Squeezed Quark Matter

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Abstract

We discuss recent progress in understanding the phase structure of Quantum Chromodynamics(QCD) at very high baryon density.

I. INTRODUCTION

Quantum Chromodynamics (QCD) is the theory of quarks and gluons and their interactions.

The underlying equations of QCD are deceptively simple. QCD generalizes the familiar $U(1)$ gauge invariance of electrodynamics to a $SU(3)$ symmetry that involves all possible transformations of three colored quarks. We will refer to the three color charges of quarks as red, blue and green. Color gauge invariance implies the existence of colored gauge fields, called gluons, which mediate interactions between quarks. There are eight different gluons, corresponding to the eight non-trivial ways in which a quark can change its color.

Even though the elementary laws that govern QCD are formulated in terms of quarks and gluons, neither quarks nor gluons have ever been observed directly. On the other side, QCD is the theory of the strong interaction, but none of the strongly interacting particles, protons, neutrons, pions, kaons, etc. appear explicitly in the laws of QCD. The key to understanding these facts is a phenomenon called asymptotic freedom. Asymptotic freedom implies that the effective coupling between quarks and gluons becomes weak as we go to shorter and shorter distance scales. If we use high momentum probes that can resolve phenomena at very short length scales then quarks and gluons are the correct degrees of freedom. This strategy is used in interpreting the results of deep inelastic scattering and jet production experiments that helped to establish QCD as the correct theory of the strong interactions.

At long distances, on the other hand, the effective coupling between colored objects is very strong and quarks and gluons are not the correct degrees of freedom. Instead, the spectrum of QCD contains color neutral hadrons. The structure of these hadrons is very complicated. We can construct a state with the quantum numbers of the proton by taking two up quarks and a down quark and arranging their wave function to give a state that is color neutral and has total spin 1/2. However, the true wave function of the proton is much richer and contains admixtures of many quark-anti-quark pairs and gluons.

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Strong interactions also lead to another important phenomenon, the spontaneous breakdown of chiral symmetry. To a very good approximation we can consider the light quark flavors up, down and strange as massless. In this limit the chirality of the quarks, which is the projection of the quark spin onto its momentum, is conserved. As a result, the $SU(3)$ flavor symmetry is enlarged to a chiral $SU(3)_L \times SU(3)_R$ symmetry. In the QCD ground state quark-anti-quark pairs of all flavors condense. The $\bar{q}q$ condensate $(\bar{u}_L u_R+\bar{d}_L d_R+\bar{s}_L s_R)+(L \leftrightarrow R)$ selects a preferred direction in flavor space and fixes the relative phase of left and right handed fields. This implies that the chiral $SU(3)_L \times SU(3)_R$ symmetry breaks to its diagonal subgroup, the SU(3) flavor symmetry.

Chiral symmetry breaking is intimately related to the generation of "mass" in QCD. Without chiral symmetry breaking there is no coupling between left and right handed fields and all states have to be massless. Indeed, about 95% of the proton mass is generated by QCD effects, while only the remaining 5% are related to the Higgs field. On the other hand, chiral symmetry breaking also leads to the appearance of a special class of very light states. The chiral condensate selects a direction among a large set of (almost) degenerate states. This means that fluctuations in the flavor orientation of the chiral condensate correspond to low energy states. In

QCD, these states are known as pion, kaons, and eta mesons. The masses of these particles are indeed smaller than the masses of all other hadrons.

II. QCD MATTER IN EXTREME ENVIRONMENTS

There are several motivations for considering QCD matter in extreme environments, such as very high temperature or very large baryon density.

- These conditions exist in the universe : About 10^{-5} sec after the big bang the universe passed through a state in which the temperature was comparable to QCD scales. Much later, matter condensed into stars. Some of these stars, having exhausted their nuclear fuel, collapse into compact objects called neutron stars. The density at the center of a neutron star is not known very precisely, but almost certainly greater or equal to the density where quark degrees of freedom become important.
- Exploring the entire phase diagram is important to understanding the phase that we happen to live in : We cannot properly understand the structure of hadrons and their interactions without understanding the underlying QCD vacuum state. And we cannot understand the vacuum state without understanding how it can be modified.
- QCD simplifies in extreme environments: At scales relevant to hadrons QCD is strongly coupled and we have to rely on numerical simulations in order to test predictions of QCD. In the case of large temperature or large baryon density there is a large external scale in the problem. Asymptotic freedom implies that the bulk of the system is governed by weak coupling. As a result, we can study QCD matter in a regime where quarks and gluons are indeed the correct degrees of freedom.

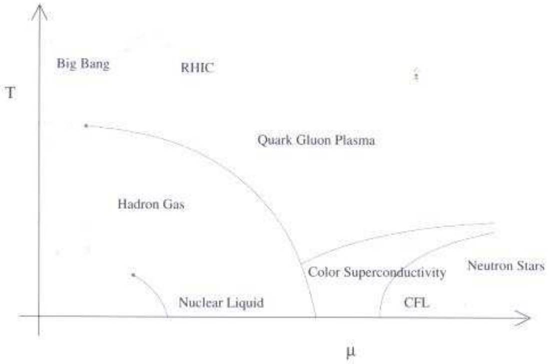

FIG. 1: Schematic phase diagram of QCD at finite temperature and density.

An important goal of the study of QCD in extreme environments is the elucidation of the QCD phase diagram. A conjectured, most likely oversimplified, view of the QCD phase diagram in the temperature-baryon chemical potential (T - μ) plane is shown in Fig. 1. At $T = \mu = 0$ chiral symmetry is broken and quarks and gluons are confined into hadrons. If the density is very small but the temperature is very large we expect that the natural starting point is a thermal gas of weakly interacting quarks and gluons, called the quark-gluon plasma (QGP). In a plasma, interactions are screened and the weak coupling description is self-consistent. This picture of the high density phase has been confirmed in numerical lattice QCD simulations and is currently under active experimental investigation using collisions of heavy nuclei.

III. NEW PHASES OF SQUEEZED MATTER

In the opposite limit of very high baryon density but small temperature the natural starting point is a Fermi liquid of quarks. The relevant degrees of freedom are quark excitations and holes in the vicinity of the Fermi surface. Since the Fermi momentum is large, asymptotic freedom implies that the interaction between quasiparticles is weak. This does not mean that a weakly interacting quark liquid is the end of the story. Indeed, we know from the theory of superconductivity that the Fermi surface is unstable with respect to the formation of pairs whenever there in an attractive interaction, no matter how weak. In QCD at large density the attraction is provided by gluon exchanges between quarks in a color anti-symmetric state. This is where the non-abelian character of QCD comes into play. Whereas the photon mediated interaction between two electrons is always repulsive, the gluon induced interaction between two quarks is attractive or repulsive, depending on the relative color orientation of the two quarks. High density quark matter therefore has a pairing instability. As a consequence, quark matter is a color superconductor [1-5].

Color superconductivity is described by a pair condensate of the form

$$\phi = \langle \psi^T C \Gamma_D \lambda_C \tau_F \psi \rangle \tag{1}$$

Here, C is the charge conjugation matrix, and Γ_D, λ_C, τ_F are Dirac, color, and flavor matrices. Except in the case of only two colors, the order parameter cannot be a color singlet. Color superconductivity is characterized by the breakdown of color gauge invariance. This implies that the gluon acquires a mass by the Higgs mechanism. In addition to that, superconductivity leads to the formation of a gap in the spectrum of quarks. At very high density the gap can be computed in weak coupling perturbation theory. We find [6 - 9]

$$\Delta \sim \mu \exp\left(-\frac{3\pi^{3/2}}{\sqrt{8}\,\sqrt{\alpha_s}}\right), \tag{2}$$

where α_s is the QCD coupling constant. What is remarkable about this formula is the fact that the gap is not exponentially small in the α_s as is the case in BCS superconductors, but in $\sqrt{\alpha_s}$. This is a consequence of the long range gauge forces in QCD, and contributes to large gaps and robust superfluidity.

The exact nature of the superconducting phase depends on the number of quark flavors and their masses. A particularly interesting situation arises in QCD with three light flavors. In this case the order parameter which describes the energetically favored phase is color-flavor-locked [10]. This means that the color and flavor orientation of the Cooper pairs are coupled. This is a strange situation: Ordinarily, the exact local gauge symmetry which governs the dynamics of QCD and the approximate global flavor symmetry of QCD are completely separate. In the color-flavor-locked (CFL) phase both the left and right handed flavor symmetries are locked to color, and as a result they are effectively locked to each other. This implies that chiral symmetry is broken and that the spectrum of the CFL phase contain light pions, kaons and eta mesons.

The properties of quarks and gluons are also dramatically modified. Quarks acquire a gap and gluons become massive due to the Higgs mechanism. Because color and flavor are entangled the electric shares of both gluons and quarks are modified. In fact, both quarks and gluons now carry charges that are integral multiples of the electron's charge. Thus the essential features of confinement, the absence of long range gauge forces, integer electric charges, and chiral symmetry breaking, arise as simple consequences of superconductivity [11].

IV. OUTLOOK

There are many challenges that lie ahead. We would like to improve our calculational tools and improve theoretical predictions for the high density phase. Effective field theory techniques have proved very useful in this context [12, 13]. In particular we would like to provide definite calculations that can be compared to observational signatures of neutron stars. We would like to overcome the sign problem, the computational stumbling block that still hampers numerical lattice studies of the high density phase. A number of interesting proposals have been made, but much work remains to be done [14 - 16]. And finally we would like to identify observables, both in the physics of heavy ion collisions and the physics of neutron stars, that will allow us experimentally identify the many phases of hot and dense matter. These problems have been discussed at a series of workshops on dense QCD matter organized by the APCTP.

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