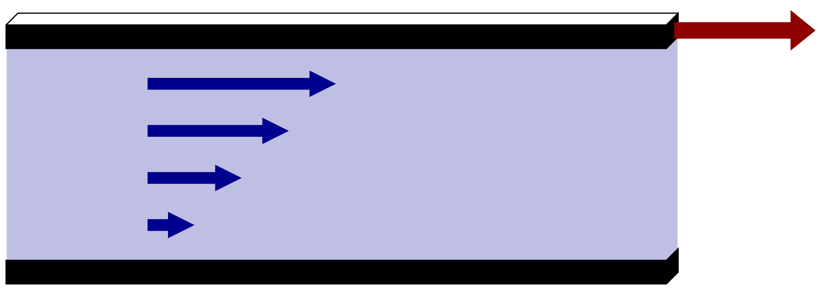
Shear viscosity and friction

Momentum conservation at $O(\nabla v)$

$$\rho \left(\frac{\partial}{\partial t}\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} \right) = -\vec{\nabla} P + \eta \nabla^2 \vec{v}$$

Navier-Stokes equation

Viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

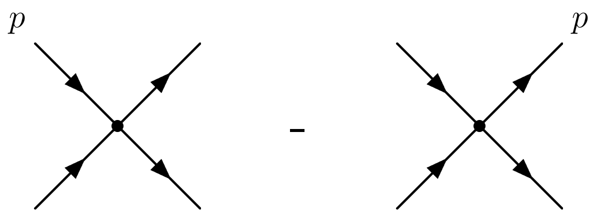
Kinetic theory

Kinetic theory: conserved quantities carried by quasi-particles.

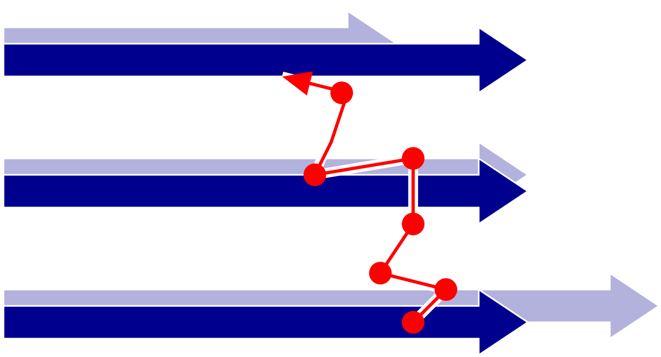
Quasi-particles described by distribution functions $f(x,p,t)$.

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = -C[f_p]$$

$$C[f_p] \;=$$



Shear viscosity corresponds to momentum diffusion



$$\eta \sim \frac{1}{3} n \, \bar{p} \, l_{mfp}$$

Shear viscosity: Additional properties

Weakly interacting gas, $l_{mfp} \sim \dfrac{1}{n\sigma}$:

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

shear viscosity independent of density

Non-interacting gas ($\sigma \to 0$):

$$\eta \to \infty$$

non-interacting and hydro limit ($T \to \infty$) limit do not commute

strongly interacting gas:

$$\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$$

but: kinetic theory not reliable!

Historical digression: Mott's minimal conductivity

(Sir) Nevill Mott predicted that the metal-insulator transition cannot be continuous; there is a minimal conductivity.

Conduction in Non-crystalline Systems

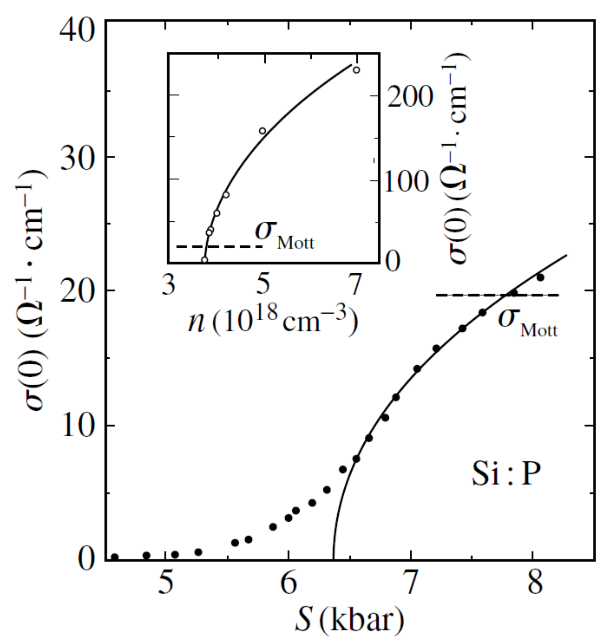
IX. The Minimum Metallic Conductivity

By N. F. Mott
Cavendish Laboratory, Cambridge

[Received 27 July 1972]

$$\frac{\sigma}{n^{1/3}} \geq \frac{1}{(3\pi^2)^{2/3}} \frac{e^2}{\hbar}$$

This idea is not correct,
the metal-insulator transition can
be continuous.



Historical digression: Minimal shear viscosity

Gyulassy & Danielewicz argue that the shear viscosity cannot be zero.

PHYSICAL REVIEW D VOLUME 31, NUMBER 1 1 JANUARY 1985

Dissipative phenomena in quark-gluon plasmas

P. Danielewicz* and M. Gyulassy
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
(Received 12 April 1984; revised manuscript received 24 September 1984)

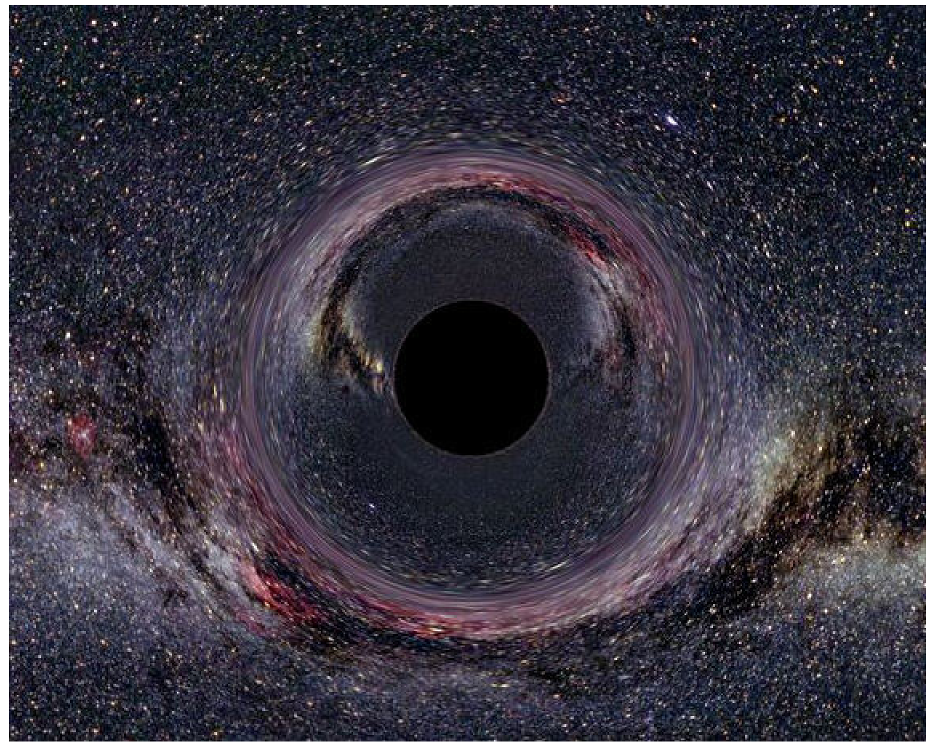
than $\langle p \rangle^{-1}$. Requiring $\lambda_i \gtrsim \langle p \rangle_i^{-1}$ leads to the lower bound

$$\eta \gtrsim \tfrac{1}{3} n \ , \tag{3.3}$$

where $n = \sum n_i$ is the total density of quanta. What seems amazing about (3.3) is that it is independent of dynamical details. There is a finite viscosity regardless of

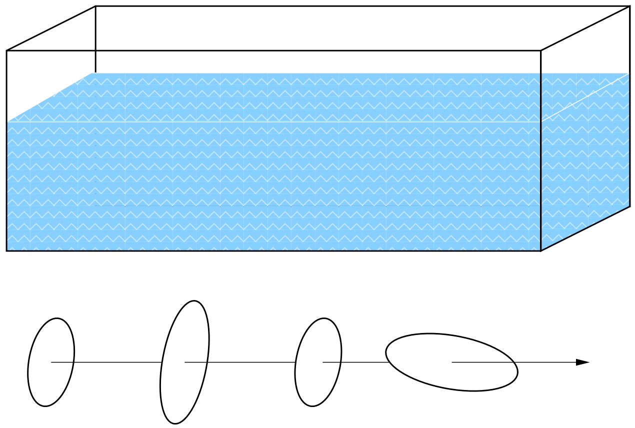
Is this idea correct?

And now for something completely different . . .

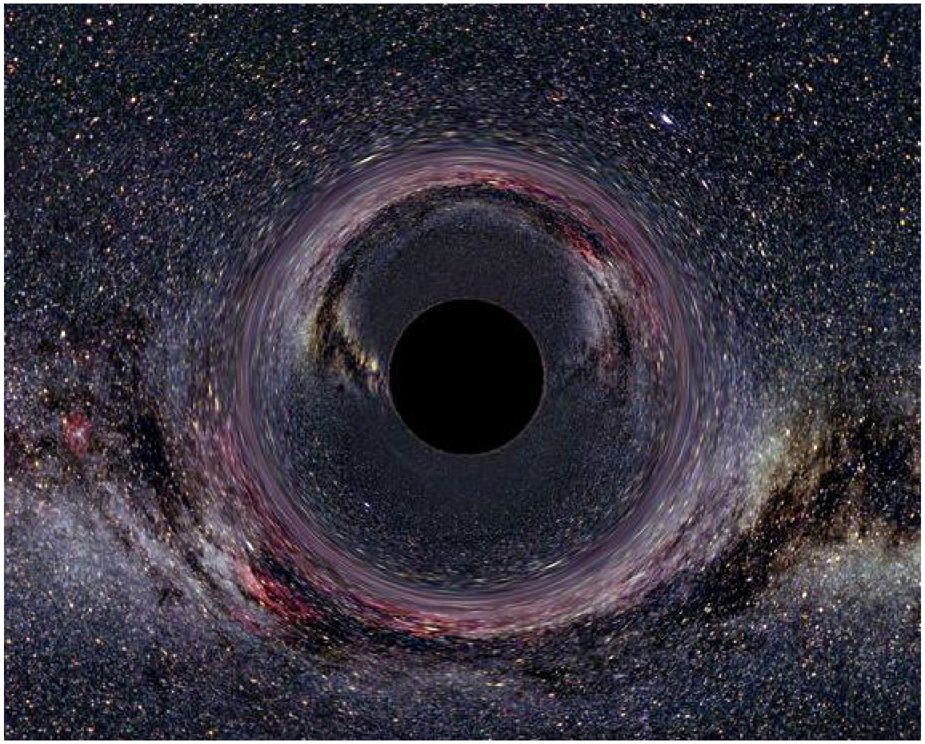


This is an irreversible process, $\Delta S > 0$.

And now for something completely different . . .

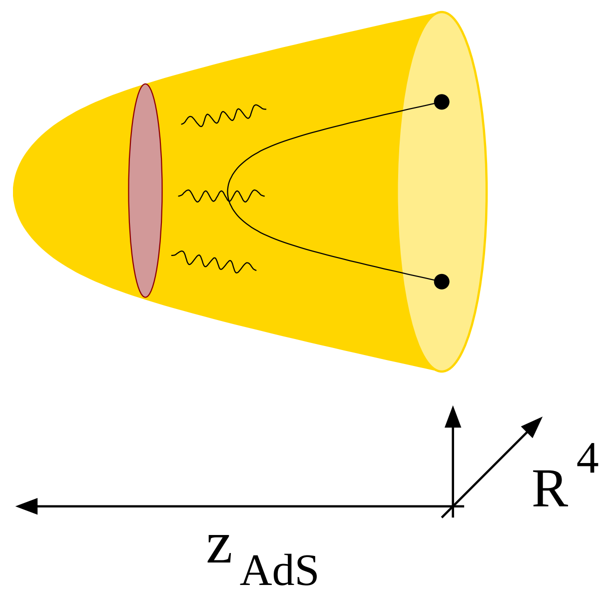


gravitational wave shears fluid



Idea can be made precise using the "AdS/CFT correspondence"

Strongly coupled thermal field theory on R^4	$\Leftrightarrow$	Weakly coupled string theory on AdS_5 black hole
CFT temperature	$\Leftrightarrow$	Hawking temperature of black hole
CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon



Holographic duals: Transport properties

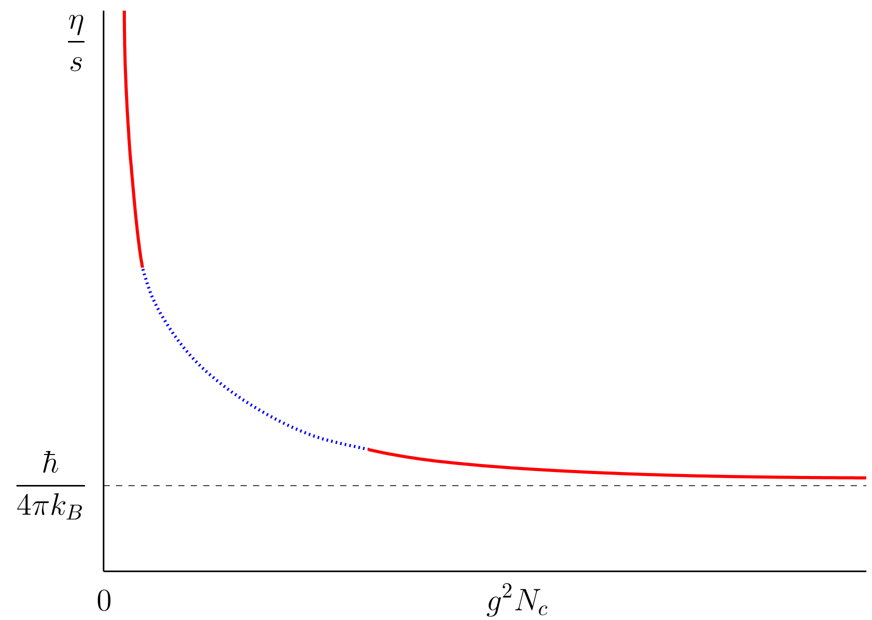
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

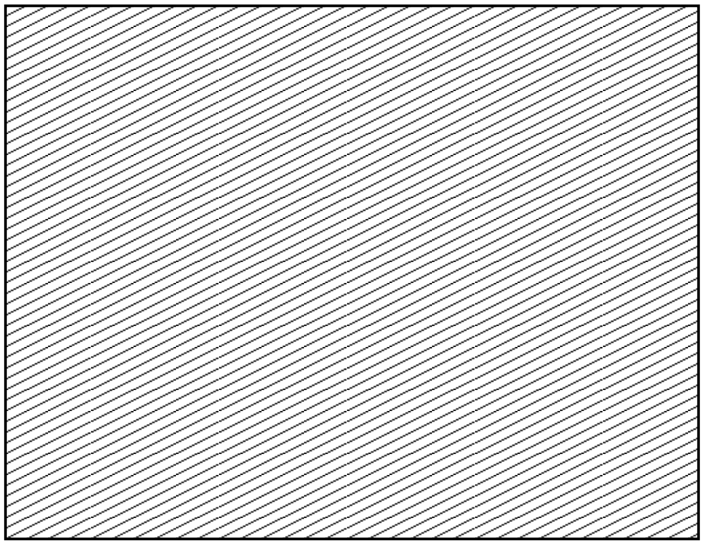
Son and Starinets (2001)



Strong coupling limit universal? Provides lower bound for all theories?

Answer appears to be no; e.g. theories with higher derivative gravity duals.

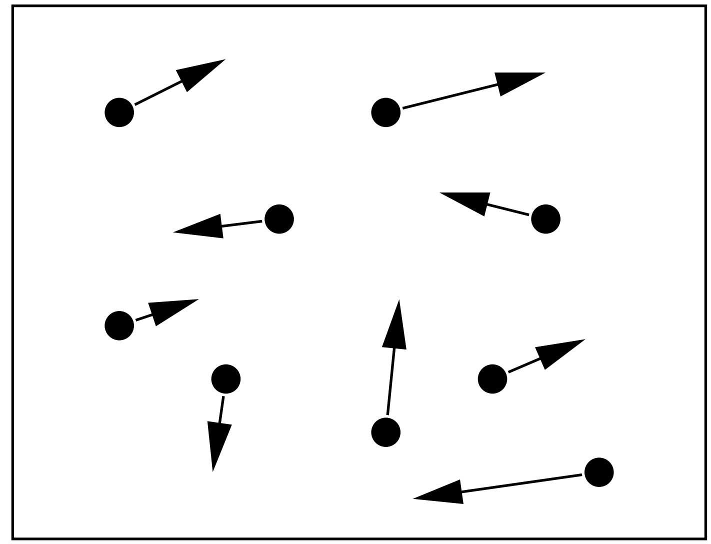
Kinetics vs no-kinetics



low viscosity goo

gravity dual

$\eta/s \simeq 1/(4\pi)$

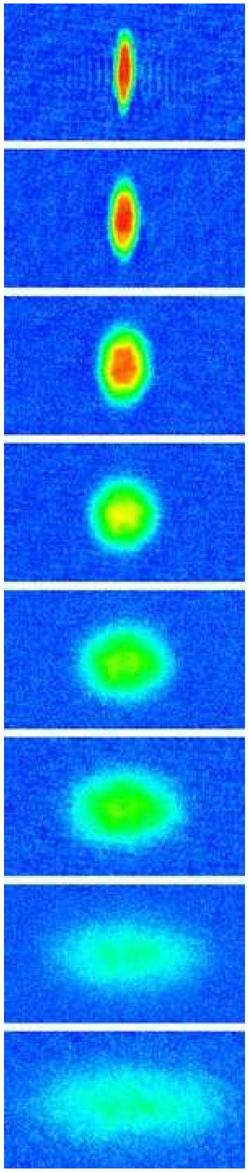


pQCD plasma

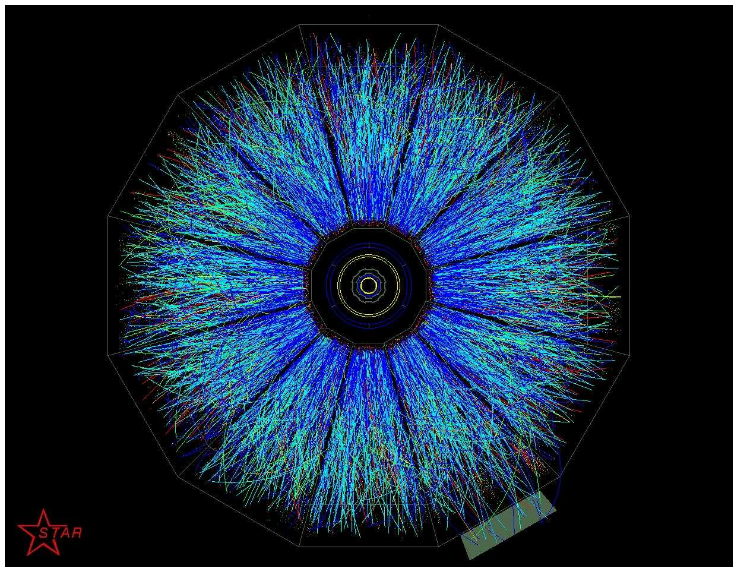
quasi-particles

$\eta/s \sim 1/\alpha_s^2 \gg 1$

Perfect Fluids: The contenders



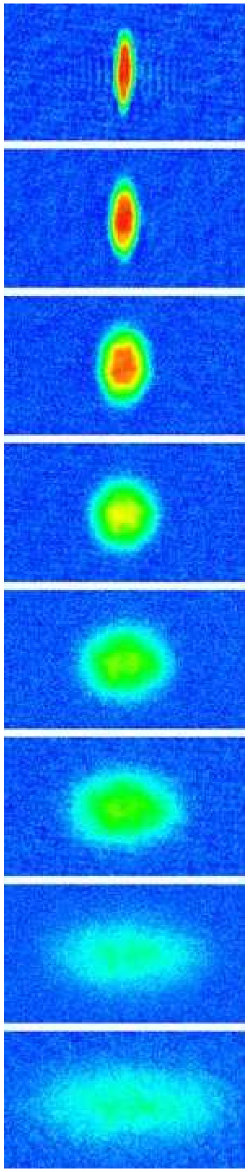
Trapped Atoms
(T=0.1 neV)

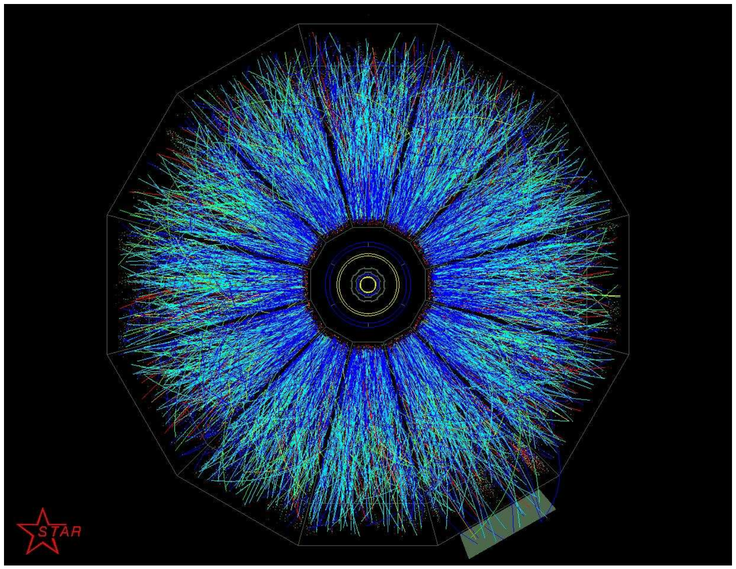


QGP (T=180 MeV)

Liquid Helium
(T=0.1 meV)

Perfect Fluids: The contenders

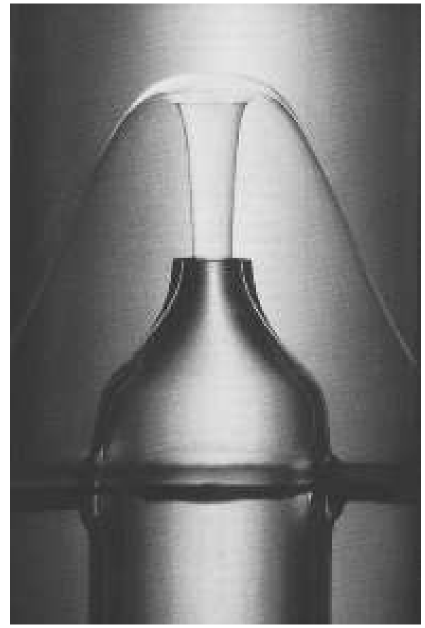




QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η / s

Perfect Fluids: Not a contender

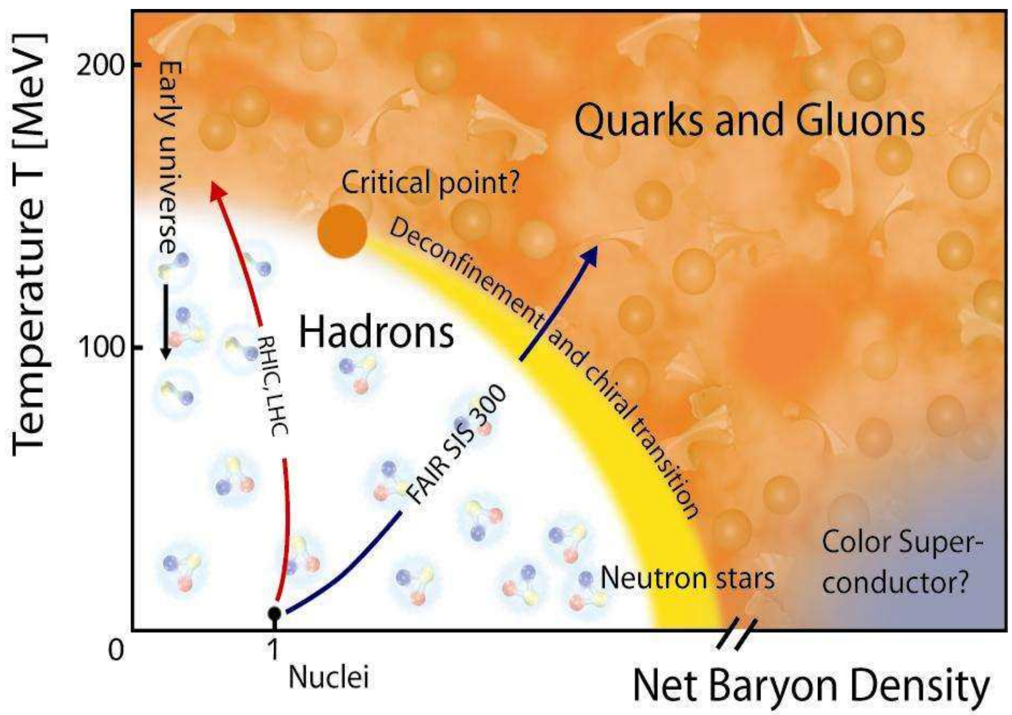
Queensland pitch-drop experiment

1927-2011 (8 drops)

$$\eta = (2.3 \pm 0.5) \cdot 10^8 \, Pa \, s$$

I. QCD and the Quark Gluon Plasma

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4g^2}G^a_{\mu\nu}G^a_{\mu\nu}$$



Quantumchromodynamics (QCD)

Elementary fields: Quarks Gluons

$$(q_\alpha)^a_f \begin{cases} \text{color} & a = 1, \dots, 3 \\ \text{spin} & \alpha = 1, 2 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$$

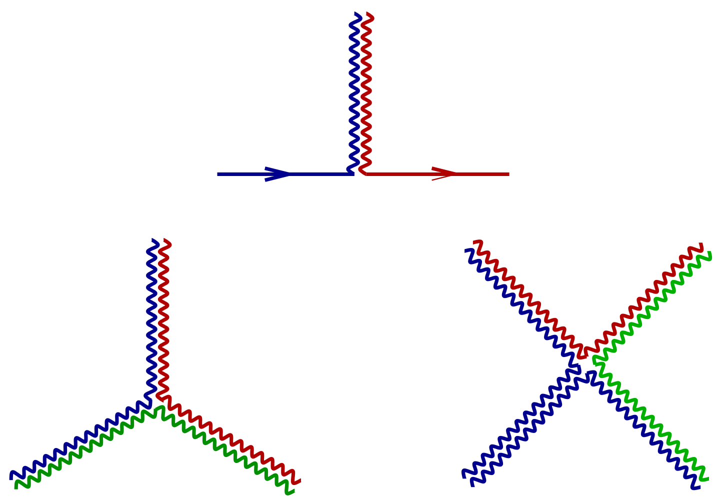
$$A^a_\mu \begin{cases} \text{color} & a = 1, \dots, 8 \\ \text{spin} & \epsilon^\pm_\mu \end{cases}$$

Dynamics: Generalized Maxwell (Yang-Mills) + Dirac theory

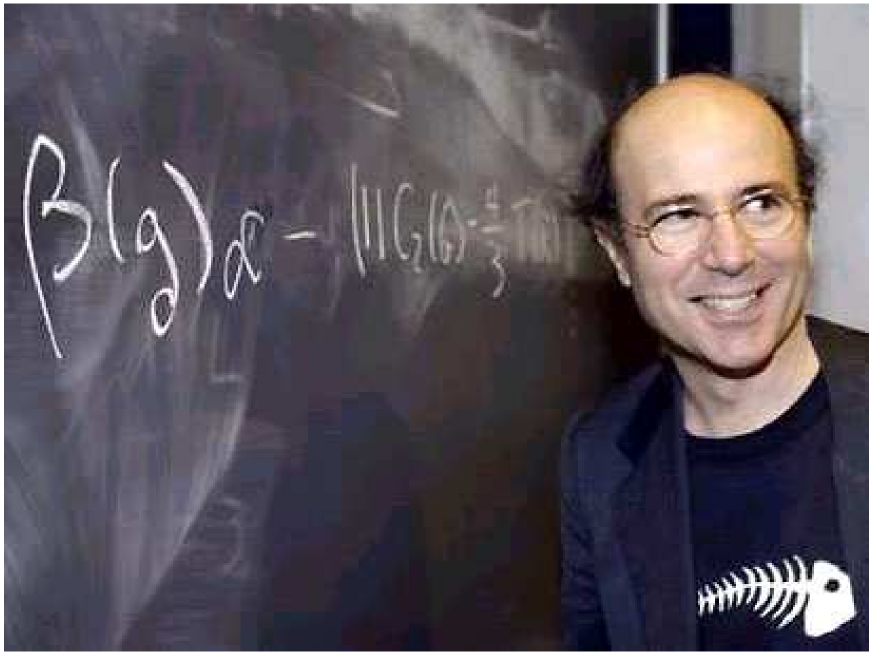
$$\boxed{\mathcal{L} = \bar{q}_f (i \not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}}$$

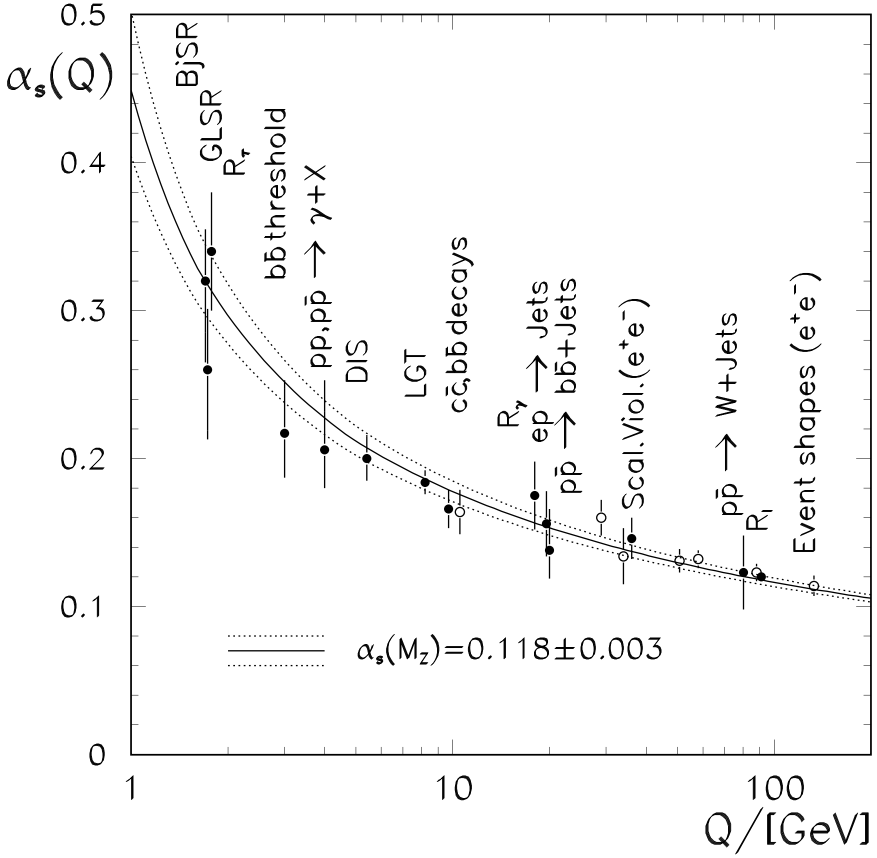
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$i \not{D} q = \gamma^\mu \left(i \partial_\mu + g A^a_\mu t^a \right) q$$

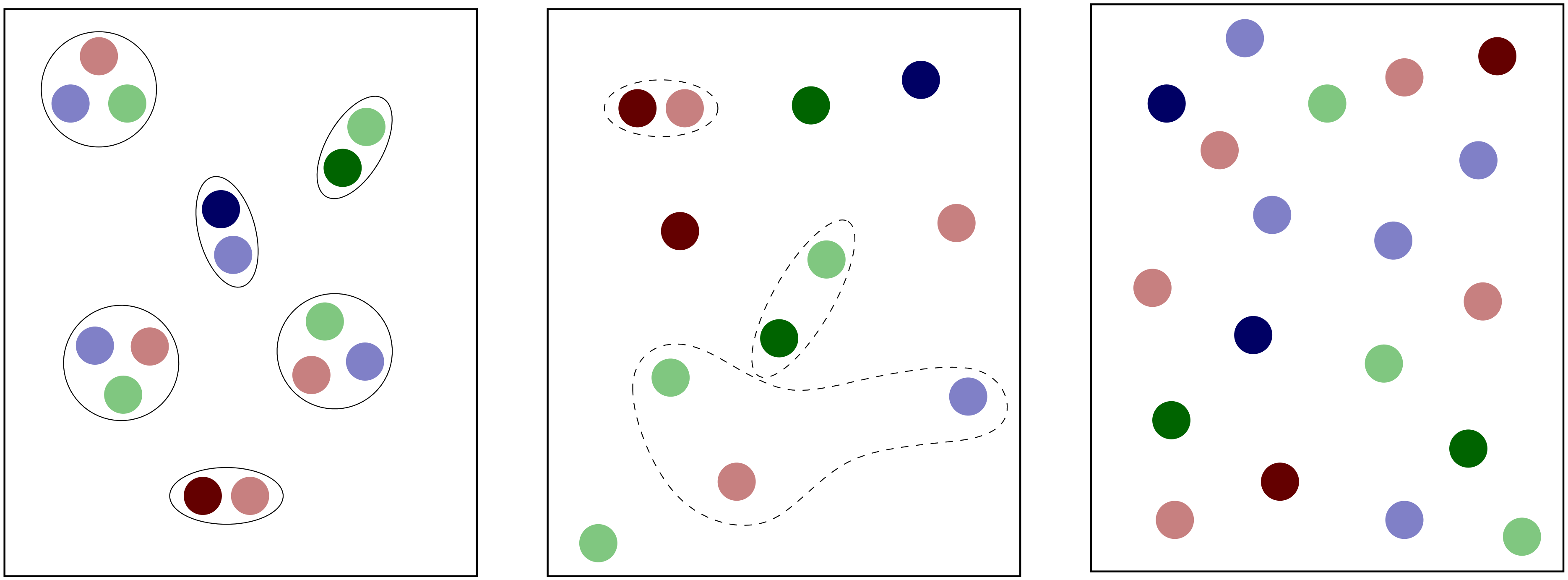


Running coupling constant





From hadrons to quarks



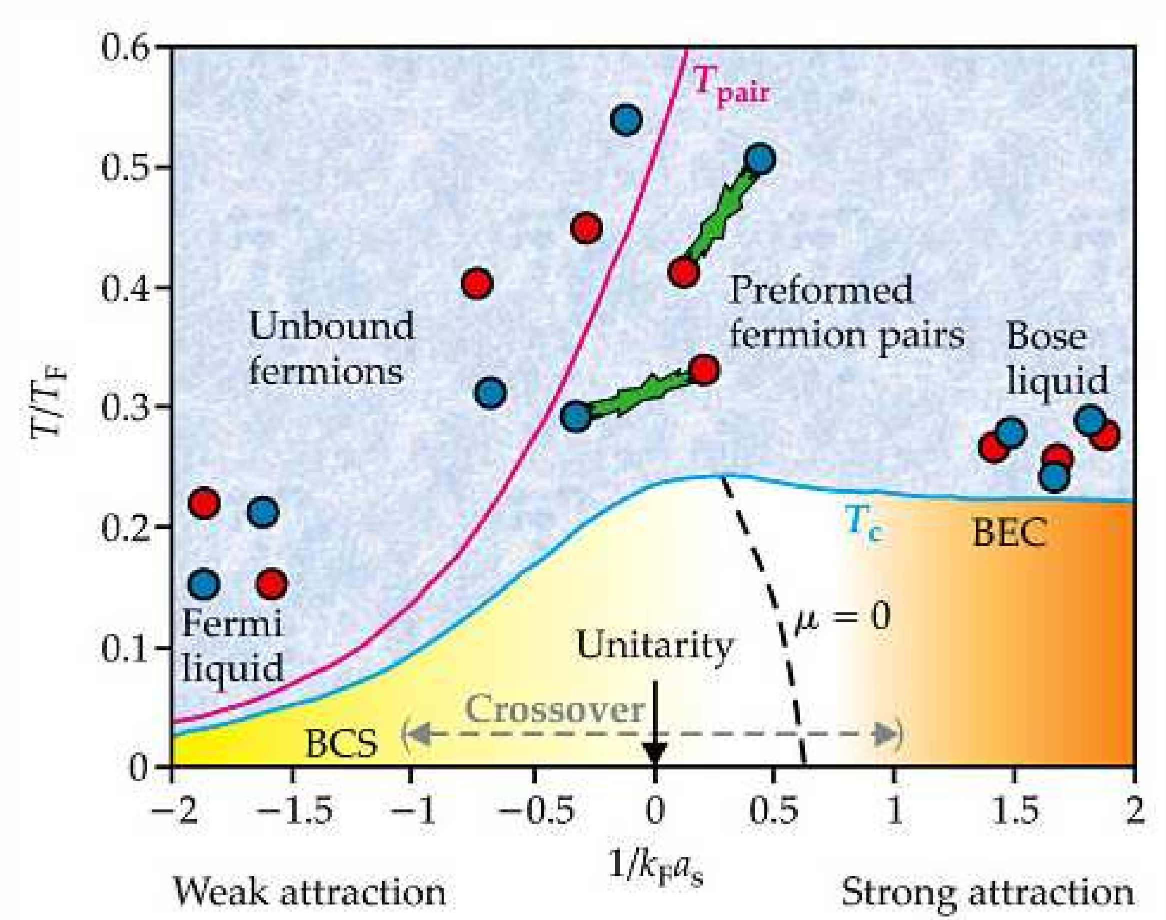
weakly coupled
hadron gas

strongly correlated
fluid

weakly coupled
quark gluon plasma

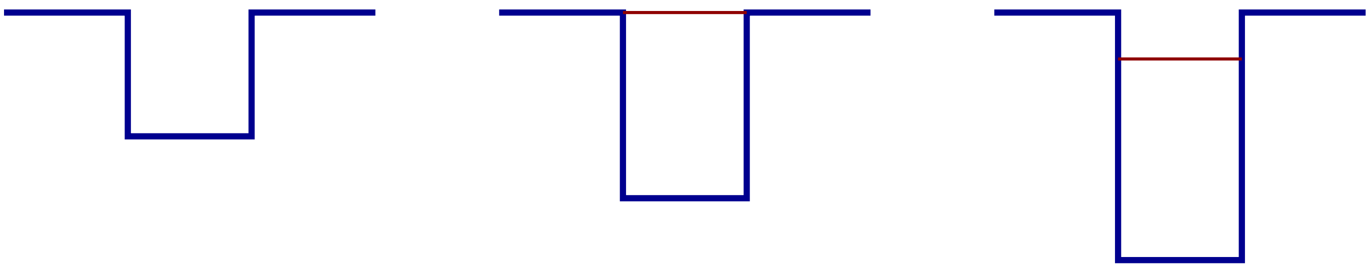
II. Dilute Fermi gas: BCS-BEC crossover

$$\mathcal{L}_{\rm eff} = \psi^\dagger\left(i\partial_0 + \frac{\nabla^2}{2M}\right)\psi - \frac{C_0}{2}(\psi^\dagger\psi)^2$$



Unitarity limit

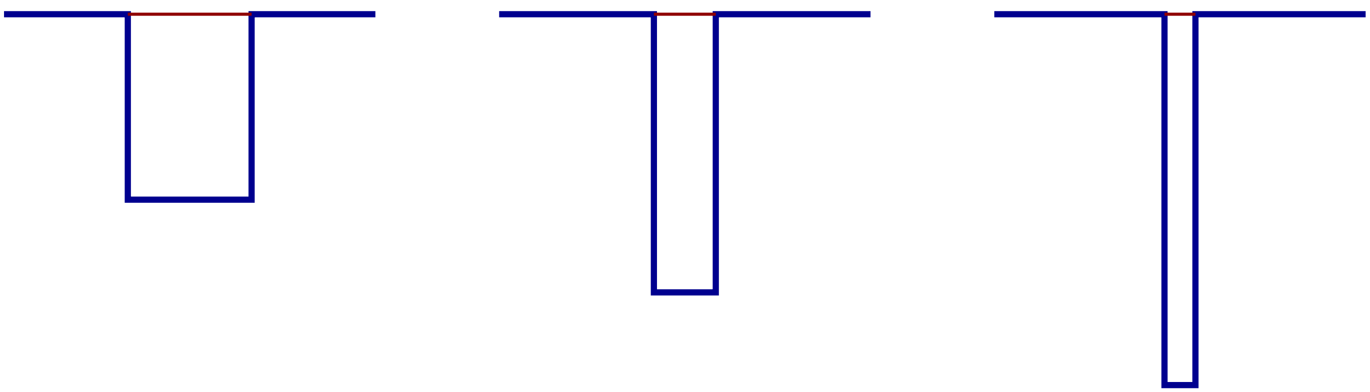
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Unitarity limit

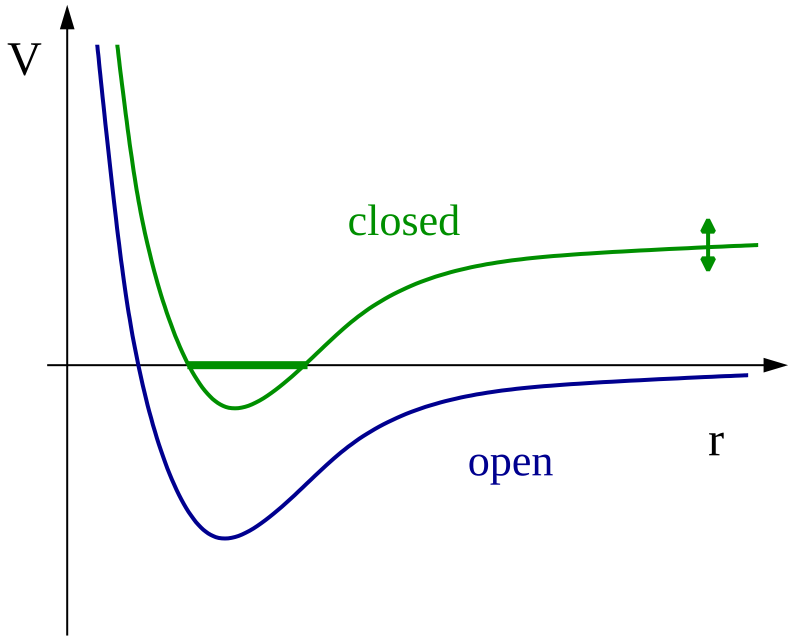
Now take the range to zero, keeping $\epsilon_B \simeq 0$

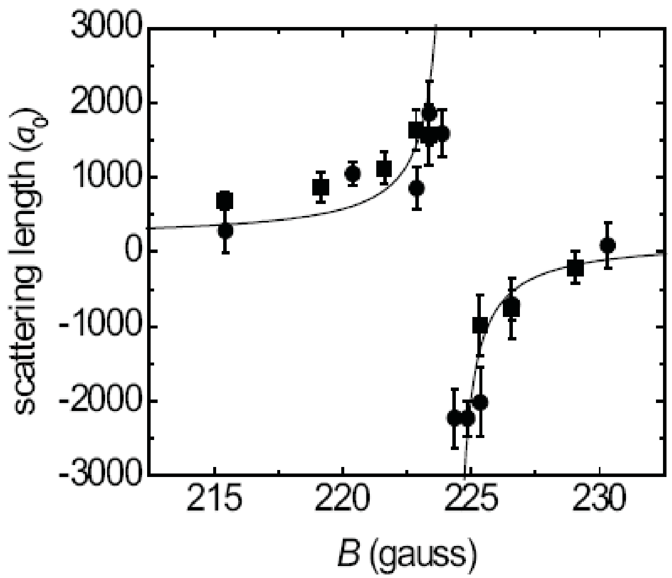


Universal scattering amplitude $\mathcal{T} = \dfrac{1}{ik}$

Feshbach resonances

Atomic gas with two spin states: "↑" and "↓"

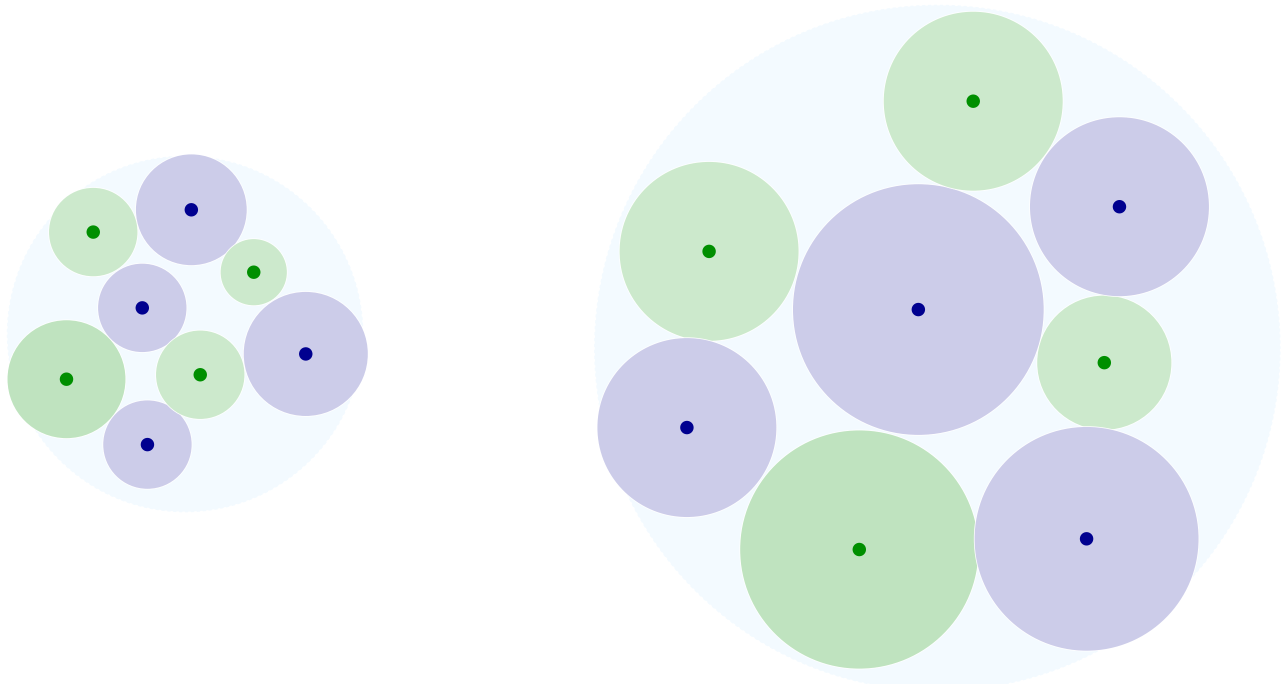




Feshbach resonance $$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

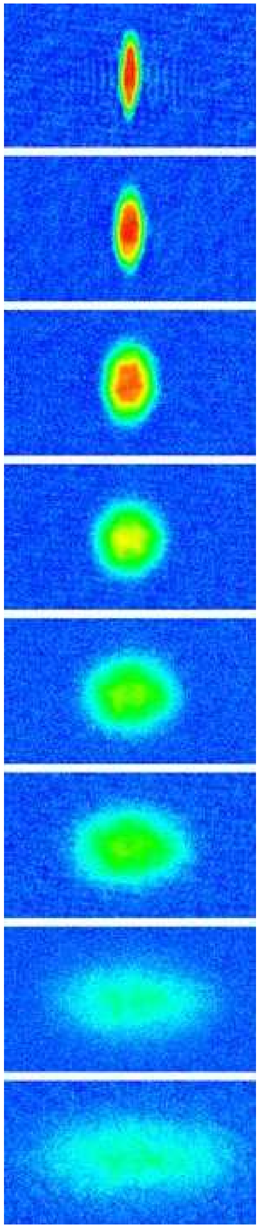
Universal fluid dynamics

Many body system: Effective cross section $\sigma_{tr} \sim n^{-2/3}$ (or $\sigma_{tr} \sim \lambda^2$)

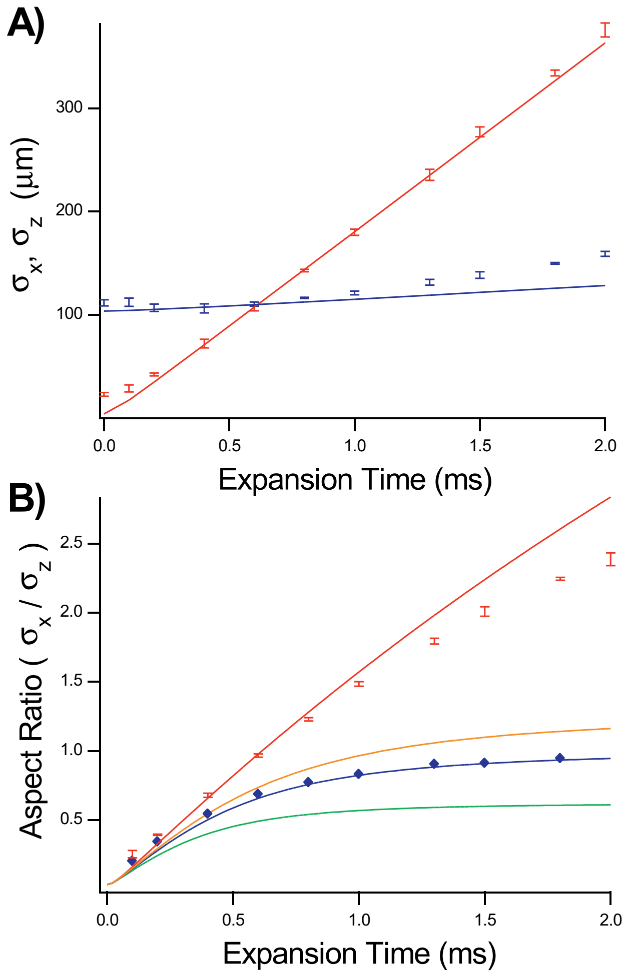


Systems remains hydrodynamic despite expansion

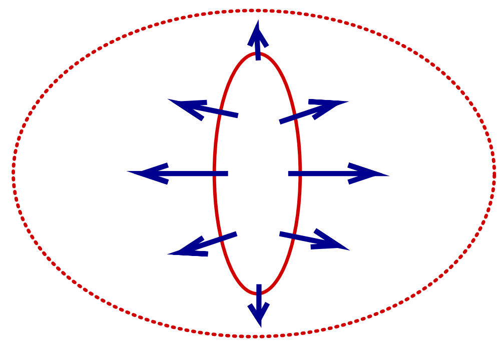
III. Almost ideal fluid dynamics (cold Fermi gas)



O'Hara et al. (2002)

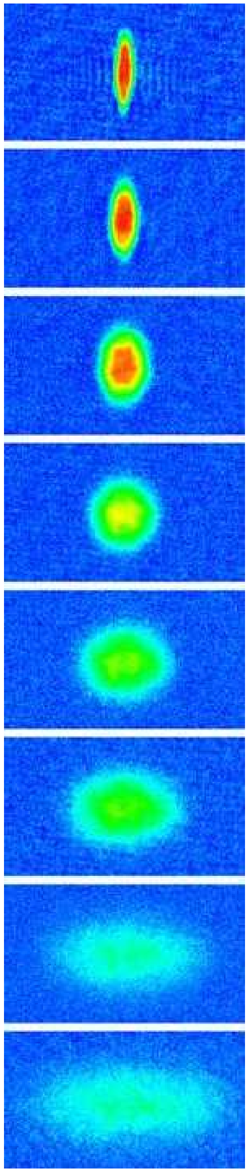


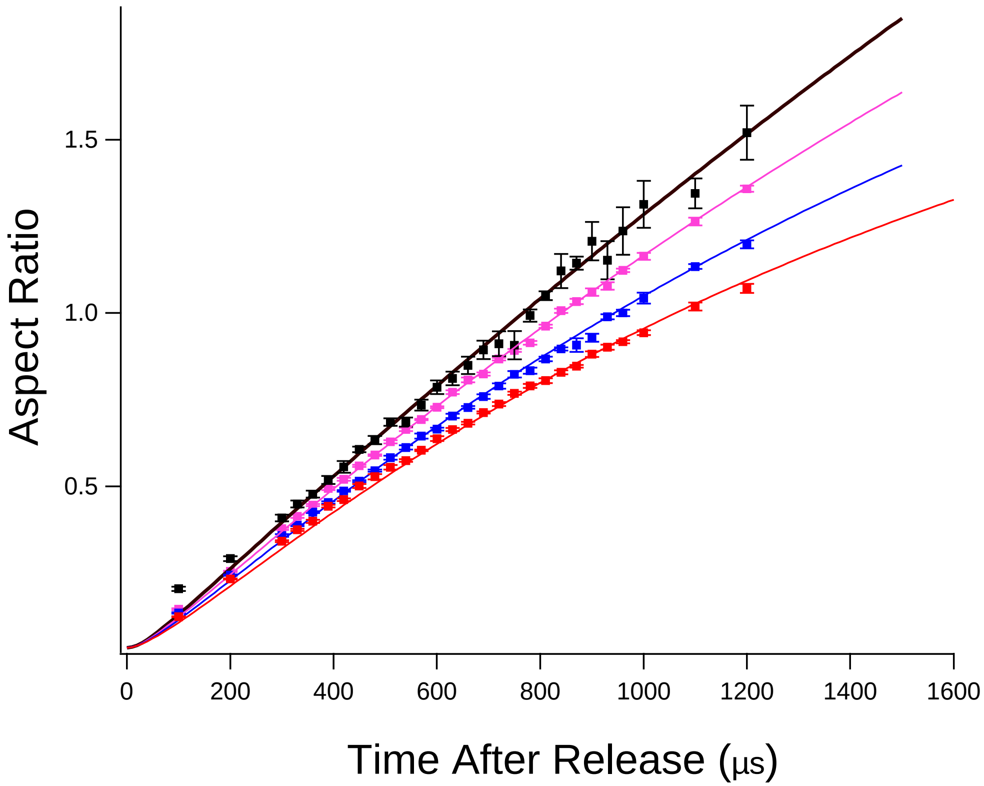
Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy



Elliptic flow: High T limit

Quantum viscosity $\eta = \eta_0 \frac{(mT)^{3/2}}{\hbar^2}$





$\eta = \eta_0 (mT)^{3/2}$

$\tau_R = \eta / P$

Cao et al., Science (2010)

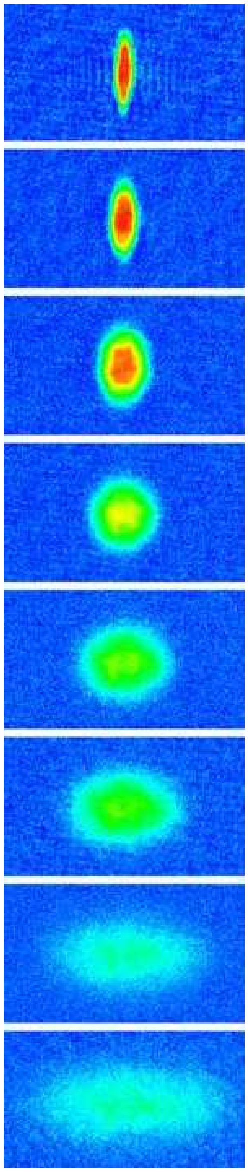
fit: $\eta_0 = 0.33 \pm 0.04$

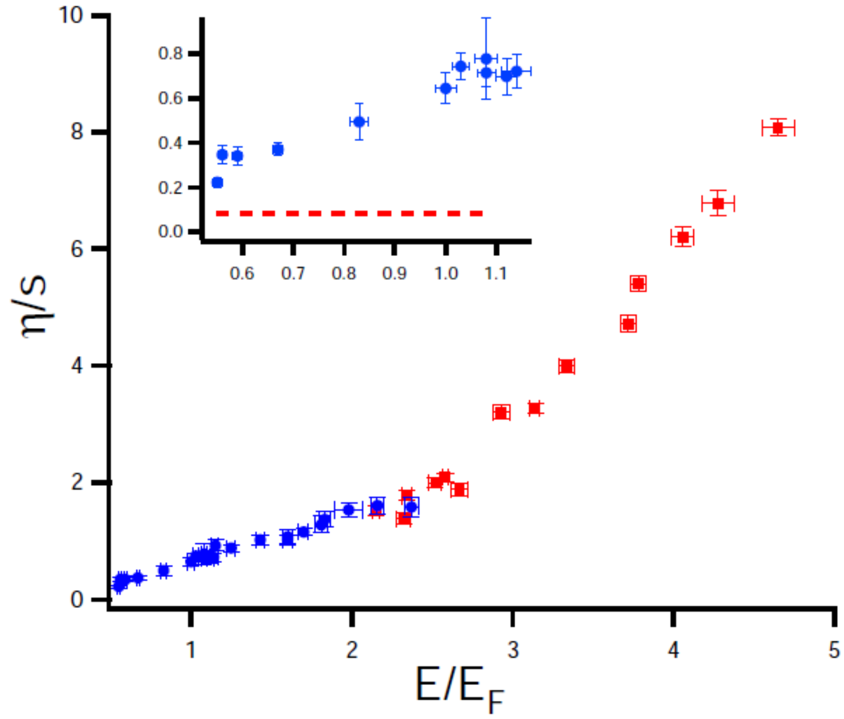
theory: $\eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$

Viscosity to entropy density ratio

consider both collective modes (low T)

and elliptic flow (high T)



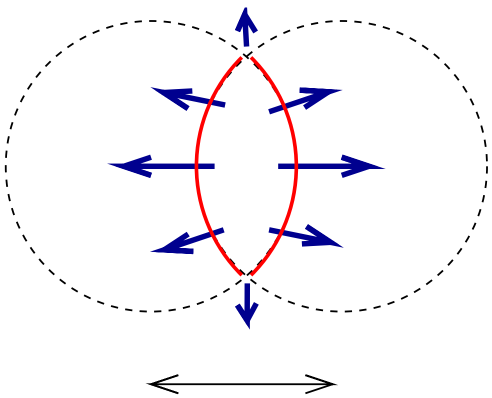


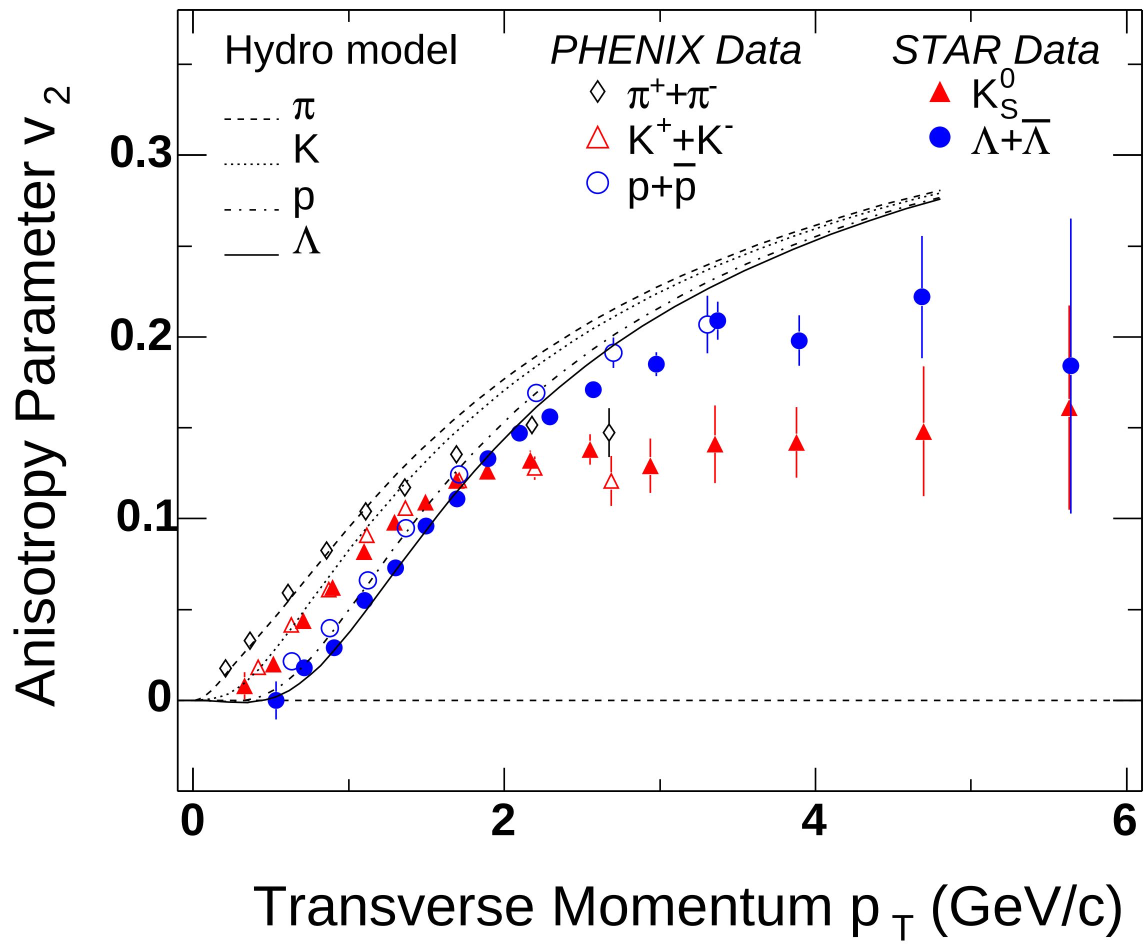
Cao et al., Science (2010)

$$\eta/s \leq 0.4$$

IV. Elliptic Flow (QGP)

Hydrodynamic expansion converts coordinate space anisotropy to momentum space anisotropy

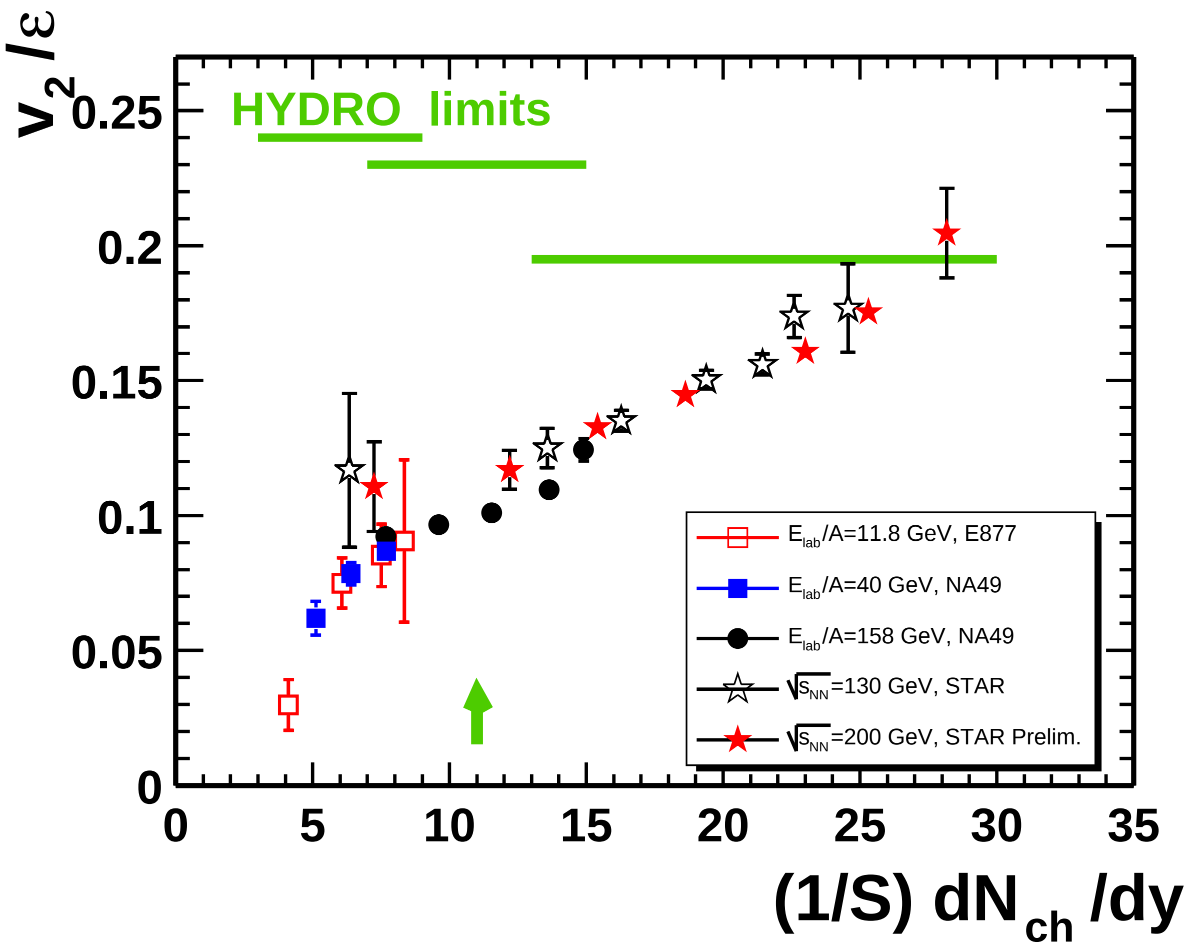




source: U. Heinz (2005)

$$p_0 \frac{dN}{d^3p}\bigg|_{p_z=0} = v_0(p_\perp)\,(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots)$$

Elliptic flow: initial entropy scaling



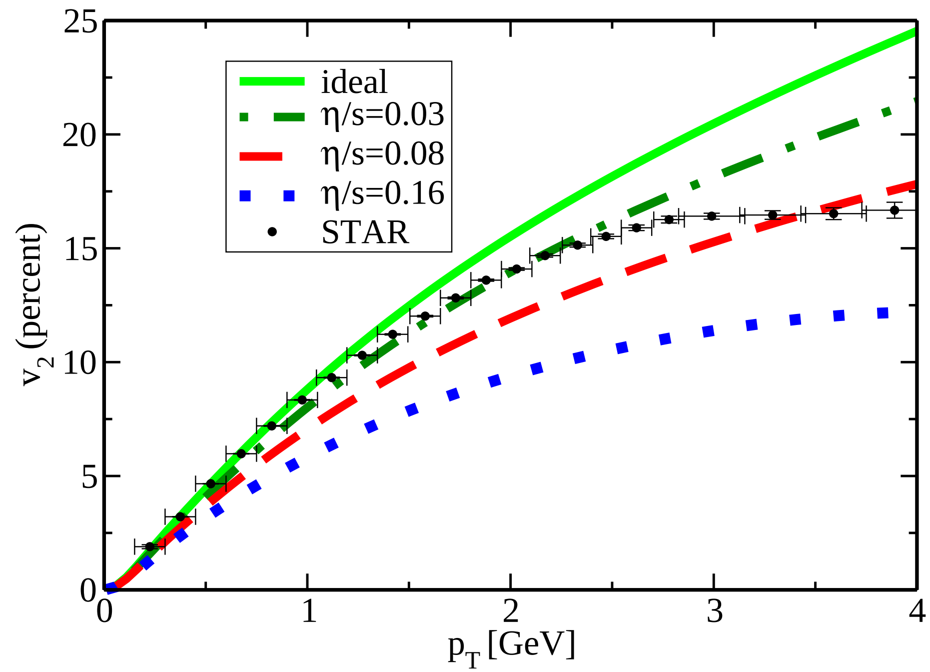
source: U. Heinz (2005)

Viscosity and Elliptic Flow

Viscous correction to v_2 (blast wave model)

$$\frac{\delta v_2}{v_2} = -\frac{1}{3}\frac{1}{\tau_f T_f}\left(\frac{\eta}{s}\right)\left(\frac{p_\perp}{T_f}\right)^2$$

Grows with $p_\perp$, decreases with system size



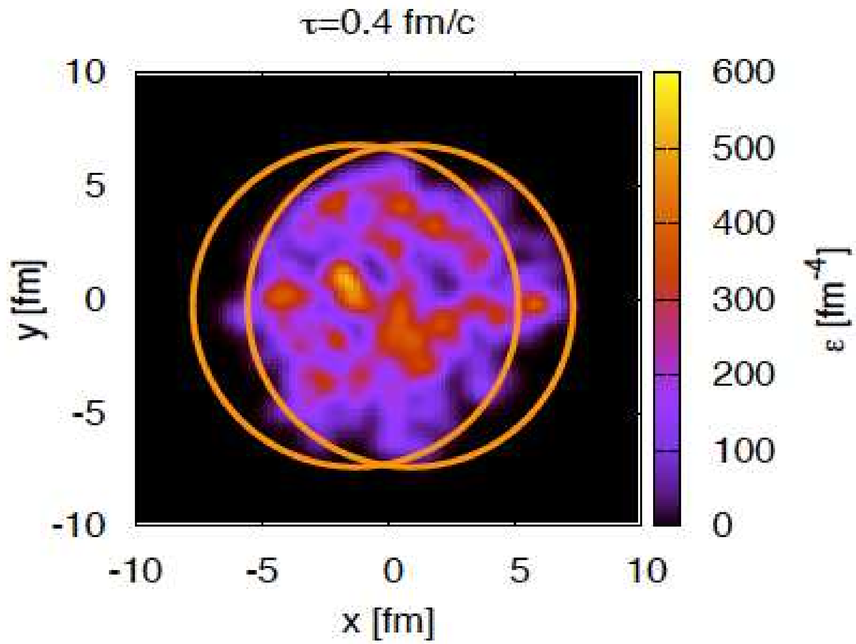
Romatschke (2007), Teaney (2003)

Many details: Dependence on initial conditions, freeze out, etc.

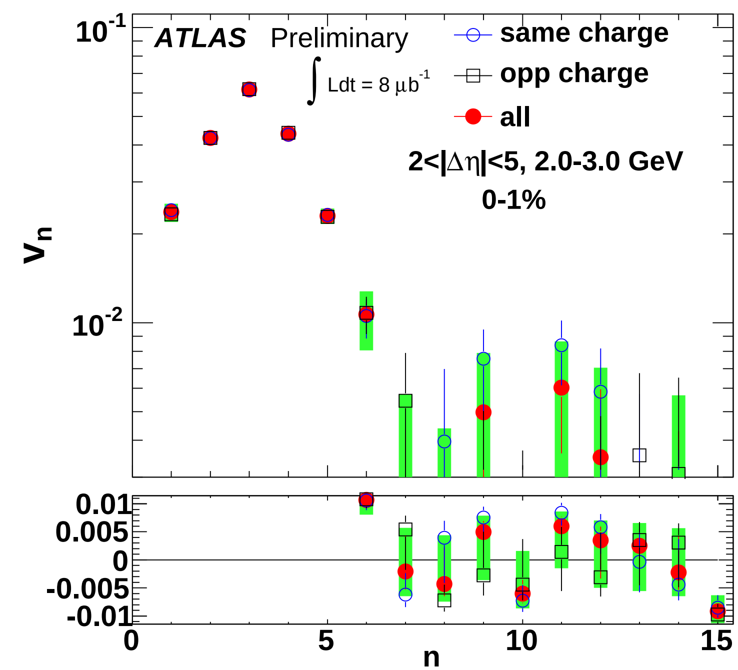
conservative bound $\frac{\eta}{s} < 0.25$

Higher moments of flow

Hydro converts moments of initial deformation to moments of flow



B. Schenke



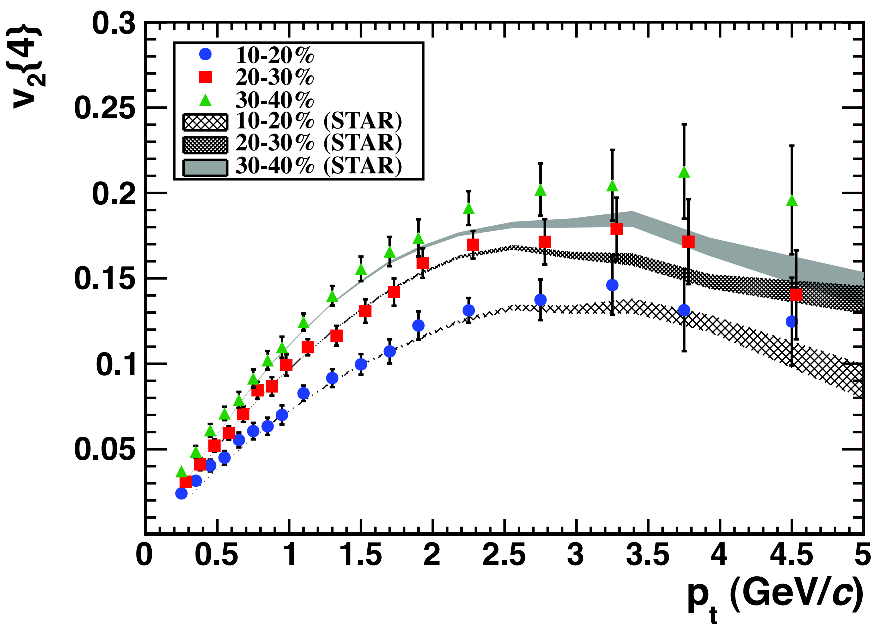
Atlas (J. Jia, QM 2011)

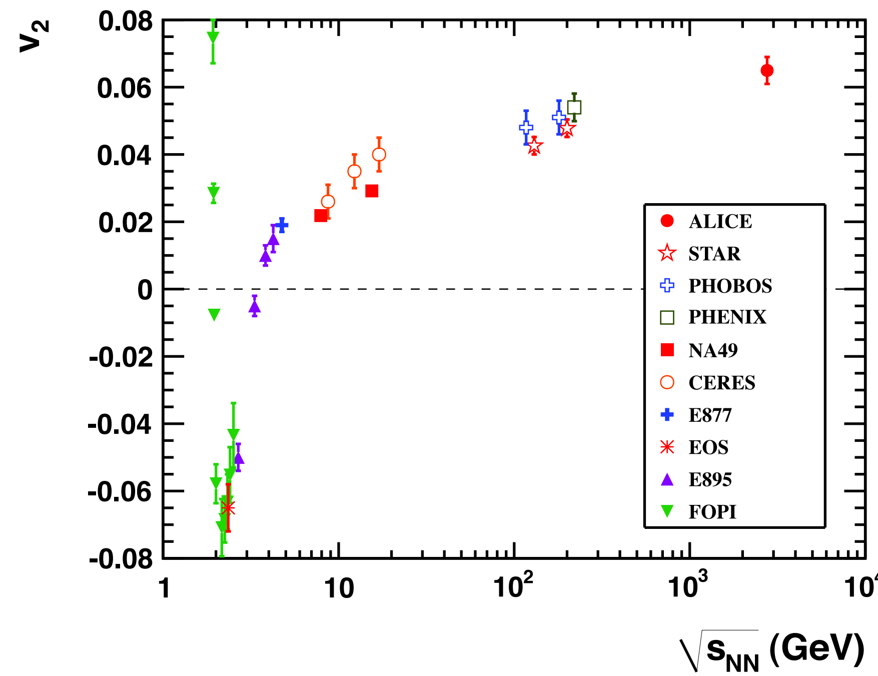
Glauber predicts flat initial spectrum ($n \geq 3$). Observed flow spectrum consistent with sound attenuation

$$\delta T^{\mu\nu}(t) = \exp\left(-\frac{2}{3}\frac{\eta}{s}\frac{k^2 t}{T}\right)\delta T^{\mu\nu}(0)$$

Nearly perfect fluidity at the LHC?

Yes, but some questions remain.





Alice (2011)

Differential v_2 equal to RHIC

Coincidence? Freezeout?

Integrated v_2 somewhat high

Mean p_T increase?

The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.

We still do not know whether there is a fundamental lower bound on η.