Mass without Mass from Instantons in QCD

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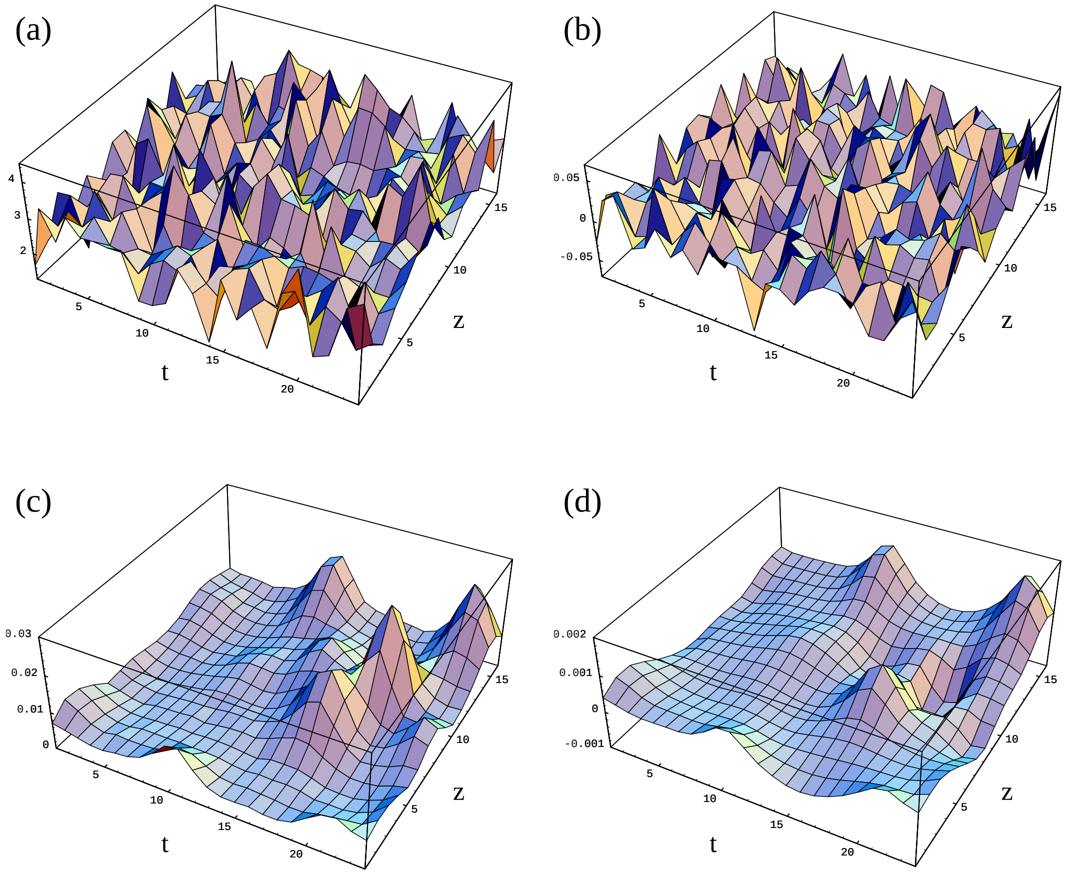


QuantumChromoDynamics

- elementary fields

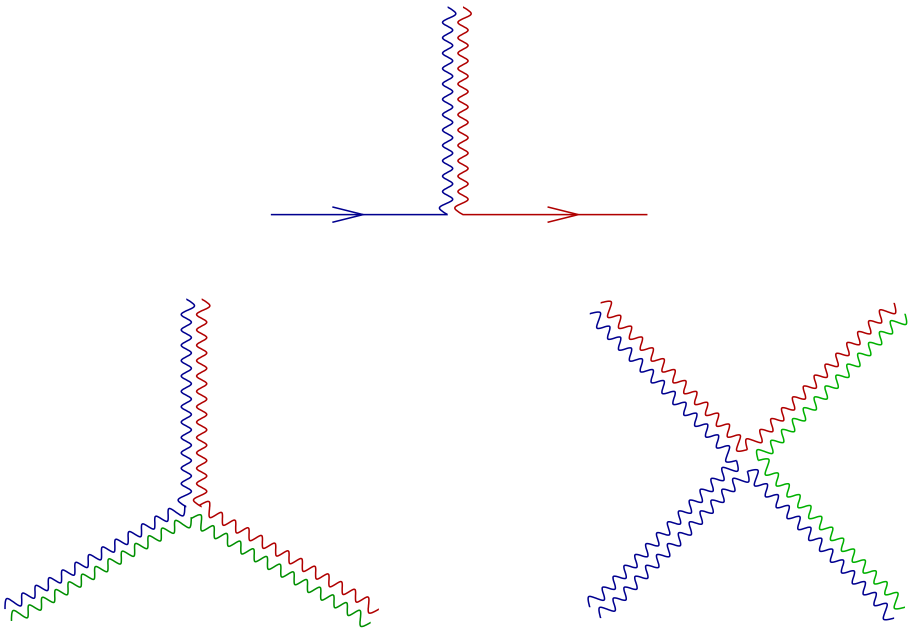
quarks	$(q_\alpha)^a_f$	$\begin{cases} \text{color} & a = 1, \ldots, 3 \\ \text{spin} & \alpha = 1, 2 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$
gluons	A^a_μ	$\begin{cases} \text{color} & a = 1, \ldots, 8 \\ \text{polarization} & \epsilon^\pm_\mu \end{cases}$

- dynamics: gauge theory

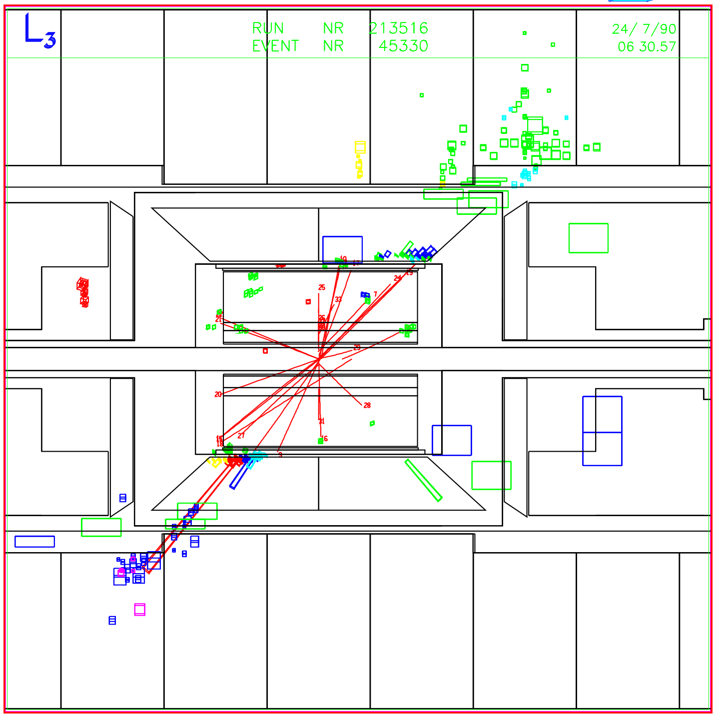
$$\mathcal{L} = \sum_f^{N_f} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}$$

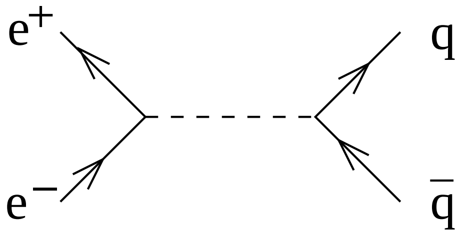
$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

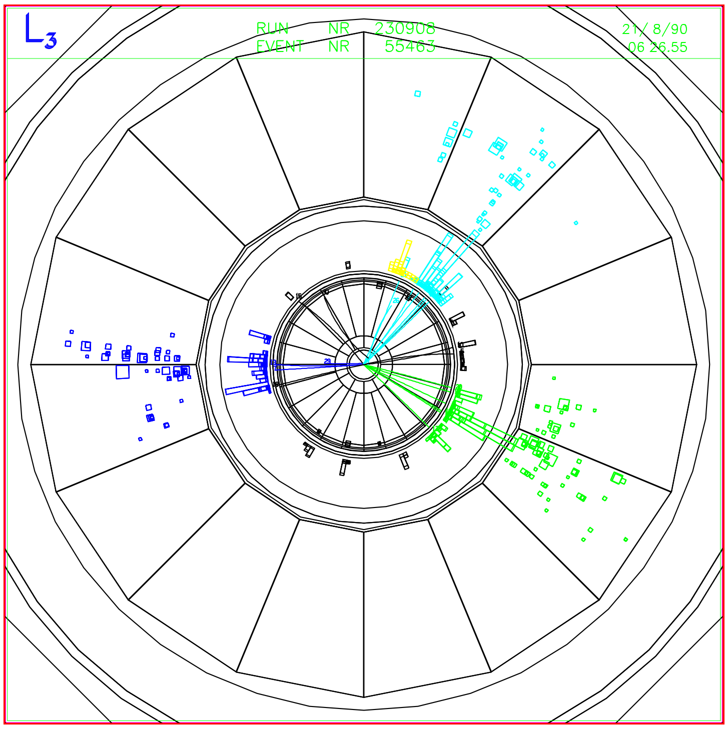
$$i\not{D} q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu \frac{\lambda^a}{2} \right) q$$

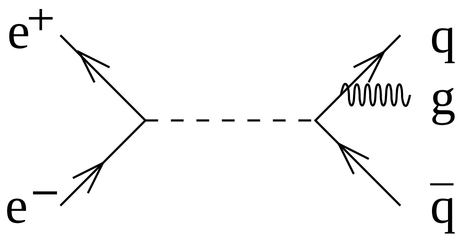


“Seeing” Quark and Gluons

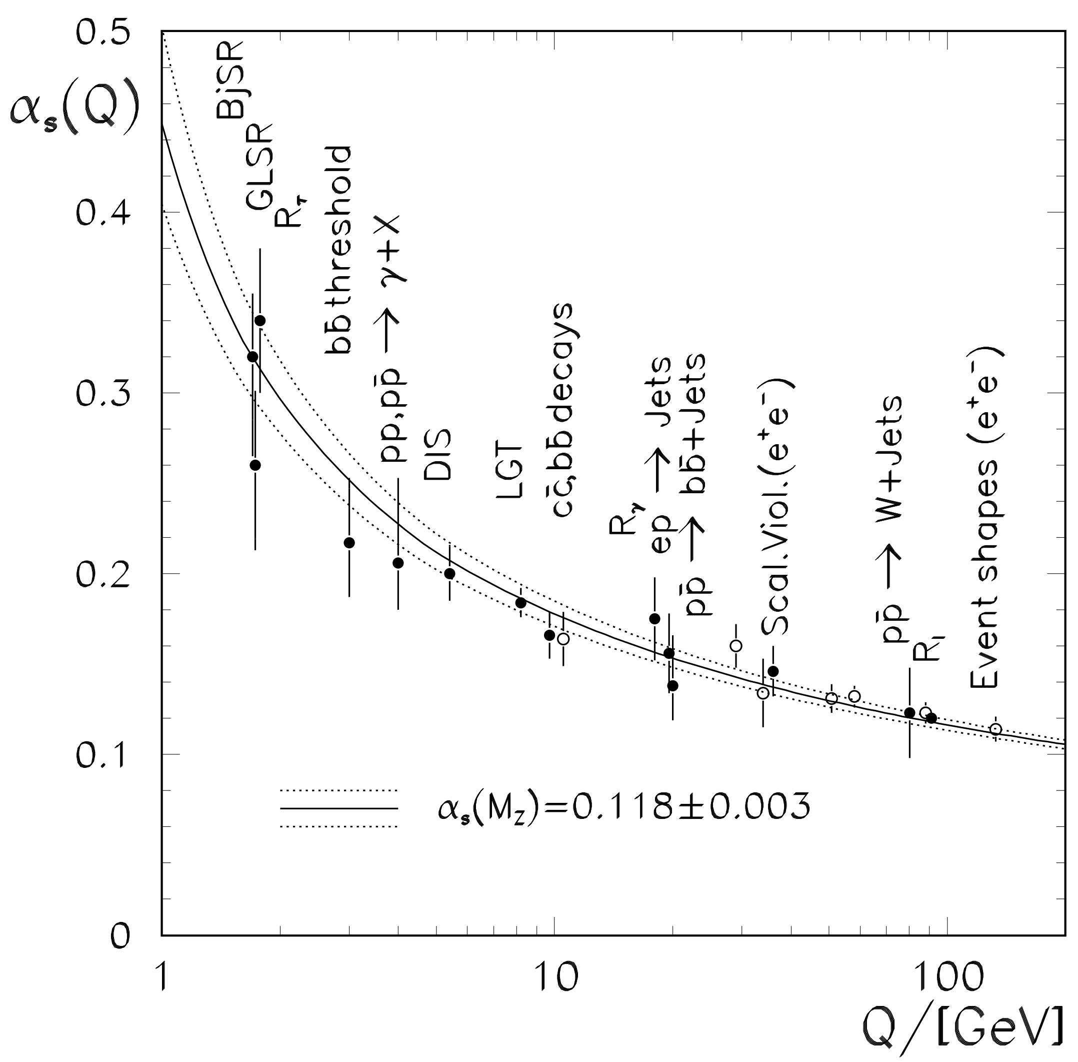








Interaction Strength



About Units

QCD Lite* is a parameter free theory

The lagrangian has a coupling constant, g,
but no scale.

- After renormalization g becomes scale dependent

g is traded for a scale parameter Λ

- Λ is the only scale, the QCD "standard kilogramm"

$$\Lambda_{QCD} \simeq 200\,\mathrm{MeV} \simeq 1\,\mathrm{fm}^{-1}$$

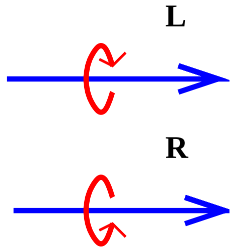
- $\Lambda^{-1} \simeq 1\,\mathrm{fm}$ = distance where the coupling becomes strong

*QCD Lite is QCD in the limit where the masses of the light quarks are taken to zero, and the masses of the heavy quarks are taken to infinity

Chiral Symmetry

- define L/R fields

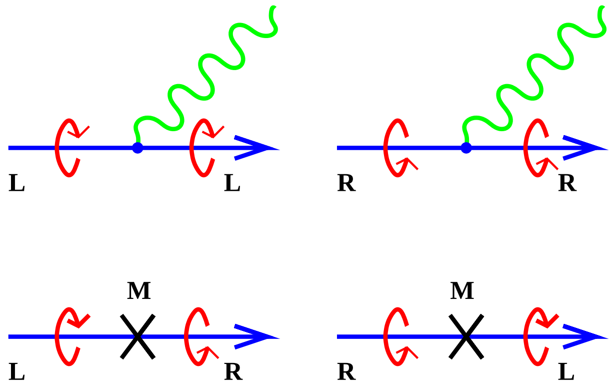
$$\psi_{L,R} = \frac{1}{2}(1 \pm \gamma_5)\psi$$



- fermionic lagrangian

$$\mathcal{L} = \bar{\psi}_L(i\not{D})\psi_L + \bar{\psi}_R(i\not{D})\psi_R$$

$$+ \bar{\psi}_L M \psi_R + \bar{\psi}_R M \psi_L$$



$$M = \mathrm{diag}(m_u, m_d, m_s)$$

- $m_u = m_d = m_s$: Flavor symmetry $V \in SU(3)$

$$\psi_L \to V\psi_L, \qquad \psi_R \to V\psi_R$$

- $m_u = m_d = m_s = 0$: Chiral symmetry

$$(L, R) \in SU(3)_L \times SU(3)_R$$

$$\psi_L \to L\psi_L, \qquad \psi_R \to R\psi_R$$

The Fate of Chiral Symmetry

- Chiral symmetry implies parity doubling, e.g.

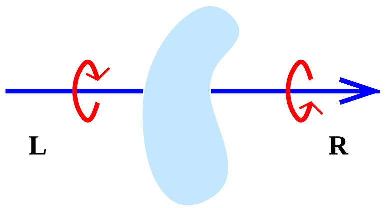
$$\rho(LL+RR) \quad \leftrightarrow \quad a_1(LL-RR)$$

$$\text{But}: \quad m_\rho = 770\,\text{MeV} \qquad m_{a_1} = 1230\,\text{MeV}$$

compare: $m_u, m_d \sim 10\,\text{MeV}$

- chiral symmetry is spontaneously broken

$$\langle \bar{\psi}_L \psi_R + \bar{\psi}_L \psi_R \rangle \simeq -(230\,\text{MeV})^3$$



- consequences: dynamical mass $m_Q = 300\,\text{MeV} \gg m_q$

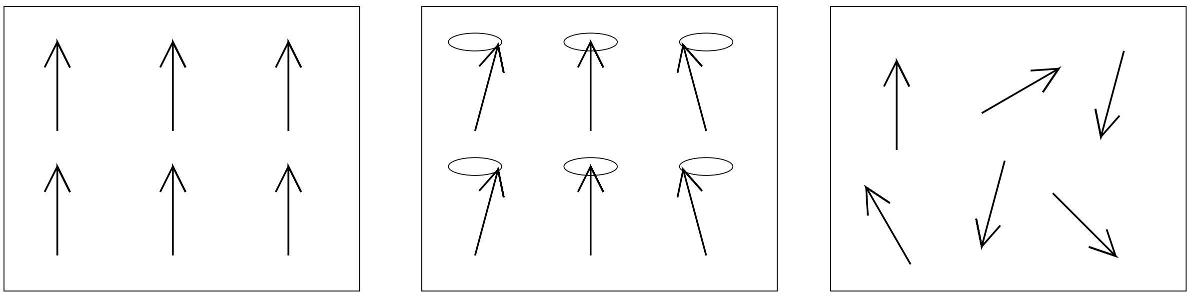
$$m_p = \quad 890\,\text{MeV} \quad + \quad 45\,\text{MeV}$$

$$(\text{QCD}, 95\%) \qquad (\text{Higgs}, 5\%)$$

"Mass without Mass"

existence of Goldstone modes

$$\langle \bar{\psi}_L^f \psi_R^g \rangle = B\,(\sigma + i\vec{\tau}\vec{\pi})^{fg}$$



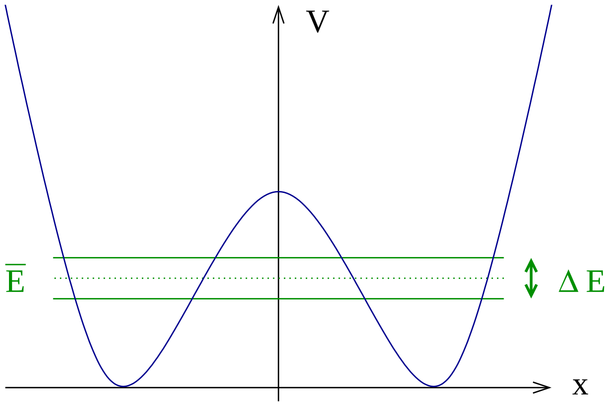
ordered — chiral symmetry broken

spin wave — pion

disordered — chiral symmetry restored

Instantons in Quantum Mechanics

- Consider particle moving in a potential



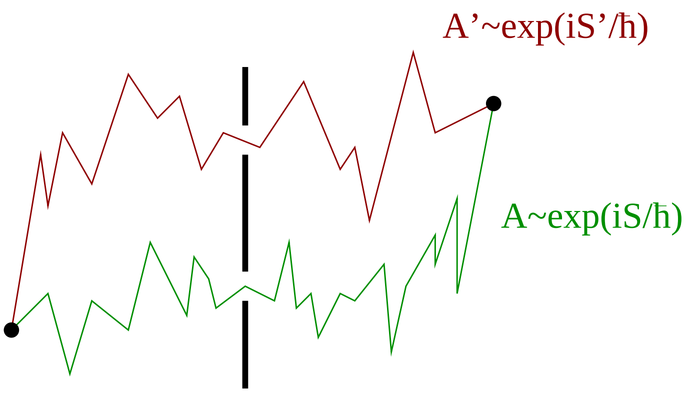
$$H = T + V$$

$$V = \lambda(x^2 - \eta^2)^2$$

classically degenerate states
mixed by tunneling

$$\boxed{E_{1,2} \simeq E_0 \pm \Delta E}$$

- Feynman path integral



$$\langle x|e^{-iHt/\hbar}|y\rangle \sim \sum_{paths} e^{iS/\hbar}$$

$$S = \int dt\,(T - V)$$

- Use quantum mech ↔ statistical mech correspondence

$$e^{iS} \quad \leftrightarrow \quad e^{-\beta H}$$

quantum fluctuations ↔ thermal fluctuations

formal trick: continuation to euclidean time

$$t \to it = \tau, \qquad e^{iS} \to e^{-S_E}$$

- tunneling amplitude

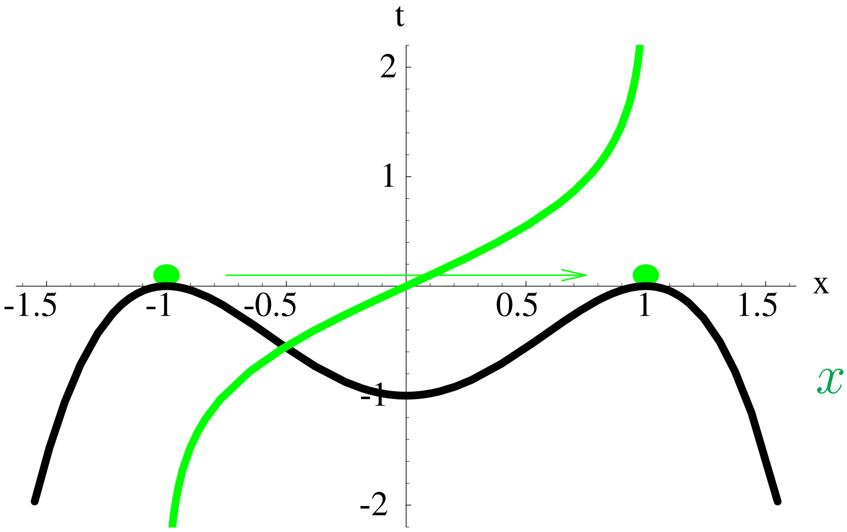
$$\langle -\eta | e^{-\beta H} | \eta \rangle = \int Dx(\tau)\, e^{-S_E}$$

$$S_E = \int_0^\beta d\tau \left(\frac{1}{2} m \dot{x}^2 + V(x) \right)$$

- Tunneling is (classically) forbidden in Minkowski space, but (classically) allowed in Euclidean space

semi-classical(WKB) approximation

- classical path follows equation of motion: $m\ddot{x} = V'$



$$\frac{1}{2} m \dot{x}^2 - V(x) = 0$$

$$x_{cl}(\tau) = \eta \tanh \left[\frac{\omega}{2} (\tau - \tau_0) \right]$$

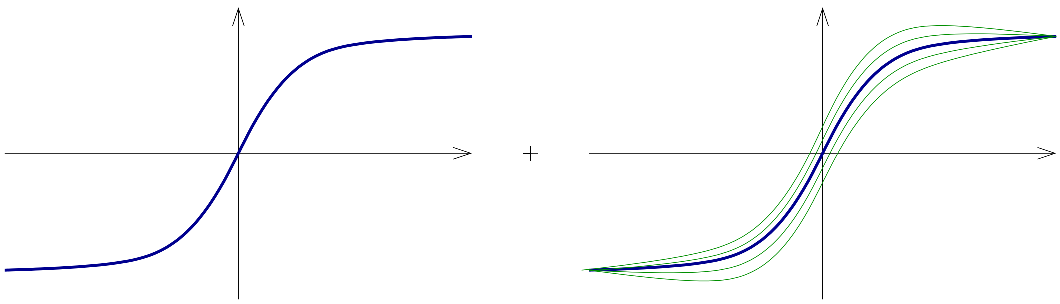
solution has euclidean action (barrier height)

$$\boxed{S = \frac{\omega^3}{12\lambda}}$$

THE INSTANTON

- $S \gg 1$ ($S \gg \hbar$): semi-classical expansion

$$\langle -\eta | e^{-H\tau} | \eta \rangle = e^{-S_0} \int Dx(\tau)\ \exp\left(-\frac{1}{2} \delta x \left. \frac{\delta^2 S}{\delta x^2} \right|_{x_{cl}} \delta x + \ldots \right)$$



Compare to

$$\langle -\eta | e^{-H\tau_m} | \eta \rangle = 2\psi_0^\dagger(-\eta)\psi_0(\eta) \exp(-\tau_m(\bar{E} - \Delta E))$$

read off the level spacing

$$\boxed{\Delta E = \sqrt{\frac{6S_0}{\pi}} \omega \exp\left(-S_0\right)}$$

Lattice Quantum Mechanics

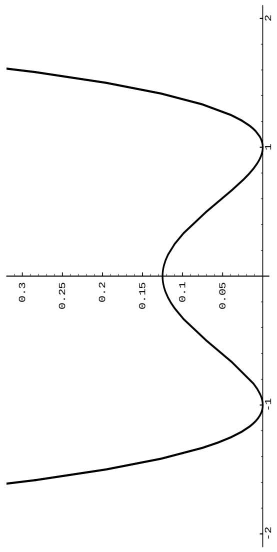
- Partition Function (euclidean space)

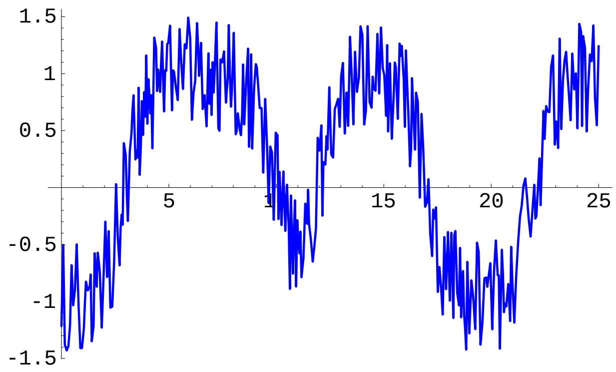
$$Z = \mathrm{tr}(e^{-\beta H}) = \int Dx(\tau)\, e^{-S} \qquad S = \int d\tau \left(\frac{1}{2}m\dot{x}^2 + V(x)\right)$$

- Lattice version: discretize time $x(\tau = na) = x_n$

$$S = \sum_n \left[\frac{m}{2a}(x_n - x_{n-1})^2 + aV(x_n)\right]$$

- Consider double well potential



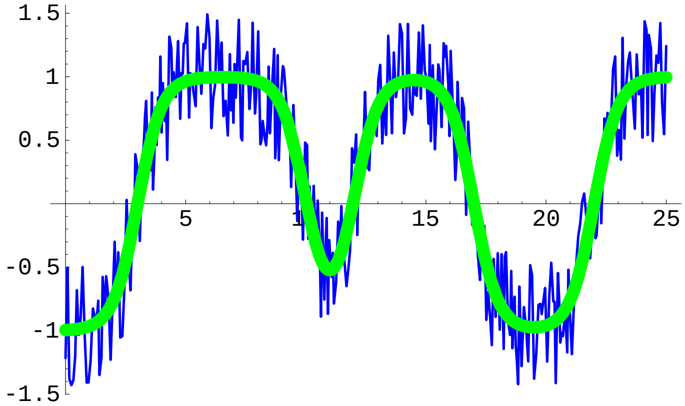


typical lattice configuration

- Metropolis updates

$$\{x_i\}_n \to \{x_i\}_{n+1} \qquad p(n \to n+1) = \exp(-\Delta S)$$

- Effective description: Tunneling events (instantons) plus small fluctuations



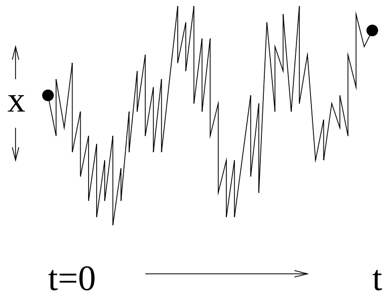
- Classical component can be extracted using cooling (or RG blocking)

$$\{x_i\}_n \to \{x_i\}_{n+1} \qquad S_n > S_{n+1} \;\; (\hbar \to 0)$$

Correlation Functions

- consider coordinate correlation function

$$\Pi(\tau) = \langle x(\tau) x(0) \rangle$$



- spectral representation

$$\Pi(\tau) = \sum_n |\langle 0|x|n\rangle|^2 e^{-\Delta E_n \tau}$$

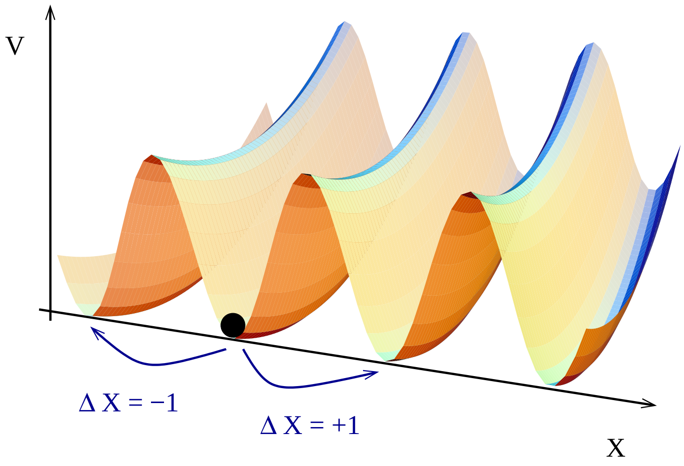
dipole matrix element $\times$ exponential decay

- energy gap

$$\Delta E_1 = -\frac{d}{d\tau} \log(\Pi(\tau))\Big|_{\tau\to\infty}$$

Tunneling in QCD

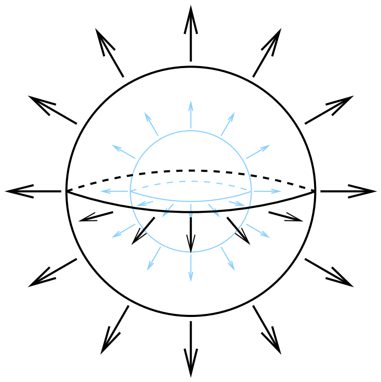
- Classical potential is periodic in variable X



$$X = \int d^3x \, K_0(x,t)$$

$$\partial^\mu K_\mu = \frac{1}{32\pi^2} G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}$$

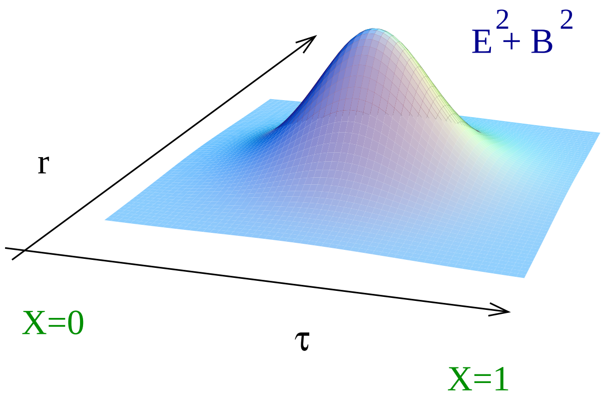
- classical minima correspond to pure gauge configurations



$$A_i(x) = iU^\dagger(x)\partial_i U(x)$$

$$E^2 = B^2 = 0$$

- classical tunneling paths: Instantons



$$A^a_\mu(x) = 2\frac{\eta_{a\mu\nu}x_\nu}{x^2+\rho^2},$$

$$G^a_{\mu\nu}\tilde{G}^a_{\mu\nu} = \frac{192\rho^4}{(x^2+\rho^2)^4}.$$

Phenomenology

- tunneling lowers the ground state energy

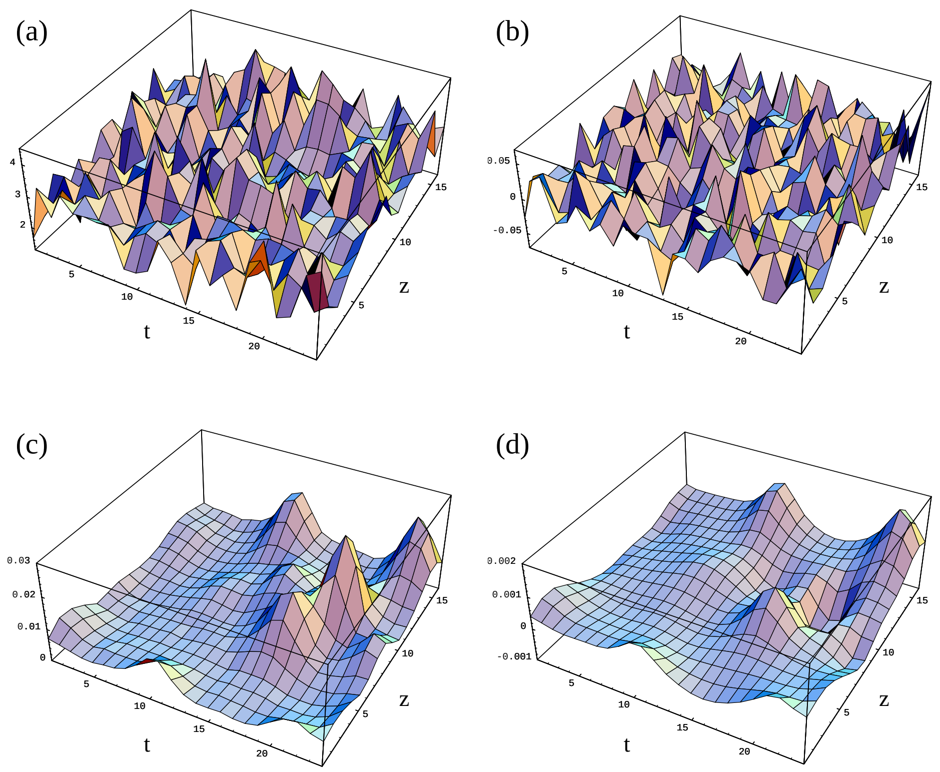
$$\epsilon = -\frac{b}{4}\left(\frac{N}{V}\right), \qquad b = \frac{11}{3}N_c - \frac{2}{3}N_f$$

- QCD vacuum energy $\epsilon \simeq -0.5\,\mathrm{GeV}/\mathrm{fm}^3$. Expect

$$\left(\frac{N}{V}\right) \simeq 1\,\mathrm{fm}^{-4}$$

- typical instanton size $\rho \simeq 0.3\,\mathrm{fm}$

$$\left(\frac{N}{V}\right)\rho^4 = 0.01, \quad \Rightarrow \text{inhomogeneous vacuum!}$$



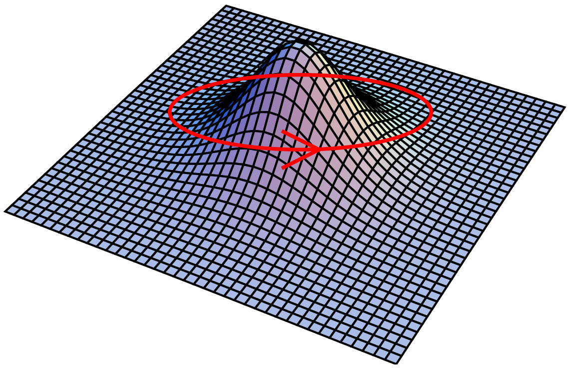
hot

cool

field strength   topological charge

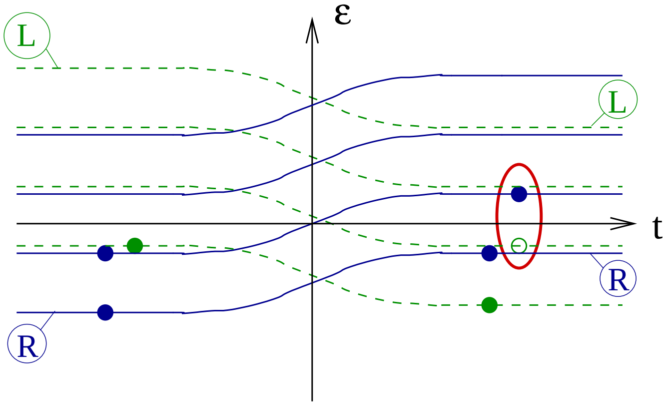
Instantons and Light Quarks

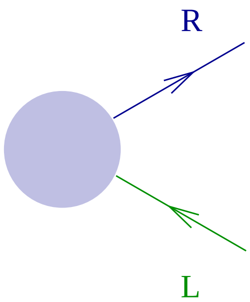
- (Anti)Instantons: Dirac operator has a left (right) handed zero mode.



$$\gamma \cdot (\partial + A_{I,A})\, \psi^0_{L,R} = 0$$

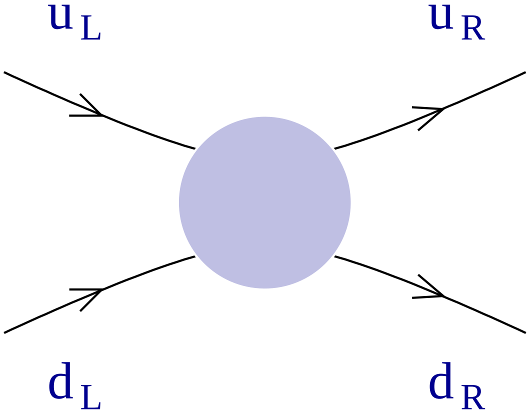
- spectrum as a function of euclidean time





axial charge violation: $\Delta Q_A = 2$

- Instanton induced quark interaction ($N_f = 2$)



$$\mathcal{L} = G \det_f(\bar\psi_{L,f}\psi_{R,g})$$

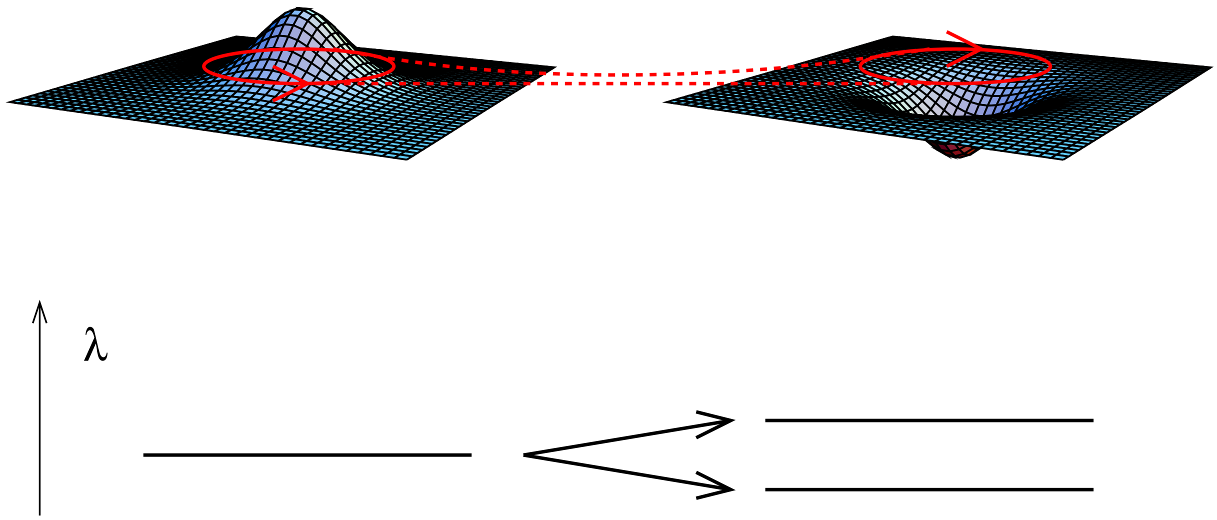
$$G = \int d\rho\, n(\rho)$$

Chiral Symmetry Breaking

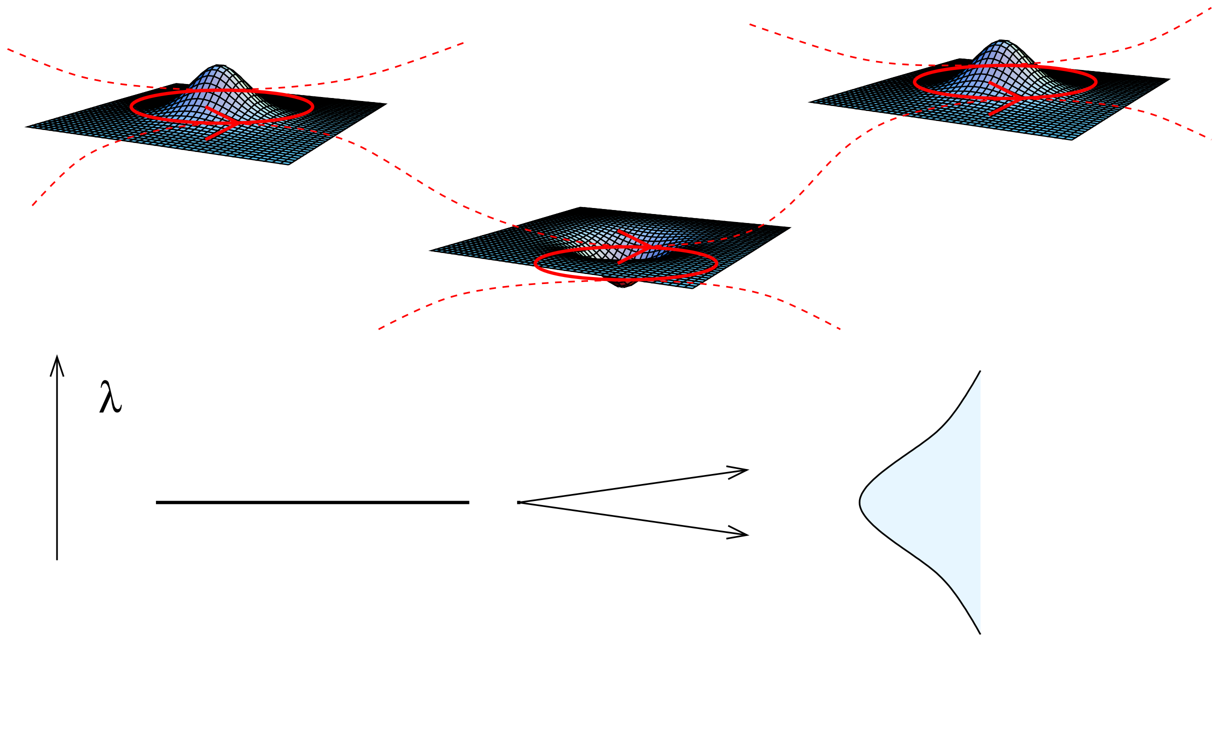
- Banks-Casher: $\langle \bar{q}q \rangle$ related to density $\rho(0)$ of zero modes

$$\boxed{\langle \bar{q}q \rangle = -\pi \rho(\lambda = 0)}$$

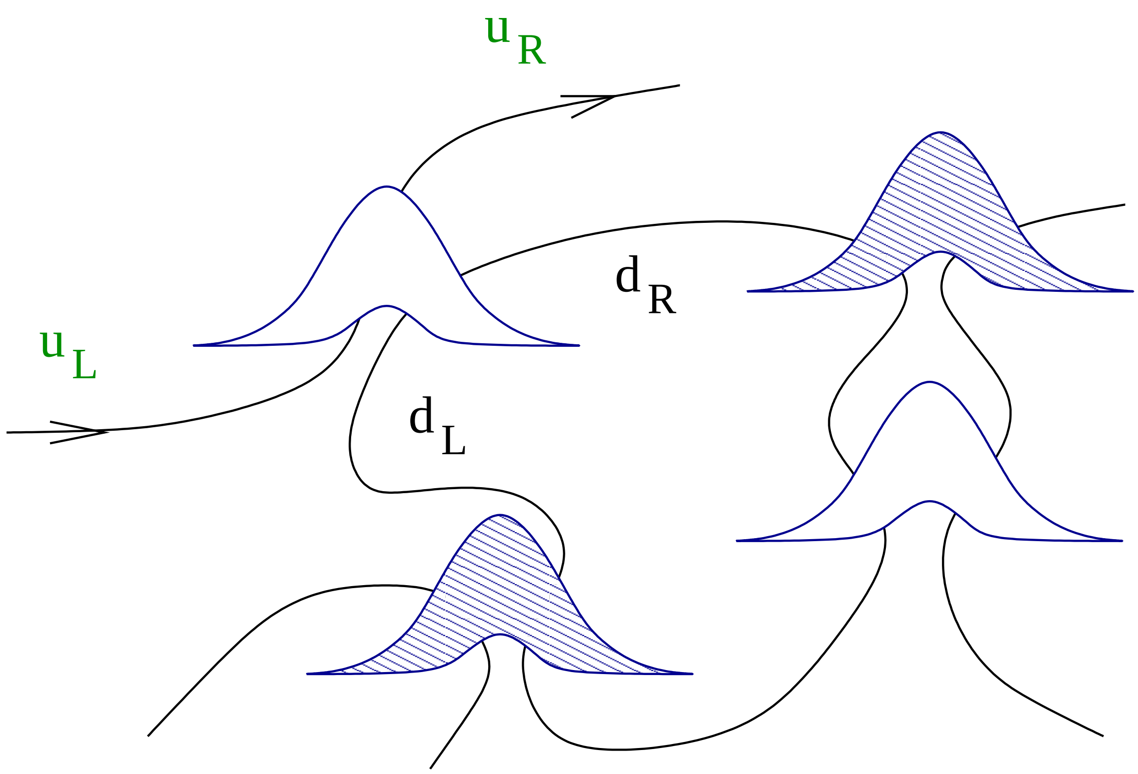
- Instanton-Anti-Instanton configurations: zero modes mix



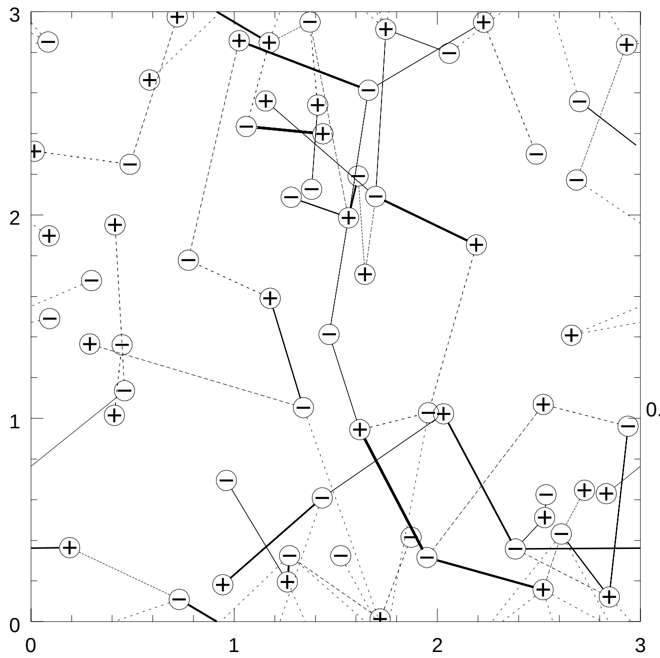
- Instanton Liquid: zero modes zone



quark condensate $\equiv$ delocalized zero modes

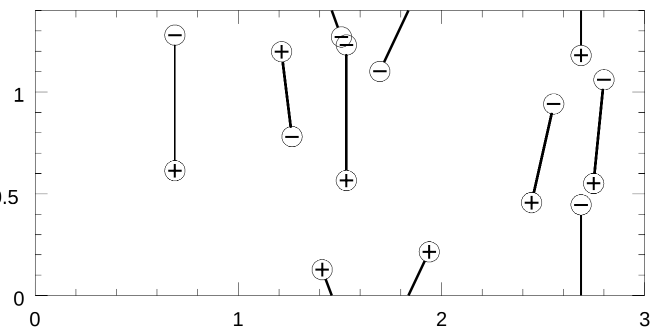


$$\langle \bar{q}q \rangle = -\frac{1}{\pi\rho}\left(\frac{3N_c}{2}\frac{N}{V}\right)^{1/2} \simeq -(240\,\mathrm{MeV})^3,$$



$\langle \bar{q}q \rangle \neq 0$

hadronic phase

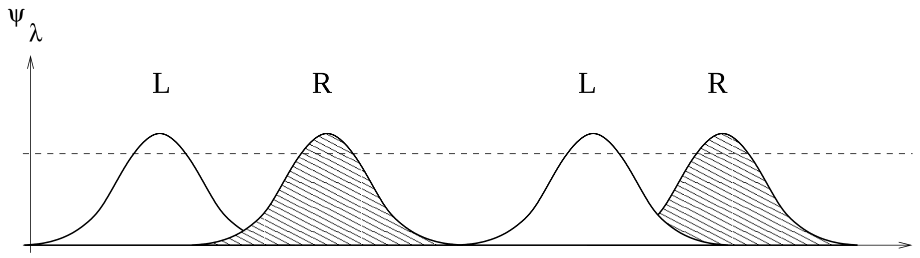


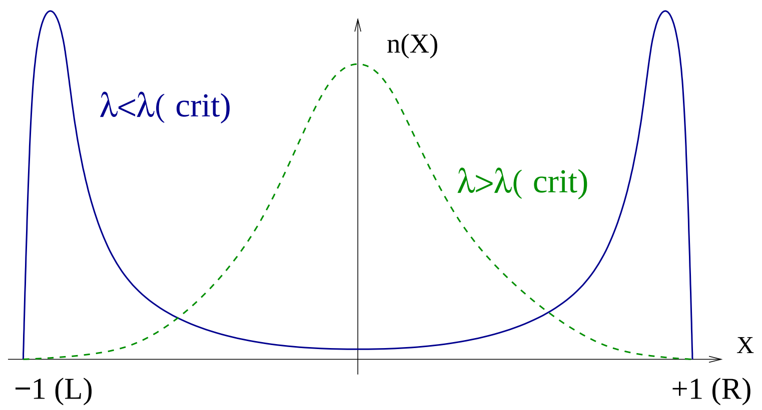
$\langle \bar{q}q \rangle = 0$

quark gluon plasma

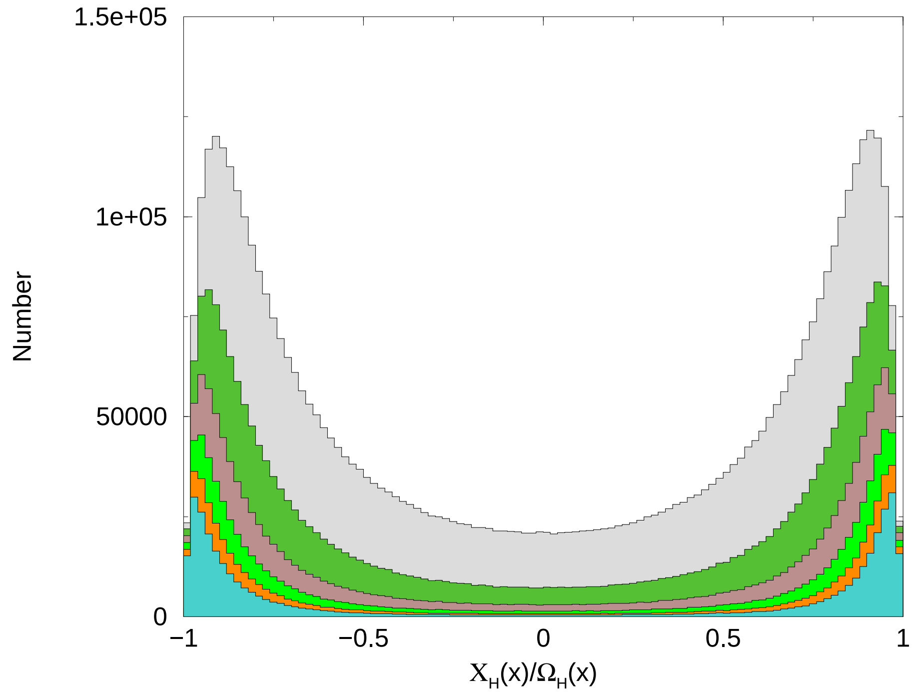
Chiral Symmetry Breaking on the Lattice

- Chirality distribution function



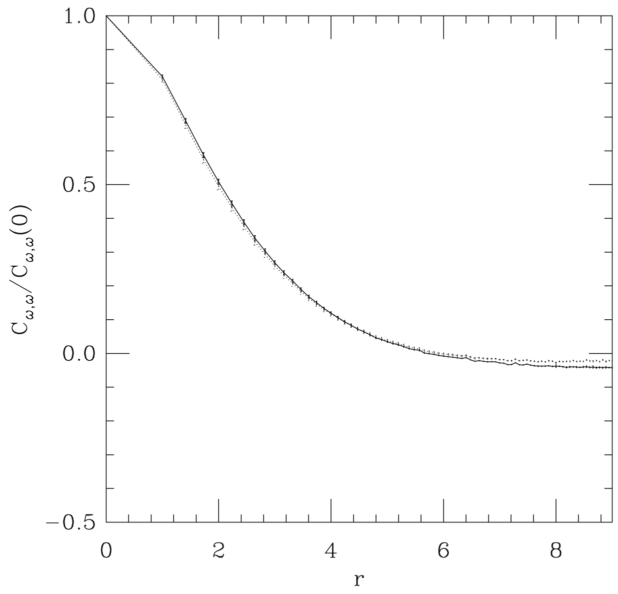


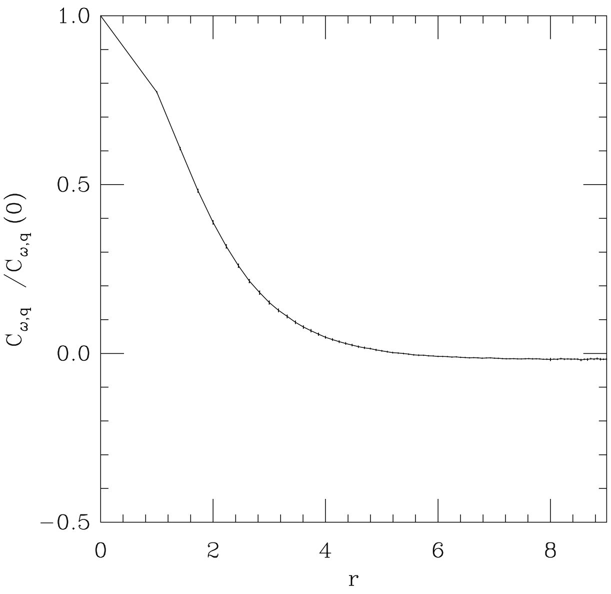
- Chirality distribution measured using overlap fermions



T. Blum et al., [hep-lat/0105006]

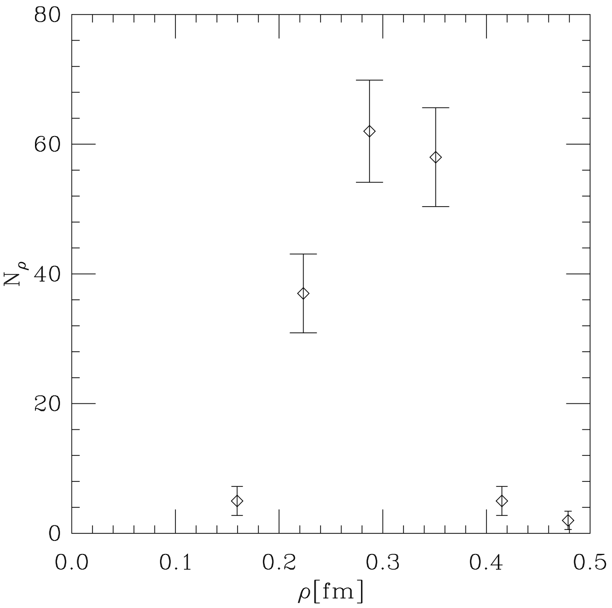
chirality-chirality and chirality-topology correlation





T. DeGrand, A. Hasenfratz, Phys.Rev.D64:034512,2001, [hep-lat/0012021]

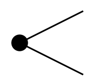
size distribution



Correlation Functions in QCD

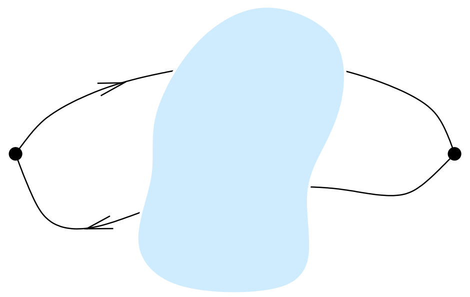
- pion source function

$$j(x) = \bar{u}(x)\gamma_5 d(x)$$

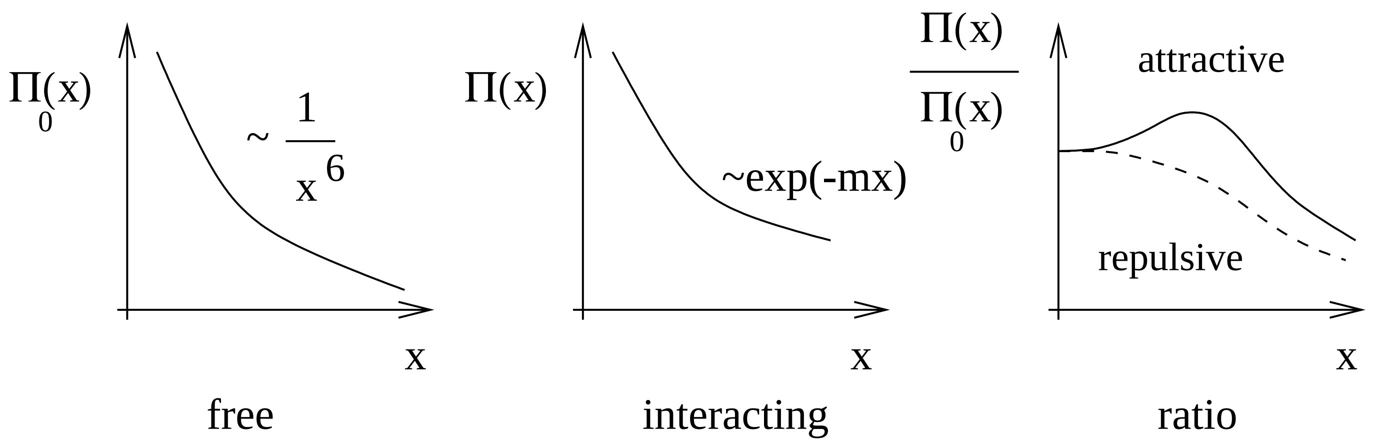


- correlation function

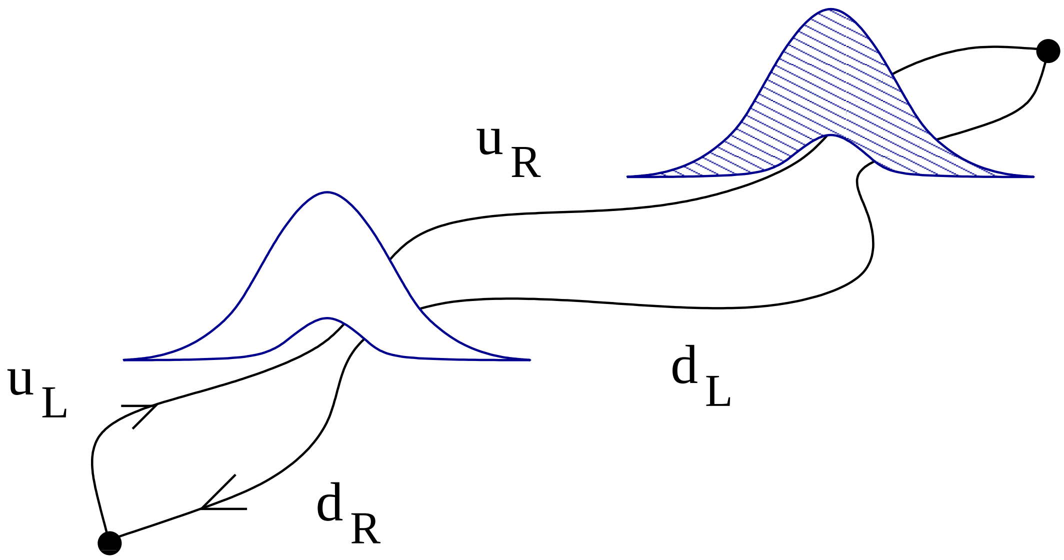
$$\Pi(x) = \langle j(0)j(x)\rangle$$
$$= P(\text{pion propagates from } 0 \text{ to } x)$$
$$\sim \exp(-m_\pi x)$$



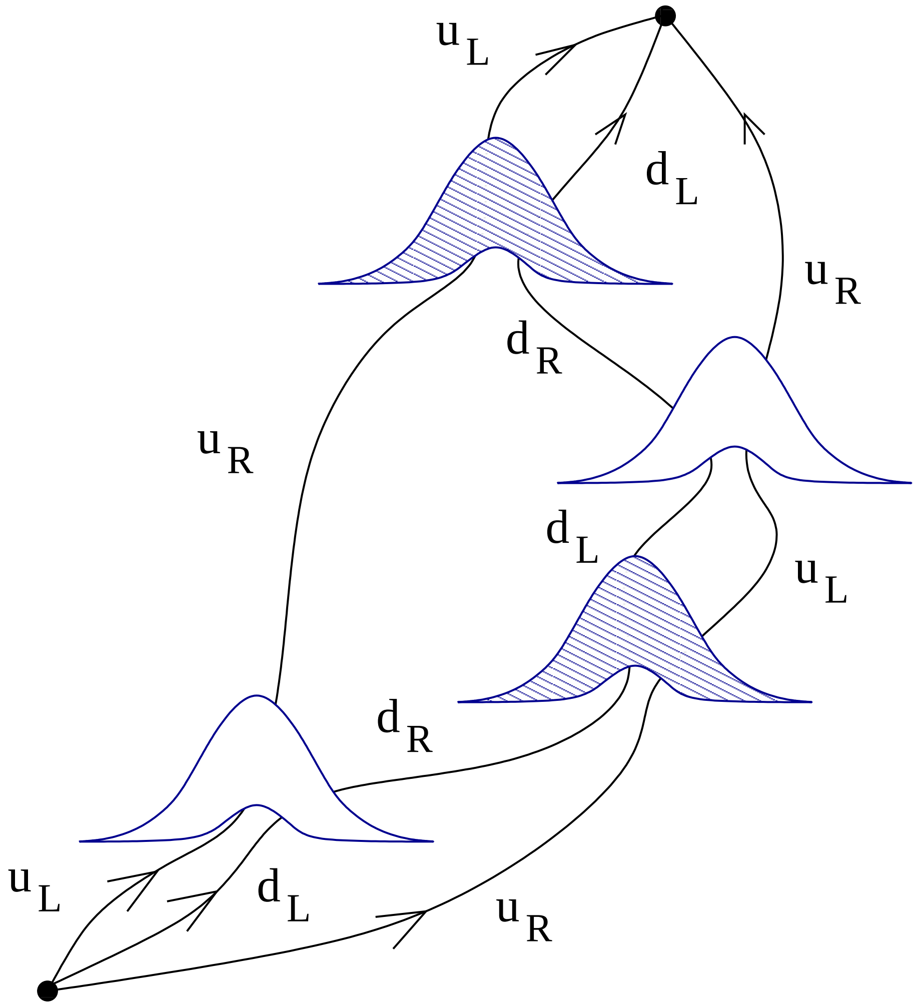
- normalize to non-interacting quarks



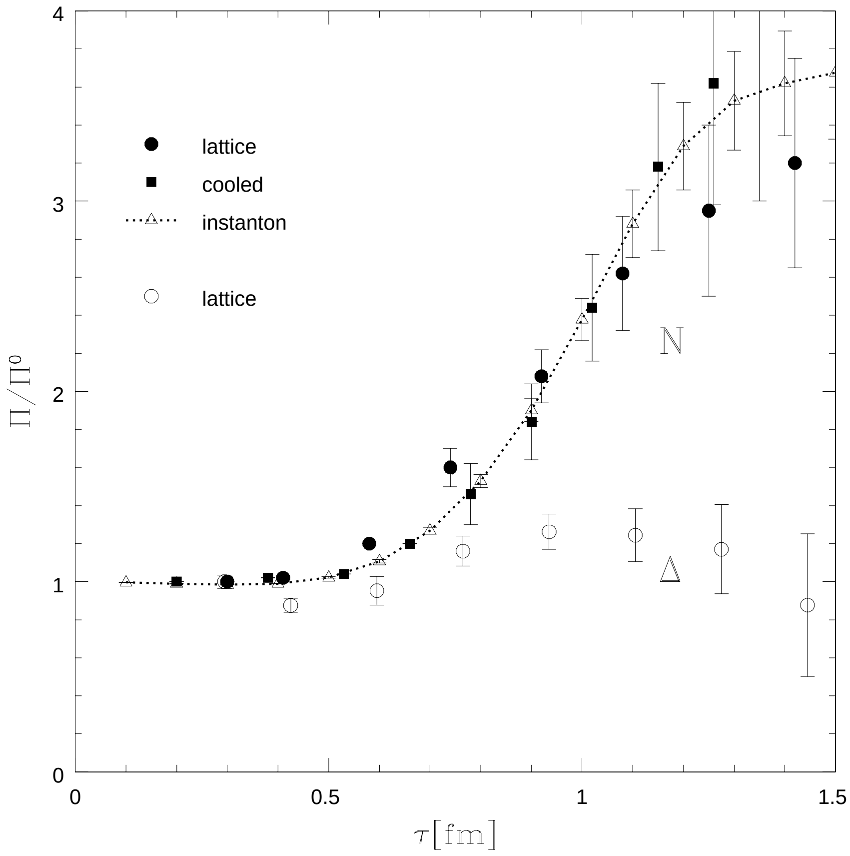
Pion Propagator



Nucleon Propagator



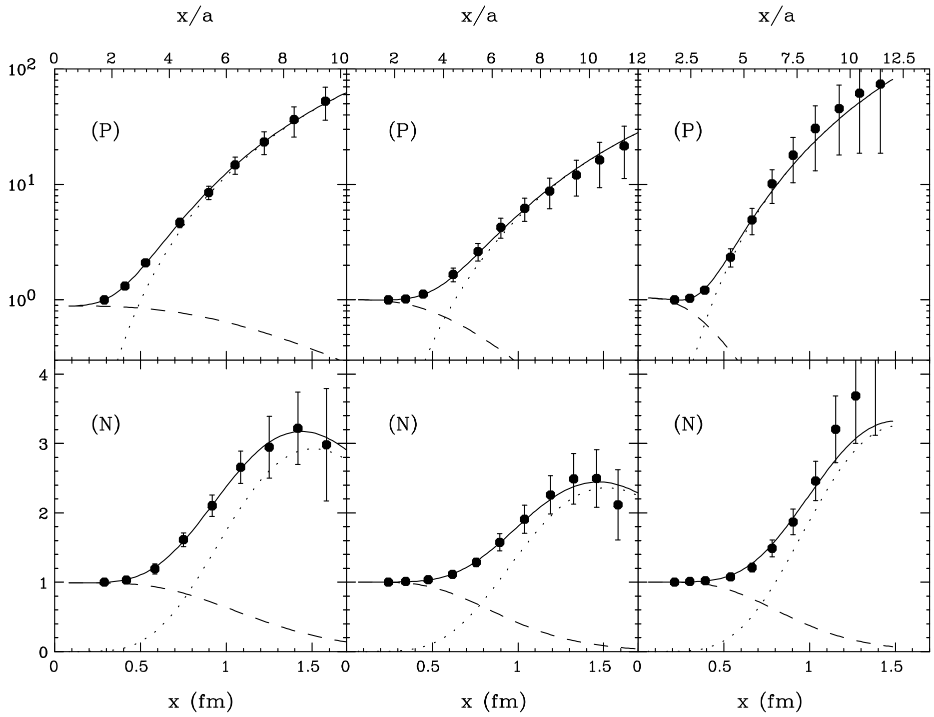
Baryons: Nucleon and Delta



$$m_p = 960\,\mathrm{MeV} \qquad \lambda_1 = 0.035\,\mathrm{GeV}^3$$

$$\langle 0|\eta_1|N(p)\rangle = \lambda_1 u(p)$$
$$\eta_1 = \epsilon^{abc}(u^a C\gamma_\mu u^b)\gamma^\mu d^c$$

Cooled Correlation Functions: Pion and Proton



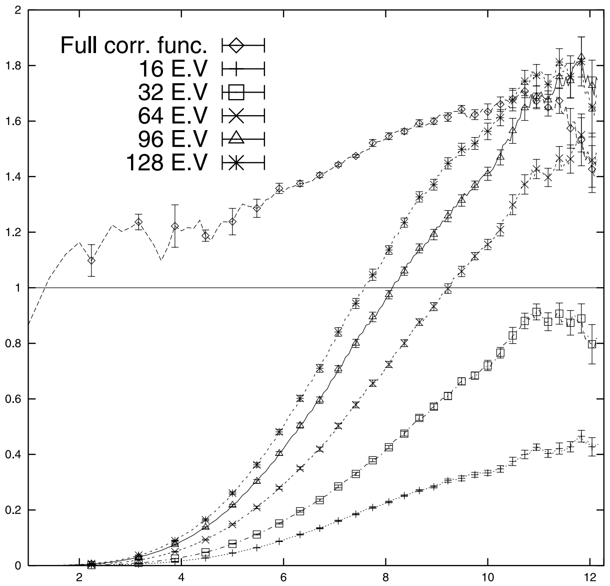
$$m_p(\text{full}) = 950\,\text{MeV} \qquad m_p^*(\text{cool}) = 950\,\text{MeV}$$
$$\lambda_1(\text{full}) = 0.293\,\text{GeV}^3 \qquad \lambda_1(\text{cool}) = 0.297\,\text{GeV}^3$$

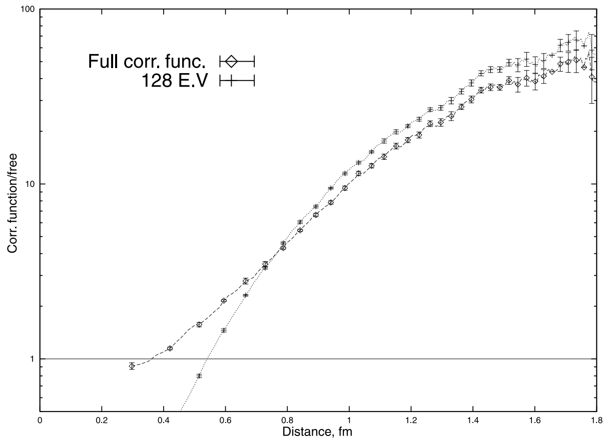
* m_p used to set the scale ($\Delta m_\rho/m_\rho = 2\%$)

M.C. Chu et al., Phys.Rev.D49:6039-6050,1994, [hep-lat/9312071]

Mode Expansion

T. Ivanenko, J. Negele, Nucl.Phys.Proc.Suppl.63:504,1998, [hep-lat/9709130]





$$\Pi(x) = \langle \mathrm{Tr}[S(x,y)\Gamma S(y,x)\Gamma] \rangle$$

$$S(x,y) = \sum_{\lambda=1}^{\nu_{max}} \frac{\phi_\lambda(x)\phi_\lambda^\dagger(y)}{\lambda + im}$$

zero mode zone

$$\lambda < \sqrt{|T_{IA}|^2} = \rho\pi\sqrt{\frac{2}{3N_c}\frac{N}{V}} \simeq 100\,\mathrm{MeV}$$

Outlook: Structure of the Proton

- The mass of the Proton (from DIS, trace anomaly)

$$\begin{aligned} E_q &= \langle p|\int d^3x\,(-i\vec{\alpha}\cdot\vec{D})|p\rangle && \simeq 310\,\mathrm{MeV} \\ E_g &= \langle p|\int d^3x\,\tfrac{1}{2}\left(E^2+B^2\right)|p\rangle + \ldots && \simeq 545\,\mathrm{MeV} \\ E_m &= \langle p|\int d^3x\,(m_u\bar{u}u + m_d\bar{d}d)|p\rangle && \simeq 45\,\mathrm{MeV} \end{aligned}$$

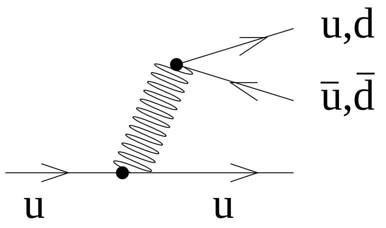
- Gluon field strength is large

$$\langle p|E^2|p\rangle \simeq 1700\,\mathrm{MeV} \qquad \langle p|B^2|p\rangle \simeq -1050\,\mathrm{MeV}$$

- number of quark-anti-quark pairs is large

$$\langle p|\bar{u}u+\bar{d}d|p\rangle = \frac{\Sigma_{\pi N}}{\overline{m}} \simeq 6$$

... and not flavor symmetric



$$\frac{\bar{d}(x)}{\bar{u}(x)} \simeq 2 \qquad \text{(NuSea,}\ldots\text{)}$$

$$x \sim 0.1$$

- quark contribution to proton spin is small

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s = (0.2\pm 0.1) \qquad \text{(SMC, SLAC, Hermes)}$$

... and strange quarks are polarized $\Delta s = -0.12$

- pentaquarks, tetraquarks, ...

$$(ud)(ud)\bar{s}, \quad (ud)(\overline{ud}), \quad (ud)(us)(ds)$$

Outlook: Structure of Hadronic Matter