In Search of the Perfect Fluid

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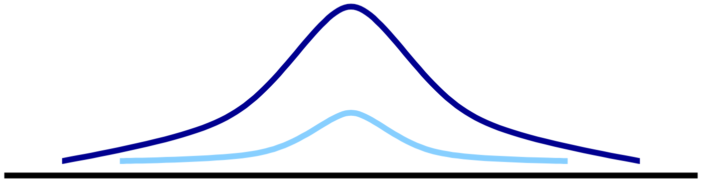
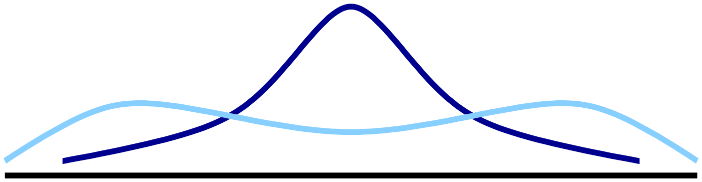
Fluids: Gases, liquids, plasmas, . . .

Hydrodynamics: Long-wavelength, low-frequency dynamics of conserved or spontaneously broken symmetry variables.



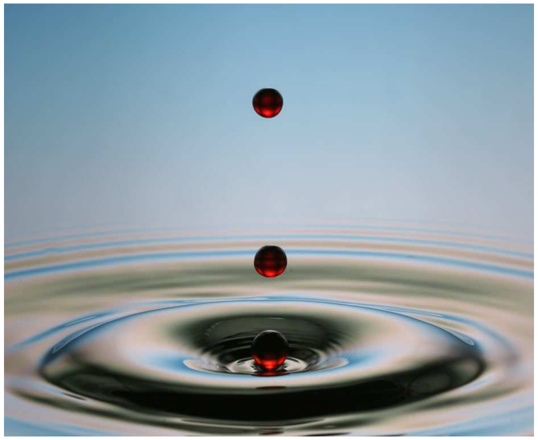
$\tau \sim \tau_{micro}$



$\tau \sim \lambda$

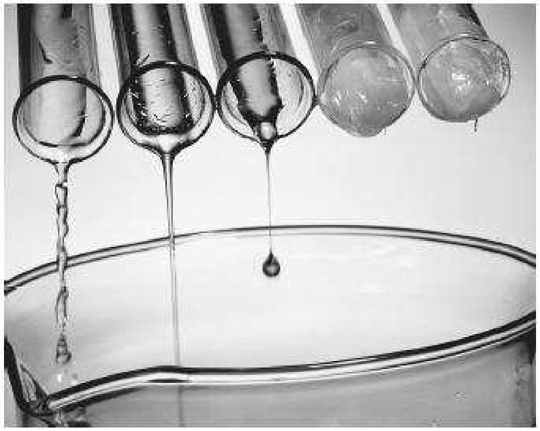
Historically: Water

$(\rho, \epsilon, \vec{\pi})$



Simple non-relativistic fluid

Simple fluid: Conservation laws for mass, energy, momentum



$$\frac{\partial \rho}{\partial t} + \vec{\nabla}(\rho \vec{v}) = 0$$

$$\frac{\partial \epsilon}{\partial t} + \vec{\nabla} \vec{j}^{\,\epsilon} = 0$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j} \Pi_{ij} = 0$$

Constitutive relations: Energy momentum tensor

$$\Pi_{ij} = P\delta_{ij} + \rho v_i v_j + \eta \left(\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k \right) + O(\partial^2)$$

reactive dissipative 2nd order

Expansion $\Pi^0_{ij} \gg \delta\Pi^1_{ij} \gg \delta\Pi^2_{ij}$

Regime of applicability

Expansion parameter $Re^{-1} = \frac{\eta(\partial v)}{\rho v^2} = \frac{\eta}{\rho L v} \ll 1$

$$Re \quad = \quad \underset{\text{fluid property}}{\frac{\hbar n}{\eta}} \quad \times \quad \underset{\text{flow property}}{\frac{m v L}{\hbar}}$$

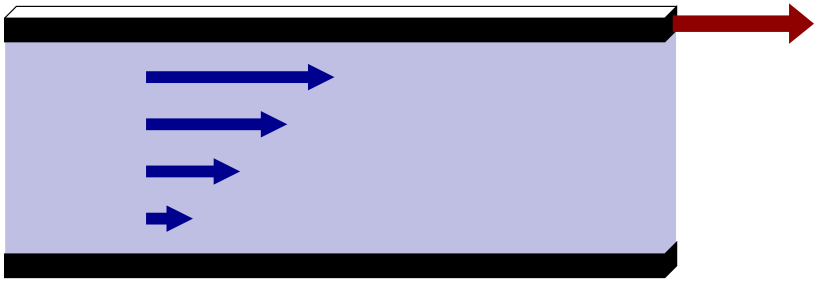
Kinetic theory estimate: $\eta \sim n p l_{mfp}$

$$Re^{-1} = \frac{v}{c_s} Kn \qquad\qquad Kn = \frac{l_{mfp}}{L}$$

expansion parameter $Kn \ll 1$

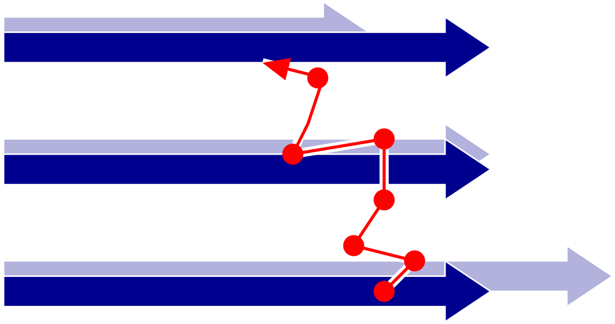
Shear viscosity

Viscosity determines shear stress ("friction") in fluid flow



$$F = A\,\eta\,\frac{\partial v_x}{\partial y}$$

Kinetic theory: conserved quantities carried by quasi-particles



$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p + \vec{F}\cdot\vec{\nabla}_p f_p = C[f_p]$$

$$\eta \sim \frac{1}{3} n\,\bar{p}\,l_{mfp}$$

Dilute, weakly interacting gas: $l_{mfp} \sim 1/(n\sigma)$

$$\eta \sim \frac{1}{3}\frac{\bar{p}}{\sigma}$$

independent of density!

Shear viscosity

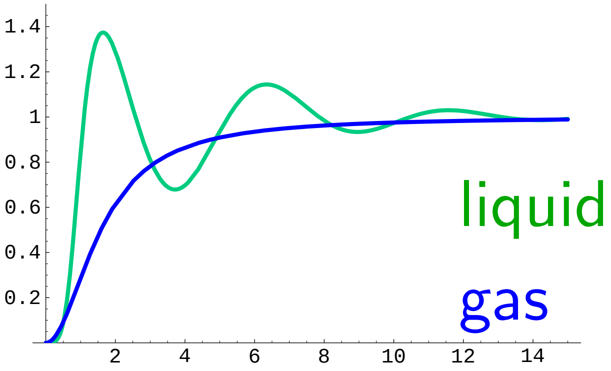
non-interacting gas $(\sigma \rightarrow 0)$: $\boxed{\eta \rightarrow \infty}$

non-interacting and hydro limit $(T \rightarrow \infty)$ limit do not commute

strongly interacting gas: $\boxed{\frac{\eta}{n} \sim \bar{p} l_{mfp} \geq \hbar}$

but: kinetic theory not reliable!

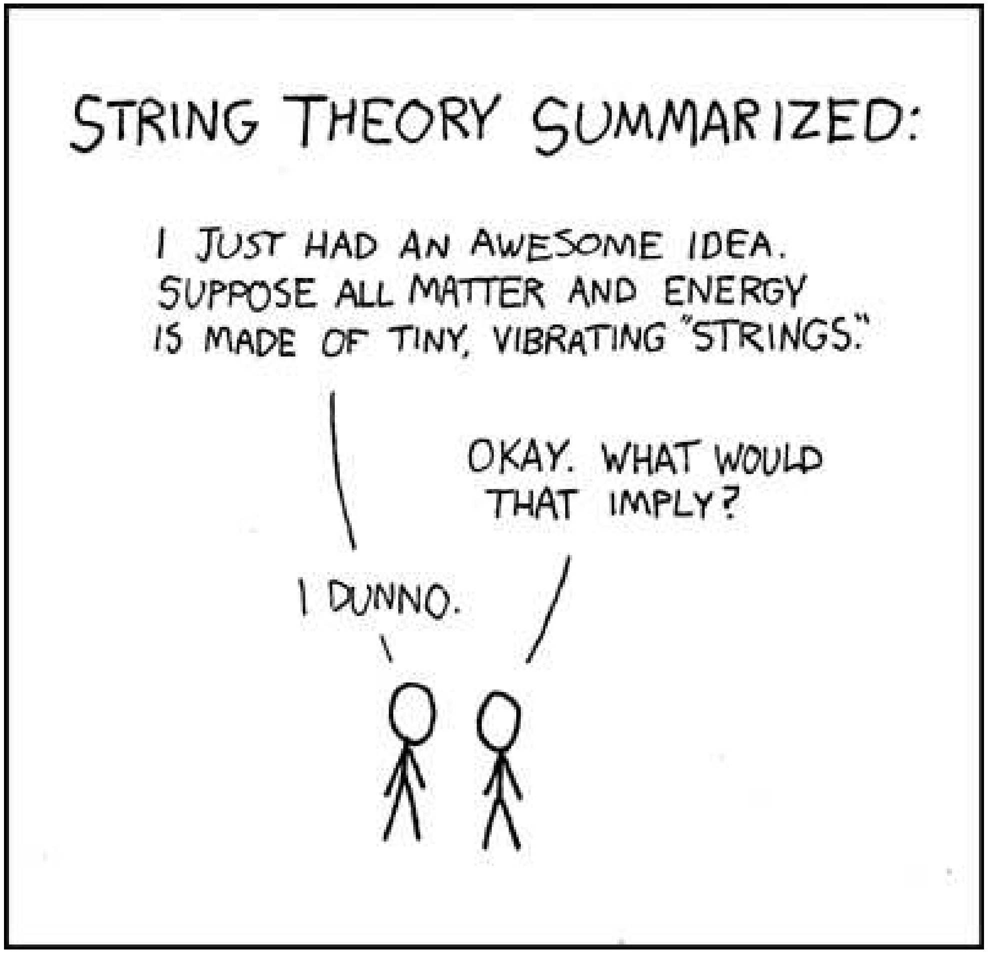
what happens if the gas condenses into a liquid?



Eyring, Frenkel:

$$\boxed{\eta \simeq hn \exp(E/T) \geq hn}$$

And now for something completely different . . .



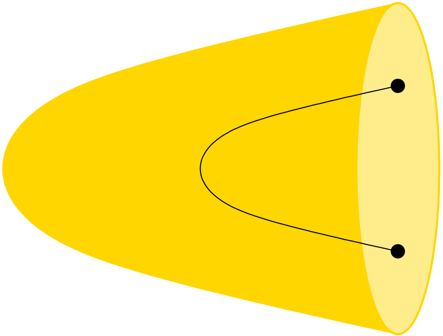
Gauge theory at strong coupling: Holographic duality

The AdS/CFT duality relates

large N_c (conformal) gauge theory in 4 dimensions	$\Leftrightarrow$	string theory on 5 dimensional Anti-de Sitter space $\times S_5$
correlation fcts of gauge invariant operators	$\Leftrightarrow$	boundary correlation fcts of AdS fields

$$\langle \exp \int dx \; \phi_0 \mathcal{O} \rangle =$$

$$Z_{string}[\phi(\partial AdS) = \phi_0]$$



The correspondence is simplest at strong coupling $g^2 N_c$

strongly coupled gauge theory $\Leftrightarrow$ classical string theory

Holographic duals: Transport properties

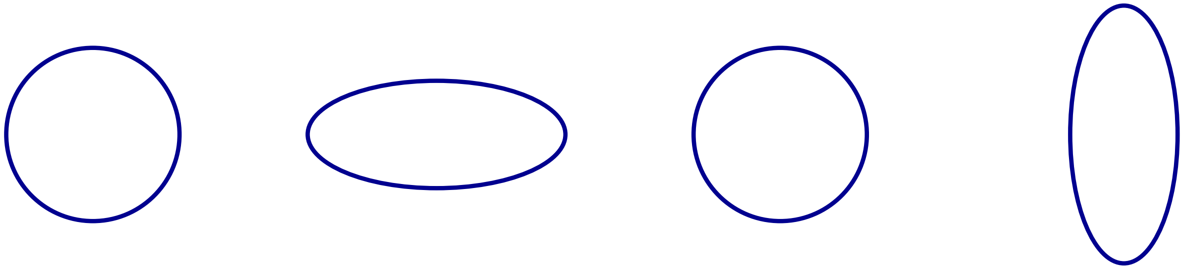
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT temperature $\Leftrightarrow$ Hawking temperature

CFT entropy $\Leftrightarrow$ Hawking-Bekenstein entropy $\sim$ area of event horizon

shear viscosity $\Leftrightarrow$ Graviton absorption cross section $\sim$ area of event horizon

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \qquad g_{\mu\nu} = g^0_{\mu\nu} + \gamma_{\mu\nu}$$



Holographic duals: Transport properties

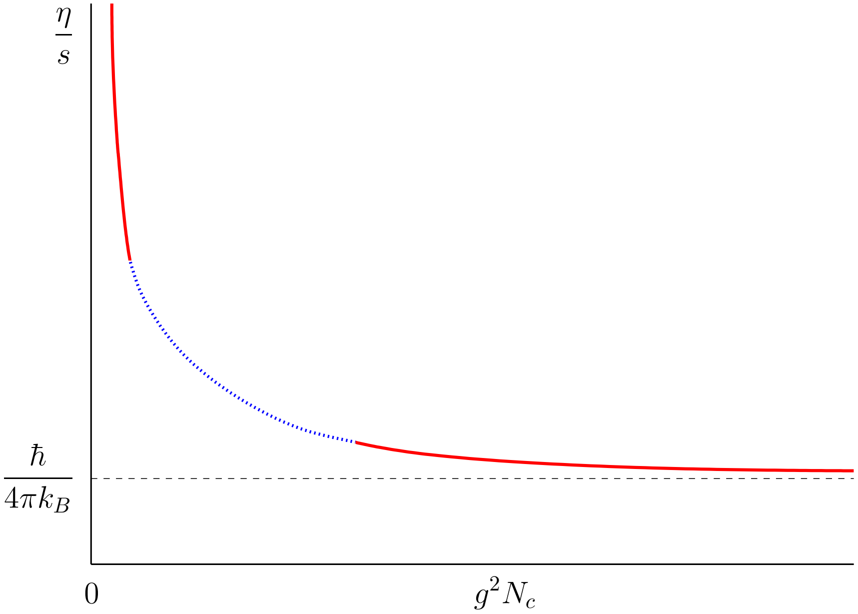
Thermal (conformal) field theory $\equiv AdS_5$ black hole

CFT entropy	$\Leftrightarrow$	Hawking-Bekenstein entropy $\sim$ area of event horizon
shear viscosity	$\Leftrightarrow$	Graviton absorption cross section $\sim$ area of event horizon

Strong coupling limit

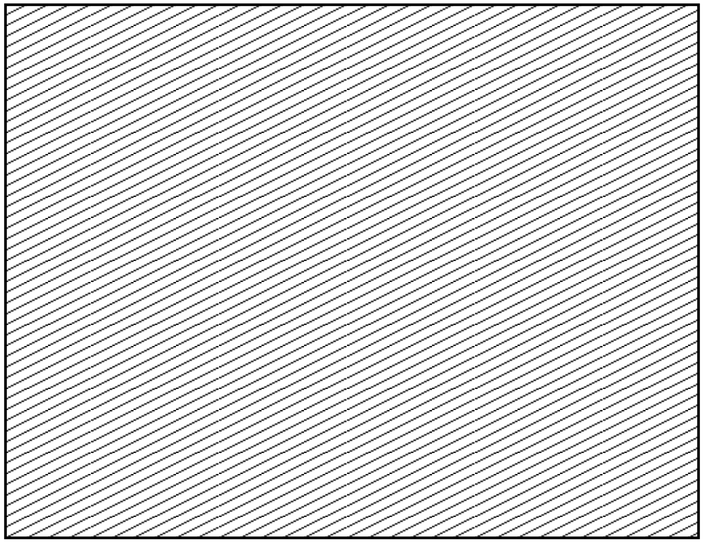
$$\boxed{\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}}$$

Son and Starinets (2001)

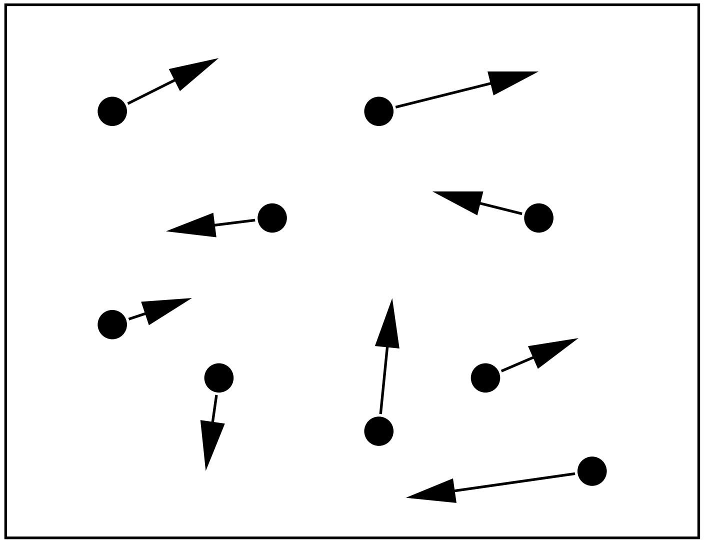


Strong coupling limit universal? Provides lower bound for all theories?

Kinetics vs no-kinetics



AdS/CFT low viscosity goo



pQCD kinetic plasma

Effective theories for fluids (Here: Weak coupling QCD)

$$\mathcal{L} = \bar{q}_f(i\not{D} - m_f)q_f - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}$$

$$\frac{\partial f_p}{\partial t} + \vec{v}\cdot\vec{\nabla}_x f_p = C[f_p] \qquad (\omega < T)$$

$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \qquad (\omega < g^4 T)$$

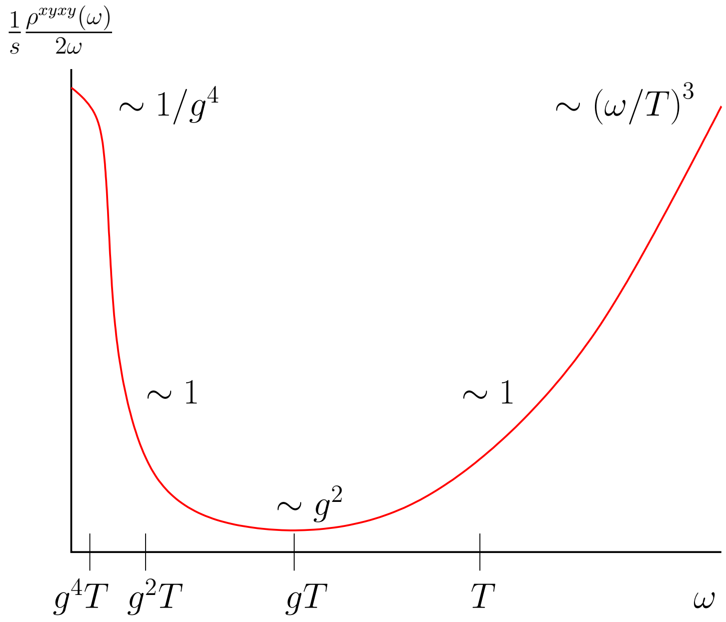
Effective theories (Strong coupling)

$$\mathcal{L} = \bar{\lambda}(i\sigma \cdot D)\lambda - \frac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu} + \ldots \Leftrightarrow S = \frac{1}{2\kappa_5^2}\int d^5x\sqrt{-g}\mathcal{R} + \ldots$$

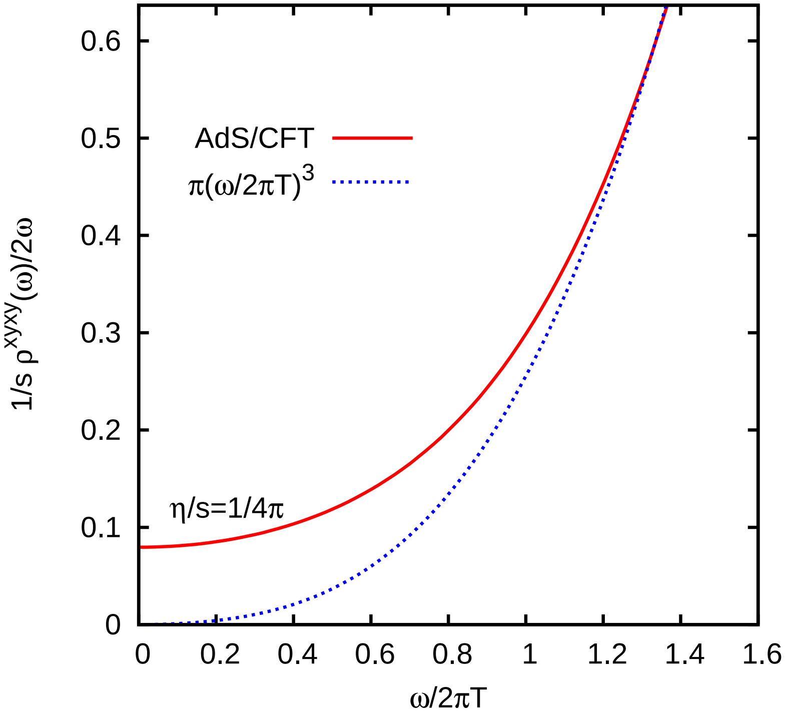
$$\frac{\partial}{\partial t}(\rho v_i) + \frac{\partial}{\partial x_j}\Pi_{ij} = 0 \quad (\omega < T)$$

Kinetics vs no-kinetics

Spectral function $\rho(\omega) = \mathrm{Im}G_R(\omega, 0)$ associated with T_{xy}



weak coupling QCD



strong coupling AdS/CFT

transport peak vs no transport peak

Perfect Fluids: How to be a contender?

Bound is quantum mechanical

need quantum fluids

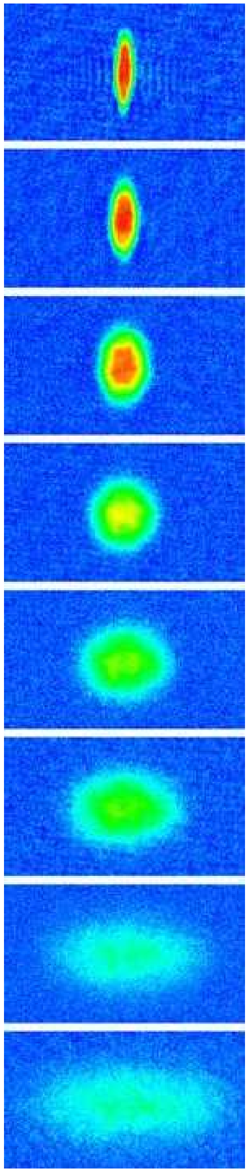
Bound is incompatible with weak coupling and kinetic theory

strong interactions, no quasi-particles

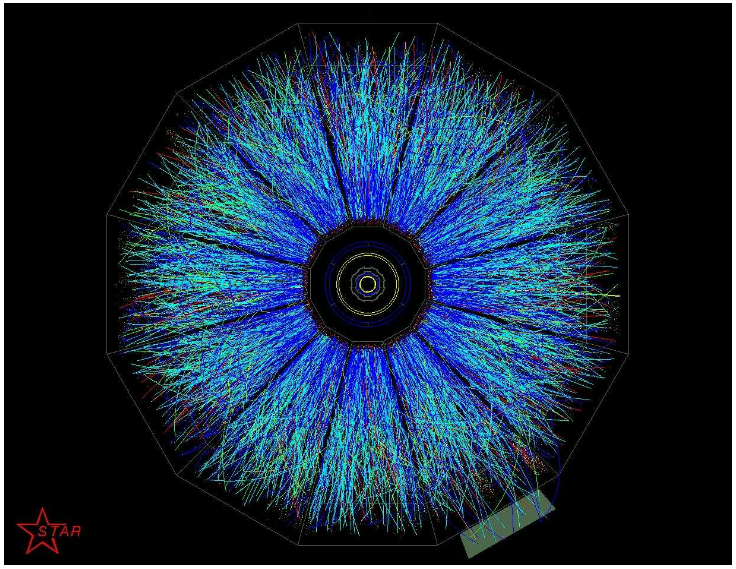
Model system has conformal invariance (essential?)

(Almost) scale invariant systems

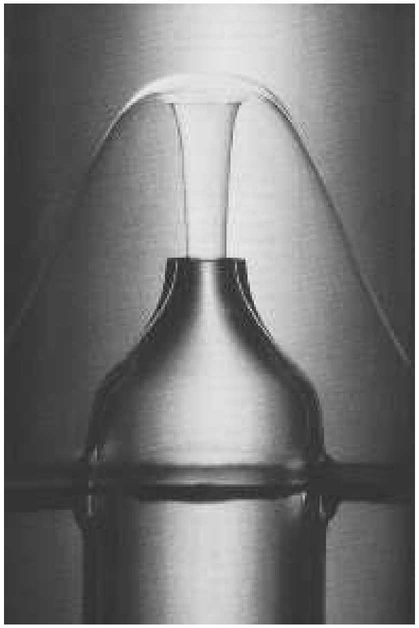
Perfect Fluids: The contenders



Trapped Atoms
(T=0.1 neV)

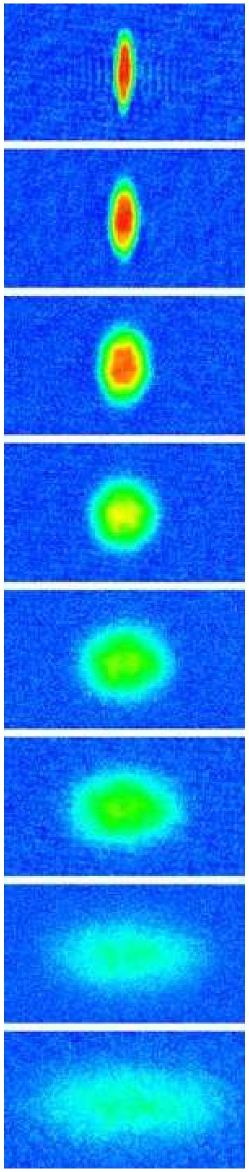


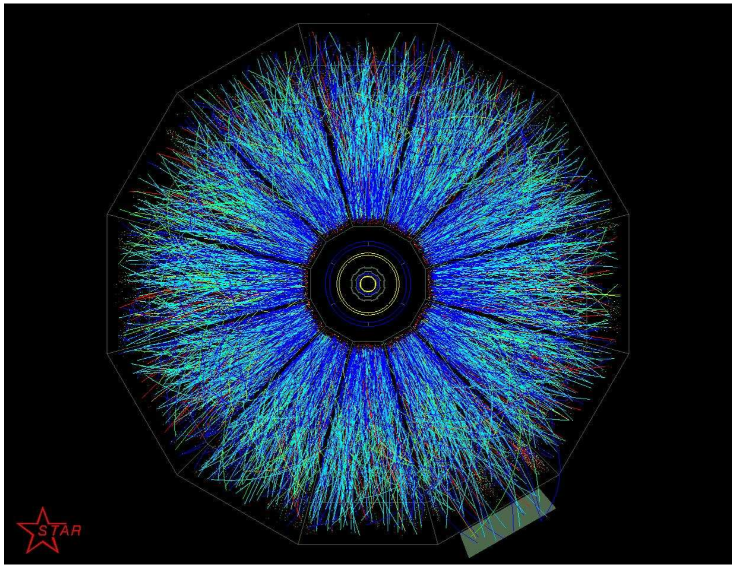
QGP (T=180 MeV)



Liquid Helium
(T=0.1 meV)

Perfect Fluids: The contenders

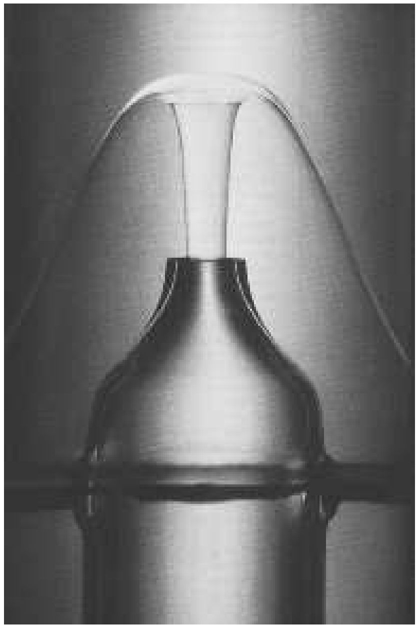




QGP $\eta = 5 \cdot 10^{11} Pa \cdot s$

Trapped Atoms

$\eta = 1.7 \cdot 10^{-15} Pa \cdot s$



Liquid Helium

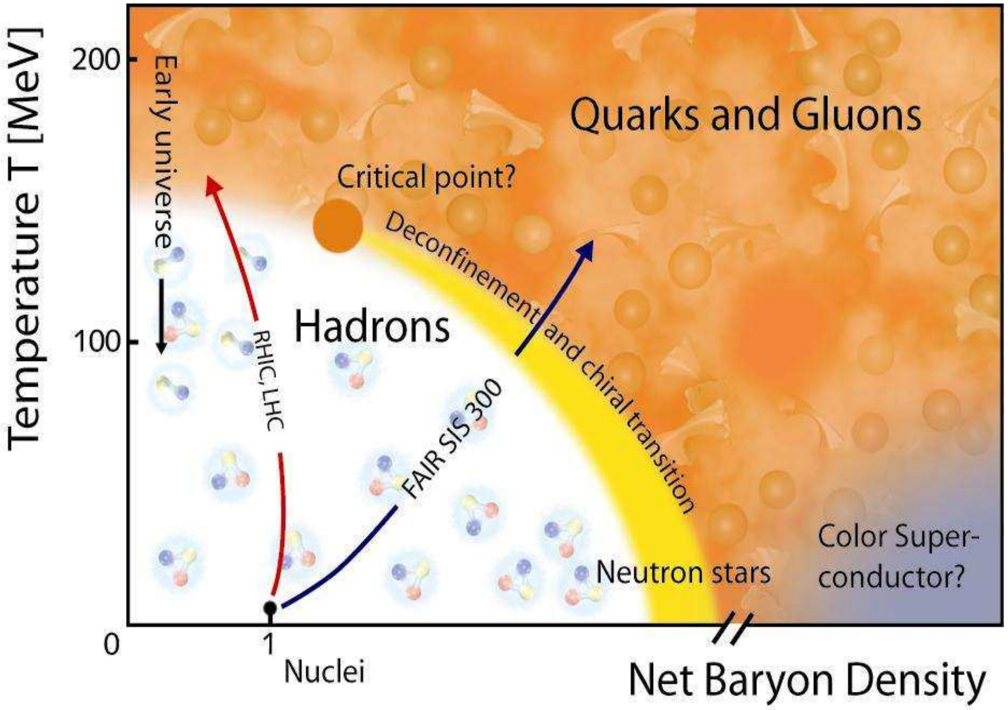
$\eta = 1.7 \cdot 10^{-6} Pa \cdot s$

Consider ratios

η/s

QCD and the Quark Gluon Plasma

$$\boxed{\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4g^2} G^a_{\mu\nu} G^a_{\mu\nu}}$$



Quantumchromodynamics (QCD)

Elementary fields: Quarks Gluons

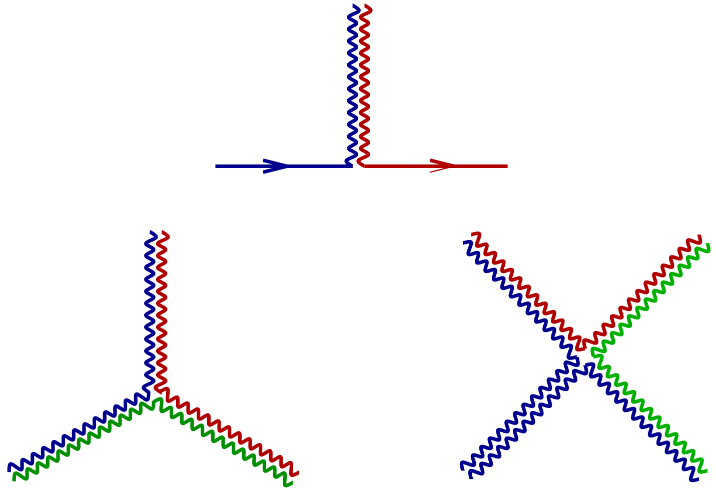
$$(q_\alpha)^a_f \begin{cases} \text{color} & a = 1,\ldots,3 \\ \text{spin} & \alpha = 1,2 \\ \text{flavor} & f = u,d,s,c,b,t \end{cases} \qquad A^a_\mu \begin{cases} \text{color} & a = 1,\ldots,8 \\ \text{spin} & \epsilon^\pm_\mu \end{cases}$$

Dynamics: Generalized Maxwell (Yang-Mills) + Dirac theory

$$\boxed{\mathcal{L} = \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu}}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

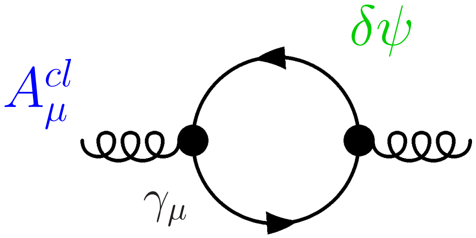
$$i\not{D} q = \gamma^\mu \left(i\partial_\mu + g A^a_\mu t^a \right) q$$



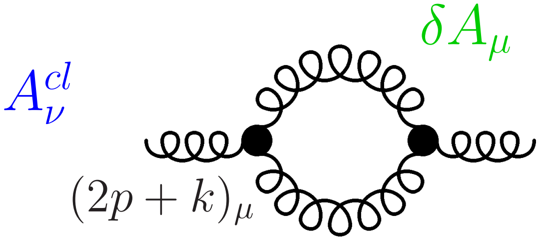
Asymptotic freedom

Modification of Coulomb interaction due to quantum fluctuations

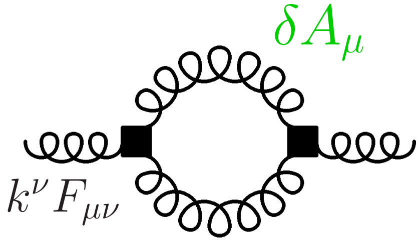
$q\bar{q}$-pairs



electric gluons



magnetic gluons



dielectric $\epsilon > 1$

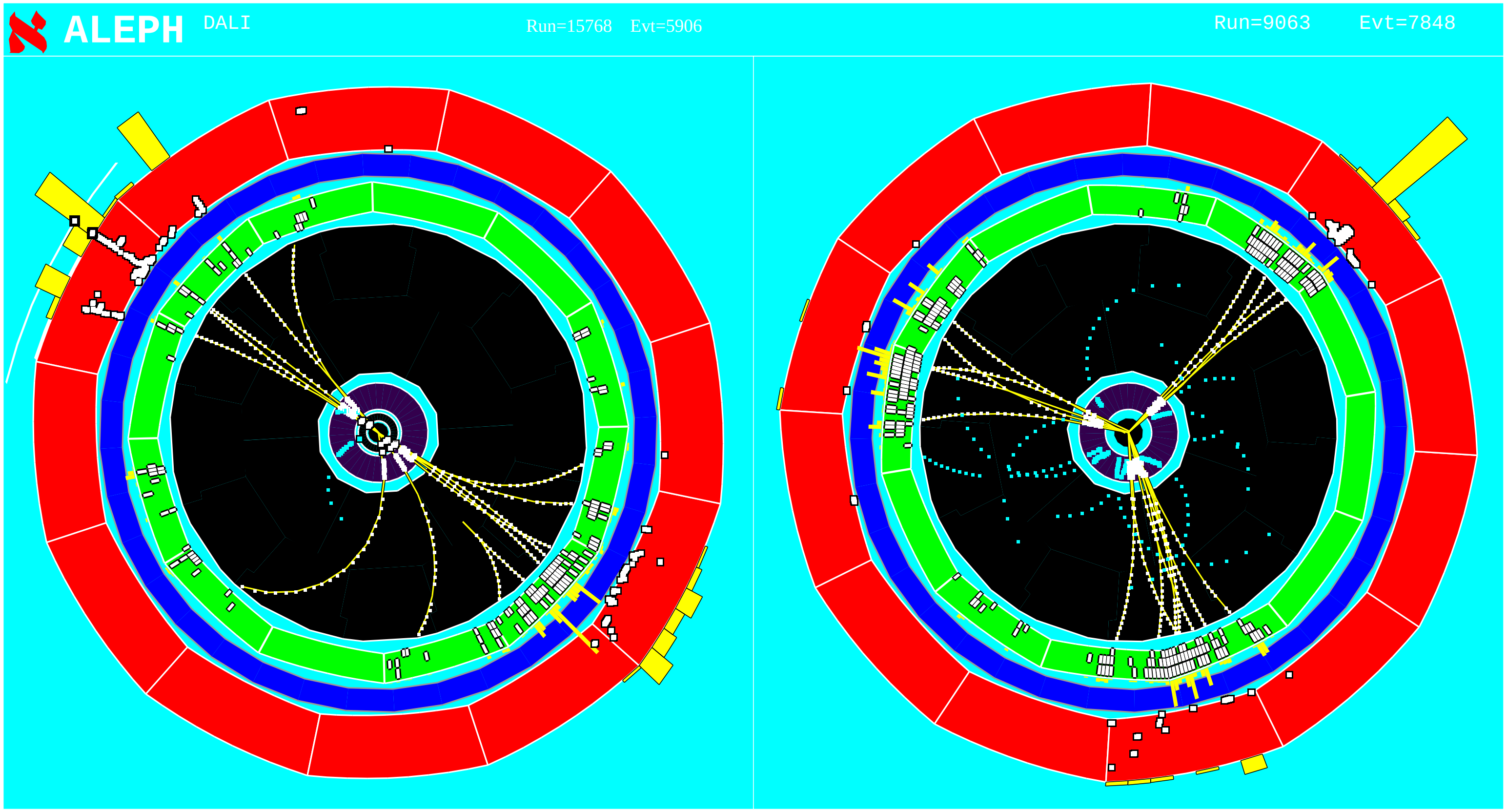
dielectric $\epsilon > 1$

paramagnetic $\mu > 1$

$$\mu\epsilon = 1 \;\Rightarrow\; \epsilon < 1$$

$$\beta(g) = -\frac{\partial g}{\partial \log(r)} = \frac{g^3}{(4\pi)^2}\left\{\left[\frac{1}{3} - 4\right]N_c + \frac{2}{3}N_f\right\} < 0$$

“Seeing” quarks and gluons



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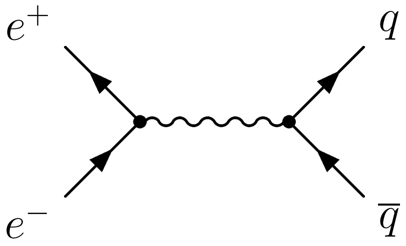
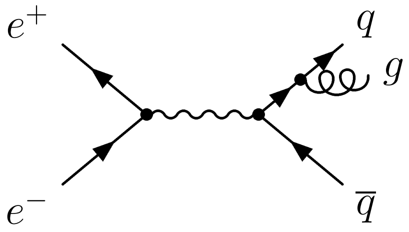
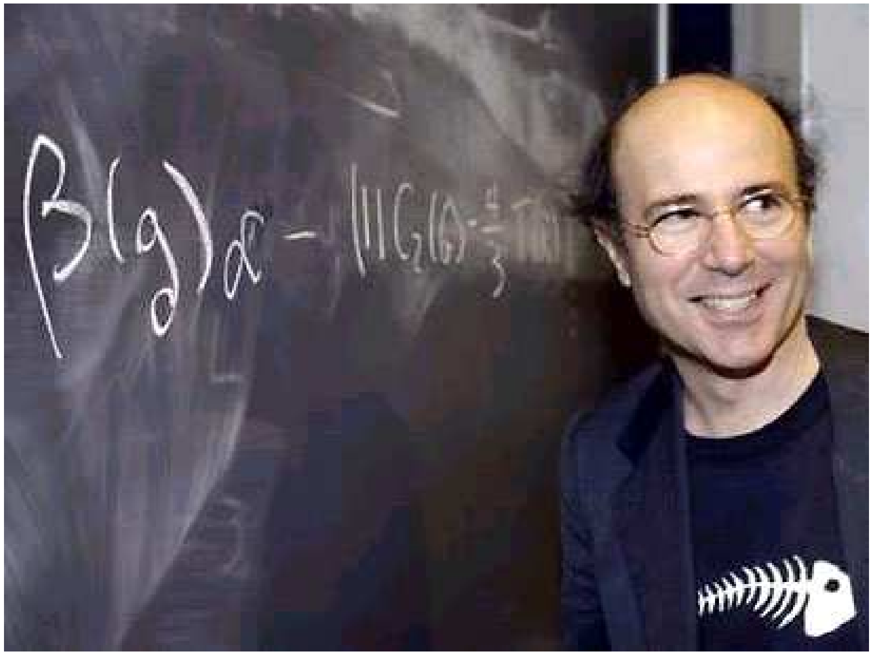
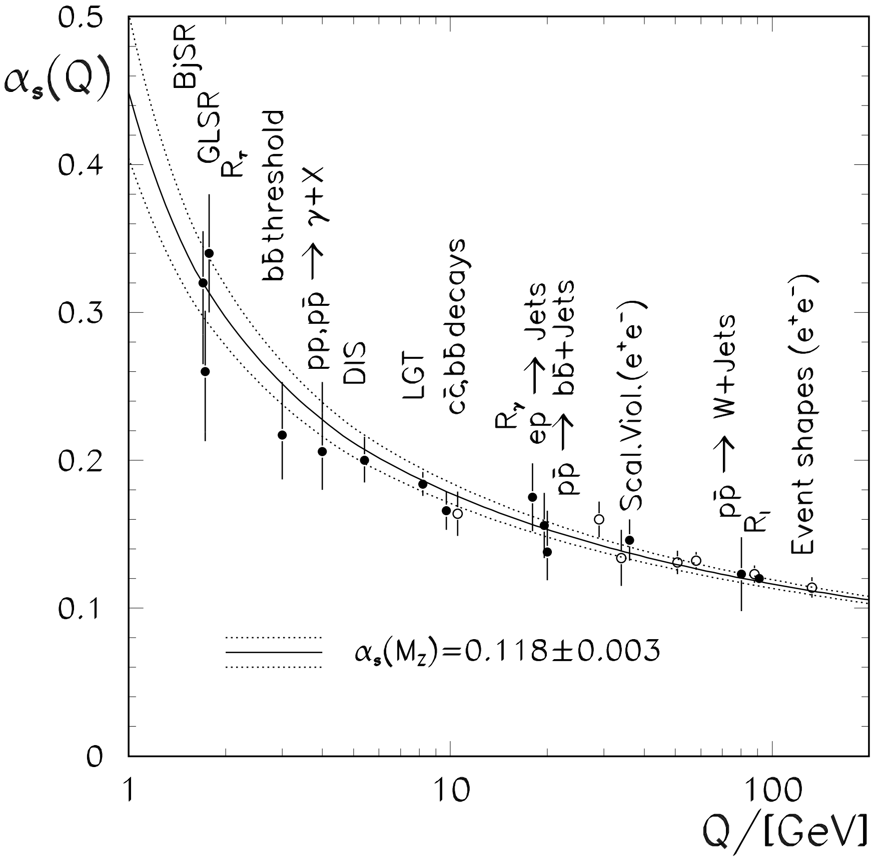




Running coupling constant





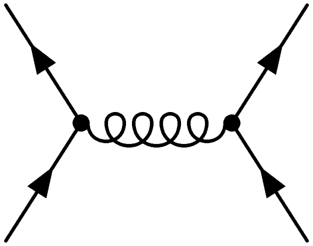
The high T phase: Qualitative argument

High T phase: Weakly interacting gas of quarks and gluons?

typical momenta $p \sim 3T$

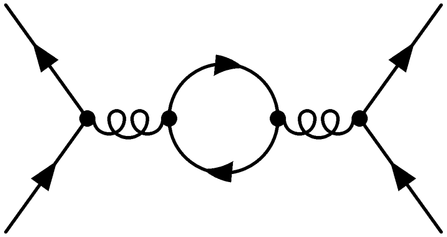
Large angle scattering involves large momentum transfer

effective coupling is small



Small angle scattering is screened (not anti-screened!)

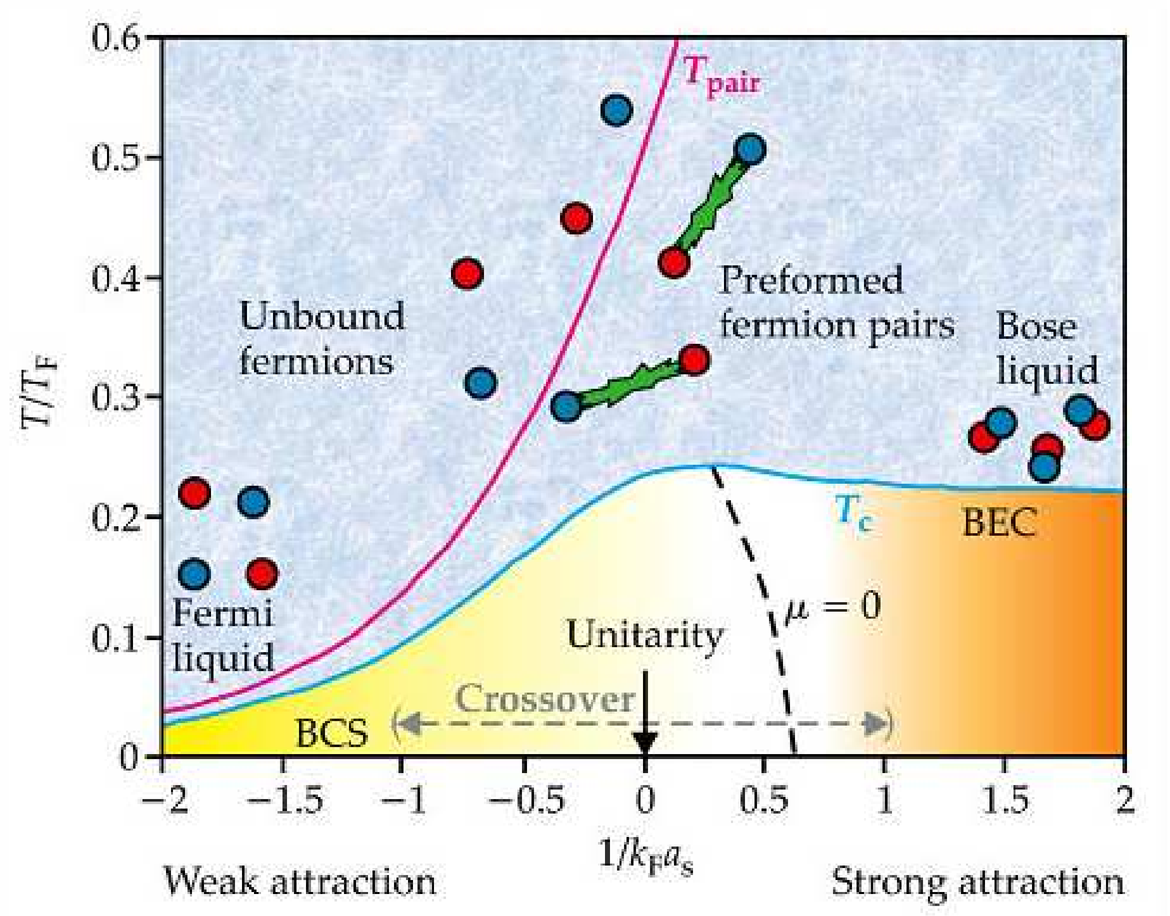
coupling does not become large



Quark Gluon Plasma

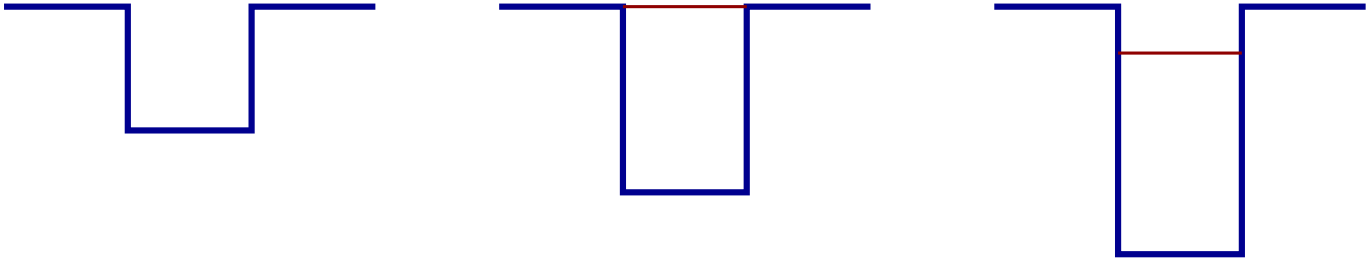
Dilute Fermi gas: BCS-BEC crossover

$$\boxed{\mathcal{L}_{\text{eff}} = \psi^\dagger \Big(i\partial_0 + \frac{\nabla^2}{2M} \Big) \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2}$$



Unitarity limit

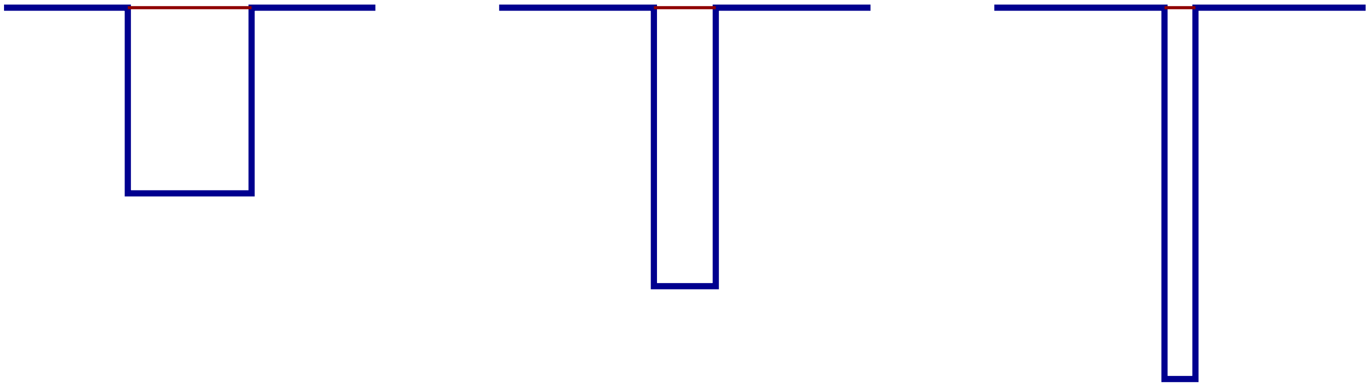
Consider simple square well potential



$a < 0$ $\qquad$ $a = \infty,\ \epsilon_B = 0$ $\qquad$ $a > 0,\ \epsilon_B > 0$

Unitarity limit

Now take the range to zero, keeping $\epsilon_B \simeq 0$

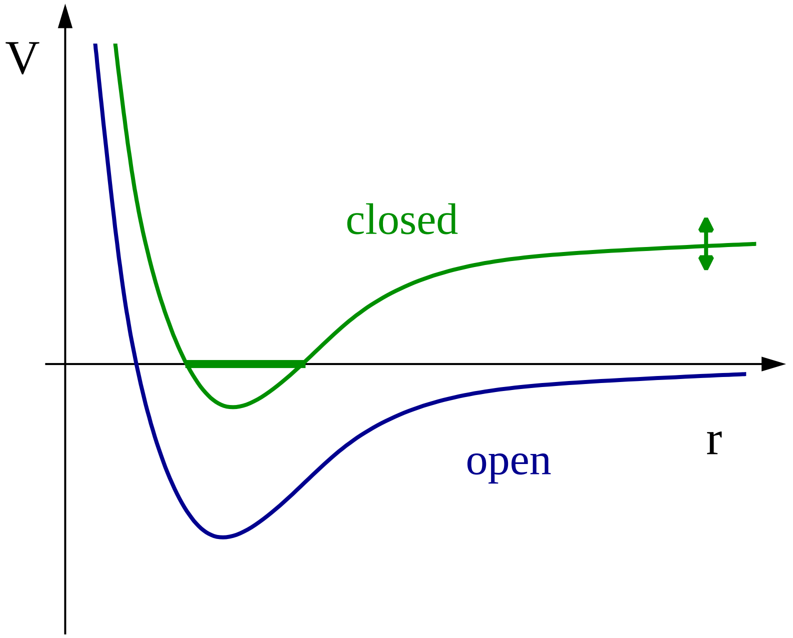


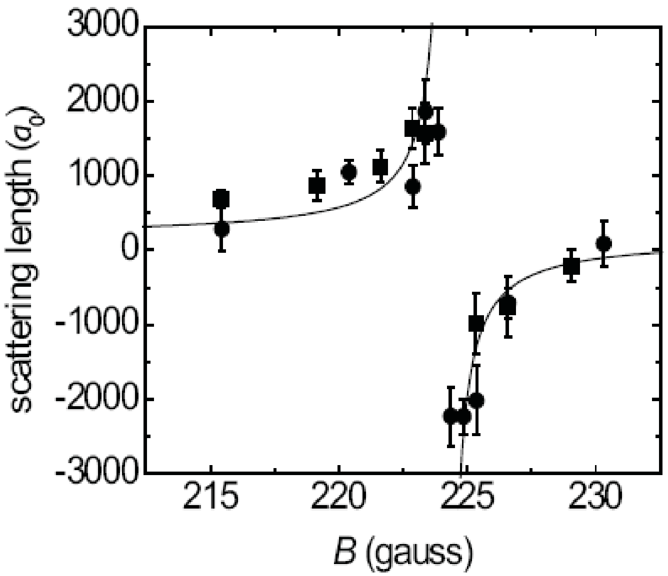
Universal relations

$$\mathcal{T} = \frac{1}{ik + 1/a} \qquad \epsilon_B = \frac{1}{2ma^2} \qquad \psi_B \sim \frac{1}{\sqrt{a}r} \exp(-r/a)$$

Feshbach resonances

Atomic gas with two spin states: “↑” and “↓”





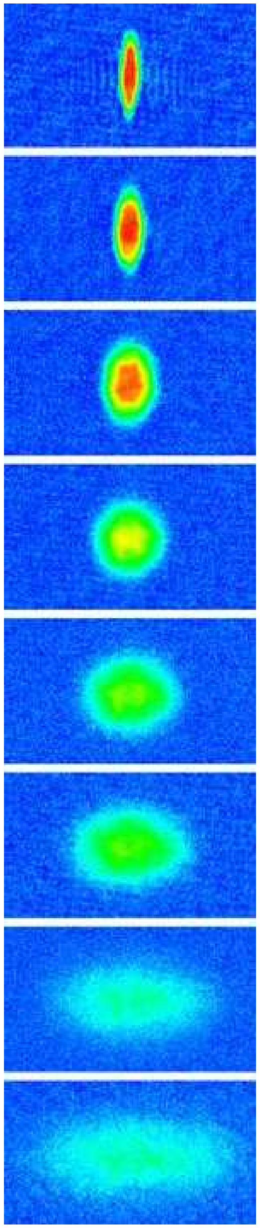
Feshbach resonance

$$a(B) = a_0\left(1 + \frac{\Delta}{B - B_0}\right)$$

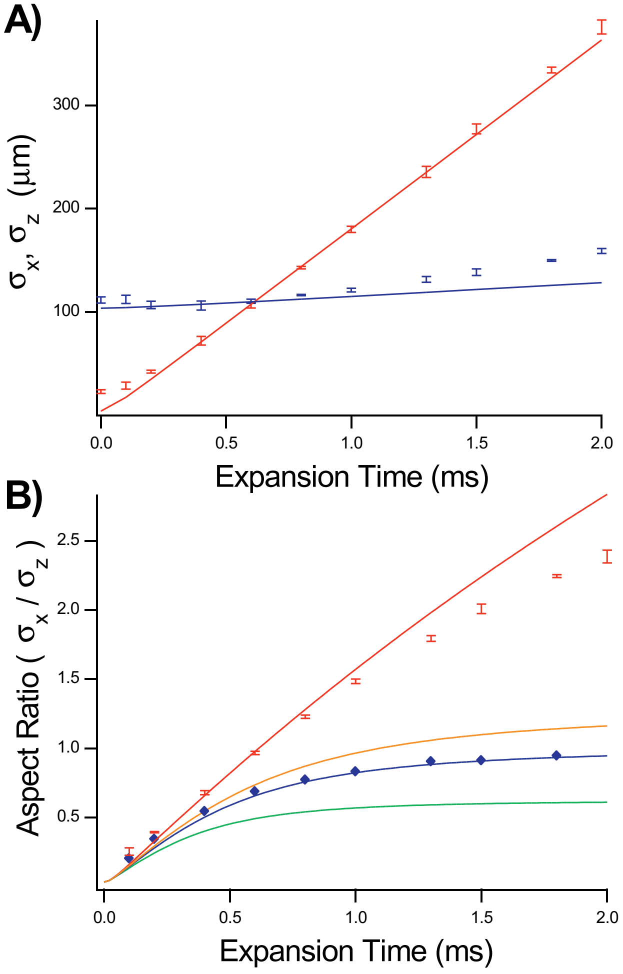
“Unitarity” limit $a \to \infty$

$$\sigma = \frac{4\pi}{k^2}$$

Almost ideal fluid dynamics (cold gases)



O'Hara et al. (2002)



Hydrodynamic

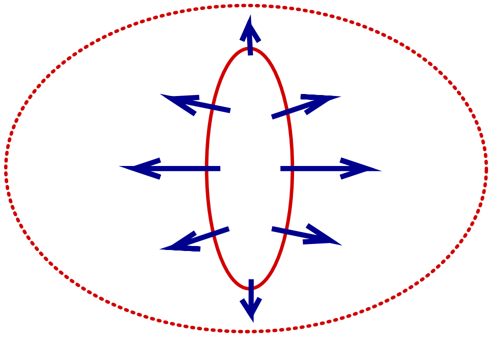
expansion converts

coordinate space

anisotropy

to momentum space

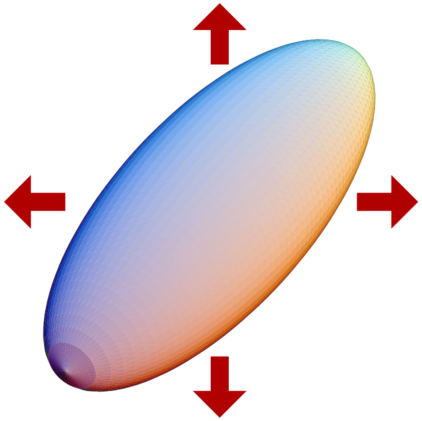
anisotropy



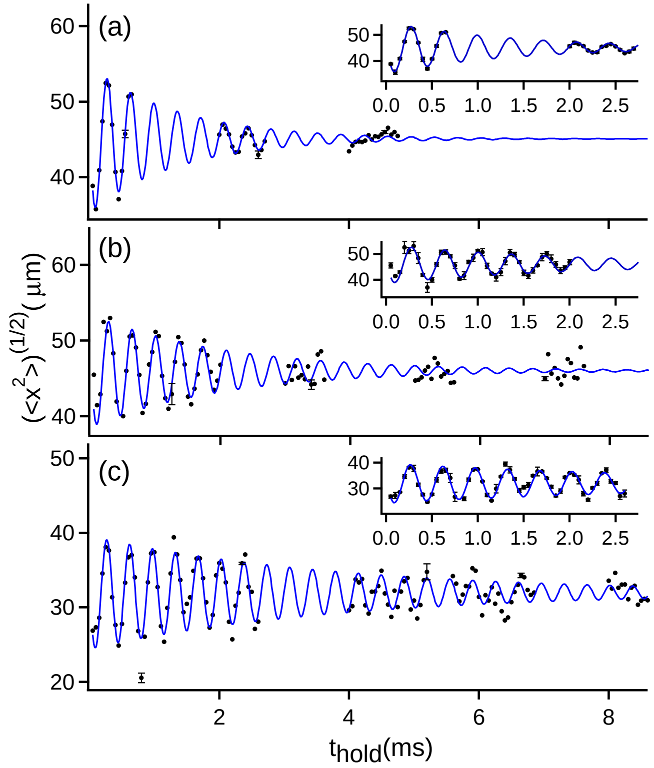
Collective oscillations

Radial breathing mode

Ideal fluid hydrodynamics ($P = \frac{2}{3}\mathcal{E}$)



$$\begin{aligned} \frac{\partial n}{\partial t} + \vec{\nabla}\cdot(n\vec{v}) &= 0 \\ \frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} &= -\frac{\vec{\nabla}P}{mn} - \frac{\vec{\nabla}V}{m} \end{aligned}$$



Hydro frequency at unitarity

$$\omega = \sqrt{\frac{10}{3}}\,\omega_\perp$$

Damping small, depends on T/T_F.

experiment: Kinast et al. (2005)

Viscous hydrodynamics

Energy dissipation (η, ζ, κ: shear, bulk viscosity, heat conductivity)

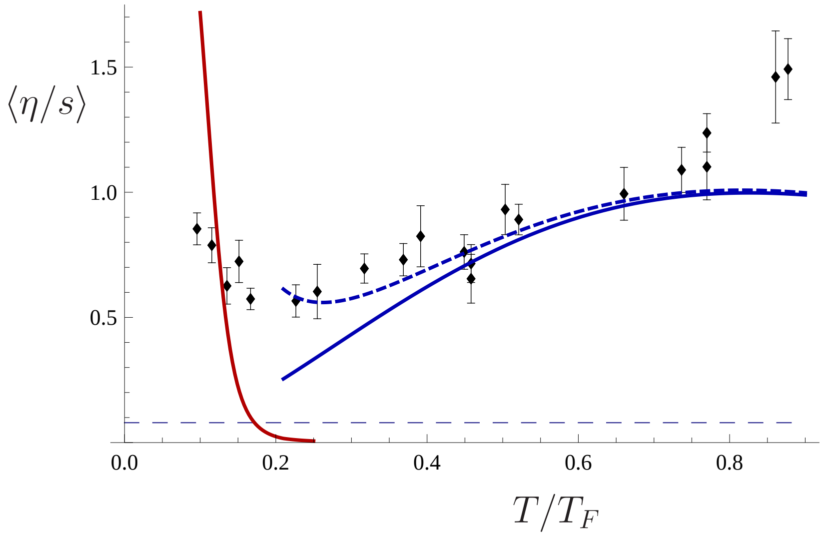
$$\begin{aligned}\dot{E} \;&=\; -\frac{1}{2}\int d^3x\,\eta(x)\left(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k\right)^2 \\ &\quad - \int d^3x\,\zeta(x)\,(\partial_i v_i)^2 - \frac{1}{T}\int d^3x\,\kappa(x)\,(\partial_i T)^2\end{aligned}$$

Shear viscosity to entropy ratio

(assuming $\zeta = \kappa = 0$)

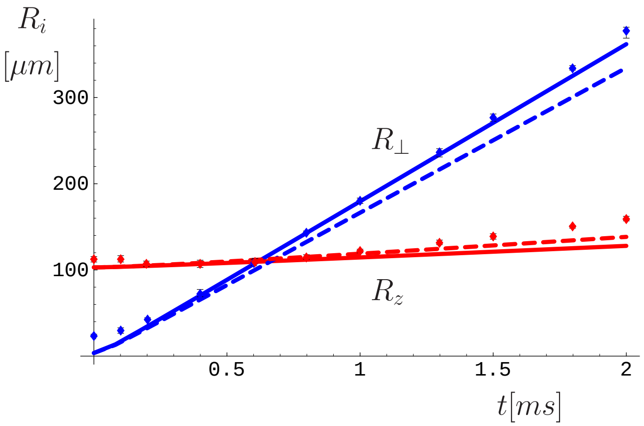
$$\boxed{\frac{\eta}{s} = (3\lambda N)^{\frac{1}{3}}\,\frac{\Gamma}{\omega_\perp}\,\frac{E_0}{E_F}\,\frac{N}{S}}$$

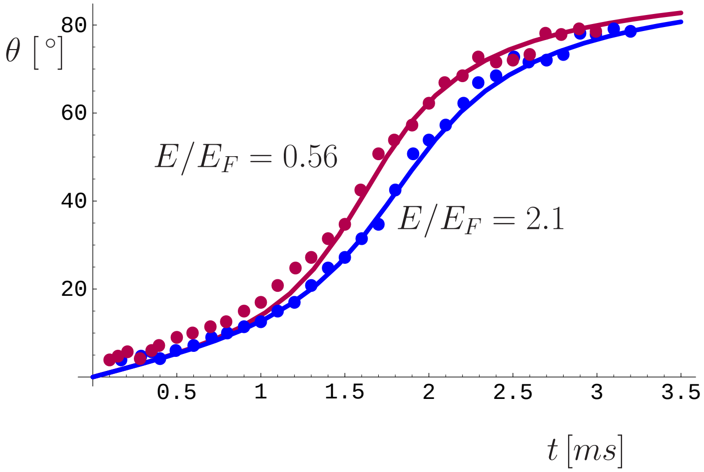
Schaefer (2007), see also Bruun, Smith

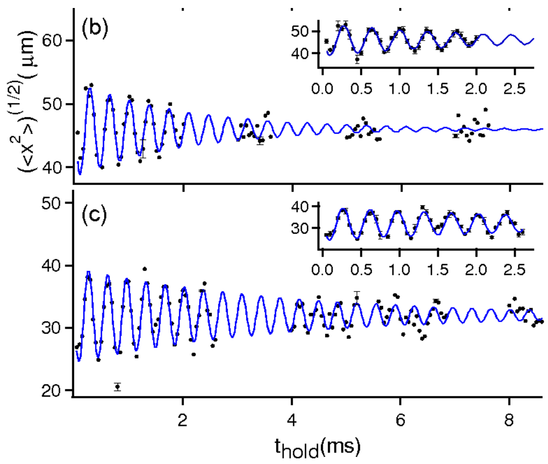


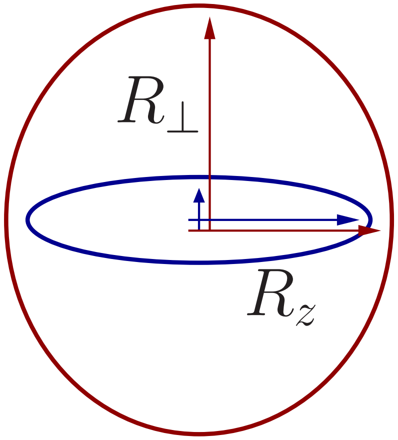
$T \ll T_F$ $\quad$ $T \gg T_F,\ \tau_R \simeq \eta/P$

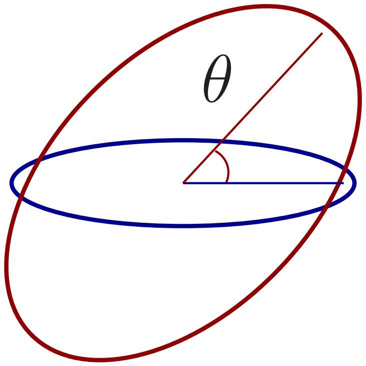
Dissipation

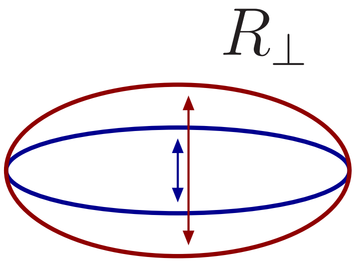






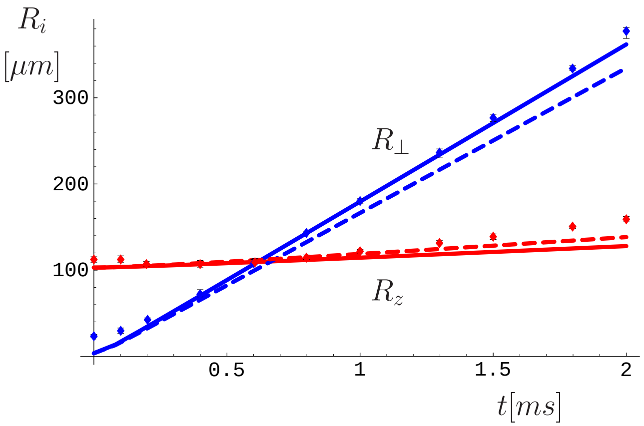


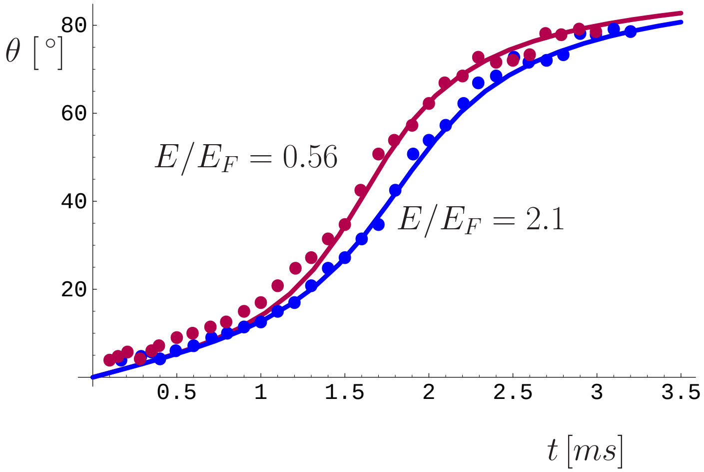


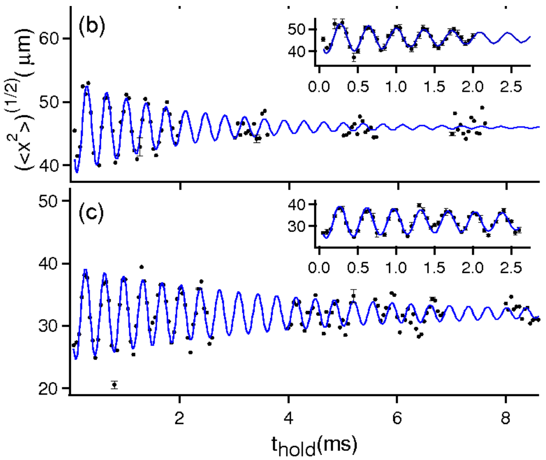


O’Hara et al (2002), Kinast et al (2005), Clancy et al (2007)

Dissipation







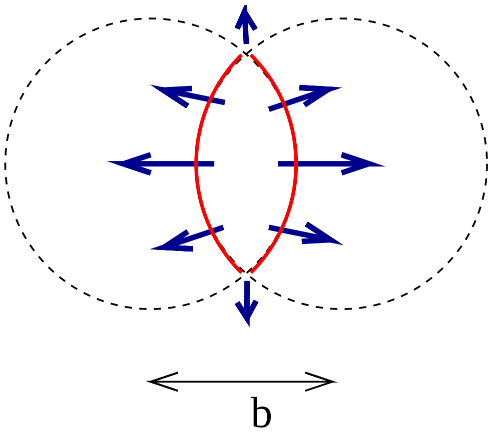
$$\left.\begin{array}{c}(\delta t_0)/t_0\\(\delta a)/a\end{array}\right\} = \left\{\begin{array}{c}0.008\\0.024\end{array}\right\}\left(\frac{\langle\eta/s\rangle}{1/(4\pi)}\right)\left(\frac{2\cdot 10^5}{N}\right)^{1/3}\left(\frac{S/N}{2.3}\right)\left(\frac{0.85}{E_0/E_F}\right)$$

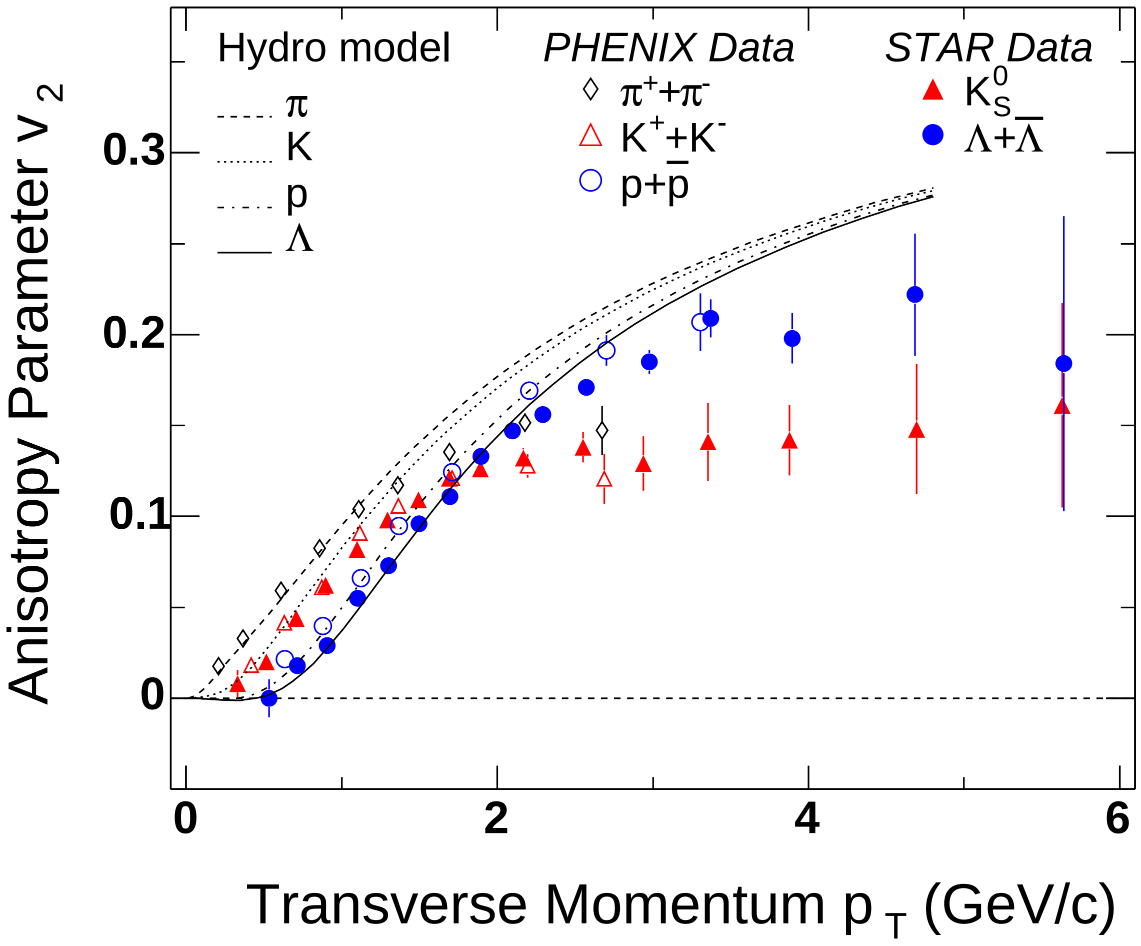
t_0: "Crossing time" $(b_\perp = b_z,\ \theta = 45^\circ)$

a: amplitude

Elliptic flow (QGP)

Hydrodynamic
expansion converts
coordinate space
anisotropy
to momentum space
anisotropy

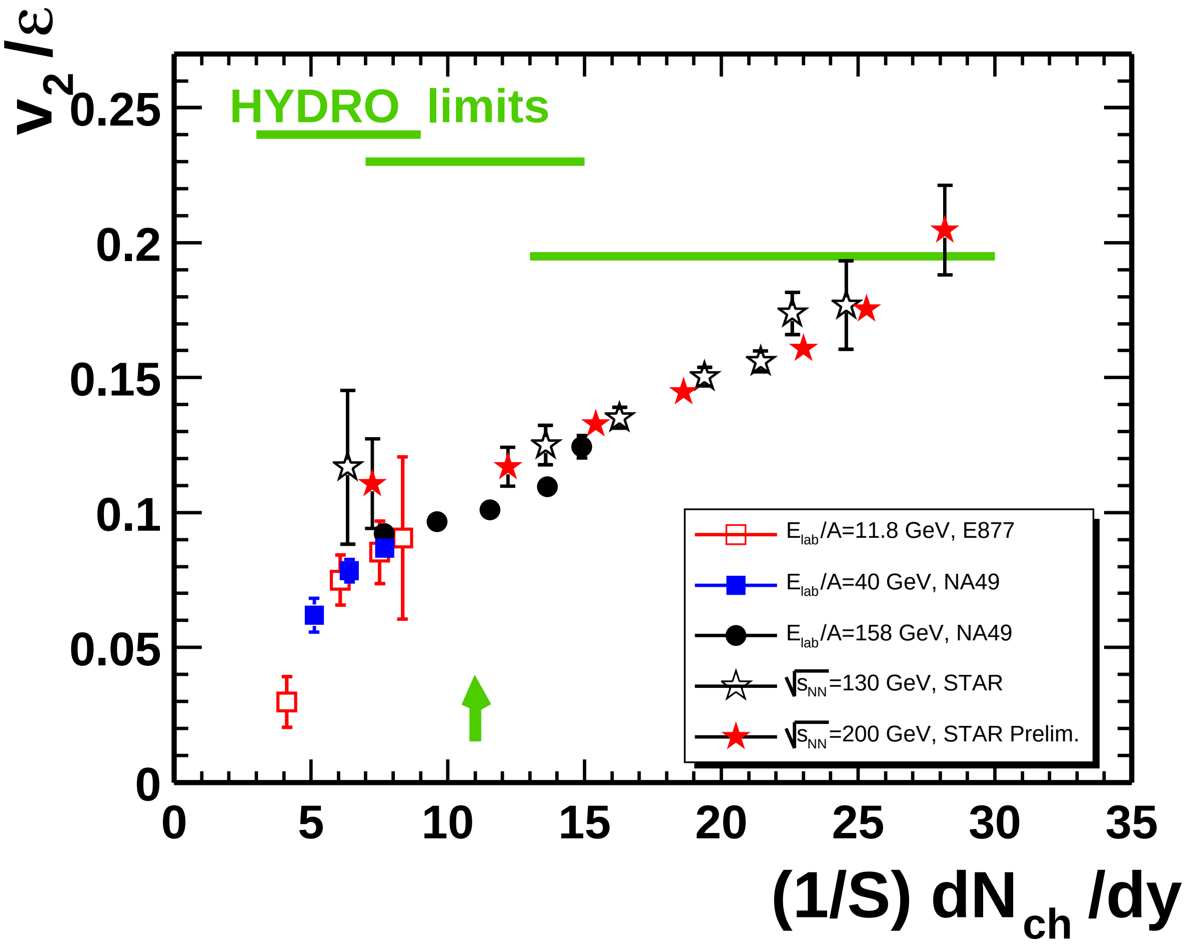




source: U. Heinz (2005)

$$p_0 \, \frac{dN}{d^3p}\bigg|_{p_z=0} = v_0(p_\perp)\,\left(1 + 2v_2(p_\perp)\cos(2\phi) + \ldots\right)$$

Elliptic flow: initial entropy scaling



source: U. Heinz (2005)

Viscosity and elliptic flow

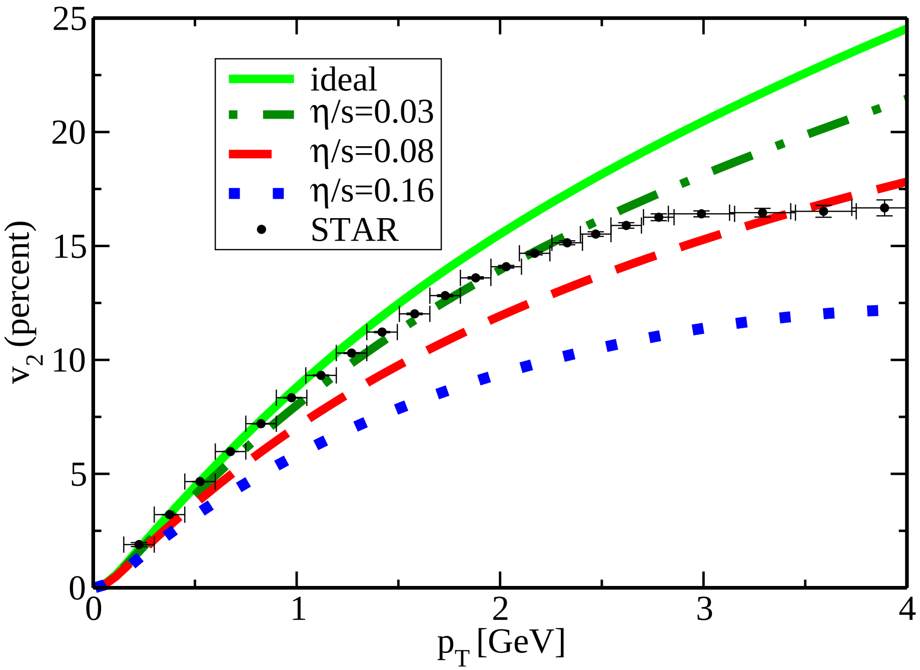
Consistency condition $T_{\mu\nu} \gg \delta T_{\mu\nu}$

(applicability of Navier-Stokes)

$$\frac{\eta + \frac{4}{3}\zeta}{s} \ll \frac{3}{4}(\tau T)$$

Danielewicz, Gyulassy (1985)

Very restrictive for $\tau < 1$ fm



Romatschke (2007), Teaney (2003)

Many questions: Dependence on initial conditions, freeze out, etc.

conservative bound $\boxed{\frac{\eta}{s} < 0.4}$

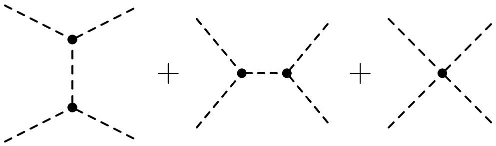
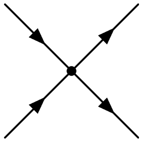
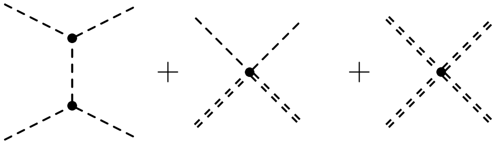
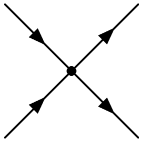
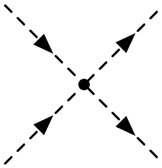
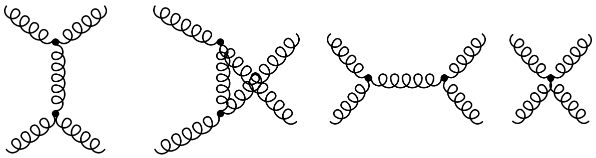
The bottom-line

Remarkably, the best fluids that have been observed are the *coldest* and the *hottest* fluid ever created in the laboratory, cold atomic gases (10^{-6}K) and the quark gluon plasma (10^{12}K) at RHIC.

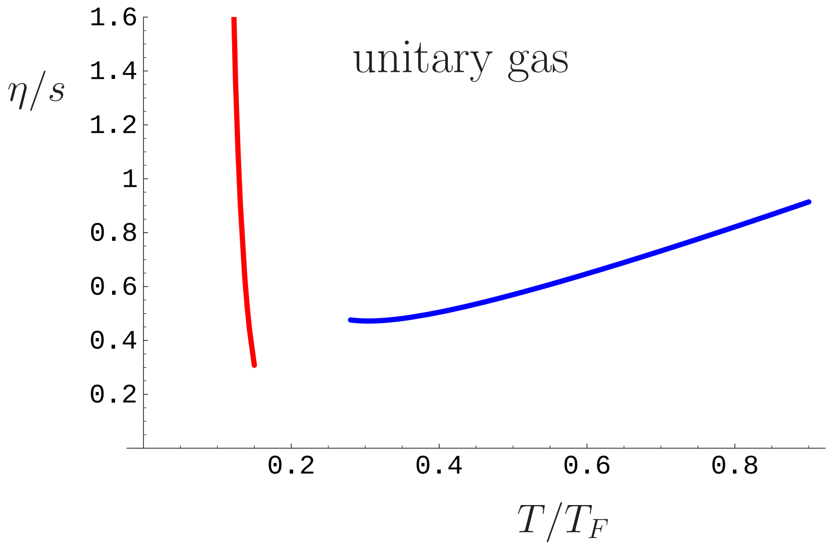
Both of these fluids come close to a bound on the shear viscosity that was first proposed based on calculations in string theory, involving non-equilibrium evolution of back holes in 5 (and more) dimensions.

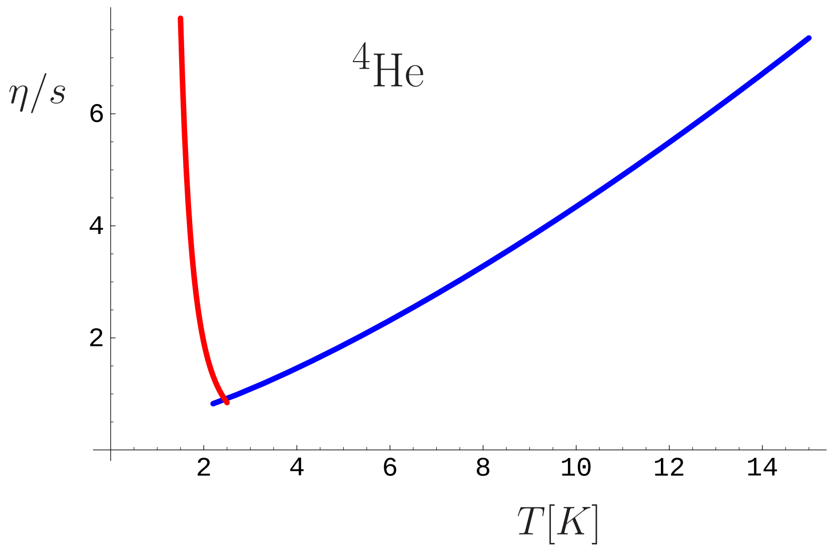
Extra Slides

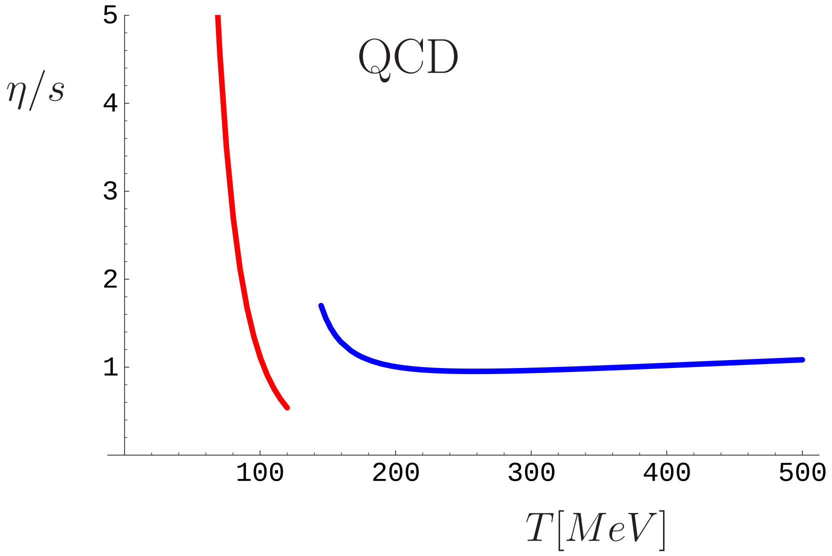
Kinetic theory: Quasiparticles

	low temperature	high temperature
unitary gas	phonons 	atoms 
helium	phonons, rotons 	atoms 
QCD	pions 	quarks, gluons 

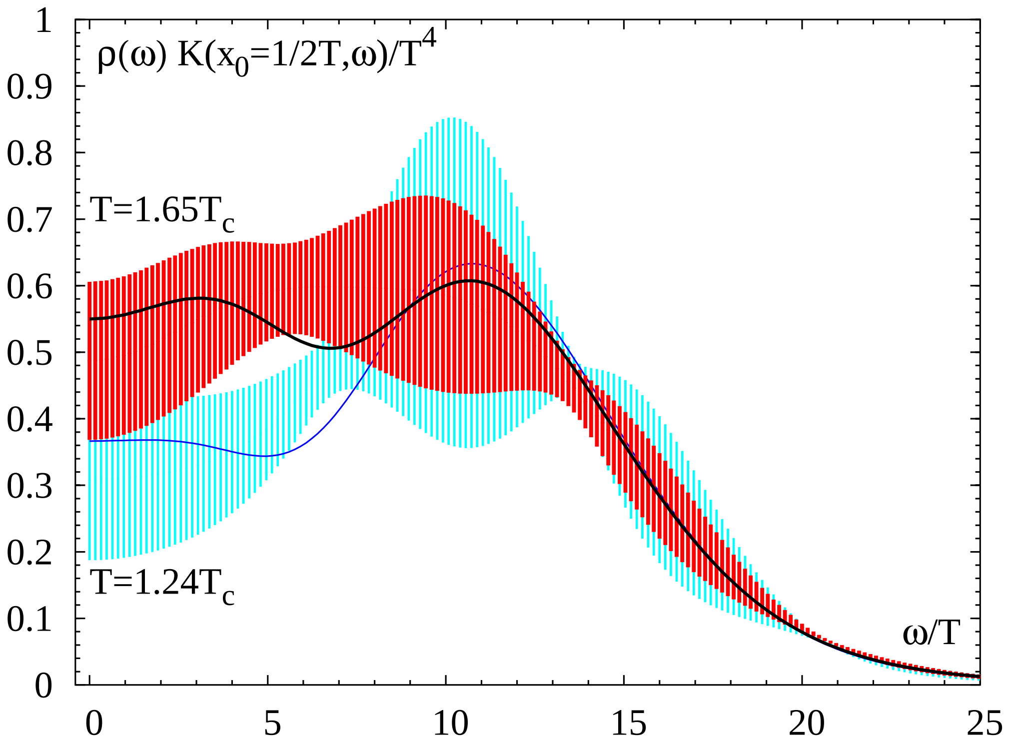
Theory Summary







Spectral function (lattice QCD)



T	$1.02\ T_c$	$1.24\ T_c$	$1.65\ T_c$
η/s		0.102(56)	0.134(33)
ζ/s	0.73(3)	0.065(17)	0.008(7)

H. Meyer (2007)

Experiment (liquid helium)

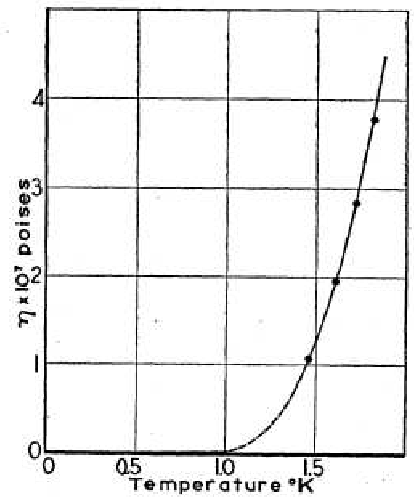
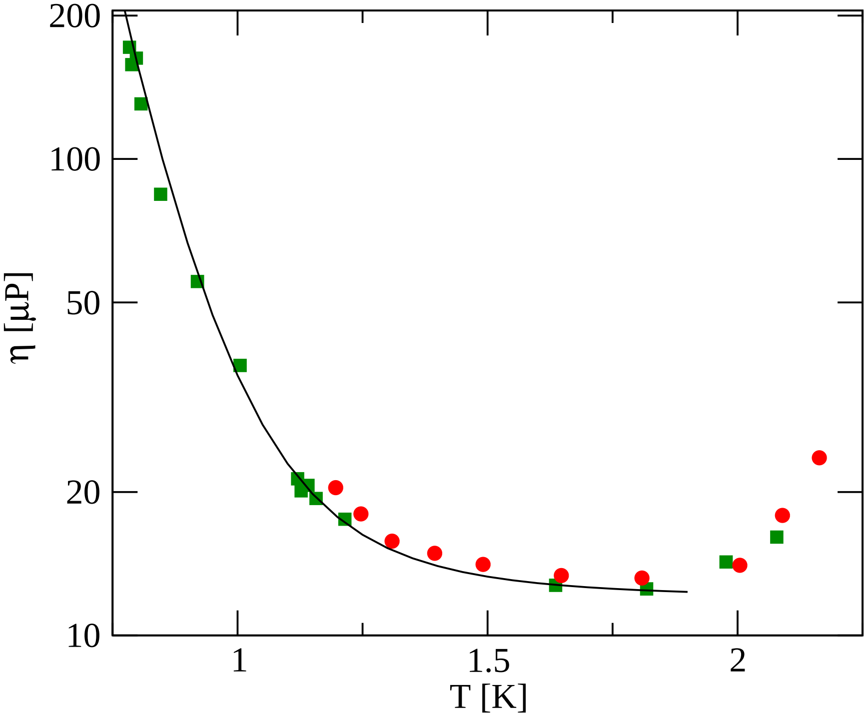


FIG. 1. The viscosity of liquid helium II measured by flow through a 10^{-4} cm channel.



Kapitza (1938)

viscosity vanishes below T_c

capillary flow viscometer

Hollis-Hallett (1955)

roton minimum, phonon rise

rotation viscometer

$$\eta/s \simeq 0.8\,\hbar/k_B$$

Time scales