

Nearly Perfect Fluidity: From Cold Atoms to Hot Quarks

Thomas Schaefer, North Carolina State University



RHIC serves the perfect fluid



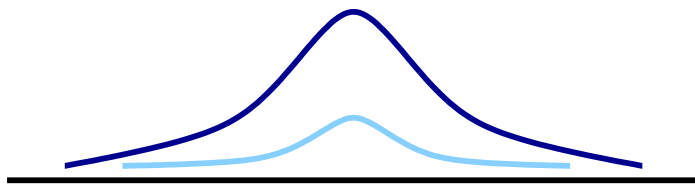
Experiments at RHIC and the LHC are consistent with the idea that a thermalized plasma is produced, and that the equation of state is that of a weakly coupled gas of quarks and gluons.

But: Transport properties of the system (primarily viscosity and energy loss) are in dramatic disagreement with expectations for a weakly coupled QGP. The plasma must be very strongly coupled.

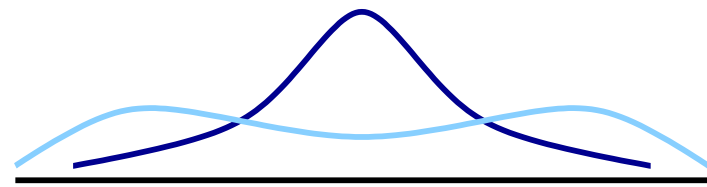
In this talk I will try to explain this statement, review the current evidence, and put the results in a broader perspective (by comparing with another strongly coupled fluid, the dilute atomic Fermi gas at “unitarity”).

Fluids: Gases, liquids, plasmas, ...

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved charges (or spontaneously broken symmetry fields).



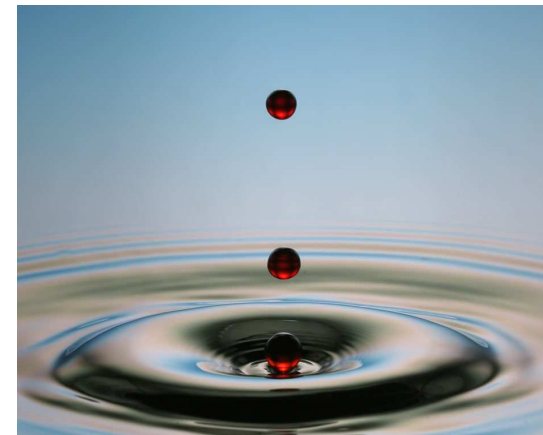
$$\tau \sim \tau_{micro}$$



$$\tau \sim \lambda$$

Historically: Water

$$(\rho, \epsilon, \vec{\pi})$$



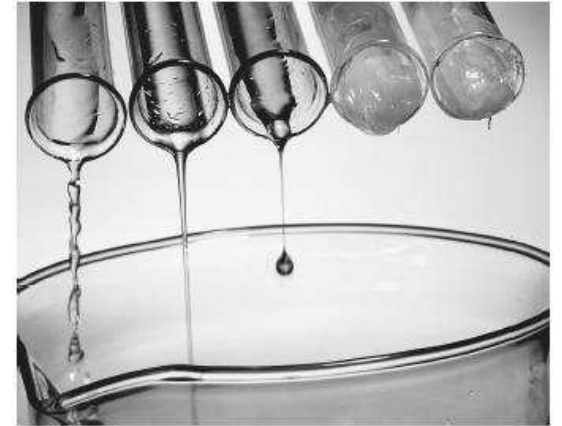
Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \cdot \vec{j}^\epsilon = 0$$

$$\frac{\partial}{\partial t} (\rho v_i) + \nabla_j \Pi_{ij} = 0$$



Constitutive relations: Stress tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\nabla_i v_j + \nabla_j v_i - \frac{2}{3} \delta_{ij} \nabla_k v_k \right) + O(\nabla^2)$$

reactive

dissipative

2nd order

$$\text{Expansion } \Pi_{ij}^0 \gg \delta \Pi_{ij}^1 \gg \delta \Pi_{ij}^2$$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$\frac{1}{Re} = \frac{\eta}{\hbar n} \times \frac{\hbar}{mvL}$$

fluid property flow property

Bath tub : $mvL \gg \hbar$ hydro reliable

Heavy ions : $mvL \sim \hbar$ need $\eta < \hbar n$

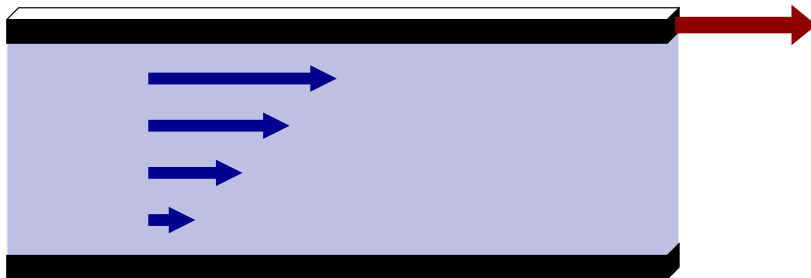
Shear viscosity and friction

Momentum conservation at $O(\nabla v)$

$$\rho \left(\frac{\partial}{\partial t} \vec{v} + (\vec{v} \cdot \nabla) \vec{v} \right) = -\nabla P + \eta \nabla^2 \vec{v}$$

Navier-Stokes equation

Viscosity determines shear stress (“friction”) in fluid flow



$$F = A \eta \frac{\partial v_x}{\partial y}$$

Kinetic theory

Kinetic theory: conserved quantities carried by quasi-particles.

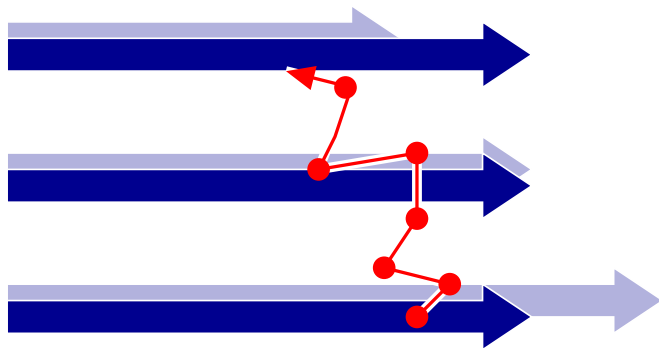
Quasi-particles described by distribution functions $f(x, p, t)$.

$$\frac{\partial f_p}{\partial t} + \vec{v} \cdot \vec{\nabla}_x f_p + \vec{F} \cdot \vec{\nabla}_p f_p = -C[f_p]$$

$$C[f_p] = \begin{array}{c} p \\ \swarrow \quad \nearrow \\ \bullet \\ \nwarrow \quad \searrow \\ p \end{array} - \begin{array}{c} \swarrow \quad \nearrow \\ \bullet \\ \nwarrow \quad \searrow \\ p \end{array}$$



Shear viscosity corresponds to momentum diffusion



$$\eta \sim \frac{1}{3} n \bar{p} l_{mfp}$$

Shear viscosity: Additional properties

Weakly interacting gas, $l_{mfp} \sim \frac{1}{n\sigma}$: $\eta \sim \frac{1}{3} \bar{p}$

shear viscosity independent of density

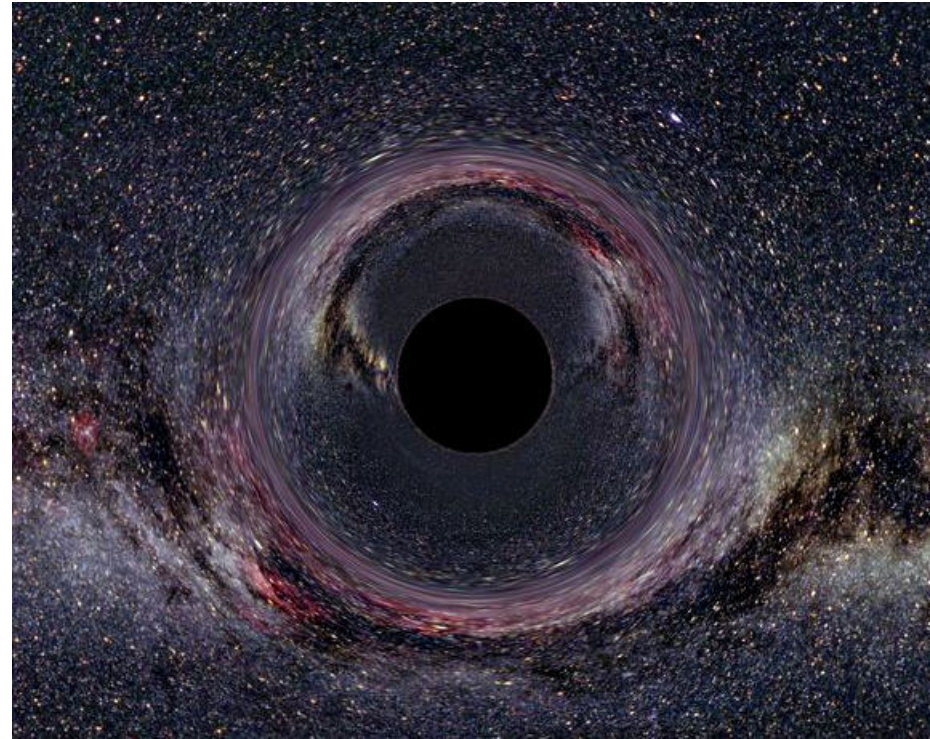
Non-interacting gas ($\sigma \rightarrow 0$): $\eta \rightarrow \infty$

non-interacting and hydro limit ($T \rightarrow \infty$) limit do not commute

strongly interacting gas: $\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar$

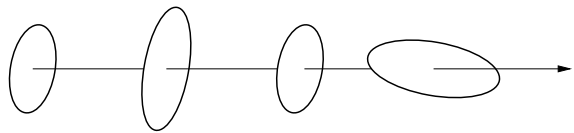
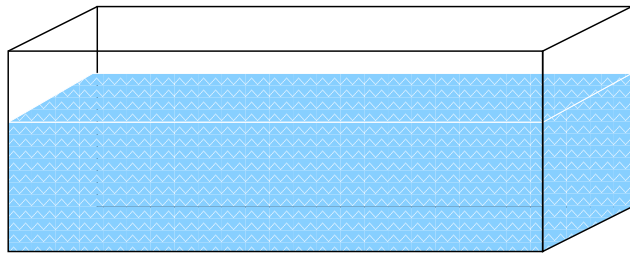
but: kinetic theory not reliable!

And now for something completely different ...

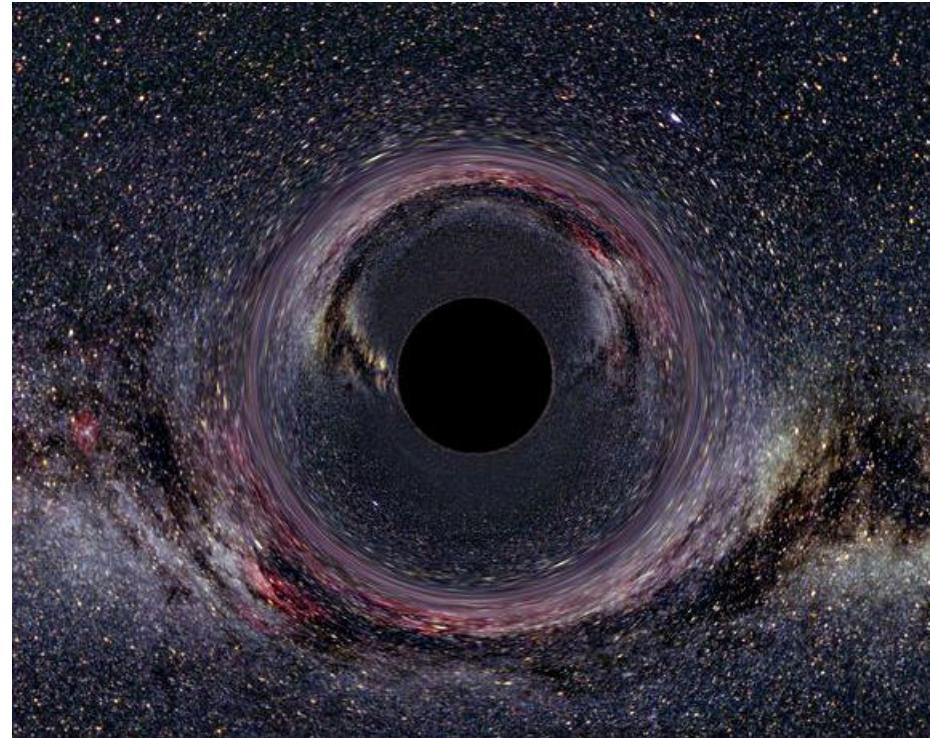


This is an irreversible process, $\Delta S > 0$.

And now for something completely different ...



gravitational wave shears
fluid



Idea can be made precise using the “AdS/CFT correspondence”

Strongly coupled thermal
field theory on R^4



Weakly coupled string theory
on AdS_5 black hole

CFT temperature

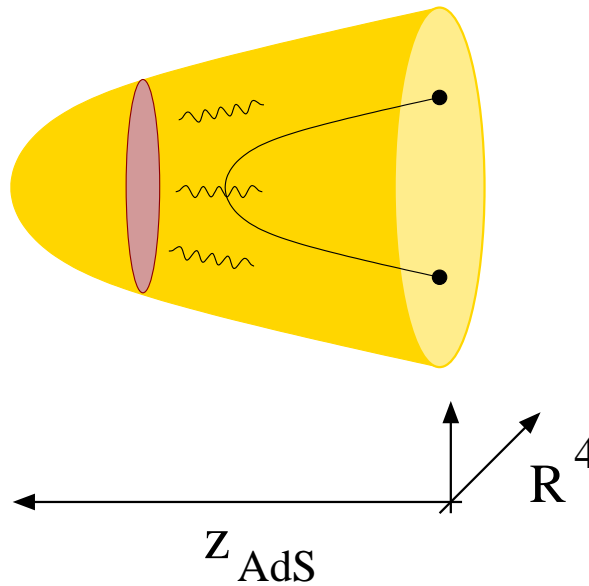


Hawking temperature of
black hole

CFT entropy



Hawking-Bekenstein entropy
 \sim area of event horizon



Holographic duals: Transport properties

Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy



Hawking-Bekenstein entropy

\sim area of event horizon

shear viscosity



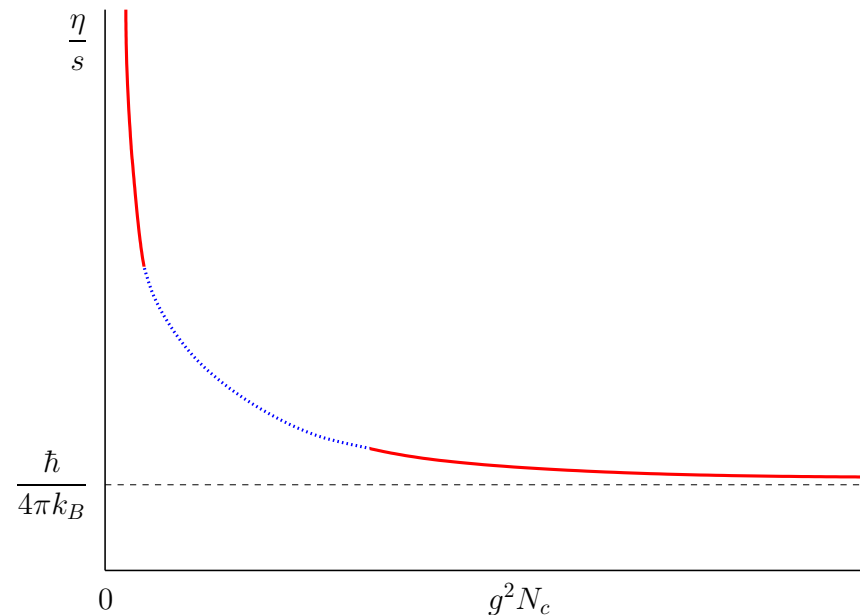
Graviton absorption cross section

\sim area of event horizon

Strong coupling limit

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

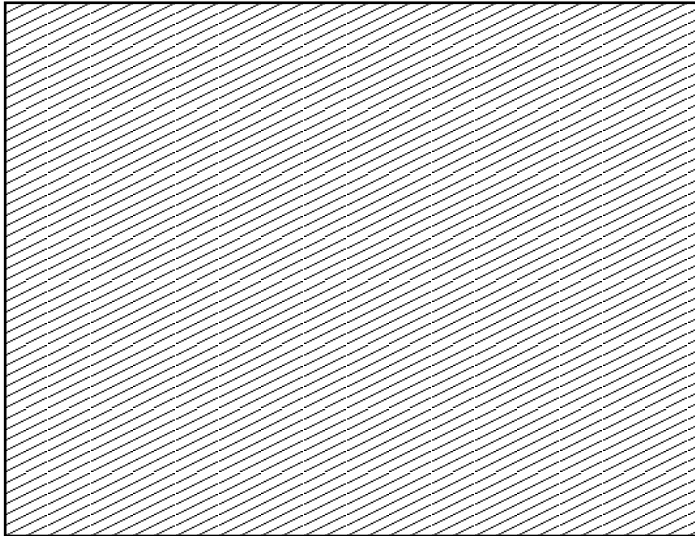
Son and Starinets (2001)



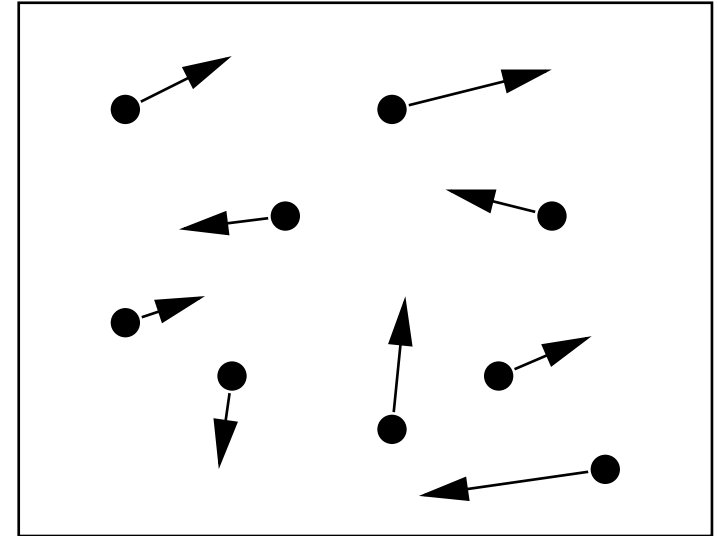
Strong coupling limit universal? Provides lower bound for all theories?

Answer appears to be no; e.g. theories with higher derivative gravity duals.

Kinetics vs no-kinetics

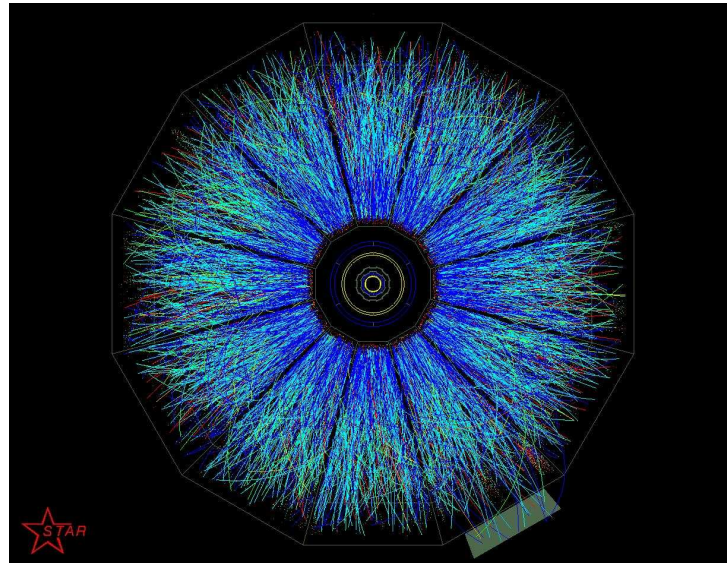
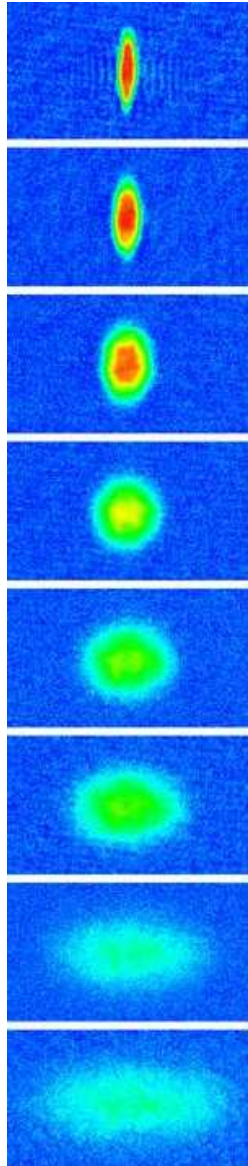


low viscosity goo
gravity dual
 $\eta/s \simeq 1/(4\pi)$



pQCD plasma
quasi-particles
 $\eta/s \sim 1/\alpha_s^2 \gg 1$

Perfect Fluids: The contenders



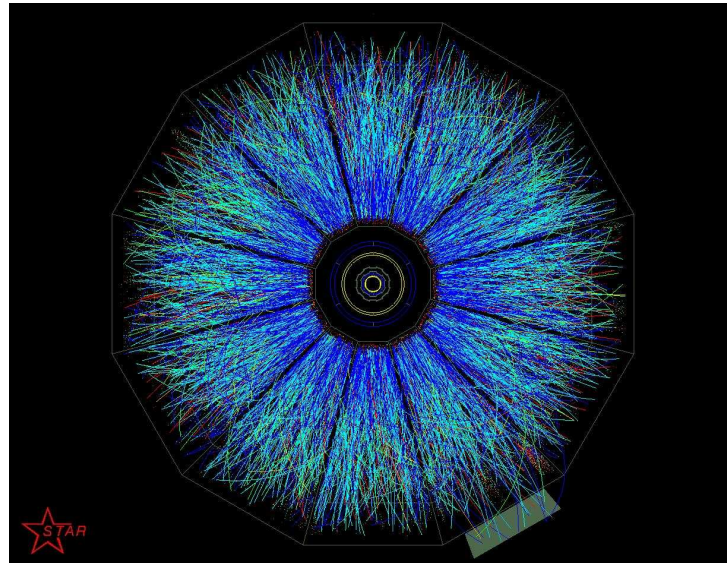
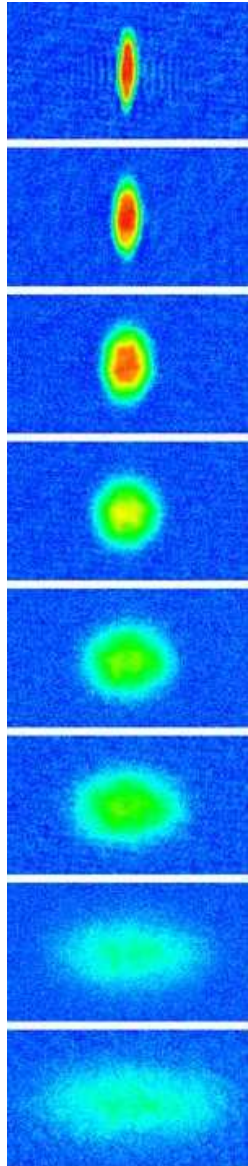
QGP ($T=180$ MeV)

Trapped Atoms
($T=0.1$ neV)



Liquid Helium
($T=0.1$ meV)

Perfect Fluids: The contenders



QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η/s

Perfect Fluids: Not a contender



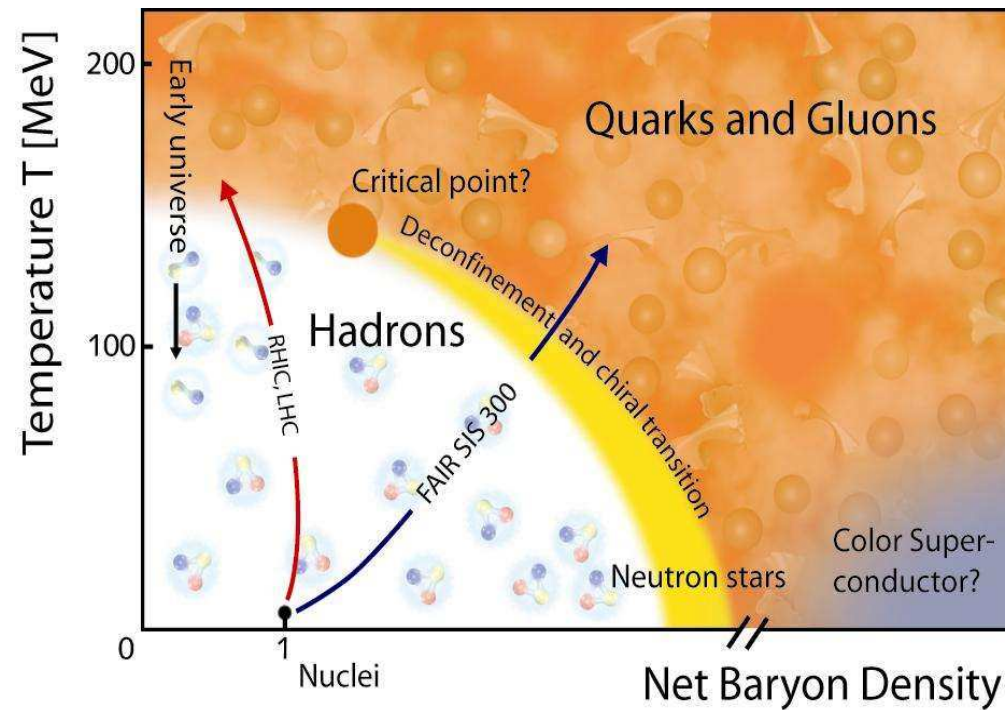
Queensland pitch-drop
experiment

1927-2011 (8 drops)

$$\eta = (2.3 \pm 0.5) \cdot 10^8 \text{ Pa s}$$

I. QCD and the Quark Gluon Plasma

$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4g^2} G_{\mu\nu}^a G_{\mu\nu}^a$$



Quantumchromodynamics (QCD)

Elementary fields:

Quarks

Gluons

$$(q_\alpha)_f^a \begin{cases} \text{color} & a = 1, \dots, 3 \\ \text{spin} & \alpha = 1, 2 \\ \text{flavor} & f = u, d, s, c, b, t \end{cases}$$

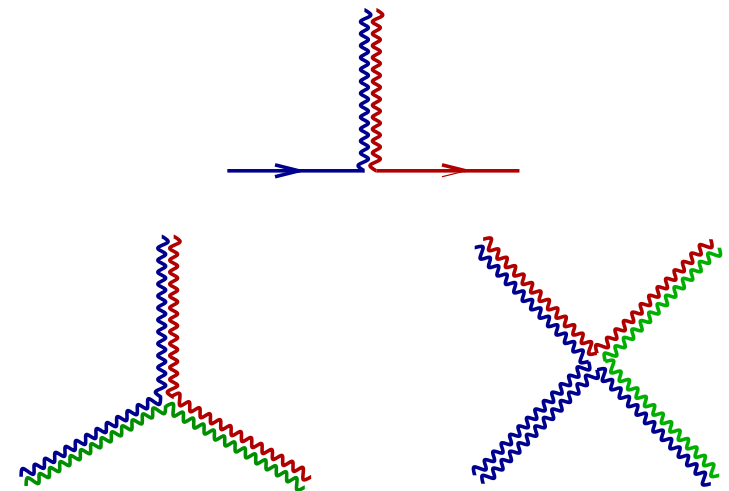
$$A_\mu^a \begin{cases} \text{color} & a = 1, \dots, 8 \\ \text{spin} & \epsilon_\mu^\pm \end{cases}$$

Dynamics: Generalized Maxwell (Yang-Mills) + Dirac theory

$$\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G_{\mu\nu}^a G_{\mu\nu}^a$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$$

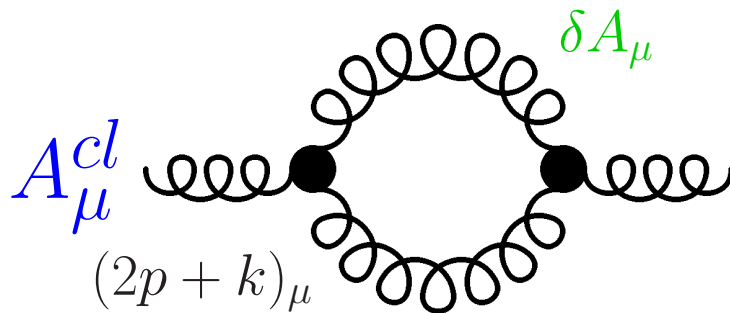
$$i\not{D}q = \gamma^\mu (i\partial_\mu + gA_\mu^a t^a) q$$



Asymptotic freedom

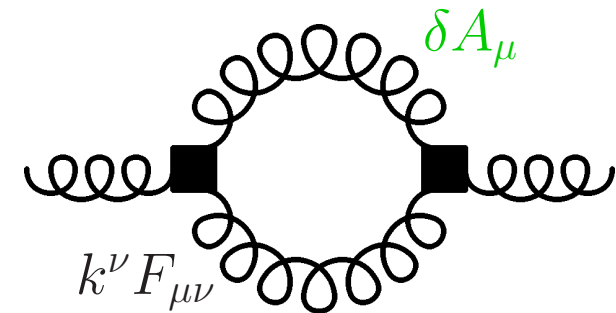
Modification of Coulomb interaction due to quantum fluctuations

electric gluons



dielectric $\epsilon > 1$

magnetic gluons



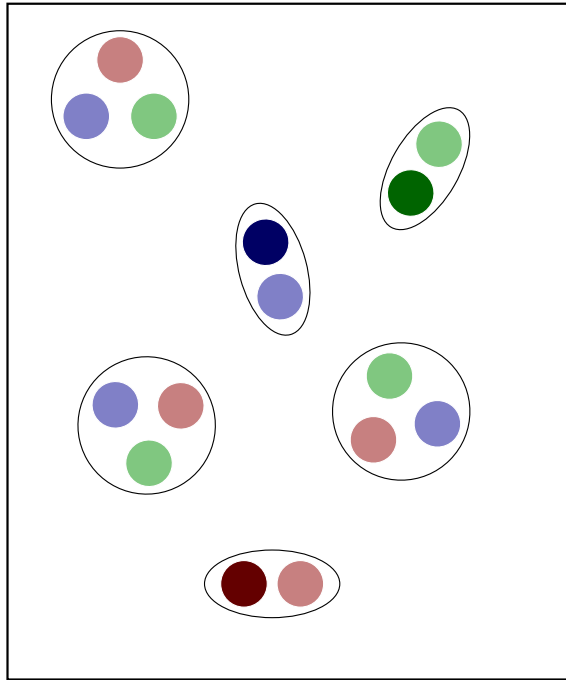
para-magnetic $\mu > 1$

vacuum: $\mu\epsilon = 1$

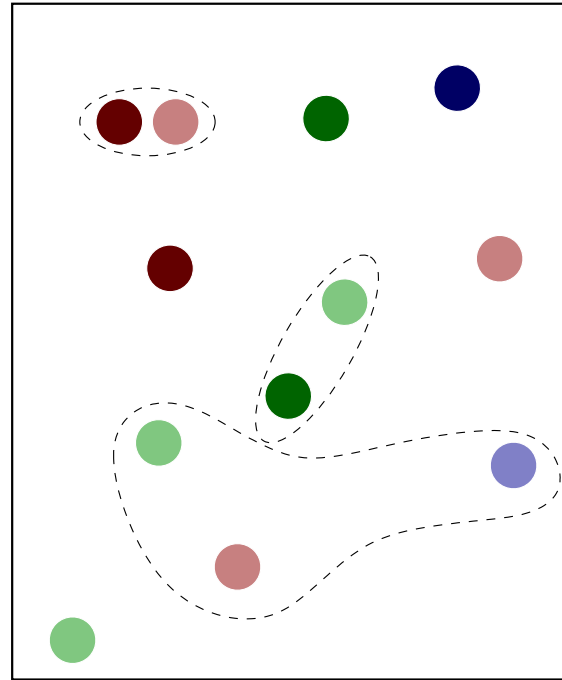
anti-screening $\epsilon < 1$

$$\beta(g) = -\frac{\partial g}{\partial \log(r)} = \frac{g^3}{(4\pi)^2} \left\{ \frac{1}{3} - 4 \right\} N_c < 0$$

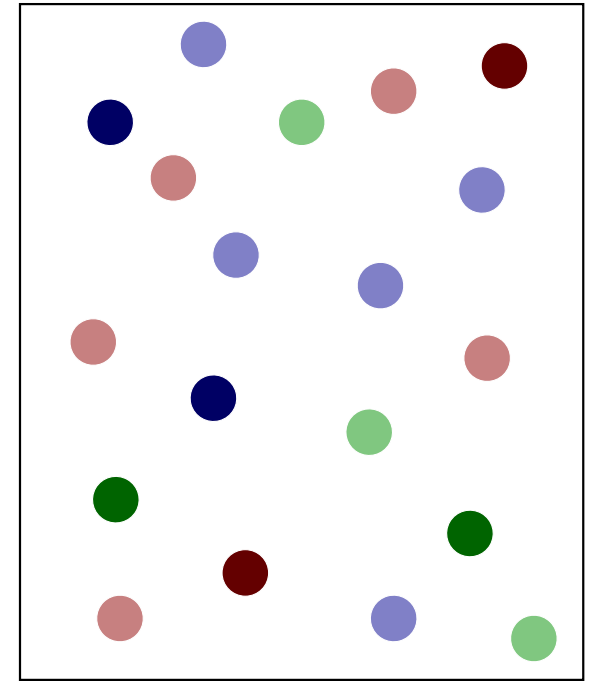
From hadrons to quarks



weakly coupled
hadron gas



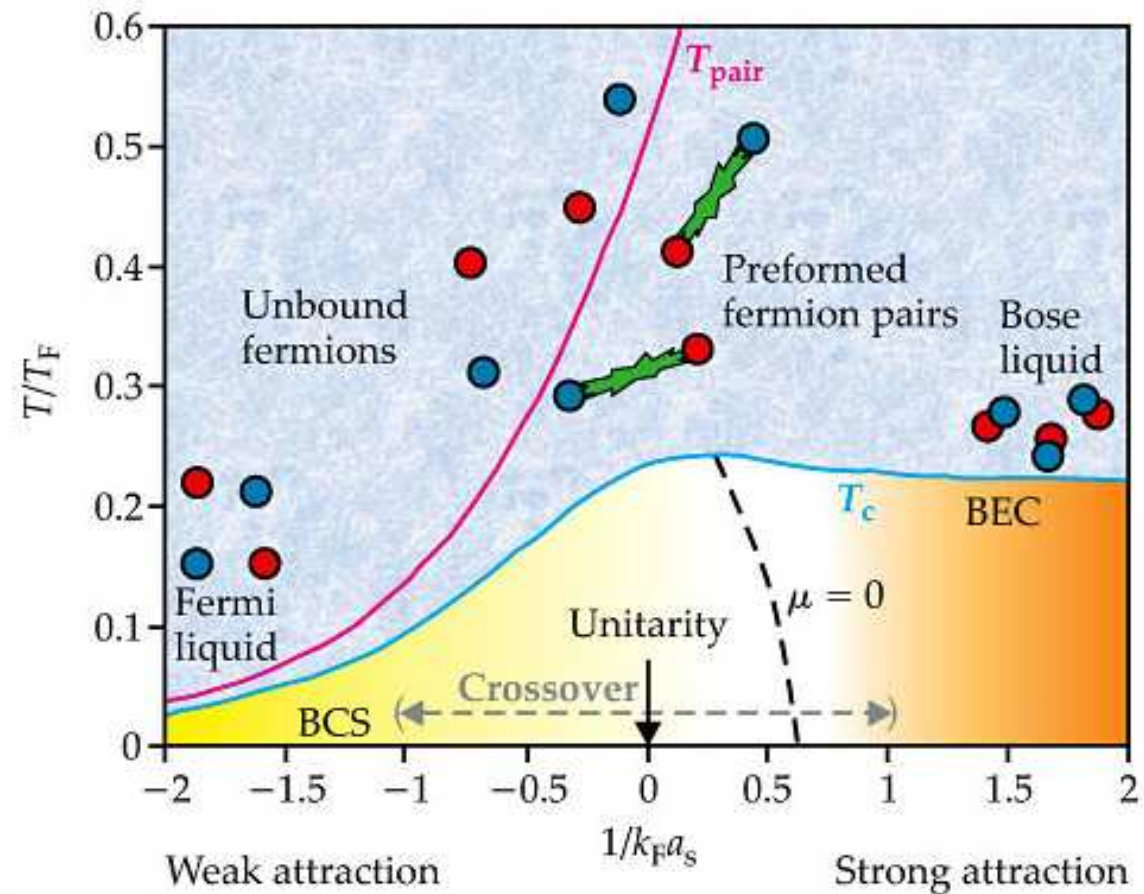
strongly correlated
fluid



weakly coupled
quark gluon plasma

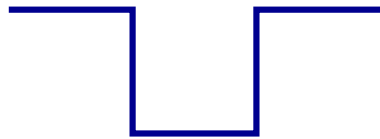
II. Dilute Fermi gas: BCS-BEC crossover

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_0 + \frac{\nabla^2}{2M} \right) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2$$

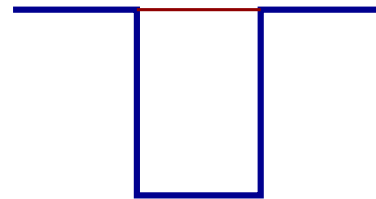


Unitarity limit

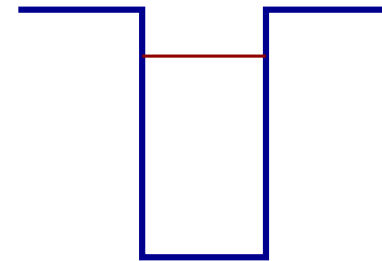
Consider simple square well potential



$$a < 0$$



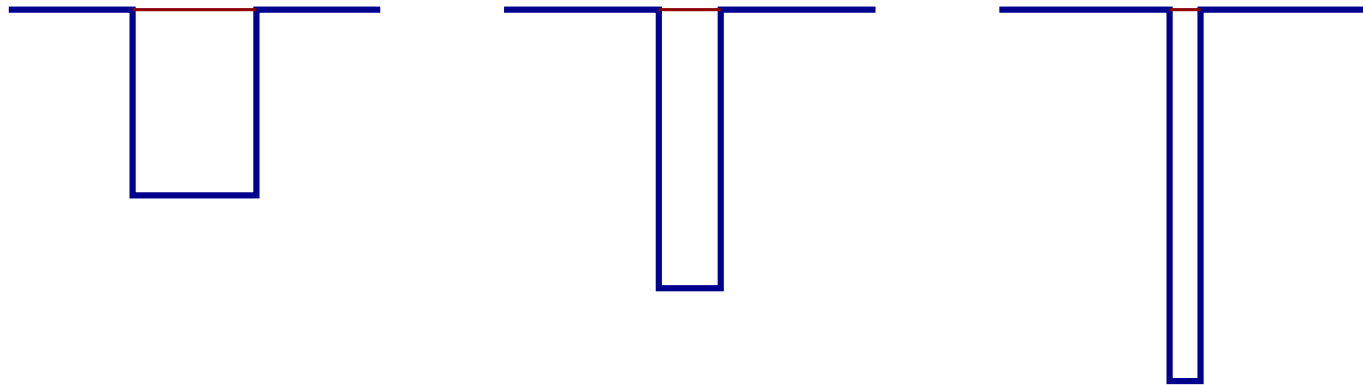
$$a = \infty, \epsilon_B = 0$$



$$a > 0, \epsilon_B > 0$$

Unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$



Universal relations

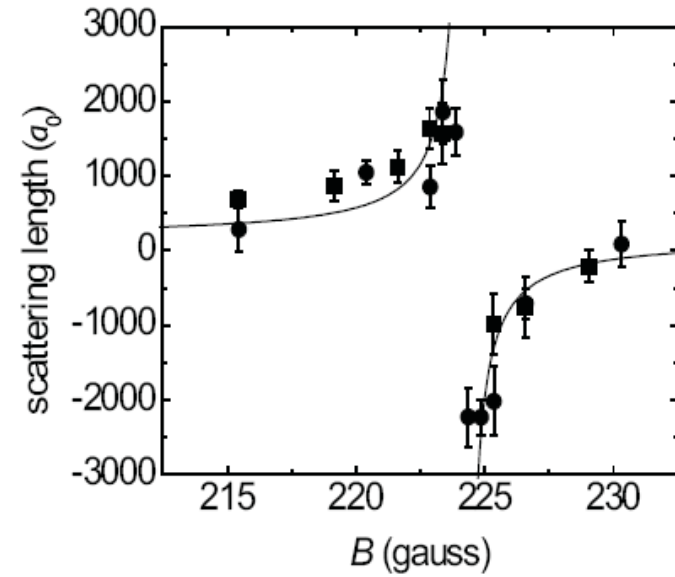
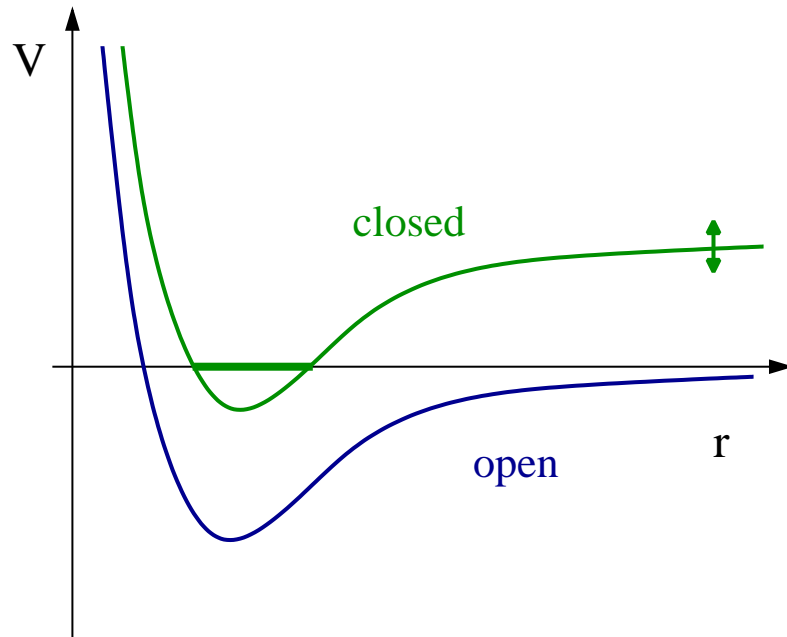
$$\mathcal{T} = \frac{1}{ik + 1/a}$$

$$\epsilon_B = \frac{1}{2ma^2}$$

$$\psi_B \sim \frac{1}{\sqrt{ar}} \exp(-r/a)$$

Feshbach resonances

Atomic gas with two spin states: “↑” and “↓”

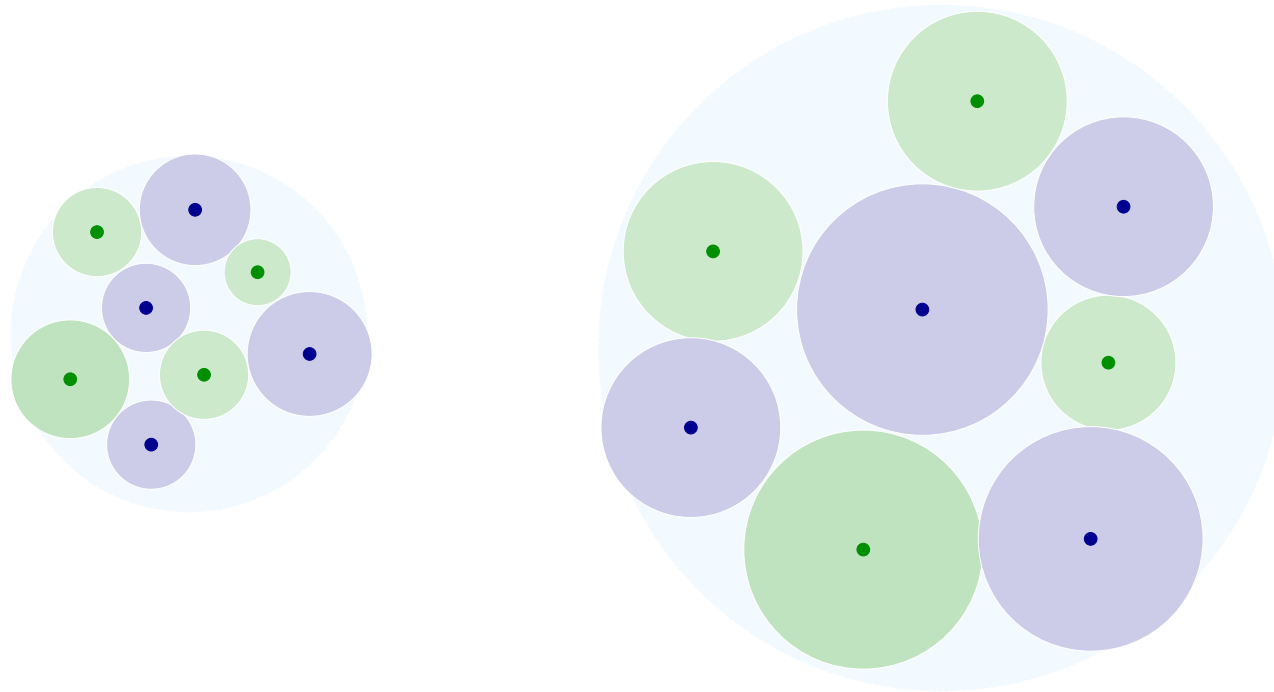


Feshbach resonance

$$a(B) = a_0 \left(1 + \frac{\Delta}{B - B_0} \right)$$

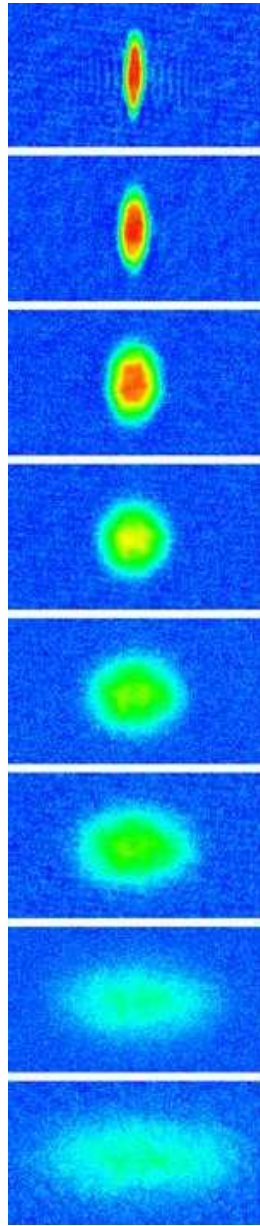
Universal fluid dynamics

Many body system: Effective cross section $\sigma_{tr} \sim n^{-2/3}$ (or $\sigma_{tr} \sim \lambda^2$)

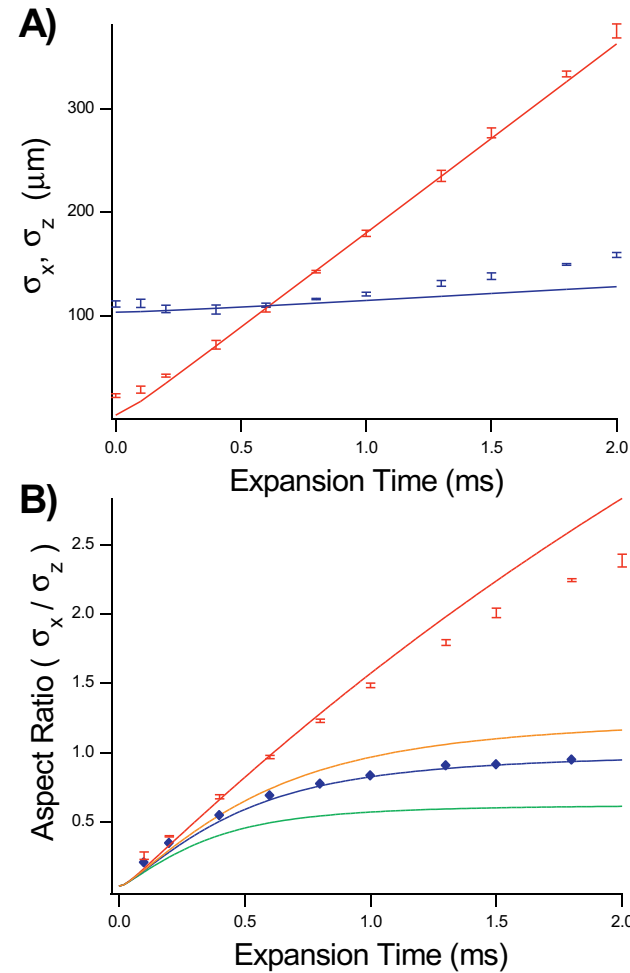


Systems remains hydrodynamic despite expansion

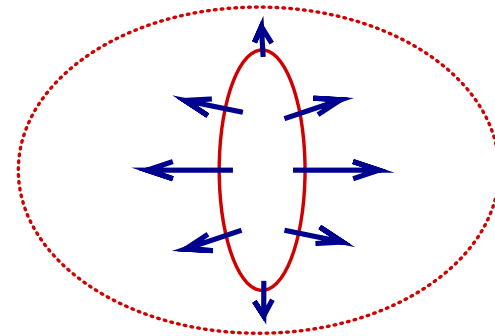
III. Almost ideal fluid dynamics (cold Fermi gas)



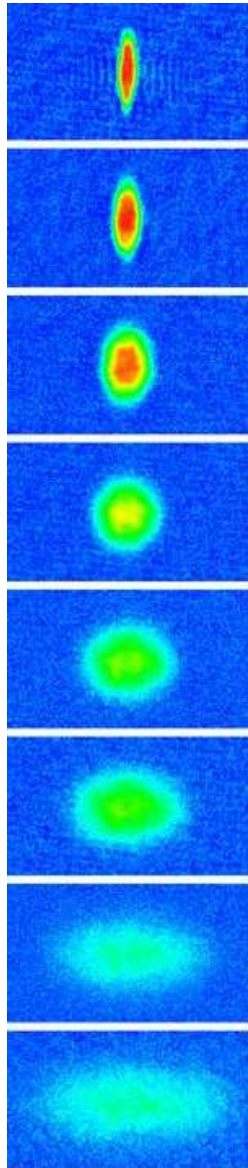
O'Hara et al. (2002)



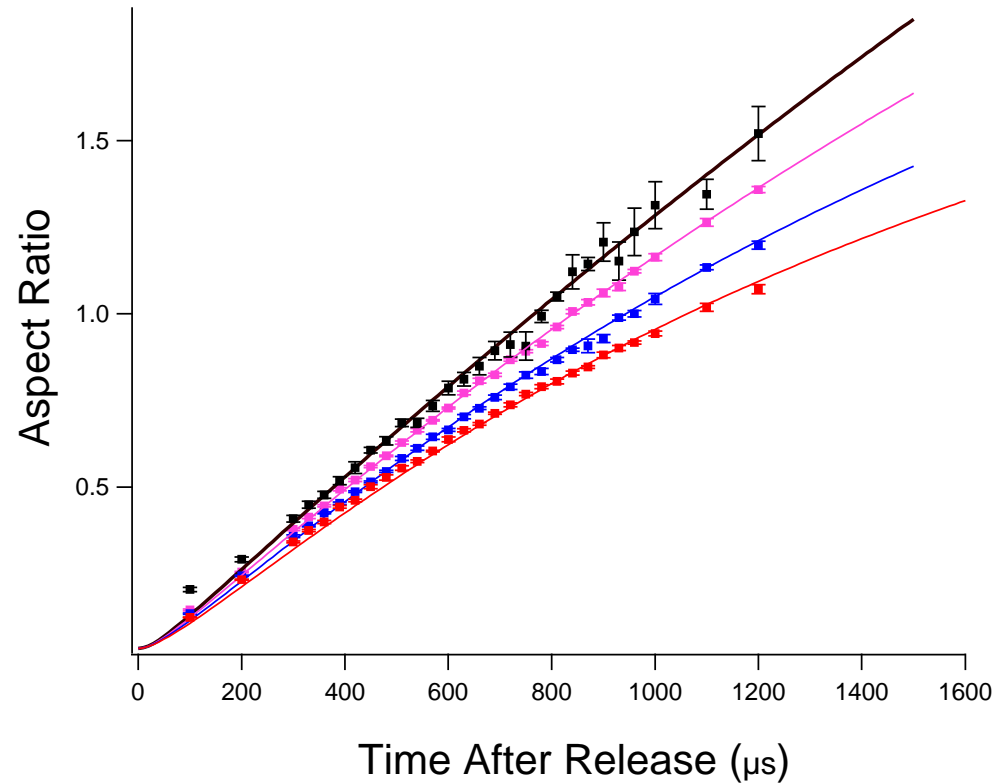
Hydrodynamic
expansion converts
coordinate space
anisotropy
to momentum space
anisotropy



Elliptic flow: High T limit



$$\text{Quantum viscosity } \eta = \eta_0 \frac{(mT)^{3/2}}{\hbar^2}$$



$$\eta = \eta_0 (mT)^{3/2}$$

$$\tau_R = \eta / P$$

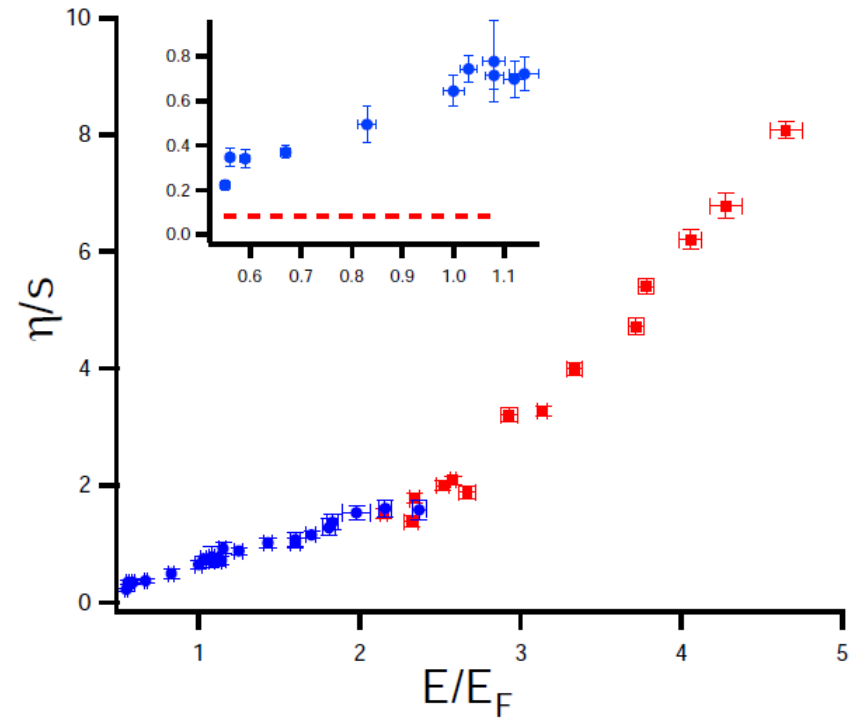
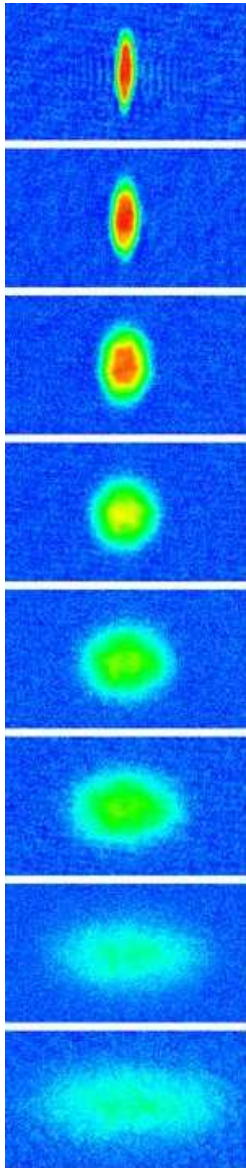
Cao et al., Science (2010)

$$\text{fit: } \eta_0 = 0.33 \pm 0.04$$

$$\text{theory: } \eta_0 = \frac{15}{32\sqrt{\pi}} = 0.26$$

Viscosity to entropy density ratio

consider both collective modes (low T)
and elliptic flow (high T)



Cao et al., Science (2010)

$$\eta/s \leq 0.4$$

IV. Elliptic Flow (QGP)

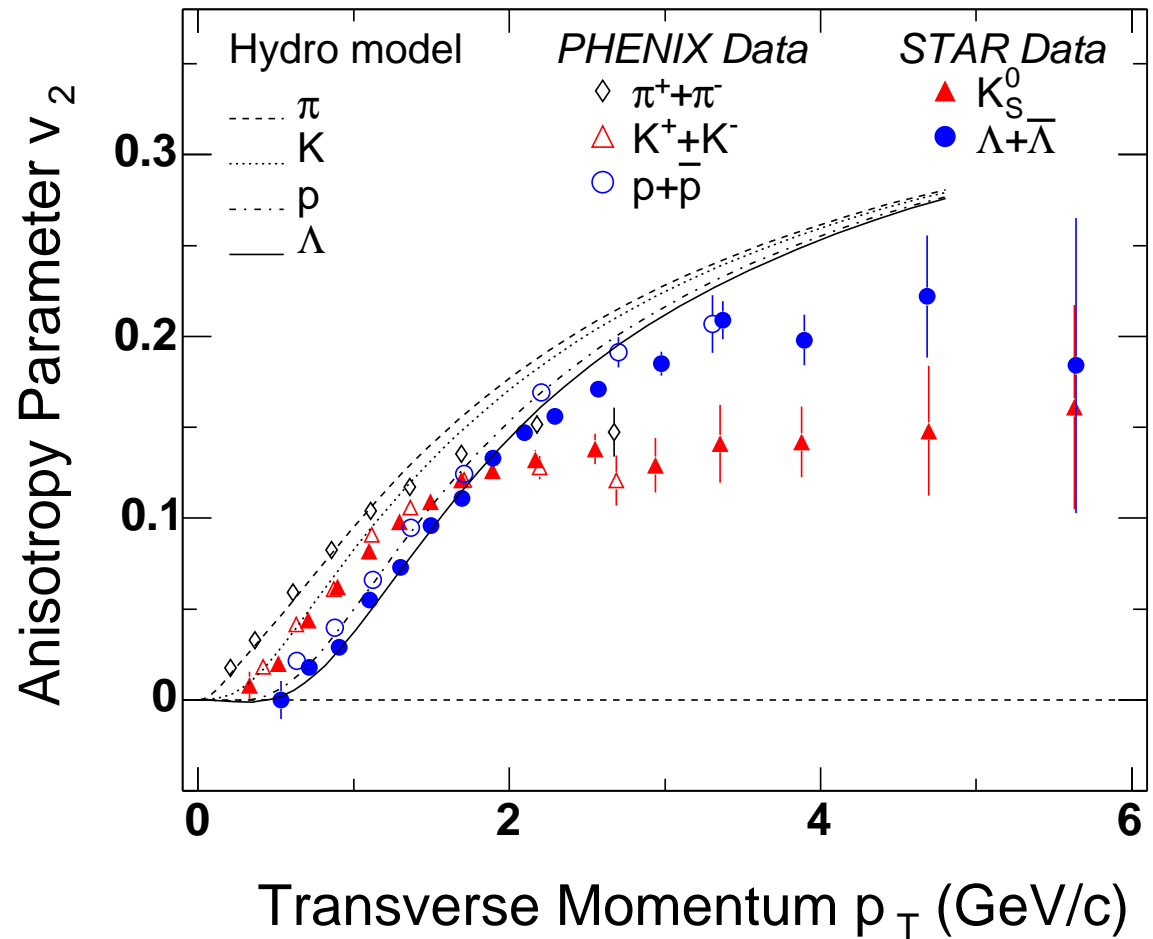
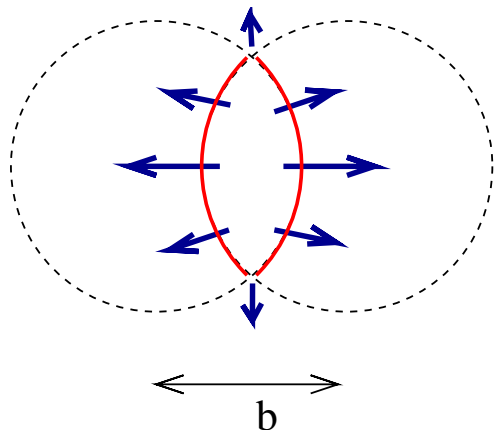
Hydrodynamic expansion converts

coordinate space

anisotropy

to momentum space

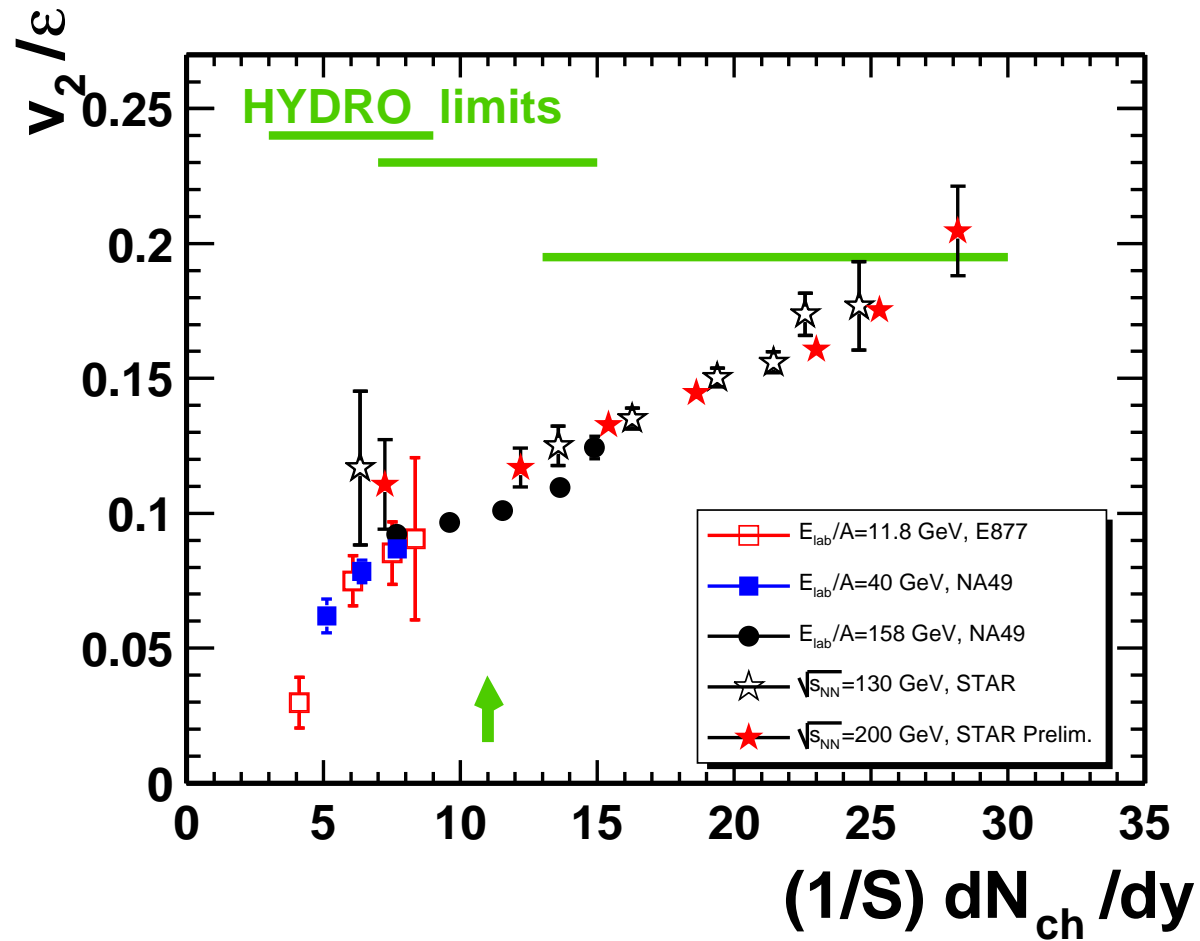
anisotropy



source: U. Heinz (2005)

$$p_0 \left. \frac{dN}{d^3p} \right|_{p_z=0} = v_0(p_\perp) (1 + 2v_2(p_\perp) \cos(2\phi) + \dots)$$

Elliptic flow: initial entropy scaling



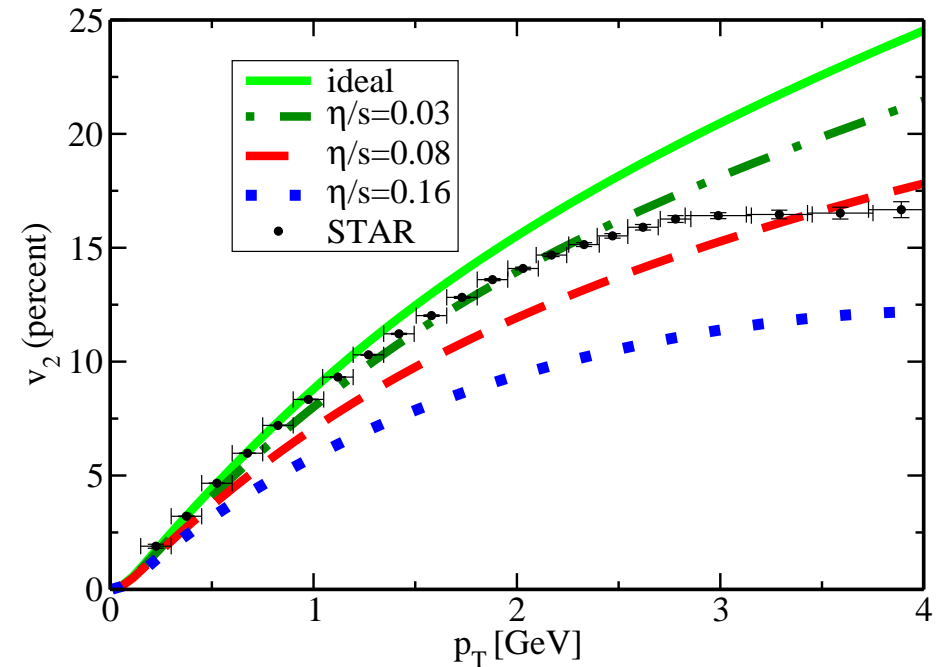
source: U. Heinz (2005)

Viscosity and Elliptic Flow

Viscous correction to v_2 (blast wave model)

$$\frac{\delta v_2}{v_2} = -\frac{1}{3} \frac{1}{\tau_f T_f} \left(\frac{\eta}{s}\right) \left(\frac{p_\perp}{T_f}\right)^2$$

Grows with p_\perp , decreases with system size



Romatschke (2007), Teaney (2003)

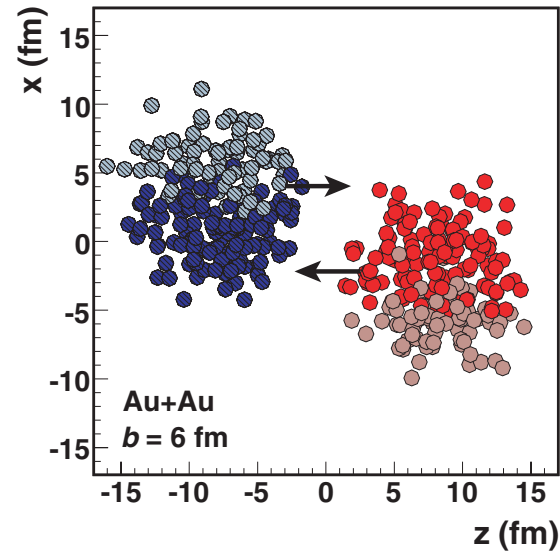
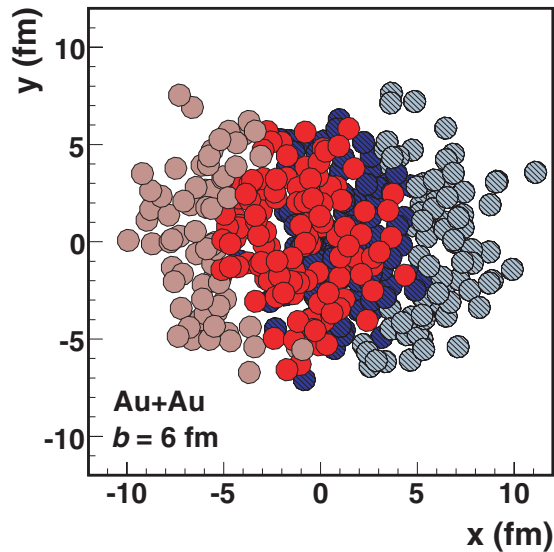
Many details: Dependence on initial conditions, freeze out, etc.

conservative bound

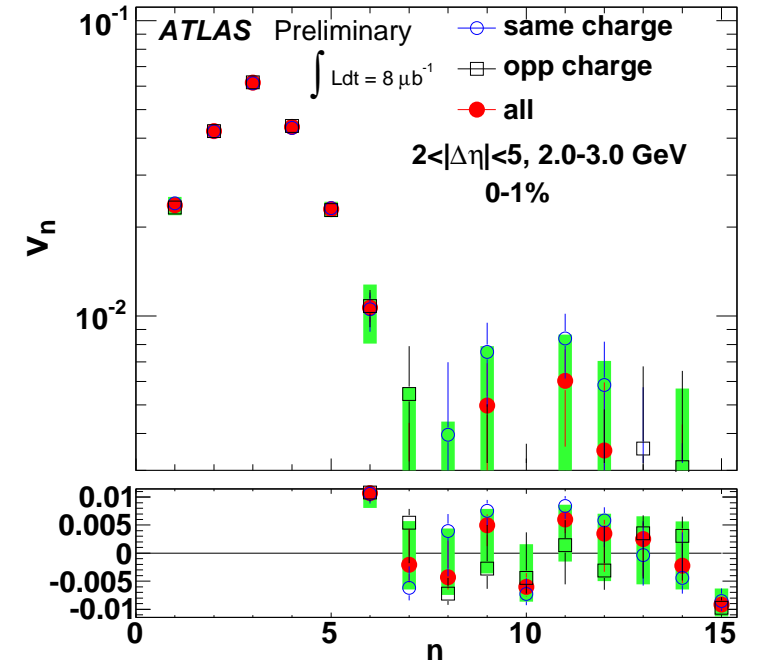
$$\frac{\eta}{s} < 0.25$$

Higher moments of flow

Hydro converts moments of initial deformation to moments of flow



Glauber MC (P. Steinberg)

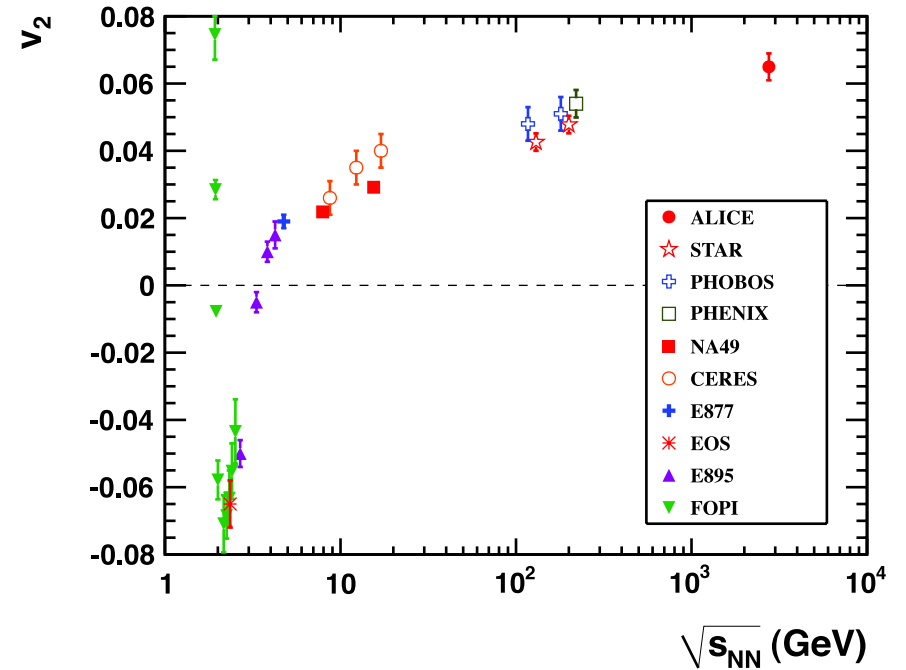
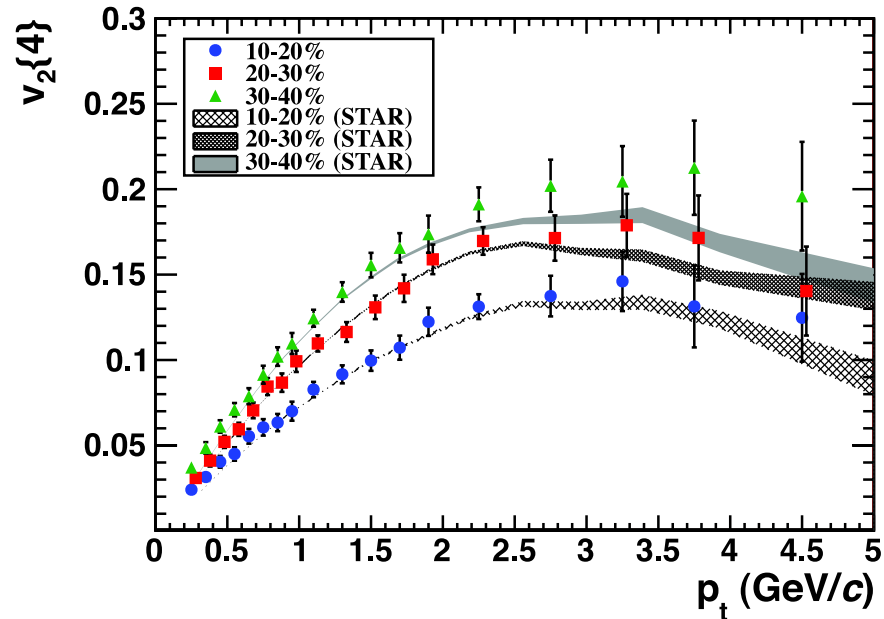


Atlas (J. Jia, QM 2011)

Glauber predicts flat initial spectrum ($n \geq 3$). Observed flow spectrum consistent with sound attenuation

$$\delta T^{\mu\nu}(t) = \exp\left(-\frac{2}{3} \frac{\eta}{s} \frac{k^2 t}{T}\right) \delta T^{\mu\nu}(0)$$

Alice flow: Nearly perfect fluidity at the LHC?



Differential v_2 equal to RHIC

Integrated v_2 somewhat high

accidental cancellation? freezeout?

mean p_T increase? acceptance?

The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of black holes in 5 (and more) dimensions.